

Discrete Structures.

lecture 1

- Basics of this Course:
- Sets
 - Sequence
 - Integers
 - Matrix
 - Logic
 - Relations

* Sets *

$$A = \{a, b, c\}, \quad B = \{1, 2, 3\}, \quad C = \{\text{"apple"}, \text{"orange"}\}$$
$$D = \{\#, \&, !\}$$

→ The Set is Well defined Collection of elements.

Another way to define a Set:

→ $X = \{x \mid x \text{ is a positive integer less than } 10\}$
 $\therefore X = \{1, 2, 3, \dots, 9\}$

→ $Z = \{x \mid x \text{ is a letter in Car}\}$
 $\therefore Z = \{C, a, r\}$

* Note: In Sets, No order and No Repetition.

Example: $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 1, 2\}$

$$\{1, 2, 3, 1\} = \{1, 2, 3\} \quad \text{"repetition is useless"}$$

* \emptyset "Phi" = $\{ \}$ empty set

- Some operations:

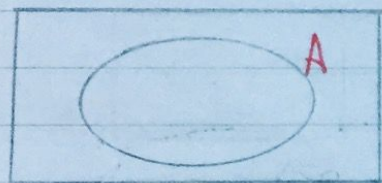
$$x = \{1, 2, 3, 4, 5\}$$

- $1 \in x$
- $7 \notin x$
- $\{2, 3\} \subseteq x$

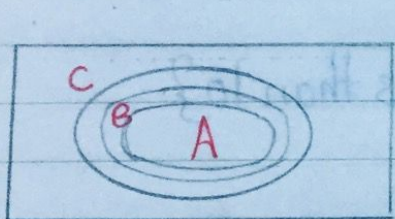
- $\{1, 2\} \subseteq x$
- $\{1, 7\} \not\subseteq x$

- Note: \emptyset is subset " \subseteq " of any set but it doesn't belong " \in " to any set unless it is an element in it.

* $\{See\} \subseteq \{yes\}$



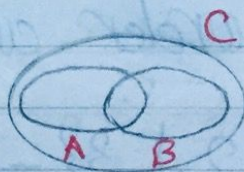
U "universe"



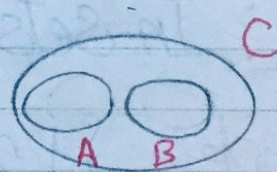
$$A \subseteq B \text{ \& } B \subseteq C \\ \therefore A \subseteq C$$

* if $A \subseteq C$ & $B \subseteq C$

\therefore it can be represented by



or



→ $A = \{2, 4, 6\}$, $|A| = 3$ "Cardinality" عناصر المجموعة

, $p(A)$ "powerset of A " = $\{\emptyset, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}\}$

$$|p(A)| = 8 = 2^{|A|}$$

$$\rightarrow |p(x)| = 2^{|x|} \leftarrow$$

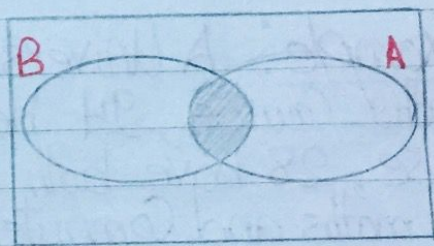
* if we can count the elements of a set, so the set is finite
but if we can't count them, so the set is infinite.

\mathbb{Z}^+ "Set of positive integers" = $\{1, 2, 3, \dots\}$

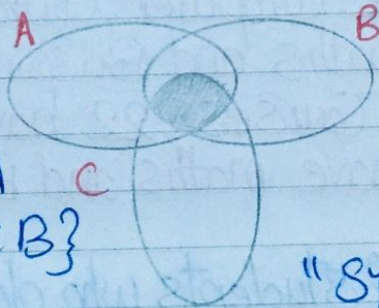
\mathbb{N} "Set of natural numbers" = $\{0, 1, 2, \dots\}$

