Numerical Analysis

Section 1

* methods of Solving «getting roots» of Equations:-

1) Bisection method:

→ $f(x)=0$

ex: $f(x)=x^2+2x+1=0$, $\sin x - x = 0$

- Interval: $[a,b]$, where: $|a-b| < \epsilon$ → error «very small number»

- Number of repitition «n»: $n+1 \geqslant \dfrac{\log(b-a)+K}{\log 2}$, where: $\epsilon = 10^{-K}$

- Example: By using Bisection method, find an approximated solution for $f(x)=x^3-x-1$ in $[1,2]$ under error $\epsilon = 10^{-2}$.

→ $n+1 \geqslant \dfrac{\log(2-1)+2}{\log 2} \Rightarrow n+1 \geqslant 6.6 \Rightarrow n \geqslant 5.6$ «repeat 6 times» bec. $n \simeq 6$

, $x_0 = \dfrac{1+2}{2} = 1.5 \Rightarrow f(1.5) = (1.5)^3 - 1.5 - 1 = 0.875 \rightarrow +$ $[1, 1.5]$

$x_1 = \dfrac{1.5+1}{2} = 1.25 \Rightarrow f(1.25) = -0.297 \rightarrow -$ $[1.25, 1.5]$

$x_2 = \dfrac{1.25+1.5}{2} = 1.375 \Rightarrow f(1.375) = +$ $[1.25, 1.375]$

$x_3 = \dfrac{1.25+1.375}{2} = 1.312 \Rightarrow f(1.312) = -$ $[1.312, 1.375]$

$x_4 = \dfrac{1.312+1.375}{2} = 1.344 \Rightarrow f(1.344) = +$ $[1.312, 1.344]$

$x_5 = \dfrac{1.344+1.312}{2} = 1.328 \Rightarrow f(1.328) = +$ $[1.312, 1.328]$

$\therefore |x_{n+1} - x_n| = x_5 - x_4 = 0.017 \simeq 0.02$

$\therefore$ Root $= \dfrac{x_5+x_4}{2} = 1.336$

2) Newton Raphson method:-

→ $f(x)=0$, $[a,b]$, where: $f(a).f(b)<0$

, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ ex: $x_1 = x_0 - \frac{f(x_0)}{f''(x_0)}$

, $|x_{n+1} - x_n| < \epsilon$

, Condition of choosing x_0: $f(x_0).f''(x_0) > 0$

* Example: $f(x) = x.e^x - 1 = 0$, under 3 digits.

→ $f(0) = -1$ ⊖
$f(1) = 1.7$ ⊕

, $\because f(0).f(1) < 0$

$\therefore$ The interval $= [0,1]$

, $\because x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\therefore f'(x) = x.e^x + e^x$, $f''(x) = x.e^x + 2e^x$

$f''(0) = 2$, $f''(1) = 3e$

, $\because f(1).f''(1) > 0$

$\therefore x_0 = 1$

$x_1 = 1 - \frac{1.73}{5.44} = 0.682$

, $x_2 = 0.682 - \frac{f(0.682)}{f''(0.682)} = 0.577$

, $x_3 = 0.577 - \frac{f(0.577)}{f''(0.577)} = 0.566$

, $x_4 = 0.566 - \frac{f(0.566)}{f''(0.566)} = 0.567$

, $|x_4 - x_3| = 0.001$

, $\therefore$ Root $= \frac{x_4 + x_3}{2}$

* Try By yourself:

1) $f(x)$ where $x = \tan x$, interval $[4, 4.5]$, $\epsilon = 10^{-2}$ « By 2 methods.»