* Lecture 2 *

- The Derivative :-

$\rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- examples:

1) $\frac{dK}{dx} = 0$, $f(x) = K$

$\rightarrow f'(x) = \lim_{h \to 0} \frac{K-K}{h} = \lim_{h \to 0} \frac{0}{h} = 0$

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2) $f(x) = \sin x$

$\rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x \cos h + \sinh \cos x - \sin x}{h}$

$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\sinh \cos x}{h}$

$= \sin x \lim_{h \to 0} \cancel{\frac{\cos h - 1}{h}}^{0} + \cos x \lim_{h \to 0} \cancel{\frac{\sin h}{h}}$

$= \cos x$

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3) $f(x) = \cos x$

$$\rightarrow f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \cos x \lim_{h \to 0} \frac{\cosh - 1}{h}^{0} - \sin x \lim_{h \to 0} \frac{\sinh}{h}^{1}$$

$$= -\sin x$$

4) $f(x) = x^n$

$$\rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{x+h \to x} \frac{(x+h)^n - x^n}{(x+h) - x} = n(x)^{n-1}$$

5) $y = \tan x = \frac{\sin x}{\cos x}$

$$\rightarrow y' = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

6) $y = \sec x = \frac{1}{\cos x}$

$$\rightarrow y' = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

7) $y = \csc x = \dfrac{1}{\sin x}$

$\rightarrow y' = \dfrac{-\cos x}{\sin^2 x} = \dfrac{-\cos x}{\sin x} \cdot \dfrac{1}{\sin x} = -\cot x \csc x$

8) $y = \cot x = \dfrac{\cos x}{\sin x}$

$\rightarrow y' = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \dfrac{-1}{\sin^2 x} = -\csc^2 x$

- Find the following derivatives:-

1) $y = 4x^3 - 3x^2 + 2x + 10$

$\rightarrow y' = 12x^2 - 6x + 2$

2) $y = 4\sin x - 3\cos x$

$\rightarrow y' = 4\cos x + 3\sin x$

3) $y = x\sin x + x^2\cos x$

$\rightarrow y' = \sin x + x\cos x + 2x\cos x - x^2\sin x$

4) $y = \dfrac{x^3+1}{x^2+4}$

$\rightarrow y' = \dfrac{3x^2(x^2+4) - 2x(x^3+1)}{(x^2+4)^2}$

* Chain rule *

if : $y = y(u)$, $u = u(x)$

$\therefore \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

Examples:- Find $\frac{dy}{dx}$

1) $y = u^2 + 1 \quad , \quad u = x^3 + 4$

$$\rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \times 3x^2 = 6ux^2 = 6x^2(x^3+4)$$

2) $y = \sin(u^2+3) \quad , \quad u = x^2 + 3$

$$\rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cos(u^2+3) \times 2x = 4(x^2+3)\cos((x^2+3)^2+3) . x$$

3) $y = w^2 \quad , \quad w = \sin u \quad , \quad u = 4x+1$

$$\rightarrow \frac{dy}{dx} = \frac{dy}{dw} \cdot \frac{dw}{du} \cdot \frac{du}{dx}$$

$$= 2w \times \cos u \times 4$$

$$= 8 w \cos u$$

$$= 8 \sin(4x+1) \cos(4x+1)$$

4) $y = (\sin(4x+1) + 3)^2$

$$\rightarrow y' = 2(\sin(4x+1)+3) \times 4 \times \cos((4x+1))$$

$$= 8 \sin((4x+1)+3) \cos(4x+1)$$

5) $y = (\cos(3x+1))^5$

$$\rightarrow y' = -\sin(3x+1) \times 5(\cos(3x+1))^4 \times 3$$

$$= -15 \sin(3x+1)(\cos(3x+1))^4$$

6) $y = \tan^3(2x^2+1)$

$$\rightarrow y' = 3\tan^2(2x^2+1) \times \sec^2(2x^2+1) \times 4x$$

$$= 12x \tan^2(2x^2+1) \sec^2(2x^2+1)$$

7) $y = \cot\left(\frac{x+1}{x^2+1}\right)$

$\rightarrow y' = -\csc^2\left(\frac{x+1}{x^2+1}\right) \times \frac{x^2+1-2x(x+1)}{(x^2+1)^2}$

8) $y = \sec\left(\frac{x^2+1}{x^4+2}\right)^3$

$\rightarrow y' = \sec\left(\frac{x^2+1}{x^4+2}\right)^3 \tan\left(\frac{x^2+1}{x^4+2}\right)^3 \times 3\left(\frac{x^2+1}{x^4+2}\right)^2 \times \frac{2x(x^4+2)-4x^3(x^2+1)}{(x^4+2)^2}$

9) $y = \csc(2x+5)^4$

$\rightarrow y = -\csc(2x+5)^4 \cot(2x+5)^4 \times 4(2x+5)^3 \times 2$