الدرس : Quizz 1 اليوم ..... التاريخ / /

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1) $\hat{\theta} = ?$ using $L_x(\theta)$

① $L_x(\theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} (\theta+1)\, x_i^{\theta-1}$

② $\ell_x(\theta) = \ln L_x(\theta) = \ln \left(\prod_{i=1}^{n} (\theta+1)\, x_i^{\theta-1} \right)$

$= \sum_{i=1}^{n} \ln \left[(\theta+1)\, x_i^{\theta-1} \right]$

$= \sum_{i=1}^{n} \ln(\theta+1) + \sum_{i=1}^{n} \ln\left(x_i^{\theta-1}\right)$

$= n \ln(\theta+1) + (\theta-1) \sum_{i=1}^{n} \ln(x_i)$

③ $\frac{\partial}{\partial \theta} \left(\ell_x(\theta) \right) = \frac{n}{\theta+1} + \sum_{i=1}^{n} \ln(x_i)$

④ $\frac{\partial}{\partial \theta} \left(\ell_x(\theta) \right) = 0$

$\frac{n}{\theta+1} + \sum_{i=1}^{n} \ln(x_i) = 0 \Rightarrow$

$$\therefore \frac{n}{\hat{\theta}+1} + \sum_{i=1}^{n} \ln(x_i) = 0$$

$$\Rightarrow \frac{n}{\hat{\theta}+1} = -\sum_{i=1}^{n} \ln(x_i)$$

$$\Rightarrow \frac{\hat{\theta}+1}{n} = \frac{-1}{\sum_{i=1}^{n} \ln(x_i)} \Rightarrow \boxed{\hat{\theta} = \frac{-n}{\sum_{i=1}^{n} \ln(x_i)} - 1}$$

$$\therefore \hat{\theta} = \frac{-n}{\sum_{i=1}^{n} \ln(x_i)} - 1$$