Discrete Structures CS2023

Tutorial III

Question 1

Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

a. If n is a real number such that $n > 1$, then $n^2 > 1$.

Suppose that $n^2 > 1$.

Therefore, $n > 1$.

b. If n is a real number with $n > 3$, then $n^2 > 9$.

Suppose that $n^2 \leq 9$.

Therefore, $n \leq 3$.

c. If n is a real number with $n > 2$, then $n^2 > 4$.

Suppose that $n \leq 2$.

Therefore, $n^2 \leq 4$.

Question 1

Solution

a. If n is a real number such that $n > 1$, then $n^2 > 1$.

Suppose that $n^2 > 1$.

Therefore, $n > 1$.

Let p represents $n > 1$ and q represents $n^2 > 1$

The argument is in the form

$p \rightarrow q$

q

$\therefore p$

Which is invalid (the fallacy of affirming the conclusion)

Question 1

Solution

b. If n is a real number with $n > 3$, then $n^2 > 9$.

Suppose that $n^2 \leq 9$.

Therefore, $n \leq 3$.

Let p represents $n > 3$ and q represents $n^2 > 9$

The argument is in the form

$p \rightarrow q$

$\neg q$

$\therefore \neg p$

Which is valid(Modus Tollens)

Question 1

Solution

c. If n is a real number with $n > 2$, then $n^2 > 4$.

Suppose that $n \leq 2$.

Therefore $n^2 \leq 4$.

Let p represents $n > 2$ and q represents $n^2 > 4$

The argument is in the form

$p \rightarrow q$

$\neg p$

$\therefore \neg q$

Which is invalid(the fallacy of denying the hypothesis)

Question 2

Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

Proof

Let n be any even number then:

$$\exists k \in \mathbb{Z}, \mathrm{n} = 2\mathrm{k}$$

The additive inverse of n is $-n$ which is $-2k = 2(-k)$ and is also an even number by definition of even numbers.

Question 3

Use a direct proof to show that the product of two odd numbers is odd.

Proof

Let m, n be any two odd numbers such then:

$$\exists k \in \mathbb{Z}, m = 2k + 1$$
$$\exists l \in \mathbb{Z}, n = 2l + 1$$

Now $mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$ is also an odd number by definition of odd numbers.

Question 4

Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

Proof

Assume that x is a rational number, y is an irrational number, and their sum is rational.

That means $x = \frac{a}{b}$ for some integers $a, b \neq 0$.

y cannot be written as the division of two integers,

And the sum $x + y = \frac{k}{l}$ for some integers $k, l \neq 0$.

Solving the last equation for y gives:

$$y = \frac{k}{l} - x$$

$$y = \frac{k}{l} - \frac{a}{b} = \frac{kb - al}{bl}$$

Which contradicts that y cannot be written as the division of two integers.

Therefore, that the sum of an irrational number and a rational number is irrational.

Question 5

Find a counterexample to the statement that every positive integer can be written as the sum of the squares of three integers.

Solution

$1 = 0^2 + 0^2 + 1^2$
$2 = 0^2 + 1^2 + 1^2$
$3 = 1^2 + 1^2 + 1^2$
$4 = 0^2 + 0^2 + 2^2$
$5 = 0^2 + 1^2 + 2^2$
$6 = 1^2 + 1^2 + 2^2$

$7 = 0 + 0 + 7$
$7 = 0 + 1 + 6$
$7 = 0 + 2 + 5$
$7 = 0 + 3 + 4$
$7 = 1 + 1 + 5$
$7 = 1 + 2 + 4$
$7 = 1 + 3 + 3$
$7 = 2 + 2 + 3$

7 is a counter example!

Assignment III

1. Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

2. Prove that these four statements about the integer *n* are equivalent:
(*i*) n^2 is odd, (*ii*) $1 - n$ is even, (*iii*) n^3 is odd, (*iv*) n^2 + 1 is even.

3. Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.

Assignment III

- The submission deadline is: **Saturday, June 29th 2024, 23:59:59 GMT+2.**
- Upload a clearly captured photocopy of your answer-sheet to: https://forms.gle/zKwBG7oeinWdncw4A
- In cases of cheating, the student will suspect themselves to **strict** cheating penalties.