Digital Signal

Section 1

* Complex Numbers:-

→ Form: $z = \underbrace{a}_{\text{real number}} + \underbrace{bi}_{\text{imaginary number}}$

- ex: $\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1} = \pm 2i \rightarrow b = \pm 2 \;,\; a = 0$

* Notice that:

→ $i^0 = 1$ → $i^1 = i$

→ $i^2 = -1$ → $i^3 = -i$

$\therefore\; i^4 = i^2 \cdot i^2 = -1 \times -1 = 1 \;,\; i^5 = i^4 \cdot i = 1 \times i = i \;,\; i^6 = i^4 \cdot i^2 = 1 \times -1 = -1$ and so on...

* Operations on Complex Numbers:-

1) Addition: $(2+3i) + (1-2i) = (3+i)$

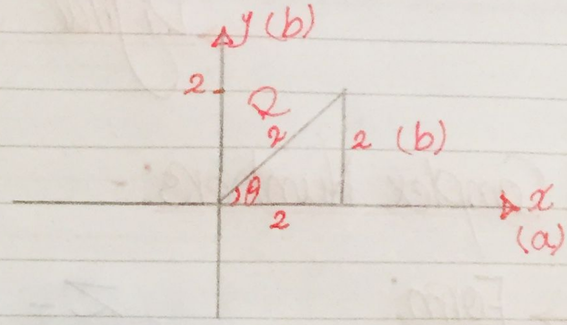
2) Subtraction: $(2+3i) - (1-2i) = (1+5i)$

3) Multiplication: $(2+3i) * (1-2i) = 2 - 4i + 3i - 6i^2 = 2 - i + 6 = 8 - i$

4) Division: $\frac{2+3i}{1-2i}$ « X Conjugate »

$$= \frac{2+3i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{2+4i+3i+6i^2}{1+\cancel{2i}-\cancel{2i}-4i^2} = \frac{-4+7i}{5}$$

* Rectangular form:-



→ $x = R\cos\theta$, $y = R\sin\theta$

$\therefore z = R\cos\theta + R\sin\theta$

$= R(\cos\theta + \sin\theta)$

* Polar form:-

→ $r = \sqrt{a^2 + b^2}$ → $\theta = \tan^{-1}\dfrac{b}{a}$

, $z = r < \theta$

- Example:

$z = 2 + 2i$ « Convert to polar, rectangular forms. »

→ polar: $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

, $\theta = \tan^{-1}\dfrac{2}{2}$ $\Rightarrow \theta = 45°$

$\therefore z = 2\sqrt{2} < 45$

→ rectangular: $z = 2\sqrt{2}(\cos 45 + \sin 45)$

* Determine the Complex Solution: « $z = a + bi$ »

1) $z^2 + 36 = 0$

→ $z^2 = -36$

$\therefore z = \sqrt{36} . \sqrt{-1} = \pm 6i$

2) $z^2 + 8z + 20 = 0$ « Can't be factorized. »

→ « $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ » $= \dfrac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 20}}{2 \times 1} = \dfrac{-8 \pm \sqrt{64 - 80}}{2} = -4 \pm 2i$

* Roots in Complex numbers:-

- polar: $z = r\angle\theta$

$\therefore \sqrt[n]{z} = \sqrt[n]{r\angle\theta}$

roots:

$$r_1 = r^{1/n} \angle \frac{\theta}{n} \;,\; r_2 = r^{1/n} \angle \frac{\theta + 360}{n} \;,\; r_3 = r^{1/n} \angle \frac{\theta + (2 \times 360)}{n}$$

- examples:

1) $z = 5 \angle 53.13$ «Three Cube roots.»

$\rightarrow \sqrt[3]{z} = \sqrt[3]{5 \angle 53.13}$

$$\therefore r_1 = 5^{1/3} \angle \frac{53.13}{3} \;,\; r_2 = 5^{1/3} \angle \frac{53.13 + 360}{3} \;,\; r_3 = 5^{1/3} \angle \frac{53.13 + (2 \times 360)}{3}$$

2) $z = 2\sqrt{2} \angle 45$ «Three Cube roots.»

$\rightarrow \sqrt[3]{z} = \sqrt[3]{2\sqrt{2} \angle 45}$

$$\therefore r_1 = (2\sqrt{2})^{1/3} \angle \frac{45}{3} \;,\; r_2 = (2\sqrt{2})^{1/3} \angle \frac{45 + 360}{3} \;,\; r_3 = (2\sqrt{2})^{1/3} \angle \frac{45 + (2 \times 360)}{3}$$