عنوان الدرس Ass 1 اليوم .... التاريخ / /

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1. $f_Y(y) = ?$

$\because Y = X^2 \Rightarrow X = \pm\sqrt{Y}$

ولكن $-\sqrt{Y}$ مرفوضة من دالة توزيع X

$\therefore X = \sqrt{Y}$

$$f_X(g^{-1}(y)) = f_X(\sqrt{y}) = \frac{3!}{(\sqrt{y})!\,(3-\sqrt{y})!} \left(\frac{2}{3}\right)^{\sqrt{y}} \left(\frac{1}{3}\right)^{3-\sqrt{y}}$$

When X	0	1	2	3
Y	0	1	4	9

$\therefore Y = 0, 1, 4, 9$

$$\therefore f_Y(y) = \begin{cases} \frac{3!}{(\sqrt{y})!\,(3-x)!} \left(\frac{2}{3}\right)^{\sqrt{y}} \left(\frac{1}{3}\right)^{3-\sqrt{y}}, & y = 0, 1, 4 \\ 0 & \text{elsewhere} \end{cases}$$

2. $f_y(y) = ?$

- $\because Y = X^3 \Rightarrow X = \sqrt[3]{Y}$

- $f_X(g^{-1}(y)) = f_X(\sqrt[3]{y}) = \frac{(\sqrt[3]{y})^2}{9} = \frac{y^{\frac{2}{3}}}{9}$

- $J = \frac{d}{dy} f_X(\sqrt[3]{y}) = \frac{d}{dy}\left(\frac{y^{\frac{2}{3}}}{9}\right) = \frac{2}{27 \cdot \sqrt[3]{y}}$

- $f_Y(y) = f_X(g^{-1}(y)) \cdot |J| = \frac{(\sqrt[3]{y})^{\cancel{2}}}{9} \cdot \frac{2}{27\cancel{\sqrt[3]{y}}}$

$= \frac{2\sqrt[3]{y}}{243}$

- $X \to 0 \quad , Y \to 0$
  $X \to 3 \quad , Y \to 9$ $\Big\}\ 0 < y < 9$

$\therefore f_Y(y) = \begin{cases} \frac{2\sqrt[3]{y}}{243} & 0 < y < 9 \\ 0 & \text{otherwise} \end{cases}$

3. $f_{Y_1,Y_2}(y_1,y_2) = ?$ $f_{Y}(y) = ?$

$\because Y_1 = 2X_1 \Rightarrow X_1 = \frac{Y_1}{2}$

$\because Y_2 = X_2 - X_1 \Rightarrow X_2 = Y_2 + X_1 = Y_2 + \frac{Y_1}{2}$

$- f_{X_1,X_2}(y_1,y_2) = 2e^{-\frac{y_1}{2}} e^{-y_2 - \frac{y_1}{2}} = 2\exp(-(y_1+y_2))$

$- J = \begin{vmatrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}$

$- f_{Y_1,Y_2}(y_1,y_2) = f_{X_1,X_2}(y_1,y_2) \cdot |J| = \cancel{2}\exp(-(y_1+y_2)) \cdot |\frac{1}{\cancel{2}}|$

$= \exp(-(y_1+y_2))$

$- \because 0 < x_1 < x_2 < \infty \Rightarrow 0 < \frac{Y_1}{2} < Y_2 + \frac{Y_1}{2} < \infty$

$\therefore 0 < Y_2 < \infty \quad , \quad 0 < Y_1 < \infty$

$\therefore f_{Y_1,Y_2}(y_1,y_2) = \begin{cases} e^{-(y_1+y_2)} & 0 < y_1, y_2 < \infty \\ 0 & \text{otherwise} \end{cases}$

4. $f_{Y_1,Y_2}(y_1,y_2) = ?$

~~X_1~~ $\because X_1, X_2$ are Gamma distributed.

$\therefore f_{X_1}(x_1) = \dfrac{x_1^{\alpha-1} e^{-x_1}}{\Gamma(\alpha)} \qquad 0 < x_1 < \infty$

$f_{X_2}(x_2) = \dfrac{x_2^{\beta-1} e^{-x_2}}{\Gamma(\beta)} \qquad 0 < x_2 < \infty$

$\because Y_1 = \dfrac{x_1}{x_1+x_2} \Rightarrow x_1 = Y_1(x_1+x_2) = Y_1 Y_2$

$Y_2 = x_1 + x_2 \Rightarrow x_2 = Y_2 - Y_1 Y_2 = Y_2(1-Y_1)$

$\because X_1, X_2$ are independent

$\therefore f_{X_1X_2}(x_1,x_2) = f_{X_1}(x_1) \cdot f_{X_2}(x_2) = \dfrac{x_1^{\alpha-1} e^{-x_1}}{\Gamma(\alpha)} \cdot \dfrac{x_2^{\beta-1} e^{-x_2}}{\Gamma(\beta)}$

