V.A 1

/ / التاريخ .... اليوم Ass 1 .... عنوان الدرس

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1] $\underline{r} = \underline{A}\cos(wt) + \underline{B}\sin(wt)$; $\underline{A}, \underline{B}, w$ are const.

(i) Show $\frac{d^2\underline{r}}{dt^2} = -w^2\underline{r}$

$$\frac{d\underline{r}}{dt} = \underline{A}\,(-w\sin(wt)) + \underline{B}\,(w\cos(wt))$$

$$\frac{d^2\underline{r}}{dt^2} = -\underline{A}\,w^2\cos(wt) + (-\underline{B}\,w^2\sin(wt))$$

$$= -w^2\left(\underline{A}\cos(wt) + \underline{B}\sin(wt)\right)$$

$$= -w^2\underline{r} \quad *$$

(ii) Show $\underline{r} \times \frac{d\underline{r}}{dt} = w\underline{A} \times \underline{B}$

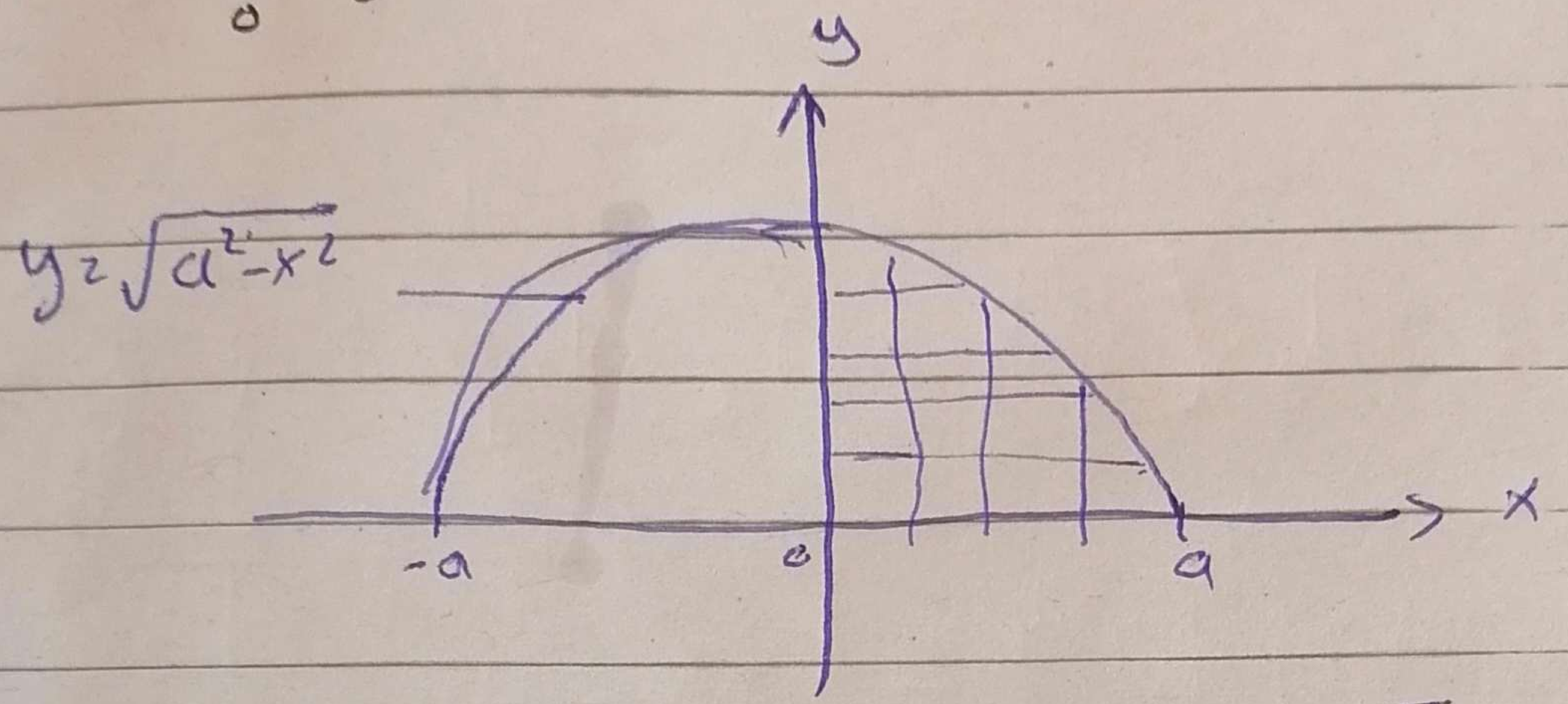
$$\underline{r} \times \frac{d\underline{r}}{dt} = \begin{vmatrix} \underline{A} & \underline{B} & \underline{A} \times \underline{B} \\ \cos(wt) & \sin(wt) & 0 \\ -w\sin(wt) & w\cos(wt) & 0 \end{vmatrix}$$

