09 Ch2 Boolean Algebra & logic Gates:

2.1 Demonstrate The Validity of The Following by means of Truth table:

a) De Morgan's Theorem for Three Variables

$(x+y+z)' = x'y'z'$

$(xyz)' = x'+y'+z'$

x	y	z	(x+y+z)	(x+y+z)'	x'	y'	z'	x'y'z'
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

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x	y	z	xyz	(xyz)'	x'	y'	z'	x'+y'+z'
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

(b) The Distributive law:

$$x + yz = (x+y)(x+z)$$

x	y	z	yz	x+yz	(x+y)	(x+z)	(x+y)(x+z)

(c) The Distributive law = $x(y+z) = xy + xz$

x	y	z	(y+z)	x(y+z)	xy	xz	xy+xz

(d) The Associative law: $x + (y+z) = (x+y) + z$

x	y	z	(y+z)	x+(y+z)	(x+y)	(x+y)+z

(e) The Associative law: $x(yz) = (xy)z$

x	y	z	yz	x(yz)	(xy)	( )z

$\boxed{2.2}$ Simplify The Following Boolean Expressions to a Minimum number of literals

(a) $xy + xy'$

$= x(y+y')$

$= x \cdot 1 = \boxed{x}$

$y + y' = 1$

(b) $(x+y)(x+y')$

$= x + yy'$

$= x + 0$

$= x + 0$

$= \boxed{x}$

$x + yz = (x+y)\cdot(x+z)$

$y \cdot y = 0$

(c) $xyz + x'y + xyz'$

$= xy(z+z') + x'y$

$= xy(1) + x'y$

$= xy + x'y$

$= y(x+x')$

$= y \cdot 1 = \boxed{y}$

$z + z' = 1$

d) $(A+B)'(A'+B')'$

$= (A'B')\cdot(AB)$

$= \boxed{0}$

$(x+y)' = x'y'$

$(x'+y')' = xy$

$(x'+y')' = xy$

e) $xyz' + x'yz + xyz + x'yz'$

$= xy(z+z') + x'y(z+z')$

$= xy + x'y$

$= y(x+x') = \boxed{y}$

f) $(x+y+z')(x'+y'+z)$

$= xx' + xy' + xz + yx' + yy' + yz + z'x' + z'y' + z'z$

$= 0 + xy' + xz + yx' + 0 + yz + z'x' + z'y' + 0$

$= xy' + xz + yx' + yz + z'x' + z'y'$

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2.3 Simplify the Following Boolean Expressions to minimum Number of literals:

(a) $ABC + A'B + ABC'$

$= AB(C+C') + A'B$

$= AB + A'B$

$= B(A+A')$

$= \boxed{B}$

(b) $x'yz + xz$

$= z(x'y + x)$

$= z[(x+x')(x+y)]$

$= z[1 \cdot (x+y)]$

$= \boxed{z(x+y)}$

(c) $(x+y)'(x'+y')$

$= x'y'(x'+y')$

$= x'y'$

* Absorption Theorem

$x(x+y) = x$

(d) $xy + x(wz + wz')$

$= x\,(y + wz + wz')$

$= x\,(y + w(z + z'))$

$= \boxed{x\,(y + w)}$

(e) $(BC' + A'D)(AB' + CD')$

$= BC'AB' + BC'CD' + A'DAB' + A'DCD'$

$= \boxed{0}$

(f) $(x + y' + z')(x' + z')$

$= xx' + xz' + y'x' + y'z' + z'x' + z'z'$

$= 0 + xz' + y'x' + y'z' + z'x' + z'z'$

$= z'(x + x') + y(x' + z') + z'z'$

$= z' + y(x' + z') + z'z'$

$\boxed{2.4}$ Reduce The Following Boolean Expressions to The indicated number of literals.

ⓐ $A'C' + ABC + AC' \longrightarrow$ To Three literals.

$= C'(A + A') + ABC$

$= C' + ABC$

Distributative Theorem:

$X + YZ = (X+Y)(X+Z)$

$= (C' + C)(C' + AB)$

$= 1 \cdot (C' + AB)$

$= \boxed{C' + AB} \longrightarrow$ Three literals #

ⓑ $(x'y' + z)' + z + xy + wz \longrightarrow$ Three literals.

