"Physics"

Lecture «5»

Gauss's Law

* **Electric Flux «Φ_E» الفيض الكهربي :-** it is the number of electric field lines that penetrates a given Surface.

$$\phi = \int E \cdot dA$$

- if Closed Surface: $\phi_{closed} = \oint E \cdot dA \quad \rightarrow (1)$
- You must also know that: $\phi_{closed} = \dfrac{\sum Q_{en}}{\varepsilon_0} \quad \rightarrow (2)$

Where: $Q_{en} \rightarrow$ total Charge enclosed by the surface. مجموع الشحنات داخل السطح.
$\varepsilon_0 \rightarrow$ Constant.

$\rightarrow$ From (1), (2): $\phi_{closed} = \oint E \cdot dA = \dfrac{\sum Q_{en}}{\varepsilon_0}$

«Gauss's Law»

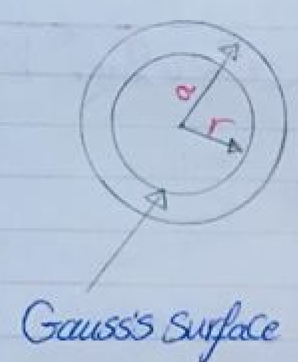
* Note: The electric field «E» must be uniform to apply Gauss's law.

- How to apply Gauss's Law to get «E»:-

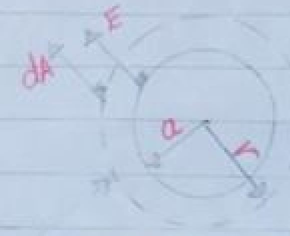
* Case «1»: $r \leq a$ «inside the shell»

$\because Q_{en}$ on Gauss's Surface $= 0$

$, \therefore E = 0 \quad$ «for $r \leq a$»



* Case «2»: $r \geq a$ «outside the Shell»



Gauss's Surface «sphere»

$$\longrightarrow \oint E \cdot dA = \oint E\, dA \cos 0$$

$$= E \oint dA = EA$$

$$= E \cdot 4\pi r^2$$

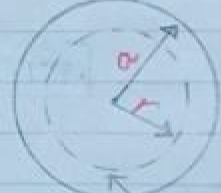
«L.H.S»

$$\longrightarrow \frac{\Sigma Q_{en}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \quad \text{«R.H.S»}$$

$$\therefore E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Longrightarrow E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

* A Solid sphere Charged uniformally with volumetric charges «$\rho\ C/m^3$»

* Case «1»: $r \leq a$ «inside the sphere»



Gauss's Surface

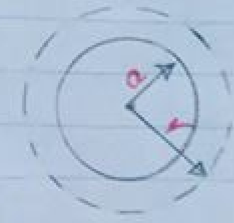
$$\oint E \cdot dA = \frac{\Sigma Q_{en}}{\varepsilon_0}$$

$$\longrightarrow Q_{en} = \rho \cdot Vol = \rho \cdot \frac{4}{3}\pi r^3 = \frac{Q}{4/3\, \pi a^3} \cdot \frac{4}{3}\pi r^3 = Q \frac{r^3}{a^3}$$

$$\longrightarrow \oint E \cdot dA = E\, 4\pi r^2$$

$$\therefore E \cdot 4\pi r^2 = \frac{Q \frac{r^3}{a^3}}{\varepsilon_0} \Longrightarrow E = \frac{Q r}{4\pi \varepsilon_0 a^3}$$

* Case «2»: $r \geq a$ «outside the sphere»



$$\oint E \cdot dA = \frac{\Sigma Q_{en}}{\varepsilon_0}$$

$$\therefore E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Longrightarrow E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

* Maxwell's Equations *

$\rightarrow EA = \frac{q}{\varepsilon_0}$ $\quad , \therefore E = \frac{q}{\varepsilon_0 A}$

$\therefore dE = \frac{dq}{\varepsilon_0 A}$ $\quad , \because I = \frac{q}{t}$ $\quad , \therefore dI = \frac{dq}{dt}$

$$\therefore dE = \frac{I\,dt}{\varepsilon_0 A} \Longrightarrow I = \varepsilon_0 A \frac{dE}{dt}$$

displacement current

تيار الإزاحة.