$$= (\underline{A} \times \underline{B})\left(w\cos^2(wt) + w\sin^2(wt)\right) =$$

$$= (\underline{A} \times \underline{B})\,w(1) = w\underline{A} \times \underline{B} \quad *$$

$\underline{A} \times \underline{B} \perp \underline{A} \cap \underline{A} \times \underline{B} \perp \underline{B}$

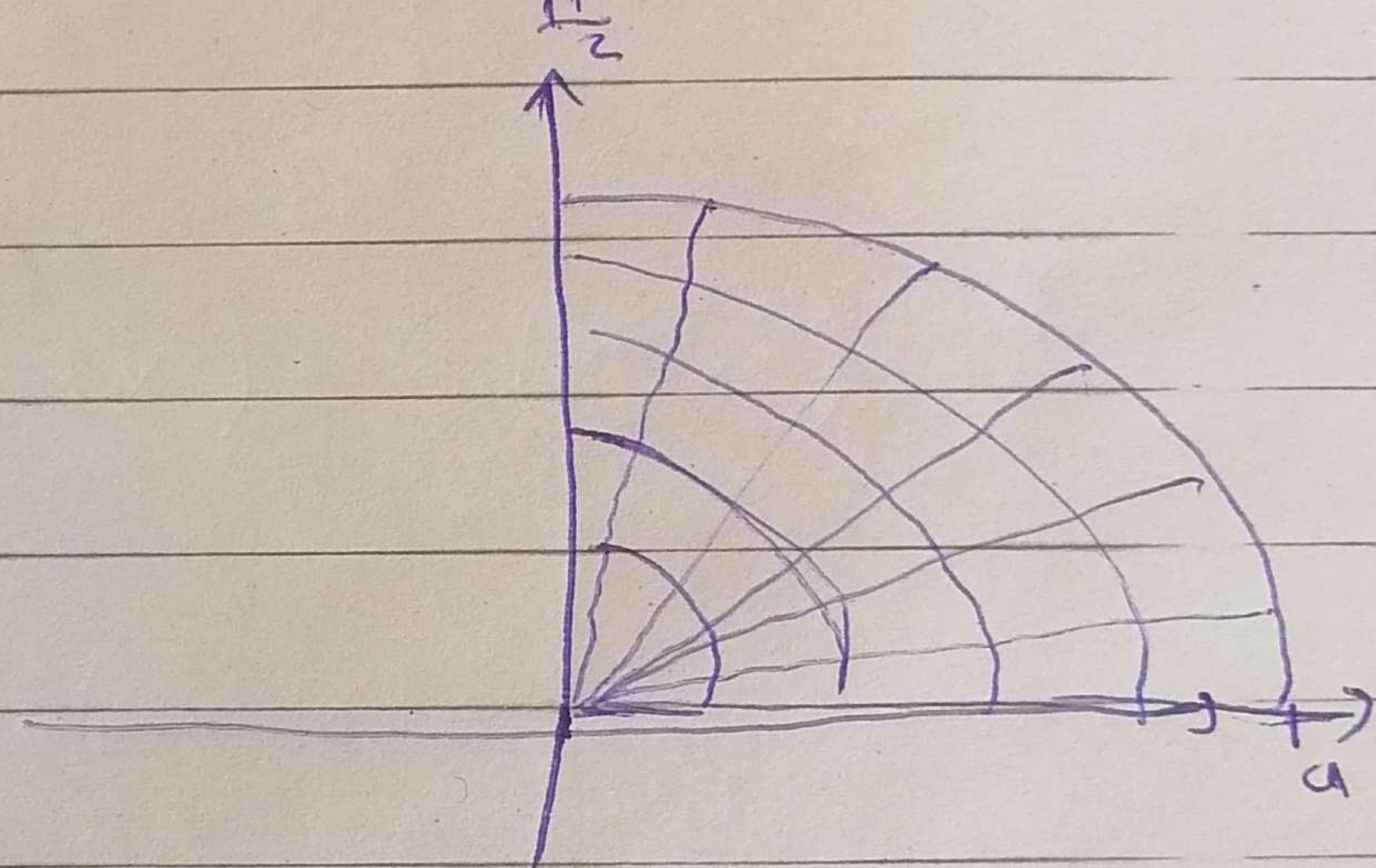
② $I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dy\, dx$