$= (x'y')'z' + z + xy + wz$

$= (x+y)z' + z + xy + wz$

$X + YZ = (X+Y)(X+Z)$

$= (z+z')(z+x+y) + xy + wz$

$= 1 \cdot (z+x+y) + xy + wz$

$= z + x + y + xy + wz$

$= z + wz + x + y(1+x)$

$= z + wz + x + y$

$= z(1+w) + x + y$

$= \boxed{z + x + y} \longrightarrow$ Three literals

(c) $A'B(D' + C'D) + B(A + A'CD) \rightarrow$ one literal

$= A'BD' + A'BC'D + AB + A'BCD$

$= A'BD' + AB + A'BD(C' + C)$

$= A'BD' + AB + A'BD$

$= A'B(D + D') + AB$

$= A'B + AB$

$= B(A' + A) = \boxed{B} \rightarrow$ one literal

(d) $(A' + C)(A' + C')(A + B + C'D)$

$= (A' + CC')(A + B + C'D)$

$= A'(A + B + C'D)$

$= A'A + A'B + A'C'D$

$= A'B + A'C'D$

$= A'(B + C'D) \rightarrow$ Four literals //

(e) $ABCD + A'BD + ABC'D$

$\longrightarrow$ Two literals

$= ABD(C + C') + A'BD$

$= ABD + A'BD$

$= BD(A + A')$

$= \boxed{BD} \rightarrow$ Two literals //

[2.5] Draw logic Diagrams of The Circuits That implement The orginal & Simplified Expressions in [2.2]

[2.6] Draw logic Diagrams of The Circuits that implement The orginal & Simplified Expressions in [2.3]

[2.7] Draw logic Diagrams of the Circuits that implement The orginal & Simplified Expressions in [2.4]

$\boxed{2.8}$ Find The Complement of $F = wx + yz$

Then show $FF' = 0$

$F + F' = 1$

Note
De Morgan Theorem

$F' = (wx + yz)'$

$F' = (wx)'(yz)'$

$F' = (w' + x')(y' + z')$

* $FF' = 0$

$FF' = (wx + yz)[(w' + x')(y' + z')]$

$= \overset{0}{\overbrace{wx(w' + x')}}(y' + z') + \overset{0}{\overbrace{yz(w' + x')(y' + z')}}$

$= wxw'y' + wxx'z' + yzw'y' + yzx'z'$

$= \boxed{0}$

* $F + F' = 1$

$F + F' = wx + yz + (wx + yz)'$

$= 1$

2.9 Find The Complement of The Following Expressions:

(a) $xy' + x'y$

$= (xy' + x'y)' = (xy')'(x'y)'$

$= (x'+y)(x+y')$

$= \cancel{xx'} + x'y' + xy + \cancel{yy'}$

$\boxed{= xy + x'y'}$

(b) $(A'B+CD)E' + E$

$= [(A'B+CD)E' + E]'$

$= [(A'B+CD)E']' \cdot E'$

$= [(A+B')(C'+D') + E] \cdot (E)'$

$= [(A+B')(C'+D') + E] \cdot E'$

$= (A+B')(C'+D')E' + E \cdot E'$

$= (A+B')(C'+D')E' + 0$

$= (A+B')(C'+D')E'$

$= AC'E' + AD'E' + B'C'E' + B'D'E'$

ⓒ $(x'+y+z')(x+y')(x+z)$

$= [(x'+y+z')(x+y')(x+z)]'$

$= [(x'+y+z)' + (x+y')' + (x+z)']$

$= [(xy'z') + (x'y) + (x'z')]$

$= z'(x'+xy') + (x'y)$

$\boxed{2.11}$ list The Truth table of the Function.

a) $F = xy + xy' + y'z$

b) $F = x'y' + yz$

$\boxed{2\text{-}12}$ We can perform the logical operations on strings of bits by considering each pair of corresponding bits separately (called bitwise operation).

Given two eight-bit strings.

$A = 10110001$
$B = 10101100$

Evaluate the eight-bit result after the following logical operations

a) AND.

$A = 10110001$
$B = 10101100$
$A \cdot B = 10100000$

b) OR

$A = 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1$
$B = 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0$
$A+B = 1\ 0\ 1\ 1\ 1\ 1\ 0\ 1$

c) XOR.

$A = 1\ 0\ 1\ 1\ 0\ 0\ 0\ 1$
$B = 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0$
$A \oplus B = 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1$

(c) NOT A

$A = 10110001$

$A' = 01001110$

(d) NOT B

$B = 10101100$

$B' = 01010011$

2.15 Simplify the following functions T_1 & T_2 to minimum number of literals.

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$T_1 = A'B'C' + A'B'C + A'BC'$

$T_2 = A'BC + AB'C' + AB'C + ABC' + ABC$

$$T_1 = A'B'C' + A'B'C + A'BC'$$
$$= A'B'(C'+C) + A'BC'$$
$$= A'B' + A'BC'$$
$$= A'(B' + BC')$$

$$T_2 = A'BC + AB'C' + AB'C + ABC' + ABC$$
$$= BC(A'+A) + AB'(C'+C) + ABC'$$
$$= BC + AB' + ABC'$$
$$= A(B' + BC') + BC$$

$\boxed{2.18}$ For The Boolean Function:

$$F = xy'z + x'y'z + w'xy + wxy$$

a) obtain The Truth Table of F

b) Draw The logic Diagram Using The orginal Boolean Expression.

(c) Using The Boolean Algebra to Simplify The Function to a minimum number of literals

(d) obtain The Truth Table of The Function from The Simplified Expression & show that it is the Same as The one in part (a);

(e) Draw the logic Diagram from The Simplified Expression & compare the Total number of Gates with The Diagram of part (b).