

"Probability and Statistics"

* Lecture 4 *

2) Poisson distribution:

* Conditions: The Same Conditions of binomial when: $n \rightarrow \infty$, $p \rightarrow 0$, $np = \lambda = E(x)$

→ Law: $p(x) = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, \dots$

where: x represents number of occurring events, λ represents average of events

* Example: A book contains 100 pages. 10 errors are distributed randomly throughout this book. 10 pages are randomly selected from this book. Find the probability of obtaining 2 errors in these selected pages.

⇒ $n = 10$, $p = \frac{10}{100}$

- Firstly, we'll try to find it by binomial distribution:

→ $p(2) = {}^{10}C_2 (0.1)^2 \times (0.9)^{10-2} \Rightarrow$ Not Correct

« The solution must be correct at any value, By binomial if no. of errors = 120 for example, p will be > 1 which can't occur, so we will use poisson which is correct at any value... »

⇒ $\lambda = \frac{10}{100}$ « average of errors in one page »

∴ λ in 10 pages = $0.1 \times 10 = 1$

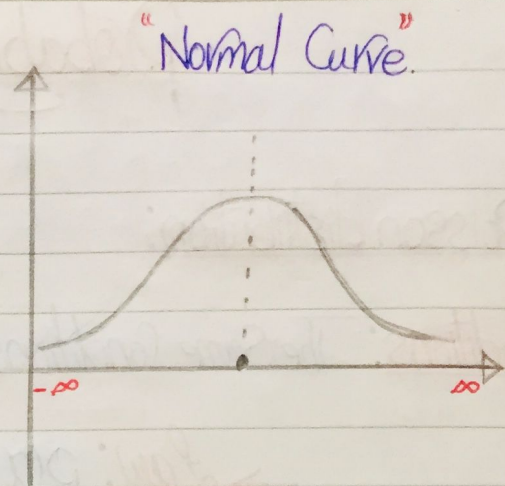
Let x be a random variable subjected to poisson distribution with $\lambda = \frac{10}{100} \times 10$, and showing the number of errors in 10 pages.

∴ $p(x=2) = \frac{e^{-1} \times (1)^2}{2!}$

3) Normal distribution:
depends on Continuous variables.

properties of normal Curve: -

1. The normal Curve is symmetric. "mean = median = mode."
 → mean: average of distribution "expectation of r.v. "E(x)"
 → median: point lying on x-axis where the Area before it equals the Area after it.
 → mode: point lying on x-axis at which the Curve is maximum.
 - As data, The mode is the most repeated value.



2. The Area under the normal Curve = 1.

3. The Curve has one Summit. "one mode"

4. The 2 ends of the normal Curve approach to x-axis and x tends to ∞ or $-\infty$.

→ Law: $f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2}$ "probability density function."

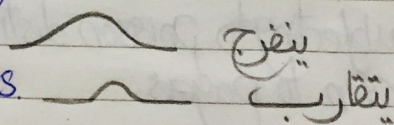
- where: $-\infty < \mu < \infty$, $\sigma > 0$, $\pi = 3.14$, $-\infty < x < \infty$
 , μ, σ are two parameters. $\mu \rightarrow$ average of x E(x), $\sigma \rightarrow$ standard deviation of x. تباین
 , $V(x) = \sigma^2$.

- on increasing μ , The Curve goes to right side.

- on decreasing μ , The Curve goes to left side.

- on increasing σ^2 , The Curve diverges

- on decreasing σ^2 , The Curve Converges.



*Example: The marks of Students were Normally distributed. The average of these marks is 70, and the deviation between marks equals 36. If a student is Chosen randomly, Find the probability that this Student has got Excellence. "where excellence starts from 85"

$\Rightarrow \mu = 70, \sigma^2 = 36$

$$P(X > 85) = \int_{85}^{\infty} f(x) dx$$

"very difficult."

$$\therefore P(X > 85) = P\left(\frac{X - \mu}{\sigma} > \frac{85 - 70}{6}\right)$$

$$= P(Z > 2.5)$$

Standard
normal r.v

$$= 1 - P(Z \leq 2.5)$$

Then, we get the answer from the given table.

$$* P(\text{Area} = 0) = \frac{1}{2}$$