Discrete Structures

* Lecture 9 *

* There are 3 methods to simplify Logical statements called: Negation, Converse and Contrapositive.

→ To simplify Logical statements means to contain only $\wedge, \vee, \sim$.

- **Example:** Simplify by Negation: $(A \wedge \sim B) \rightarrow (A \wedge B)$

$$\rightarrow \sim[(A \wedge \sim B) \rightarrow (A \wedge B)]$$

$$= \sim[\underbrace{\sim(A \wedge \sim B)}_{Z} \vee (A \wedge B)]$$

$$= \sim[(Z \vee A) \wedge (Z \vee B)]$$

$$= \sim(Z \vee A) \vee \sim(Z \vee B)$$

$$= \sim(\sim(A \wedge \sim B) \vee A) \vee \sim(\sim(A \wedge \sim B) \vee B)$$

$$= [(A \wedge \sim B) \wedge \sim A] \vee [(A \wedge \sim B) \wedge \sim B]$$

$$= \text{False} \vee (A \wedge \sim B)$$

$$= (A \wedge \sim B)$$

* $A \rightarrow B \equiv \sim A \vee B$

Commutative.

- **Simplify by Converse:** $(A \wedge \sim B) \rightarrow (A \wedge B)$

$$\rightarrow (A \wedge B) \rightarrow (A \wedge \sim B)$$

$$= \sim(A \wedge B) \vee (A \wedge \sim B)$$

$$= \underbrace{(\sim A \vee \sim B)}_{X} \vee (A \wedge \sim B)$$

$$= (X \vee A) \wedge (X \vee \sim B)$$

$$= (\sim A \vee \sim B \vee A) \wedge (\sim A \vee \sim B \vee \sim B)$$

$$= \text{true} \wedge (\sim A \vee \sim B)$$

$$= (\sim A \vee \sim B)$$

- **Simplify by Contrapositive:**

If $A \rightarrow B$

∴ Contrapositive: $\sim B \rightarrow \sim A$

$(A \wedge \neg B) \rightarrow (A \wedge B)$

$\rightarrow \neg(A \wedge B) \rightarrow \neg(A \wedge \neg B)$

$= (A \wedge B) \vee (\neg A \vee B)$, M

$= (M \vee A) \wedge (M \vee B)$

$= (\neg A \vee B \vee A) \wedge (\neg A \vee B \vee B)$

$=$ True $\wedge (\neg A \vee B)$

$= (\neg A \vee B)$

First Order Logic

Predicates «محمول» أو دالة

→ green(x) means x is green

green(pen): is pen green?

divides(1,x): 1 divides x

father(x,y): x is father of y

Quantifiers

for all «$\forall x$» كل القيم تحقق

There exist «$\exists x$» بعض القيم تحقق

→ $\exists x$ prime(x)

$\forall x$ divides(1,x)

$\exists x$ divides(2,x)

$\forall x$ Success(x) نجح جميع الطلاب

$\exists x$ Success(x) بعض الطلاب نجحوا

* $\neg \forall x\, p(x) \rightarrow$ ؟ لا قيمة تحقق $\equiv \forall x \neg p(x) \rightarrow$ ؟ جميع القيم لا تحقق

* $\neg \exists x\, p(x) \rightarrow$ لا قيمة تحقق

* $\exists x \neg p(x) \rightarrow$ يوجد على الأقل قيمة لا تحقق

Transform $\forall x\, p(x)$ to $\exists x\, p(x)$:-

1) $\neg \forall x\, p(x) \rightarrow \exists x \neg p(x)$

2) $\forall x \neg p(x) \rightarrow \neg \exists x\, p(x)$