Probability and Statistics

Lecture 1

* Chapter 1:-

I: introduction to probability
II: Random variables, probability functions and Expectations.
III: Important Discrete and Continueous distribution.
IV: Sampling Theory and Estimations.
V: Tests of hypothesis and Some Applications.

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* Important Definitions:

- Random Experiment.
- Sample Space.
- Event.
- operations on the event.
- probability of the event.
- Axioms of probability.

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1) **Random Experiment:** An Experiment whose outcomes Can't be predicted, but we Know all its possible outcomes.

- Example: - Tossing a Coin. « The result will Certainly be Head or Tail. »
  - Tossing a die. « We Know before that one number from 1 to 6 will appear. »

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2) **Sample space:** Set of all possible outcomes of a random experiment.

- Example: $S(\text{Coin}) = \{H, T\}$
  $S(\text{die}) = \{1, \dots, 6\}$
  $S(\text{birth}) = \{\text{boy}, \text{girl}\}$

- A class Contains: 50 boys, 30 girls.

→ Choosing one person: $S = \{b_1, b_2, \dots, b_{50}, g_1, g_2, \dots g_{30}\} = \{80\}$ « 80 ways. »

→ Choosing two persons: $S = {}^{80}C_2 = \dfrac{80!}{2!(80-2)!}$ « order isn't important. »

- if the order is important, we use permutation.

3) Event: Subset of Sample Space.

Example: $S = \{1, ..., 6\}$

Events: $A = \{4, 5\}$

$B = \{6\}$ Prime event.

$C = \{\ \}$ impossible event.

$D = \{1, ..., 6\}$ Certain event.

$E = \{7\}$ Not Event «Not Subset of Sample space.»

* Notice that: The prime event is that Contains only one element whatever it is prime number or not.

4) Operations on the Events: Union, intersection, Complement, Subtraction.

* Or means → union «$\cup$» , + «for mathematical operations.»
* and → intersection «$\cap$» , . «for mathematical operations.»
* Negative → Complement.
* At Least → Union «$\cup$»
* Exactly one event occurs → $(A \cap B^c) \cup (B \cap A^c) \equiv (A \cup B) - (A \cap B)$
* At most one event occurs → $(A \cap B^c) \cup (B \cap A^c) \cup (A^c \cap B^c) \equiv (A \cap B)^c$
* $A - B \equiv A \cap B^c$

5) Probability of the events: - $\dfrac{\text{N. of ways of occurring the event}}{\text{N. of ways of occurring the sample space.}}$

Example: Class Contains 50 boys, 30 girls.

, A: Event of Choosing a girl.

→ $P(A) = \dfrac{30}{80} = \dfrac{3}{8}$

, B: Event of Choosing one boy and one girl.

→ $P(B) = \dfrac{{}^{50}C_1 \times {}^{30}C_1}{{}^{80}C_2}$

6) Axioms of Probability: « 3 Axioms »

1) $0 \leq p(A) \leq 1$

2) $p(S) = 1$ Certain event.

3) if $A \cap B = \emptyset$ « mutually exclusive »
, then $p(A \cup B) = p(A) + p(B)$

* Theorem: if A and B are two events in S, then:
1) $p(\emptyset) = 0$
2) $p(A^c) = 1 - p(A)$
3) if $A \subseteq B$, then $p(A) \leq p(B)$
4) $p(A - B) = p(A) - p(A \cap B)$
5) $p(B - A) = p(B) - p(A \cap B)$
6) $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

* Conditional Probability *

→ A Class Contains 50 boy, 30 girl.
, Let A be the event that expresses that the first person is boy.
, Let B be the event that expresses that the second person is a boy.

→ $p(B|A) = \frac{49}{79}$

→ $p(B^c|A^c) = \frac{29}{79}$

→ $p(B^c|A) = \frac{30}{79}$

→ $p(B|A^c) = \frac{50}{79}$

→ $p(B^c|A) = 1 - p(B|A)$

* definition: → $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$

- Example: If the probability that a student Succeeded in IT exam = 0.8, and the probability that a student succeeded in CS exam = 0.7, and the probability that a student Succeeded in both exams = 0.6.
- A Student is chosen randomly. If the student Succeeded in CS exam, find the probability that this student Succeeded in IT exam.

⇒ Let A be the event that the student Succeeded in IT exam.

∴ $P(A) = 0.8$

, Let B be the event that the student Succeeded in CS exam.

∴ $P(B) = 0.7$

, $P(A \cap B) = 0.6$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.7} = \frac{6}{7}$$

* Partitions *

- Definition: $A_1, A_2, \ldots A_n$ make a partition to S if:

1) $A_i \cap A_j = \phi$, $i,j = 1, \ldots, n$, $i \neq j$

2) $A_1 \cup A_2 \cup \ldots \cup A_n = S$