let $x = r\cos\theta$

$y = r\sin\theta$

$\frac{\partial(x,y)}{\partial(r,\theta)} = r$



$\therefore\ 0 \le \theta \le \frac{\pi}{2}\ ,\ 0 \le r \le a$

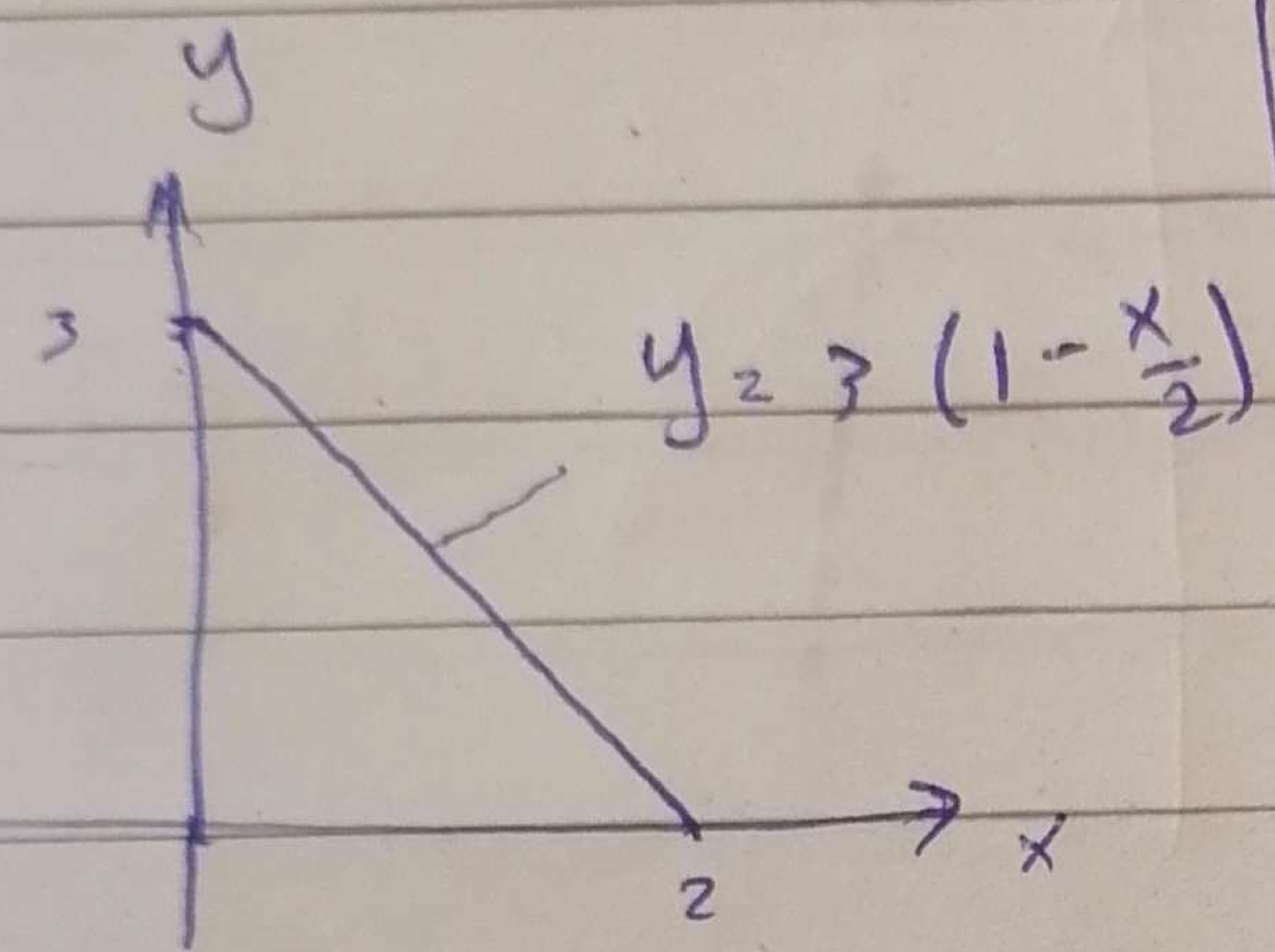
$\therefore I = \int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\, dy\, dx = \int_0^{\frac{\pi}{2}} \int_0^a \sqrt{a^2-r^2} \cdot r\, dr\, d\theta$

