"Discrete Structures"

Lecture 6

Follow GCD:-

→ If d is $GCD(a,b)$, then we want to write d in form of "$d = sa + tb$" where $st < 0$

- Example: Find $GCD(776, 342)$

→ $776 = 2(342) + 92$
$342 = 3(92) + 66$
$92 = 1(66) + 26$
$66 = 2(26) + 14$
$26 = 1(14) + 12$
$14 = 1(12) + 2$
$12 = 6(2) + 0$ GCD

- we want to Express that 2 in form of $d = sa + tb$

→ $2 = 14 - 1(12)$
$= 14 - 1(26 - 14)$
$= 2(14) - 26$
$= 2(66 - 2(26)) - 26$
$= 2(66) - 5(26)$
$= 2(66) - 5(92 - 66)$
$= 7(66) - 5(92)$
$= 7(342 - 3(92)) - 5(92)$
$= -26(92) + 7(342)$
$= -26(776 - 2(342)) + 7(342)$
s $= 59(342) - 26(776)$ t

* Follow matrices:-

$$A^{-1}A = AA^{-1} = I \quad , A^{-1} = \frac{1}{\Delta A} |A^T|$$

- Example: $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$, prove that $AA^{-1} = I$

$$\rightarrow \Delta A = 1(3-2) - 2(9-2) + 1(3-1) = 1 - 14 + 2 = -11$$

$$\overline{A^T} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \quad , |A^T| = \begin{bmatrix} 1 & -5 & 3 \\ -7 & 2 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-11} \begin{bmatrix} 1 & -5 & 3 \\ -7 & 2 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$, AA^{-1} = \frac{1}{-11} \left(\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -5 & 3 \\ -7 & 2 & 1 \\ 2 & 1 & -5 \end{bmatrix} \right)$$

$$= \frac{1}{-11} \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

* Boolean Matrix: Contains only zeros and ones.

- operations on Boolean Matrices:-

1) "$\vee$" $\rightarrow$ join «or»
2) "$\wedge$" $\rightarrow$ meet «AND»
3) "$\odot$" $\rightarrow$ Boolean product.

Examples:- $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ $A \vee B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ $A \wedge B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ $A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

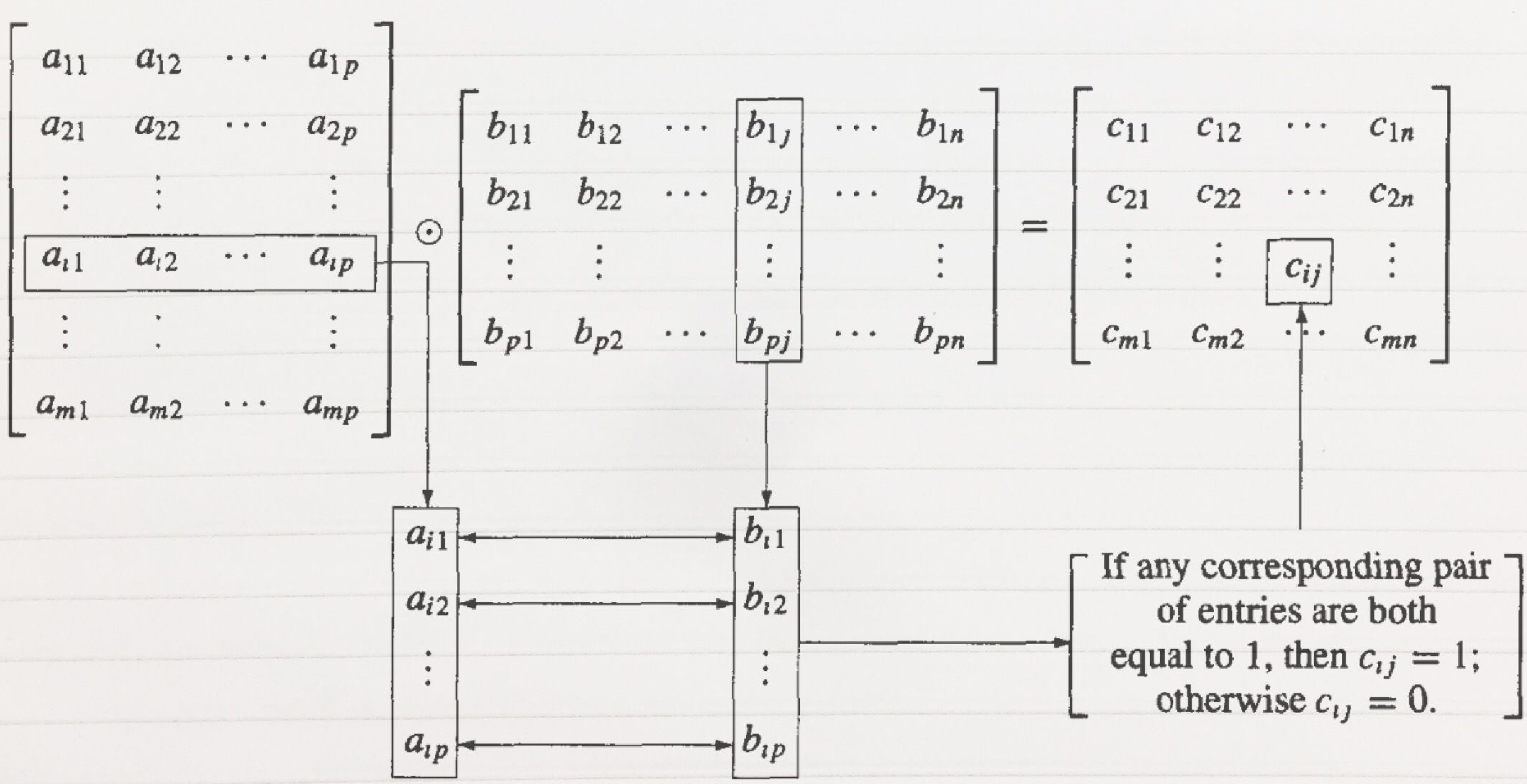

Figure 1.21