# Assignment 7

## Question 1

If $X \sim \chi^2(10)$, find the constants $a$ and $b$ such that $P(a < X < b) = 0.90$ and $P(X < a) = 0.05$.

## Question 2

The pill weight for a particular type of vitamin follows a normal distribution with a mean of 0.6 grams and a standard deviation of 0.015 grams. It is known that a particular therapy consisting of a box of vitamins with 125 pills is not effective if more than 20% of the pills are under 0.58 grams.

(a) Find the probability that the therapy with a box of vitamins is not effective.

(b) A supplement manufacturer sells vitamin bottles containing 125 vitamins per bottle with 50 bottles per box with a guarantee that at least 47 bottles per box weigh more than 74.7 grams. Find the probability that a randomly chosen box does not meet the guaranteed weight.

## Question 3

Simulate 20,000 random samples of sizes 30, 100, 300, and 500 from an exponential distribution with a mean of 1/5. Estimate the density of the sampling distribution of the sample mean with the function `density()`. Superimpose a theoretical normal density with appropriate mean and standard deviation. What sample size is needed to get an estimated density close to a normal density?

## Question 4

Consider a random sample of size $n$ from an exponential distribution with parameter $\lambda$.

(a) Use moment generating functions to show that the sample mean follows a $\Gamma(n, n\lambda)$.

(b) Graph the theoretical sampling distribution of $X$ when sampling from an $Exp(\lambda = 1)$ for $n = 30$, 100, 300, and 500. Superimpose an appropriate normal density for each $\Gamma(n, n\lambda)$. At what sample size do the sampling distribution and superimposed density virtually coincide?

## Question 5

A farmer is interested in knowing the mean weight of his chickens when they leave the farm. Suppose that the standard deviation of the chickens' weight is 500 grams.

(a) What is the minimum number of chickens needed to ensure that the standard deviation of the mean is no more than 100 grams?

(b) If the farm has three coops and the mean chicken weight in each coop is 1.8, 1.9, and 2 kg, respectively, calculate the probability that a random sample of 50 chickens with an average weight larger than 1.975 kg comes from the first coop. Assume the weight of the chickens follows a normal distribution.