Discrete Structures CS2023

Tutorial I

Question 1

a. Are the following sentences propositions? If yes, what is the truth value for each one?

1. 112358 is a prime number.
2. $2^n > 2n$
3. There is a barber in a town that shaves those, and only those who do not shave themselves in that town.

Question 1

Solution:

1. "112358 is a prime number." is a proposition, and it's false since the number is even, thus divisible by two.

2. $2^n > 2n$ is not a proposition since its truth depends on n, and n has not been assigned.

Question 1

Solution:

3. The barber sentence is a paradox. Think of the barber himself, does he shave himself?

If he shaves himself we arrive at a contradiction because the barber shaves those who do not shave themselves yet he shaves himself.

And if he doesn’t shave himself the statement says he must shave himself as he shaves those who do not shave themselves.

Question 1

b. Construct a truth table for each of these compound propositions:

1. $p \oplus (p \lor q)$
2. $\neg(p \land q \land r) \rightarrow r$

Question 1

Solution:

p	q	p ∨ q	p ⊕ (p ∨ q)
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

Question 1

Solution:

p	q	r	$p \wedge q \wedge r$	$\neg(p \wedge q \wedge r)$	$\neg(p \wedge q \wedge r) \longrightarrow r$
T	T	T	T	F	T
T	T	F	F	T	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	F

Question 1

c. Give the negation of each of the following statements:

1. It will rain tomorrow or it will snow tomorrow.
2. If today is Sunday, then I will go to the gym.

Question 1

Solution:

c.1 It will not rain tomorrow and it will not snow tomorrow.

c.2 Let p be the proposition “Today is Sunday”,
and q be the proposition “I will go to the gym today”

- The proposition “If today is Sunday, then I will go to the gym” is symbolically $p \rightarrow q$.
- $p \rightarrow q$ is equivalent to $\neg p \lor q$ according to the conjunction-implication equivalence.

Question 1

Solution:

- The negation of $p \rightarrow q$ is equivalent to $\neg(\neg p \lor q)$, which is in turn is equivalent to $p \land \neg q$ according to De Morgan's law.

- Now the negation is:
  "Today is Sunday and I will not go to the gym today."

Question 2

a. For each of the following scenarios, find a compound proposition involving the propositional variables p, q, and r that is:

1. True when p and q are true and r is false, and is false otherwise.

2. True when one of p, q or q is true, and is false otherwise.

3. True when exactly two of p, q, and r are true and is false otherwise.

Question 2

Solution:

a.1 True when p and q are true and r is false, and is false otherwise:

$p \land q \land \neg r$

a.2 True when one of p, q or q is true, and is false otherwise:

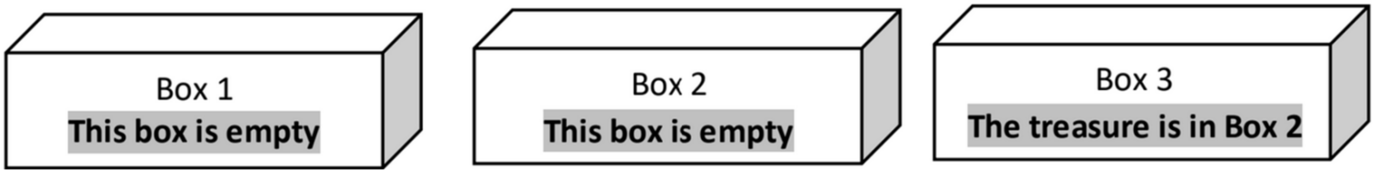
$(p \land \neg q \land \neg r) \lor (q \land \neg p \land \neg r) \lor (r \land \neg p \land \neg q)$

a.3 True when exactly two of p, q, and r are true and is false otherwise:

$[(p \land q) \land \neg r] \lor [(p \land r) \land \neg q] \lor [(q \land r) \land \neg p]$

Question 2

b. There is a treasure hidden inside one of the below three boxes, while the other two boxes are empty. Boxes 1 and 2 are each inscribed with the sign “This box is empty”, while box 3 is inscribed with the sign “The treasure is in box 2”. If you know that only one of the signs is true while the other two are false, where is the treasure hidden? Justify your answer.



Question 2

Solution:

Let:

p_1 be the proposition "the treasure is in box 1"

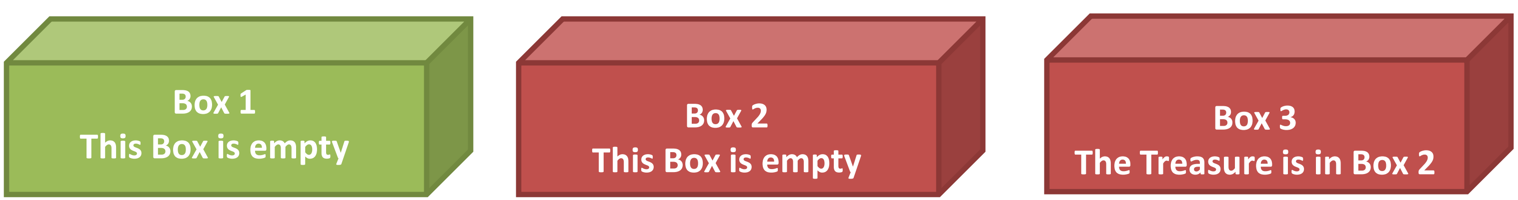
p_2 be the proposition "the treasure is in box 2"

p_3 be the proposition "the treasure is in box 3"

