

# Advanced Statistics HW8

*Due date: November 29, 2018*

## Exercises 1

Given the estimators of the mean  $T_1 = (X_1 + 2X_2 + X_3)/4$  and  $T_2 = (X_1 + X_2 + X_3)/3$ , where  $X_1, X_2, X_3$  is a random sample from a  $N(\mu, \sigma^2)$  distribution, prove that  $T_2$  is more efficient than  $T_1$ .

## Exercises 2

Let  $X$  be a  $\text{Bin}(n, p)$  random variable.

- (a) Find the mean squared error of the  $p$  parameter estimators  $T_1 = X/n$  and  $T_2 = (X + 2)/(n + 4)$ .
- (b) When  $n = 20$  and  $p = 0.4$ , which estimator,  $T_1$  or  $T_2$ , has the smaller MSE?
- (c) Plot the efficiency of  $T_2$  relative to  $T_1$  versus  $p$  values in  $(0, 1)$  for  $n$  values from 1 to 10.

## Exercises 3

Consider a random sample of size  $n$  from a geometric distribution.

- (a) Find the method of moments estimator of  $p$ .
- (b) Find the maximum likelihood estimator of  $p$ .
- (c) Use the results from (a) and (b) to compute the method of moments and maximum likelihood estimates from the sample  $\{8, 1, 2, 0, 0, 0, 2, 1, 3, 3\}$ , which represents the number of Bernoulli trials that resulted in failure before the first success in 10 experiments.

## Exercise 4

Consider the density function

$$f(x) = (\theta + 1)(1 - x)^\theta, 0 \leq x \leq 1, \theta > 0.$$

- (a) Find the maximum likelihood estimator of  $\theta$  for a random sample of size  $n$ .
- (b) Set the seed equal to 3, and generate 20,000 values from  $f(x)$  when  $\theta = 5$ . Calculate the maximum likelihood estimate of  $\theta$  from the generated values.
- (c) How close is the maximum likelihood estimate in (b) to  $\theta = 5$ ?

## Exercise 5

Consider an exponential distribution with mean  $\theta$  and the following estimators of  $\theta$ :

$$\hat{\theta}_1 = X_1, \hat{\theta}_2 = \frac{1}{2}(X_1 + X_2), \hat{\theta}_3 = \min(X_1, X_2, X_3).$$

- (a) Find the mean and variance of each estimator.
- (b) Are any of the estimators efficient?
- (c) Which estimator is the MLE?
- (d) Let  $X$  be an exponential random variable with mean  $\theta + 2$ . Which estimator is an unbiased estimator of  $\theta$ ?

## Excercise 6

Consider the density function

$$f(x) = 3\pi\theta x^2 e^{-\theta\pi x^3}, x \geq 0.$$

- (a) Set the seed equal to 102, and generate a random sample of size  $n = 20,000$  with  $\theta = 5$ .
- (b) Find the sample mean and the sample variance of the random values generated in (a).
- (c) Create a density histogram of the simulated values from (a) and superimpose the density function over the density histogram.
- (d) Find the maximum likelihood estimate of  $\theta$ .
- (e) Plot the logarithm of the likelihood function versus  $\theta$ . Use values for  $\theta$  from 0 to 15.