تمارين الفصل الخامس احصاء

[2] (i) Calculate The variance of The sample 3, 5, 8, 7, 5 and 7.

(ii) without calculating, state The variance of The sample 6, 10, 16, 14, 10 and 14

(iii) without calculating, state The variance of The sample 25, 27, 30, 29, 27 and 29

Sol

[1] $V(X) = ?$

$X_1, X_2, X_3, X_4, X_5, X_6 = 3, 5, 8, 7, 8, 7$

$n = 6$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} [x_i - \bar{X}]^2$$

$$\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{3+5+8+7+5+7}{6} = \frac{35}{6} = 5.833$$

$$\therefore s^2 = \frac{1}{5} \sum_{i=1}^{6} [x_i - \bar{x}]^2$$

$$= \frac{1}{5} \left[\left(3 - \frac{35}{6}\right)^2 + \left(5 - \frac{35}{6}\right)^2 + \left(8 - \frac{35}{6}\right)^2 + \left(7 - \frac{35}{6}\right)^2 + \left(5 - \frac{35}{6}\right)^2 + \left(7 - \frac{35}{6}\right)^2 \right]$$

$$= \frac{1}{5}\left(\frac{101}{6}\right) = \frac{101}{30} = 3.37$$

$\boxed{2}$ $s^2 = 13,47$

$\boxed{3}$ $V(X) = ?$

$x_1, x_2, x_3, x_4, x_5, x_6 = 25, 27, 30, 29, 27, 29$

$n = 6$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} [x_i - \bar{x}]^2$$

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} \quad \frac{25+27+30+29+27+29}{6}$$

$$= \frac{167}{6} = 27,83$$

$$s^2 = \frac{1}{5} \sum_{i=1}^{6} [x_i - \bar{x}]^2$$

$$= \frac{1}{5} \left[\left(25 - \frac{167}{6}\right)^2 + \left(27 - \frac{167}{6}\right)^2 + \left(30 - \frac{167}{6}\right)^2 + \left(29 - \frac{167}{6}\right)^2 + \left(27 - \frac{167}{6}\right)^2 + \left(29 - \frac{167}{6}\right)^2 \right]$$

$$= \frac{1}{5} \left[\frac{289}{36} + \frac{25}{36} + \frac{169}{36} + \frac{49}{36} + \frac{25}{36} + \frac{49}{36} \right]$$

$$= 3,37$$

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[3] A finite population consists of the numbers 2, 4 and 7

(i) Construct a frequency histogram for the sampling distribution of $\bar{x}$ when samples of size 2 are drawn with replacement

(ii) verify that $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

(iii) Between what two values would you expect the middle 68% of the sample means to fall?

Sol

(i) finit population, with replacement

$X_i = 2, 4, 7$, $n = 2$ $N = 3$

Samples $N^n = 3^2 = 9$

Samples	$\bar{X}$
(2,2)	$\frac{2+2}{2} = 2$
(2,4)	3
(2,7)	4,5
(4,2)	3
(4,4)	4
(4,7)	5,5
(7,2)	4.5
(7,4)	5.5
(7,7)	7

(ii) $\mu = \sum_{i=1}^{n} \frac{x_i}{N} = \sum_{i=1}^{3} \frac{2+4+7}{3} = \frac{13}{3}$

$$E[\bar{x}] = \mu_{\bar{x}} = \frac{39}{9} = \frac{13}{3}$$

$$\therefore \mu_{\bar{x}} = \mu \quad \#$$

$$\sigma_{\bar{x}}^2 = \frac{1}{9}\Big[\left(2-\frac{13}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(4.5-\frac{13}{3}\right)^2$$

$$\left(3-\frac{13}{3}\right)^2 + \left(4-\frac{13}{3}\right)^2 + \left(5.5-\frac{13}{3}\right)^2$$

$$+\left(4.5-\frac{13}{3}\right)^2 + \left(5.5-\frac{13}{3}\right)^2 + \left(7-\frac{13}{3}\right)^2\Big]$$

$$= \frac{1}{9}\Big[\frac{49}{9} + \frac{16}{9} + \frac{1}{36} + \frac{16}{9} + \frac{1}{9} + \frac{49}{36} + \frac{1}{36} + \frac{49}{36}$$

$$+ \frac{64}{9}\Big] = \frac{19}{9} \quad \#$$

$$\sigma^2 = \frac{1}{3}\Big[\left(2-\frac{13}{3}\right)^2 + \left(4-\frac{13}{3}\right)^2 + \left(7-\frac{13}{3}\right)^2\Big]$$

$$= \frac{1}{3}\Big[\frac{49}{9} + \frac{1}{9} + \frac{64}{9}\Big] = \frac{38}{9}$$

$$\frac{\sigma^2}{n} = \frac{38}{9} \times \frac{1}{2} = \frac{19}{9} \quad \#$$

$$\therefore \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

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(iii) $p(-a < \bar{x} < a) = 0.68$

$$\left(\frac{-a-\mu}{\sigma/\sqrt{n}} < Z < \frac{a-\mu}{\sigma/\sqrt{n}} \right) = 0.68$$

$$\left(\frac{-a-\frac{13}{3}}{\frac{\sqrt{38}}{3} \times \frac{1}{2}} < Z < \frac{a-\frac{13}{3}}{\frac{\sqrt{38}}{3} \times \frac{1}{2}} \right) = 0.68$$

$$\left(\frac{-a-\frac{13}{3}}{\frac{\sqrt{38}}{6}} < Z < \frac{a-\frac{13}{3}}{\frac{\sqrt{38}}{6}} \right) = 0.68$$

$$\phi\left(\frac{a-\frac{13}{3}}{\frac{\sqrt{38}}{6}} \right) - \phi\left(\frac{-a-\frac{13}{3}}{\frac{\sqrt{38}}{6}} \right) = 0.68$$

4. The heights of 1000 students are approximately normally distributed with a mean of 68.5 inches and a standard deviation of 2.7 inches. If 200 random samples of size 25 are drawn from this population, determine

(i) The expected mean and standard deviation of the sampling distribution of the mean.

(ii) The number of sample means that fall between 66 and 69 inclusive.

(iii) The number of sample means falling below 65.

Sol

$N = 1000$, $\mu = 68.5$, $\sigma = 2.7$

$n = 25$

1) $E[\bar{X}] = \mu = 68.5$

$\sigma_{\bar{X}}^2 = V(\bar{X}) = \frac{\sigma^2}{n} = \frac{(2.7)^2}{25} = 0.2916$

$\therefore \sigma_{\bar{X}} = \sqrt{0.2916} = 0.54$

2) $P(66 < \bar{X} < 69)$

$= P\left(\frac{66-\mu}{\sigma/\sqrt{n}} < Z < \frac{69-\mu}{\sigma/\sqrt{n}}\right)$

$= P\left(\frac{66-68.5}{2.7/5} < Z < \frac{69-68.5}{2.7/5}\right)$

$= P(-4.6296 < Z < 0.926)$

$= \Phi(0.93) - \Phi(-4.63)$

$$= \Phi(0.93) - 1 + \Phi(4.63)$$

$$= 0.82639 - 1 + 1 = 0.82639$$

The number of sample means

$$= 200 \times 0.82639 =$$

3) $p(\bar{X} < 65)$

$$p\left(Z < \frac{65 - \mu}{\sigma/\sqrt{n}}\right)$$

$$p\left(Z < \frac{65 - 68.5}{2.7/5}\right) = p(Z < -0.648)$$

$$= \Phi(-0.648)$$

$$= 1 - \Phi(0.648) = \star$$

The number of sample means falling below 65 $= 200 \times \star =$