Boolean Algebra:

* Associative Law:

$$(x * y) * z = x * (y * z)$$

* Commutative law:

$$x * y = y * x = y + x$$

* Distributative law:

$$x * (y \cdot z) = (x * y) \cdot (x * z)$$

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

* Identity law:

$$x + 0 = 0 + x$$

* Boolean Algebra:

1) - $+$ Binary operator (or)
   - $\cdot$ Binary operator (And).

2) - The element 0 is an identity element with Respect to $+$ (OR)

$$x + 0 = 0 + x = \underline{x}$$

- The element 1 is an Identity Element with Respect To • (AND)

$$X \cdot 1 = 1 \cdot X = \underline{X}$$

[3] - The structure is Commutative With Respect to (+)

$$X + Y = Y + X$$

- The Structure is Commutative With Respect to (•)

$$X \cdot Y = Y \cdot X$$

[4] - The operator • is distributive over +

$$X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$$

- The operator + is distributive over •

$$X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$$

[5] - For Every Element $x \in B$ & $x' \in B$

* $X + X' = 1$
* $X \cdot X' = 0$

الخلاصة

1. $(X * Y) * Z = X * (Y * Z)$

2. $X * Y = Y * X$

3. $X * (Y \cdot Z) = (X * Y) \cdot (X * Z)$

4. $X + 0 = 0 + X = X$

Ex $X = 1$ $\quad$ $X = 0$

$1 + 0 = 1$ $\quad$ $0 + 0 = 0$

5. $X \cdot 1 = 1 \cdot X = X$

Ex: $X = 1$ $\quad$ $X = 0$

$1 \cdot 1 = 1$ $\quad$ $1 \cdot 0 = 0$

6. $X + Y = Y + X$

7. $X \cdot Y = Y \cdot X$

Ex: $X = 0$ $\quad$ $X \cdot Y = Y \cdot X$

$Y = 0$ $\quad$ $0 \cdot 0$ $\quad$ $0 \cdot 0$

0 $\quad$ 0

$\boxed{8}$ $X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$

$\boxed{9}$ $X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$

$\boxed{10}$ $X + X' = 1$

Ex: $X = 0$ $\quad 0 + 1 = 1$
$X' = 1$

$X = 1$
$X' = 0$ $\quad 1 + 0 = 1$

$\boxed{11}$ $X \cdot X' = 0$

Ex: $X = 0$ $\quad 0 \cdot 1 = 0$ | $X = 1$ $\quad 1 \cdot 0 = 0$
$X' = 1$ | $X' = 0$

* Theorems of Boolean Algebra:

1. $X + 0 = X$
2. $X \cdot 1 = X$
3. $X + X' = 1$
4. $X \cdot X' = 0$
5. $X + 1 = 1$
6. $X \cdot 0 = 0$
7. $X + X = X$
8. $X \cdot X = X$

Note

كل النظريات نقدر
نستعين بطريقة التحقق
منها ال Truth Table.

* Involution Theorm:

$$(X')' = X$$

* Commutative Theorem:

- $X + Y = Y + X$
- $XY = YX$

★ Associative Theorem:

* $X + (y + z) = (x + y) + z$
* $X(yz) = (xy)z$

★ Distributaive Theorem:

$$X(y+z) = xy + xz$$

$$X + yz = (x+y)\cdot(x+z)$$

★ De-Morgan Theorem:

$$(X+y)' = x' \cdot y'$$

$$(xy)' = x' + y'$$

★ Absorption Theorem:

$$X + xy = X$$

$$x \cdot (X+y) = X$$

إثباتات النظريات

① $X + X = X$

$= (X + X) \cdot 1$

$= (X + X) \cdot (X + X')$

$= X \cdot (X + X')$

$= X \cdot X + X' \cdot X'$

$= X + 0 = \boxed{X}$

$X + X' = 1$

$X + X = X$

$X(y+z) = xy + xz$

$XX' = 0$

$X \cdot X = X$

② $X \cdot X = X$

$= X \cdot X + 0$

$= XX + XX'$

$= X(X + X')$

$= X \cdot 1$

$= \boxed{X}$

$XX' = 0$

$X + X' = 1$

$$X + 1 = 1$$

$$= 1 \cdot (X+1)$$
$$= (X + X') \cdot (X+1)$$
$$= X + (X' \cdot 1)$$
$$= X + X'$$
$$= \boxed{1}$$

$$X + X' = 1$$

$$X + YZ = (X+Y)(X+Z)$$

$$X + \underline{XY} = X \longrightarrow \text{Absorption Theorem.}$$

$$= X \cdot 1 + \underline{XY}$$
$$= X(1+Y)$$
$$= X \cdot 1$$
$$= \boxed{X}$$

$$X + 1 = 1$$
$$X \cdot 1 = X$$

$$X(X+Y) = X \longrightarrow \text{Absorption Theorem.}$$

$$= X \cdot X + XY$$
$$= X + XY$$
$$= X(1+Y)$$
$$= X \cdot 1 = \boxed{X}$$

$$1 + X = 1$$
$$X \cdot X = X$$