$= \dfrac{x_1^{\alpha-1} x_2^{\beta-1} e^{-(x_1+x_2)}}{\Gamma(\alpha) \cdot \Gamma(\beta)}, \qquad 0 < x_1, x_2 < \infty$

$* \; f_{X_1,X_2}(g^{-1}(y_1,y_2)) = \dfrac{(y_1y_2)^{\alpha-1} (y_2(1-y_1))^{\beta-1} e^{-y_2}}{\Gamma(\alpha)\, \Gamma(\beta)}$ ~~$= y_2^{\alpha+\beta-2}$~~

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$$* \; J = \begin{vmatrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix} = \begin{vmatrix} y_2 & y_1 \\ -y_2 & 1-y_1 \end{vmatrix} = y_2(1-y_1) + y_1 y_2$$

$$= x_2 + x_1 = \underline{\underline{y_2}}$$

$$* \; f_{Y_1,Y_2}(y_1,y_2) = f_{X_1,X_2}(g^{-1}(y_1,y_2)) \cdot |J|$$

$$= \frac{(y_1 y_2)^{\alpha-1} (y_2(1-y_1))^{\beta-1} e^{-y_2}}{\Gamma(\alpha)\,\Gamma(\beta)} \cdot y_2$$

$$= \frac{y_2^{\alpha+\beta-1} \, e^{-y_2} \, y_1^{\alpha-1} (1-y_1)^{\beta-1}}{\Gamma(\alpha)\,\Gamma(\beta)}$$

$*$ $\because \; 0 < x_1, x_2 < \infty \;,\; y_1 = \frac{x_1}{x_1+x_2} \;,\; y_2 = x_1 + x_2$

$\therefore \; 0 < y_2 < \infty \;,\; 0 \leq y_1 \leq 1$

$$\therefore \; f_{Y_1Y_2}(y_1,y_2) = \begin{cases} \dfrac{y_2^{\alpha+\beta-1} \, e^{-y_2} \, y_1^{\alpha-1} (1-y_1)^{\beta-1}}{\Gamma(\alpha)\,\Gamma(\beta)} & 0 < y_2 < \infty, \; 0 \leq y_1 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$*$

5. $f_Y(y) = ?$

$$f_X(x) = \begin{cases} \frac{1}{3} & 1 \le x \le 4 \\ 0 & \text{other wise} \end{cases}$$

$\because X \sim U[1, 4]$

$\therefore$ ~~F_X~~

$$\therefore F_X(X) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{3} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$$

$\because Y = X^{\frac{1}{2}} \Rightarrow X = Y^2$

$$F_Y(y) = P(Y \le y) = P(X^{\frac{1}{2}} \le y) = P(X \le g^{-1}(y))$$

$$= \frac{y^2 - 1}{3}$$

$$f_Y(y) = \frac{d}{dy}(F_Y(y)) = \frac{2Y}{3}$$

$\because 1 \le x \le 4 \Rightarrow 1 \le Y^2 \le 4 \Rightarrow 1 \le Y \le 2$

$$\therefore f_Y(y) = \begin{cases} \frac{2Y}{3} & 1 \le Y \le 3 \\ 0 & \text{else where} \end{cases}$$

6. $f_Y(y) = ?$

$$f_X(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{\alpha} \Gamma(\alpha)} \quad 0 < x < \infty$$

$\because X \sim$ Gamma distributed with (α, β)

$\therefore$

$$M_X = (1-\beta t)^{-\alpha}$$

$\because Y = \frac{2X}{\beta} \Rightarrow X = \frac{Y\beta}{2}$

$$\therefore M_Y = E(e^{tY}) = E(e^{t(\frac{2X}{\beta})}) = E(e^{tX} \cdot e^{\frac{2}{\beta}})$$

$$= e^{\frac{2}{\beta}} E(e^{tX}) = e^{\frac{2}{\beta}} (1-\beta t)^{-\alpha} \; ?!$$

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But using pdf.

$$f_X(\overset{g^{-1}(y)}{y}) = \frac{\left(\frac{y\beta}{2}\right)^{\alpha-1} e^{-\frac{y}{2}}}{\beta^{\alpha} \Gamma(\alpha)} = \frac{\left(\frac{y}{2}\right)^{\alpha-1} e^{-\frac{y}{2}}}{\beta \, \Gamma(\alpha)}$$

$$|J| = \left|\left|\frac{dx}{dy}\right|\right| = \left|\frac{\beta}{2}\right| = \frac{\beta}{2}$$

$$f_Y(y) = f_X(g^{-1}(y)) \cdot |J| = \frac{y^{\alpha-1} e^{-y/2}}{2^{\alpha} \Gamma(\alpha)} \quad 0 < y < \infty$$

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