Assignment 5

Question 1

In the New York State lottery game, six of the numbers 1 through 54 are chosen by a customer. Then, in a televised drawing, six of these numbers are selected. If all six of a customer's numbers are selected, then that customer wins a share of the first prize. If five or four of the numbers are selected, the customer wins a share of the second or the third prize. What is the probability that any customer will win a share of the first prize, the second prize, and the third prize, respectively?

Question 2

An urn contains 14 balls; 6 of them are white, and the others are black. Another urn contains 9 balls; 3 are white, and 6 are black. A ball is drawn at random from the first urn and is placed in the second urn. Then, a ball is drawn at random from the second urn. If this ball is white, find the probability that the ball drawn from the first urn was black.

Question 3

Two independently wealthy philatelists, Alvin and Bob, are interested in buying rare stamps at a private auction. For each stamp up for auction, given that the previous bid did not win, Alvin or Bob wins on their ith bid with probability p. Assume that Alvin always makes the first bid.

(a) Find the probability that Alvin wins the first auction.

(b) If two stamps are actually auctioned, find the probability that they are purchased by the same bidder.

(Hint: $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ if $|x| < 1$.)

Question 4

Consider an experiment where two dice are rolled. Let the random variable X equal the sum of the two dice and the random variable Y be the difference of the two dice.

(a) Find the mean, variance and skewness of X respectively.

(b) Find the mean, variance and skewness of Y respectively.

Question 5

The time, in hours, a child practices his musical instrument on Saturdays has pdf

$$f(x) = \begin{cases} k(1-x) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find k to make $f(x)$ a valid pdf.

(b) Write the cdf and find the probability the child practices more than 48 minutes on a Saturday.

(c) Find the mean and variance of X.

Question 6

(a) Construct a plot for the probability mass function and the cumulative probability distribution of a binomial random variable $Bin(8, 0.3)$.

(b) Find the smallest value of k such that $P(X \leq k) \geq 0.44$ when $X \sim Bin(8, 0.7)$.

(c) Calculate $P(Y \geq 3)$ if $Y \sim Bin(20, 0.2)$.

Question 7

Let X be a random variable with finite $E(X-a)^2$, where a is a scalar, find a such that $E(X-a)^2$ is minimized.

Question 8

Given independent random variables Y_1, Y_2, X, W, Z_1, Z_2, and Z_3,

(a) Compute $P((Y_2 \geq 3) \cap (Y_1 < 9))$ if $Y_1 \sim Bin(10, 0.3)$ and $Y_2 \sim Bin(5, 0.1)$.

(b) Compute $P(X \geq 2 | X < 6)$ if $X \sim Pois(\lambda = 4)$.

(c) If $W \sim N(\mu, \sigma^2)$, find the value of k that satisfies the equation $P(\mu < W < \mu + 2k\sigma) = 0.45$.

(d) If $Z_i \sim N(0, 1)$ for $i = 1, 2, 3$, compute $P(\sqrt{Z_1^2 + Z_2^2 + Z_3^2} > 1.5)$.