"Discrete structures".

* lecture 1o. *

→ Methods of Proof ←

1) Direct proof 2) Indirect proof. 3) by Contradiction 4) mathematical induction.

1) Direct proof

→ $P \rightarrow q$, means that if p occured, So q occured.

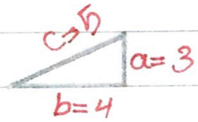
Example: Prove that if n is even, So n^2 is even.

→ $n = 2K$

$n^2 = 4K^2$

$= 2 \times 2K^2$, $\therefore n^2$ is even.

Example 2: prove that this triangle is right angled.



→ $a^2 + b^2 = c^2 \rightarrow$ right

$\because a^2 + b^2 = 9 + 16 = 25 = c^2$

$\therefore$ The triangle is right angled.

2) Indirect proof

→ To prove that: $p \rightarrow q$, we get: $\sim q \rightarrow \sim p$

Example: prove that if n^2 is odd → n is odd «$odd(n^2) \rightarrow odd(n)$»

→ we get: $\sim odd(n) \rightarrow \sim odd(n^2)$

$even(n) \rightarrow even(n^2)$

$n = 2K$

, $n^2 = 4K^2$

$= 2 \times 2K^2$

$= 2m$, $\therefore n^2$ is even.

3) Contradiction

→ To prove p, we get that $\sim p$ is not True.

Example: Prove that $\sqrt{2}$ is irrational number.

→ Let $\sqrt{2}$ is rational number « Can be written in form of $\frac{b}{a}$, where b and a has no Common factors. »

$$\sqrt{2} = \frac{b}{a} \Rightarrow 2 = \frac{b^2}{a^2} \rightarrow (1)$$

$$b^2 = 2a^2 = 2K$$

$\therefore b^2$ is even

$\therefore b = 2M$

from (1): $a^2 = \frac{b^2}{2} = \frac{4M^2}{2} = 2M^2$

$\therefore$ 2 is Common factor between a, b

$\therefore \sqrt{2}$ Can't be rational number

$\therefore \sqrt{2}$ is irrational number

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4) mathematical Induction.

→ 1) prove that n is true at the first value $n=1$

2) Suppose that the relation is true at $n=K$ « Inductive step »

3) prove that the relation is true at $n=K+1$

Example: prove that $1+2+3 \dots + n = \frac{n(n+1)}{2}$

→ a) $n=1$

→ L.H.S $=1$, R.H.S. $=1$

b) $n=K$

$\rightarrow 1+2+3 \dots + K = \frac{K(K+1)}{2}$

c) $n=K+1$

$\rightarrow 1+2+3 \dots + K + (K+1) = \frac{(K+1)(K+2)}{2}$

L.H.S. $= \frac{K(K+1)}{2} + (K+1)$

$= \frac{K(K+1)+2(K+1)}{2} = \frac{K^2+3K+2}{2} = \frac{(K+2)(K+1)}{2} =$ R.H.S.

- follow mathematical induction.

- Example 2: prove that $1+5+9+\dots+(4n-3)=n(2n-1)$

$\longrightarrow$ a) $n=1$

$\rightarrow L.H.S.=1$, $R.H.S.=1(2-1)=1$

b) $n=K$

$\rightarrow 1+5+9+\dots+(4K-3)=K(2K-1)$

c) $n=K+1$

$\rightarrow 1+5+9+\dots+(4K-3)+4(K+1)-3=(K+1)(2K+1)$

$L.H.S.=K(2K-1)+4K+1$

$=2K^2+3K+1$

$=(2K+1)(K+1)=R.H.S.$

Example 3: prove that $1^3+2^3+\dots+n^3=\dfrac{n^2(n+1)^2}{4}$

$\longrightarrow$ a) $n=1$

$\rightarrow L.H.S.=1$, $R.H.S.=\dfrac{1(1+1)^2}{4}=1$

b) $n=K$

$\rightarrow 1^3+2^3+\dots+K^3=\dfrac{K^2(K+1)^2}{4}$

c) $n=K+1$

$\rightarrow 1^3+2^3+\dots+K^3+(K+1)^3=\dfrac{(K+1)^2(K+2)^2}{4}$

$L.H.S.=\dfrac{K^2(K+1)^2}{4}+(K+1)^3$

$=\dfrac{K^2(K+1)^2+4(K+1)^3}{4}=\dfrac{(K+1)^2(K^2+4K+4)}{4}$

$=\dfrac{(K+1)^2(K+2)(K+2)}{4}$

$=\dfrac{(K+1)^2(K+2)^2}{4}=R.H.S.$

Example 4: Prove that $1+3^n<5^n$ «Important»

$\longrightarrow$ a) $n=1$

$\rightarrow L.H.S=4$, $R.H.S.=5$, $4<5$ true

b) $n=K$

$\rightarrow 1+3^K<5^K$

c) $n = k+1$

$\rightarrow 1+3^{k+1} < 5^{k+1}$

$\because 1+3^{k} < 5^{k}$, $3<5$

$\therefore 3(1+3^{k}) < 5 \times 5^{k}$

$, 3+3^{k+1} < 5^{k+1}$

$\therefore 1+3^{k+1} < 5^{k+1}$

Example 5: prove that $7^n - 2^n$ is divisible by 5

$\rightarrow$ a) $n=1$

$\rightarrow 7^1 - 2^1 = 5$, $5|5$ «True.»

b) $n=k$

$\rightarrow 7^k - 2^k = 5m$

c) $n = k+1$

$\rightarrow 7^{k+1} - 2^{k+1} = 5n$

from b: $7^k - 2^k = 5m$ $(\times 7)$

$7^{k+1} - 7.2^k = 5z$ $(+5.2^k)$

$7^{k+1} - 2.2^k = 5z + 5.2^k$

$\therefore 7^{k+1} - 2^{k+1} = 5n$