

Probability and Statistics

* Lecture 3 *

* Expectation $E(x)$:

$$\Rightarrow E(x) = \begin{cases} \sum x p(x) & , x \rightarrow \text{discrete r.v} \\ \int_{-\infty}^{\infty} x f(x) dx & , x \rightarrow \text{continuous r.v} \end{cases}$$

* Example: A box contains 10 transistors, 2 of them are defective. A man selected at random one transistor from the box until he obtains a non defective one. Find the expected number of transistors to be chosen.

→ Let x be a random variable showing the number of chosen transistors.

x	1	2	3	$\overset{8}{G} \overset{2}{D} \rightarrow G$
$p(x)$	$p(G) = \frac{8}{10}$	$p(D_1 \cap G_2) = p(D_1) \cdot p(G_2 D_1)$ $= \frac{2}{10} \times \frac{8}{9}$	$p(D_1 \cap D_2 \cap G_3)$ $= p(D_1) \cdot p(D_2 D_1) \cdot p(G_3 D_1 \cap D_2)$ $= \frac{2}{10} \times \frac{1}{9} \times \frac{8}{8}$	
$x p(x)$	$\frac{8}{10}$	$\frac{32}{90}$	$\frac{6}{90}$	

$$\therefore E(x) = \sum x p(x)$$

$$= \frac{8}{10} + \frac{32}{90} + \frac{6}{90} = \frac{11}{9}$$

"discrete r.v."

$$* \therefore p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$$\therefore p(A \cap B) = p(A|B) \cdot p(B)$$

* Example 2: A man wants to match 3 names with their professional names. He doesn't know the names and he tries to match by guess. Find the expected number of correct matches.

⇒ Let X be a random variable showing the number of correct matches.

$X=0$
 or $X=1$
 or $X=3$

"there is no true matches."
 "there is one true match and two false."
 "All matches are true."

"for illustration."

* Notice that: We'll not suppose that $X=2$ because if 2 matches are true, the third one must be true.

X	0	1	3
$P(X)$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{1}{6}$
$X P(X)$	0	$\frac{3}{6}$	$\frac{1}{2}$

$$\therefore E(X) = \sum X P(X)$$

$$= 0 + \frac{3}{6} + \frac{1}{2} = 1$$

* Explanation:

Name	prof
a	A
b	B
d	D

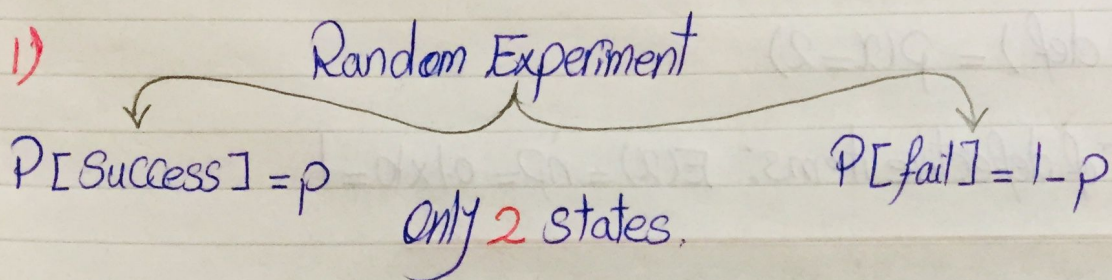
→ $S = \{(aD, bA, dB), (aB, bD, dA), (aA, bD, dB), (aD, bB, dA), (aB, bA, dD), (aA, bB, dD)\}$

, we have 2 probabilities that there is no true matches. " $\frac{2}{6}$ "
 , we have 3 probabilities that there is only one true match. " $\frac{3}{6}$ "
 , we have only one probability that all matches are true. " $\frac{1}{6}$ ".

* Chapter 3: "Some important distributions."

1) Binomial distribution: "Important."

→ There are 3 conditions must be satisfied to use Binomial dist. :-



* Success means achieving the required at the experiment.

2) The R.E is repeated n independent trials.

3) The probability of success is constant in each trial. $E(x) = np$

→ Law: $P(x) = {}^nC_x p^x (1-p)^{n-x}$

* Example: Tossing a coin 10 times. Find the prob. of appearing head 4 times.

→ $P(x) = {}^{10}C_4 \cdot \left(\frac{1}{2}\right)^4 \cdot \left(1 - \frac{1}{2}\right)^{10-4}$

* Example 2: The percentage of defective items in the production of a certain factory is 0.1. A man selected at random 10 items of the production. Find the probability of getting 2 defective items.

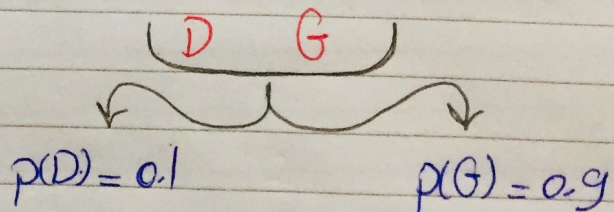
" " " " " at least 2 defective items.

" " " " " 8 good items.

→ Let x be a random variable showing number of defective items.

here: $x \sim b(n, p)$

→ x submits to binomial conditions at $n=10$, $p=0.1$.



$$\therefore p(x=2) = {}^{10}C_2 (0.1)^2 (0.9)^8$$

"1st req."

$$\begin{aligned} 2) p(x \geq 2) &= p(x=2) + p(x=3) + \dots + p(x=10) \\ \text{or } p(x \geq 2) &= 1 - p(x^0) \\ &= 1 - (p(x=0) + p(x=1)) \end{aligned}$$

$$3) p(8 \text{ Good}) = p(2 \text{ def.}) = p(x=2)$$

The expected number of defective items: $E(x) = np = 0.1 \times 10 = 1$