

Advanced Statistics HW8

Due date: November 28, 2018

Exercises 1

Given the estimators of the mean $T_1 = (X_1 + 2X_2 + X_3)/4$ and $T_2 = (X_1 + X_2 + X_3)/3$, where X_1, X_2, X_3 is a random sample from a $N(\mu, \sigma^2)$ distribution, prove that T_2 is more efficient than T_1 .

Exercises 2

Let X be a $\text{Bin}(n, p)$ random variable.

- (a) Find the mean squared error of the p parameter estimators $T_1 = X/n$ and $T_2 = (X + 2)/(n + 4)$.
- (b) When $n = 20$ and $p = 0.4$, which estimator, T_1 or T_2 , has the smaller MSE?
- (c) Plot the efficiency of T_2 relative to T_1 versus p values in $(0, 1)$ for n values from 1 to 10.

Exercises 3

Consider a random sample of size n from a geometric distribution.

- (a) Find the method of moments estimator of p .
- (b) Find the maximum likelihood estimator of p .
- (c) Use the results from (a) and (b) to compute the method of moments and maximum likelihood estimates from the sample $\{8, 1, 2, 0, 0, 0, 2, 1, 3, 3\}$, which represents the number of Bernoulli trials that resulted in failure before the first success in 10 experiments.

Exercise 4

Consider the density function

$$f(x) = (\theta + 1)(1 - x)^\theta, 0 \leq x \leq 1, \theta > 0.$$

- (a) Find the maximum likelihood estimator of θ for a random sample of size n .
- (b) Set the seed equal to 3, and generate 20,000 values from $f(x)$ when $\theta = 5$. Calculate the maximum likelihood estimate of θ from the generated values.
- (c) How close is the maximum likelihood estimate in (b) to $\theta = 5$?

Exercise 5

Consider an exponential distribution with mean θ and the following estimators of θ :

$$\hat{\theta}_1 = X_1, \hat{\theta}_2 = \frac{1}{2}(X_1 + X_2), \hat{\theta}_3 = \min(X_1, X_2, X_3).$$

- (a) Find the mean and variance of each estimator.
- (b) Are any of the estimators efficient?
- (c) Which estimator is the MLE?
- (d) Let X be an exponential random variable with mean $\theta + 2$. Which estimator is an unbiased estimator of θ ?

Excercise 6

Consider the density function

$$f(x) = 3\pi\theta x^2 e^{-\theta\pi x^3}, x \geq 0.$$

- (a) Set the seed equal to 102, and generate a random sample of size $n = 20,000$ with $\theta = 5$.
- (b) Find the sample mean and the sample variance of the random values generated in (a).
- (c) Create a density histogram of the simulated values from (a) and superimpose the density function over the density histogram.
- (d) Find the maximum likelihood estimate of θ .
- (e) Plot the logarithm of the likelihood function versus θ . Use values for θ from 0 to 15.