Question 2

Solution:

- If the first statement written in box 1 is true, while the other two are false then: $(\neg p_1 \land p_2 \land \neg p_2)$
- If the second statement written in box 2 is true, while the other two are false then: $(p_1 \land \neg p_2 \land \neg p_2)$
- If the second statement written in box 2 is true, while the other two are false then: $(p_1 \land \neg p_2 \land \neg p_2)$



Question 2

Solution:

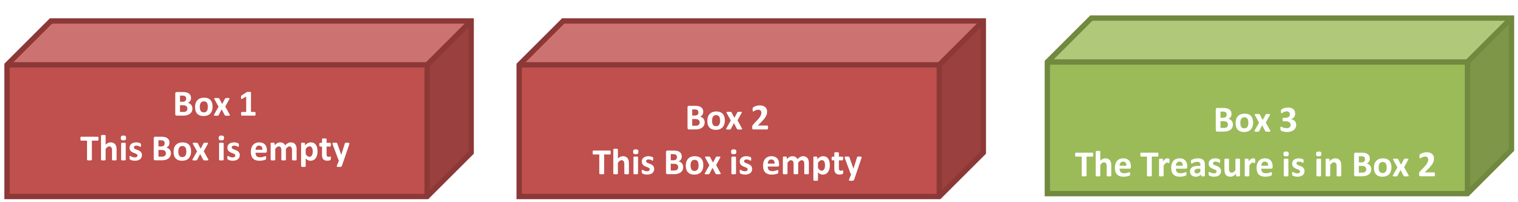
- If the first statement written in box 1 is true, while the other two are false then: $(\neg p_1 \wedge p_2 \wedge \neg p_2)$
- If the second statement written in box 2 is true, while the other two are false then: $(p_1 \wedge \neg p_2 \wedge \neg p_2)$
- If the second statement written in box 2 is true, while the other two are false then: $(p_1 \wedge \neg p_2 \wedge \neg p_2)$

Box 1	Box 2	Box 3
This Box is empty	This Box is empty	The Treasure is in Box 2

Question 2

Solution:

- If the first statement written in box 1 is true, while the other two are false then: $(\neg p_1 \land p_2 \land \neg p_2)$
- If the second statement written in box 2 is true, while the other two are false then: $(p_1 \land \neg p_2 \land \neg p_2)$
- If the third statement written in box 3 is true, while the other two are false then:$(p_1 \land p_2 \land p_2)$



Question 2

Solution:

$(\neg p_1 \land p_2 \land \neg p_2) \lor (p_1 \land \neg p_2 \land \neg p_3) \lor (p_1 \land p_2 \land p_2) \equiv \mathrm{T}$

$\equiv (\neg p_1 \land F) \lor (p_1 \land \neg p_2) \lor (p_1 \land p_2)$ *Negation law, Idempotent law*

$\equiv (p_1 \land \neg p_2) \lor (p_1 \land p_2)$ *Domination law, Identity law*

$\equiv p_1 \lor (\neg p_2 \land p_2)$ *Distributive law*

$\equiv p_1 \lor F$ *Negation law*

$\equiv p_1$ *Identity law*

The treasure is in Box 1

Question 3

a. Prove the following equivalences:

1. $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

2. $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$ are logically equivalent.

Question 3

Solution:

$p \oplus q \equiv (p \lor q) \land \neg (p \land q)$ Definition of $\oplus$

$\neg (p \oplus q) \equiv \neg(p \lor q) \lor (p \land q)$ De Morgan's law

$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$ Definition of $\leftrightarrow$

$p \leftrightarrow q \equiv (\neg p \lor q) \land (\neg q \lor p)$ Disjunction-implication twice

$p \leftrightarrow q \equiv [(\neg p \lor q) \land (\neg q)] \lor [(\neg p \lor q) \land (p)]$ Distributive law

$p \leftrightarrow q \equiv [(\neg p \land \neg q) \lor (q \land \neg q)] \lor [(\neg p \land p) \lor (q \land p)]$ Distributive law

$p \leftrightarrow q \equiv [(\neg p \land \neg q) \lor (F)] \lor [(F) \lor (q \land p)]$ Negation law

$p \leftrightarrow q \equiv (\neg p \land \neg q) \lor (q \land p)$ Identity law

$p \leftrightarrow q \equiv \neg(p \lor q) \lor (q \land p)$ De Morgan's law

$p \leftrightarrow q \equiv \neg[p \oplus q]$

Question 3

Solution:

$(p \longrightarrow r) \lor (q \longrightarrow r) \equiv (\neg p \lor r) \lor (\neg q \lor r)$ Disjunction- implication twice

$(p \longrightarrow r) \lor (q \longrightarrow r) \equiv (\neg p \lor \neg q) \lor (r \lor r)$ Commutative law

