

Discrete Structures

* Lecture 8 *

- Chapter 2: Logic

→ The processor of Computer contains ALU «arithmetic logical unit» and CU «Control unit».

ALU is responsible for arithmetic and logical operations.

- We will study Binary «mathematical» Logic «True or False»

* Propositional Statement: using a letter to represent specific sentence. «True or False»

- Examples:

P: $2+3 > 5$

T/F

q: The Doctor will come

T/F

r: The Sun is Shiny Today

T/F

* Compound Statement: more than one propositional statement in one sentence.

* Connectives *

1) Negation « \sim »:

→ p: student Succeeded

$\sim p$: Student didn't Succeed.

T F
F T

→ q: $2+3 > 4$ T

$\sim q$: F

2) AND, « \wedge », Conjunction:

Truth Table

q	p	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

r	q	p	$r \wedge q \wedge p$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

3) OR, 'v', disjunction:-

q	p	$q \vee p$
T	T	T
F	T	T
T	F	T
F	F	F

"Truth Table"

4) Conditional Statement: " $p \xrightarrow{\text{leads to}} q$ "

* antecedent $\xrightarrow{\text{leads to}}$ Conclusion
or hypothesis $\xrightarrow{\text{leads to}}$ Consequence

p	q	$p \rightarrow q$
"Study"	"Success"	
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table

p	q	r	$p \vee q \vee r$
T	T	T	T
F	T	T	T
T	F	T	T
F	F	T	T
T	T	F	T
F	T	F	T
T	F	F	T
F	F	F	F

5) Logical Equivalent: " $p \xleftrightarrow{\text{Congruent}} q$ "

* $p \leftrightarrow q$ means $(p \rightarrow q) \wedge (q \rightarrow p)$

6) XOR " $p \oplus q$ ":-

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
F	T	F
T	F	F
F	F	T

* Notice that:

$$\begin{aligned} \rightarrow p \oplus p \oplus q &= q \\ \rightarrow p \oplus q \oplus q &= p \end{aligned}$$

* If all results are **True** \rightarrow Tautology
 * If all results are **False** \rightarrow Absurdity
 * If all results are mix of True and False \rightarrow Contingency

* Example: prove that: $(p \rightarrow q) \leftrightarrow \sim q \rightarrow \sim p$

p	q	$p \xrightarrow{1} q$	$\sim q$	$\sim p$	$\sim q \xrightarrow{2} \sim p$	$1 \leftrightarrow 2$
T	T	T	F	F	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	F
F	F	T	T	T	T	T

* Notes:-

$$\rightarrow p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$$

* Associative: $p \vee (q \vee r) \equiv (p \vee q) \vee r$

* Commutative: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

* $p \vee p \equiv p, \quad p \wedge p \equiv p, \quad \sim(\sim p) \equiv p, \quad \sim(p \vee q) \equiv \sim p \wedge \sim q$
 $\sim(p \wedge q) \equiv \sim p \vee \sim q$

* Properties:

- 1) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
- 2) $p \rightarrow q \equiv \sim p \vee q$ « important »
- 3) $(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
- 4) $\sim(p \rightarrow q) \equiv p \wedge \sim q$