$= \int_0^{\frac{\pi}{2}} \left[-\frac{1}{3} (a^2-r^2)^{\frac{3}{2}} \right]_0^a d\theta = \frac{1}{3} \int_0^{\frac{\pi}{2}} a^3\, d\theta = \frac{a^3}{3} [\theta]_0^{\frac{\pi}{2}}$

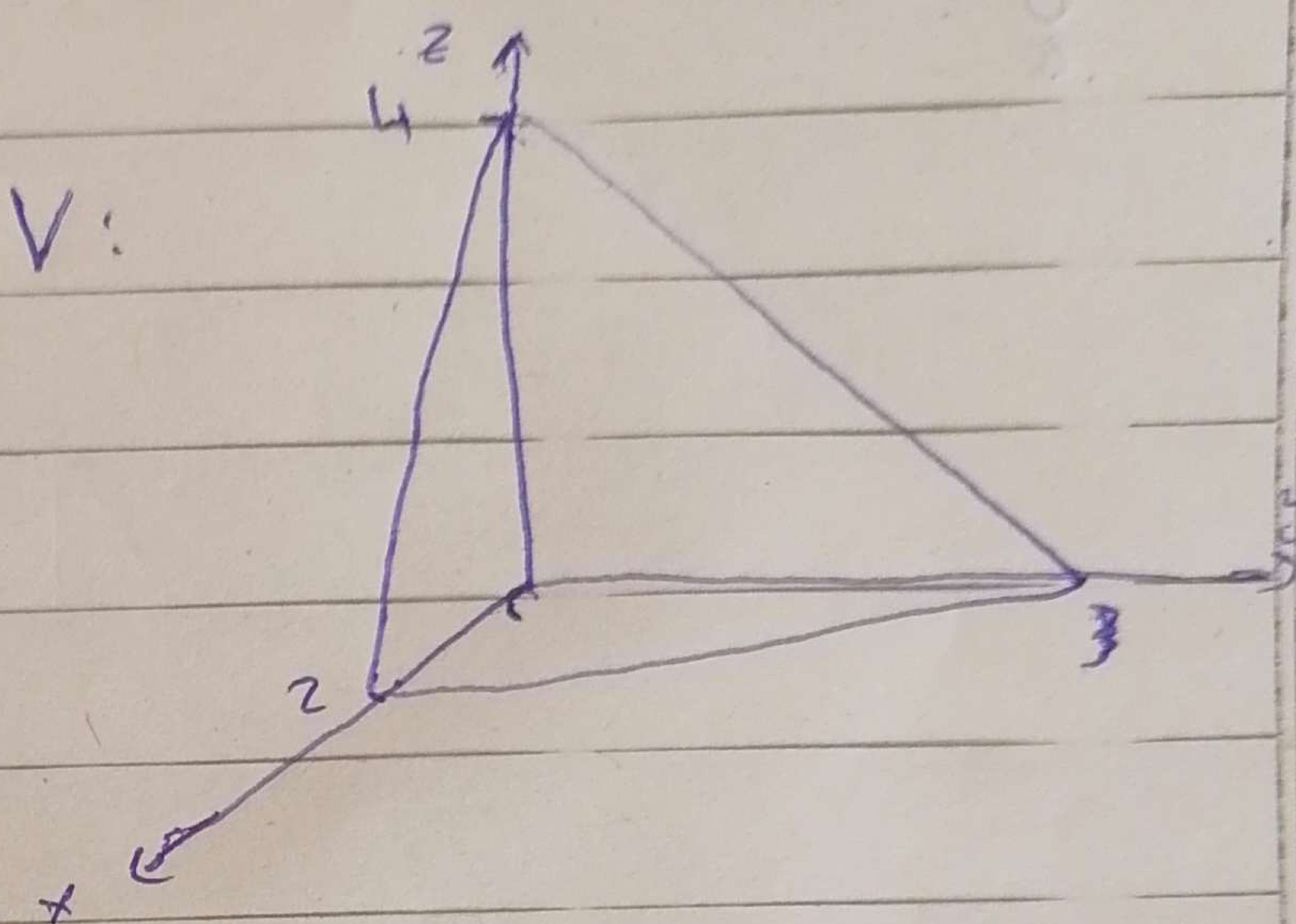
$\therefore I = \frac{a^3 \pi}{6}$

