- L'Hospital's Rule :-

$$\rightarrow \boxed{\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{0}{0} = \lim_{x \to 0} \frac{f'(x)}{g'(x)}}$$

- Find each of the following Limits using L'Hospital's rule :-

1) $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x} = \frac{0}{0}$ « unspecified   غير معرفة »

$$= \lim_{x \to 0} \frac{3\cos 3x}{5\cos 5x} = \frac{3}{5}$$

2) $\lim_{x \to 0} \frac{\tan 2x}{\tan 3x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{2\sec^2 2x}{3\sec^2 3x} = \frac{2}{3} \qquad\qquad \left\| \sec 0 = \frac{1}{\cos 0} = 1 \right.$$

3) $\lim_{x \to 0} \frac{x}{\sin x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{1}{\cos x} = 1$$

4) $\lim_{x \to 0} \frac{1 - \cos x}{x} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{\sin x}{1} = \frac{0}{1} = 0$$

5) $\displaystyle\lim_{x \to 0} x \ln x = \lim_{x \to 0} \frac{\ln x}{1/x} = \frac{\infty}{\infty}$

$\displaystyle = \lim_{x \to 0} \frac{1/x}{-1/x^2}$

$*\ \displaystyle\lim_{x \to 0^+} \ln x = -\infty$

$*\ \displaystyle\lim_{x \to 0^-} \ln x = \infty$

$*$ <u>L'Hospital's rule is applied when</u>: $\displaystyle\lim_{x \to 0} f(x) = \frac{0}{0}$ or $\frac{\infty}{\infty}$