

# Advanced Statistics HW7

*Due date: November 20, 2018*

## Exercises 1

If  $X \sim \chi^2(10)$ , find the constants  $a$  and  $b$  so that  $P(a < X < b) = 0.90$  and  $P(X < a) = 0.05$ .

## Exercises 2

The pill weight for a particular type of vitamin follows a normal distribution with a mean of 0.6 grams and a standard deviation of 0.015 grams. It is known that a particular therapy consisting of a box of vitamins with 125 pills is not effective if more than 20% of the pills are under 0.58 grams.

- (a) Find the probability that the therapy with a box of vitamins is not effective.
- (b) A supplement manufacturer sells vitamin bottles containing 125 vitamins per bottle with 50 bottles per box with a guarantee that at least 47 bottles per box weigh more than 74.7 grams. Find the probability that a randomly chosen box does not meet the guaranteed weight.

## Exercises 3

Simulate 20,000 random samples of sizes 30, 100, 300, and 500 from an exponential distribution with a mean of  $1/5$ . Estimate the density of the sampling distribution of the sample mean with the function `density()`. Superimpose a theoretical normal density with appropriate mean and standard deviation. What sample size is needed to get an estimated density close to a normal density?

## Exercise 4

Consider a random sample of size  $n$  from an exponential distribution with parameter  $\lambda$ .

- (a) Use moment generating functions to show that the sample mean follows a  $\Gamma(n, n\lambda)$ .
- (b) Graph the theoretical sampling distribution of  $X$  when sampling from an  $Exp(\lambda = 1)$  for  $n = 30, 100, 300$ , and 500. Superimpose an appropriate normal density for each  $\Gamma(n, n\lambda)$ . At what sample size do the sampling distribution and superimposed density virtually coincide?

## Exercise 5

A farmer is interested in knowing the mean weight of his chickens when they leave the farm. Suppose that the standard deviation of the chickens' weight is 500 grams.

- (a) What is the minimum number of chickens needed to ensure that the standard deviation of the mean is no more than 100 grams?
- (b) If the farm has three coops and the mean chicken weight in each coop is 1.8, 1.9, and 2 kg, respectively, calculate the probability that a random sample of 50 chickens with an average weight larger than 1.975 kg comes from the first coop. Assume the weight of the chickens follows a normal distribution.