\mathbb{Z}^- "Set of negative integers" = $\{-1, -2, -3, \dots\}$

→ $A \cap B = \{x | x \in A \text{ and } x \in B\}$ "intersection"



$$A \cap B \cap C \Rightarrow$$



* \bar{A} = all elements that $\notin A$

* $A - B = \{x | x \in A \text{ and } x \notin B\}$

* $A \oplus B = A \cup B - A \cap B$

"Symmetric difference"

→ if we have $A = \{2, 3, 5\}$, we can get another set by multiplying it by a constant, for example $B = 6A$

$$\therefore B = \{y | y = 6 * x \forall x \in A\} = \{12, 18, 30\}$$

for all

Important Notes:-

$$* A \cup B = B \cup A$$

$$* A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$* A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$* A \cup A = A$$

$$* \bar{\bar{A}} = A$$

$$* \bar{U} = \emptyset$$

$$* A \cap B = B \cap A$$

$$* A \cap (B \cap C) = (A \cap B) \cap C$$

$$* A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$* A \cap A = A$$

$$* \bar{\emptyset} = U$$

* Demorgan:

$$\rightarrow \overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

$$\rightarrow \overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

$$* A \cup U = U$$

$$* A \cup \emptyset = A$$

$$* A \cap U = A$$

$$* A \cap \emptyset = \emptyset$$

Example: A University has 260 Students, 64 of them have maths Course, 94 have Computer Course, 58 have Bussiness Course, 28 have both maths and Bussiness Courses, 26 have both maths and Computer Courses, 22 have both Computer and Bussiness Courses, 14 have maths and Bussiness and Computer Courses.

1. find number of students who don't have any Courses.

2. find number of students who have only Computer Course

→ Answer:- $|U| = 260$, $|M| = 64$, $|C| = 94$, $|B| = 58$, $|MB| = |M \cap B| = 28$, $|MC| = 26$, $|CB| = 22$, $|MBC| = 14$

$$1 \rightarrow |M \cup B \cup C| = |U| - |M \cup B \cup C|$$

- we will use "Addition principle" to get $|M \cup B \cup C|$

$$\therefore |M \cup B \cup C| = |M| + |B| + |C| - |MB| - |MC| - |CB| + |CBM| \\ = 64 + 58 + 94 - 28 - 26 - 22 + 14 = 154$$

$$\therefore |M \cup B \cup C| = 260 - 154 = 106 \text{ Students}$$

$$\begin{aligned} 2 \rightarrow \text{Students who have only Computer Course} &= |C| - |C \cap M| - |C \cap B| + |M \cap B| \\ &= 94 - 26 - 22 + 14 \\ &= 60 \text{ Students.} \end{aligned}$$

* Practically, we will use python language.

- Some python functions:-

$$x = \{1, 2, 3, 4, 5, 6\}$$

- $x = \text{print}$
- $\text{type}(x) \rightarrow \text{Set}$
- $3 \text{ in } x \rightarrow \text{true}$
- $2 \text{ not in } x \rightarrow \text{False}$
- $x.\text{function} \rightarrow \text{add, clear, remove}$
- $x.\text{add } 7 \rightarrow x = \{x, 7\}$

- $x.\text{pop} \rightarrow \text{remove 1}^{\text{st}} \text{ element}$
- $\text{alt} + B \rightarrow \text{repeat last command}$
- $x.\text{intersection}(y) = x \cap y$
- $x.\text{Union}(y) = x \cup y$
- $x.\text{difference}(y) = x - y$
- $x.\text{Symmetric}(y) = x.\text{difference}(y).\text{union}(y.\text{difference}(x))$

- To learn more about python language, you can visit Doctor's youtube channel via his facebook page "programming Community".

- Solve the practical in first two videos and send it to doctor's mail "Kareem_ahmed@hotmail.co.uk" With your name, Code, College and Subject name.

- Solve Ex 1.1, 1.2 in the book in a sheet.