* **Displacement Current**: it is the Current flowing through the Capacitor which Maxwell Said that it is equivalent to the changing electric field within the Capacitor. « I_D »

* **Conduction Current**: it is the usual Current in the Conducting wires. « The Current entering and Leaving the Capacitor. « I_C »

_ **Ampere's Law** _ A magnetic field Can be produced by a Conduction Current or a changing electric field.

$$\oint B \cdot dl = \mu_0 (I_C + I_D)$$

$$= \mu_0 I_C + \mu_0 \varepsilon_0 A \frac{dE}{dt}$$

Where: $B \rightarrow$ magnetic field

$l \rightarrow$ Length of the path.

$\mu_0 \rightarrow$ permeability

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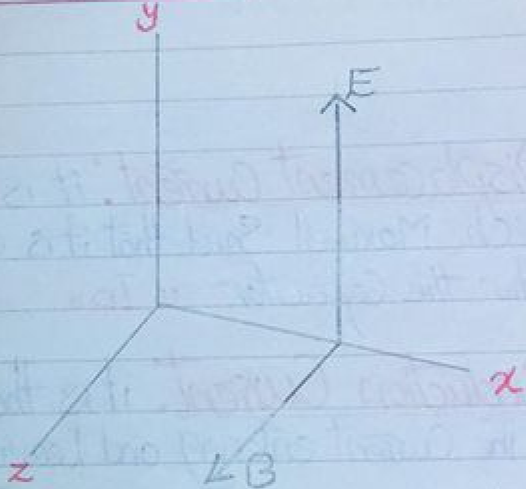
* $\vec{\nabla}$ «nabla» $= \frac{d}{dx}\vec{i} + \frac{d}{dy}\vec{j} + \frac{d}{dz}\vec{k}$

- Maxwell's equations Can also be written in differential form as follows :-

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \quad , \quad \vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla}.\vec{B} = 0 \quad , \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$$

Where : $\rho \rightarrow$ density of the free Charge.
$J \rightarrow$ density of the Conduction Current.



$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & E & 0 \end{vmatrix} = \frac{dE}{dx} \vec{k}$$

$$\therefore \frac{d\vec{E}}{dx} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 0 & 0 & B \end{vmatrix} = -\frac{dB}{dx} \vec{j}$$

$$\therefore \vec{\nabla} \times \vec{B} = \frac{-dB}{dx} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \longrightarrow (1)$$

$$, \frac{d^2E}{d^2x} = \frac{d}{dx} \cdot \frac{dE}{dx} = \frac{-d}{dx} \cdot \frac{d\vec{B}}{dt} = \frac{-d}{dt} \cdot \frac{dB}{dx}$$

$$= \frac{-d}{dt} \mu_0 \varepsilon_0 \frac{dE}{dt}$$

$$= -\mu_0 \varepsilon_0 \frac{d^2E}{dt^2}$$

$$, \frac{d^2B}{dx^2} = \mu_0 \varepsilon_0 \frac{d^2B}{dt^2} \longrightarrow (2)$$

From(1),(2): $\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2}$, $\therefore V^2 = \frac{1}{\mu_0 \varepsilon_0}$

$$\therefore V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

*Note: ψ is Constant

If: $E = E_{max} \cos(kx - \omega t)$
, $B = B_{max} \cos(kx - \omega t)$

Prove that: $C = \frac{E_{max}}{B_{max}}$

→ $\frac{dE}{dx} = \frac{-dB}{dt}$

, $\frac{dE}{dx} = -k E_{max} \sin(kx - \omega t)$

, $\frac{dB}{dx} = \omega B_{max} \sin(kx - \omega t)$

$$\therefore -k E_{max} \sin(kx - \omega t) = -\omega B_{max} \sin(kx - \omega t)$$

$$\therefore C = \frac{\omega}{k} = \frac{E_{max}}{B_{max}}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{\vec{E}\vec{B}}{\mu_0} = \frac{E^2}{\mu_0 C} = \frac{CB^2}{\mu_0}$$

$$B = \frac{E}{C}$$

Where: $\vec{S}$ is the rate of energy transfer by electromagnetic wave.