③ $z=0,\ y=0,\ x=0\ ;\ 6x+4y+3z=12\ ,\ V=?$

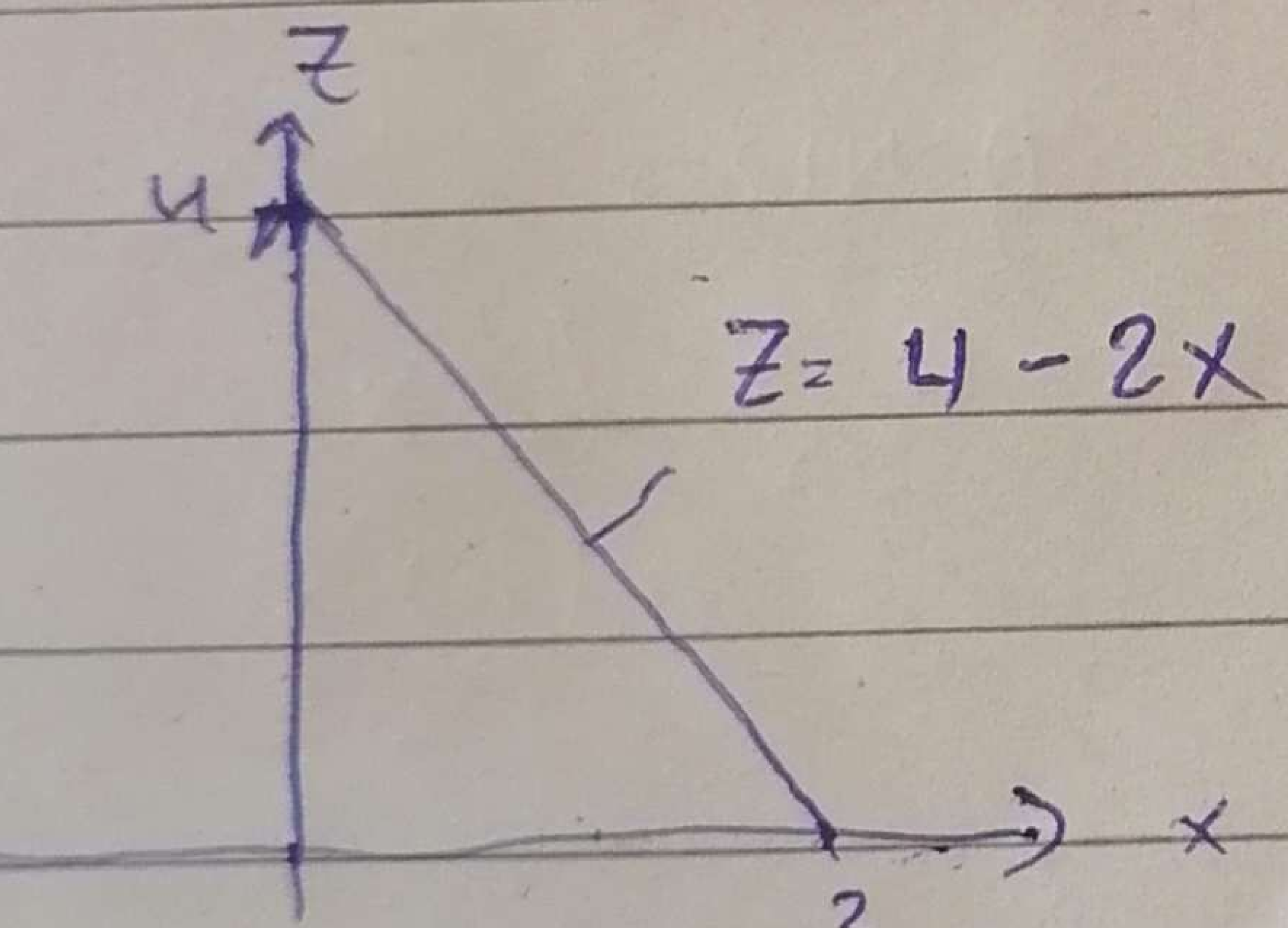
$z=0$



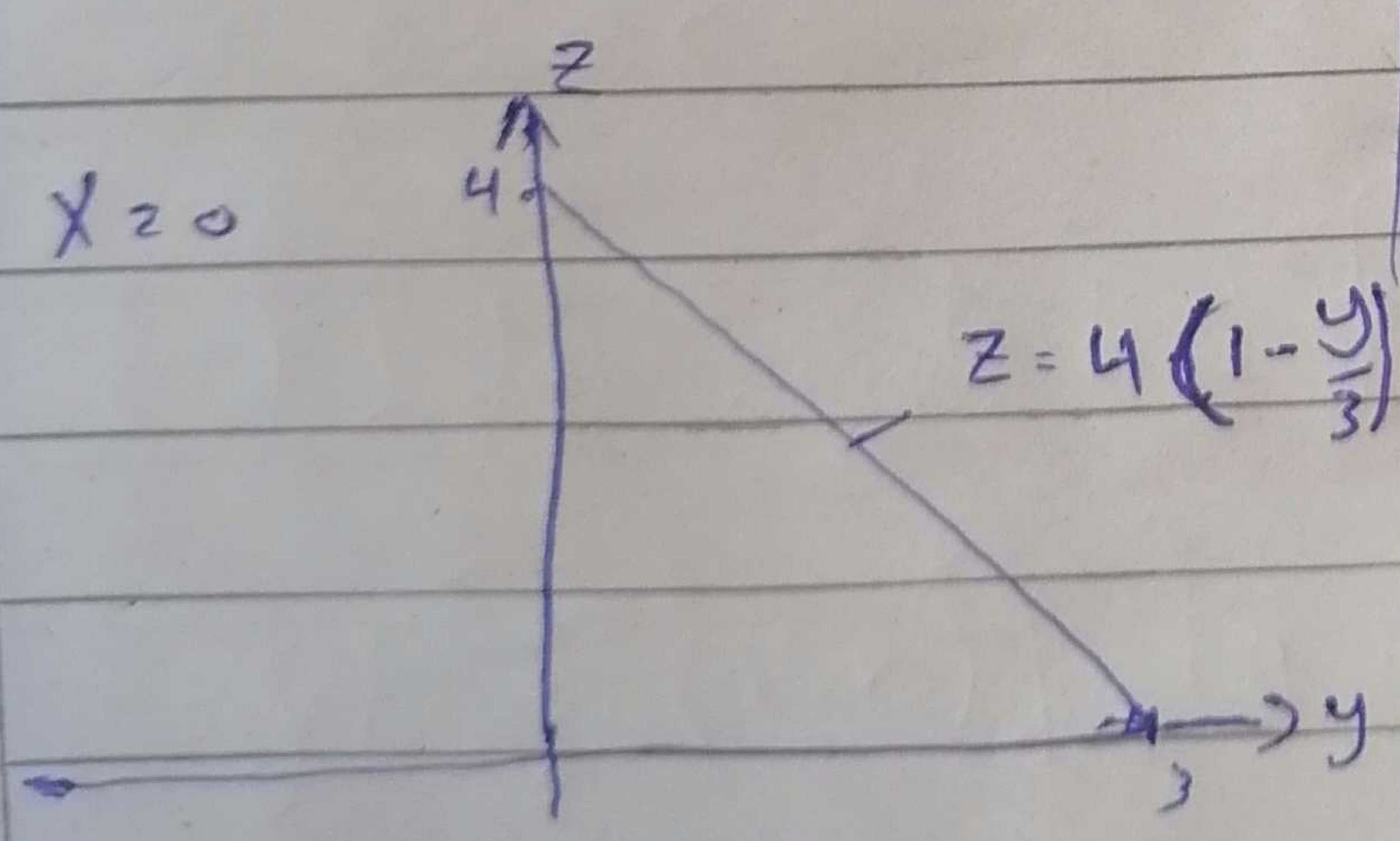
$\therefore$ V:



$y=0$



$x=0$



$\therefore\ 0 \le x \le 2$

$0 \le y \le 3\left(1-\frac{x}{2}\right)$

$0 \le z \le 4-2x-\frac{4}{3}y$

$$\therefore V = \int_0^2 \int_0^{3(1-\frac{x}{2})} \int_0^{4-2x-\frac{4}{3}y} dz\,dy\,dx$$

$$= \int_0^2 \int_0^{3(1-\frac{x}{2})} \left[4-2x-\frac{4}{3}y\right] dy\,dx$$

$$= \int_0^2 \left[4y-2xy-\frac{2}{3}y^2\right]_0^{3(1-\frac{x}{2})} dx$$

$$= \int_0^2 12-6x-6x+3x^2-6\left(1-\frac{x}{2}\right)^2 dx$$

$$= \int_0^2 12-12x+3x^2-6+6x-\frac{3}{2}x^2\,dx$$

$$= \int_0^2 6 - 6x + \frac{9}{2}x^2 \, dx$$

$$= \left[6x - 3x^2 + \frac{3}{2}x^3 \right]_0^2 = 6(2) - 3(4) + \frac{3}{\cancel{2}}(\cancel{8}^{4})$$

$$= \cancel{12} - \cancel{12} + 12$$

$$\therefore V = 12$$