$(p \longrightarrow r) \lor (q \longrightarrow r) \equiv (\neg p \lor \neg q) \lor r$ Idempotent Law

$(p \longrightarrow r) \lor (q \longrightarrow r) \equiv \neg(p \land q) \lor r$ De Morgan's law

$(p \longrightarrow r) \lor (q \longrightarrow r) \equiv (p \land q) \rightarrow r$

Question 3

b. Simplify the following Logical expression with the laws of logic:

$(p \land q \land r) \lor (-p \lor -q \lor -r)$

Question 3

Solution:

$(p \wedge q \wedge r) \vee (-p \vee -q \vee -r) \equiv (p \wedge q \wedge r) \vee \neg(p \wedge q \wedge r)$

$(p \wedge q \wedge r) \vee (-p \vee -q \vee -r) \equiv T$

Question 3

c. Classify each of the following statements into tautology, contradiction or contingency.

1. $(p \lor q) \leftrightarrow (q \oplus p)$
2. $(\neg p \lor q) \leftrightarrow (p \rightarrow q)$
3. $(p \lor q) \land (p \downarrow q)$

Question 1

Solution:

1. $(p \vee q)$ and $(q \oplus p)$ are both true when one of the two propositions is true, and the other is false. So the implication $(p \vee q) \leftrightarrow (q \oplus p)$ is true in such a case.

$(p \vee q)$ and $(q \oplus p)$ have different truth values when p and q have the same truth values. So the the implication $(p \vee q) \leftrightarrow (q \oplus p)$ is true in such a case.

Since the implication can be true or false, it’s a contingency.

Question 1

Solution:

2. $(\neg p \vee q)$ and $(p \rightarrow q)$ are logically equivalent and they have the same truth values. Therefore, the bi-conditional implication:
$(\neg p \vee q) \leftrightarrow (p \rightarrow q)$ is a tautology.

Question 1

Solution:

3. $(p \vee q)$ and $(p \downarrow q)$ are the negation of each other's. Therefore, their truth values are opposite. Hence, the conjunction $(p \vee q) \wedge (p \downarrow q)$ is a contradiction.

Question 4

As a means of optimization, Java adopts the concept of short-circuit evaluation.

1. What is the output of the following Java program? Comment on your answer.

```java

public class Main
{

    static boolean myFun(){


        System.out.println("Hey, myFun has been called!");

        return true;
    }

    public static void main(String[] args) {

        if(1>0 || myFun())
        System.out.println("yesss!");
    }
}
```

Question 4

2. If we change the logical operator in the main function from or to and. What would be the output?

```java
public class Main
{

    static boolean myFun(){


        System.out.println("Hey, myFun has been called!");

        return true;
    }

    public static void main(String[] args) {

        if(1>0 || myFun())
        System.out.println("yesss!");
    }
}
```

Question 1

Solution:

1. Since the condition 1>0 holds, and the logical operator is or, the function myFun will not be called, and the output will only be “yesss!”

```java

public class Main
{

    static boolean myFun(){


        System.out.println("Hey, myFun has been called!");

        return true;
    }

    public static void main(String[] args) {

        if(1>0 || myFun())
        System.out.println("yesss!");
    }
}
```

Question 1

Solution:

2. If the operator is changed into and, the expression 1>0 is first evaluated. Since it’s true, the other operand of the operation myFun is evaluated as well.

When myFun is called, it's print statement “Hey, myFun has been called!” will be printed and a true value will be returned.

The returned true vale alongwith the true value coming from the 1>0 expreesion will make the entire 1>0 and myFun () conjunction true, thus “yesss! Will be printed.

Assignment I

1. Slightly modify the statement “There is a barber in a town that shaves those, and only those who do not shave themselves in the town” so that it becomes a proposition while maintaining the property that the barber shaves those, and only those who do not shave themselves in the town.

2. What’s the truth value of the proposition:

$11235813^{2024}+11235813^{2010} + 11235813$ is a prime number.

3. Define a compound proposition involving the propositional variables p and q that is always true regardless of the truth values of p and q.

Assignment I

4. Coupled with a justification, what is the output of the following Java program?

```java

public class Main
{

    static boolean myFun(){


        System.out.println("Hey, myFun has been called!");

        return true;
    }

    public static void main(String[] args) {

        if(myFun() || 5>3 )
        System.out.println("yesss!");
    }
}
```

Assignment I

5. Five of the following statements are negations of the other five. Pair each statement with its negation.

a. $p \oplus q$
b. $\neg p \land q$
c. $p \rightarrow (q \rightarrow p)$
d. $p \rightarrow q$
e. $p \land \neg q$
f. $q \land (p \land \neg p)$
g. $p \lor \neg q$
h. $p \leftrightarrow q$
i. $p \land (q \lor \neg q)$
j. $(p \rightarrow q) \rightarrow p)$

Assignment I

- The submission deadline is: **Saturday, May 25th 2024, 23:59:59 GMT+2.**

- Upload a clearly captured photocopy of your answer-sheet to: https://forms.gle/zKwBG7oeinWdncw4A

- In cases of cheating, the student will suspect themselves to **strict** cheating penalties.

END