

* Lecture "6" *

"Integration"

$$* \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$* \int e^x dx = e^x + C$$

$$* \int \frac{1}{x} dx = \ln x + C$$

$$* \int e^{f(x)} f'(x) dx = e^{f(x)} + C$$

Examples

$$1) \int (x^2 + \sqrt{x} + 5) dx = \int (x^2 + x^{1/2} + 5) dx \\ = \frac{1}{3} x^3 + \frac{2}{3} x^{3/2} + 5x + C$$

$$2) \int e^{2x} dx = \frac{e^{2x}}{2} + C$$

$$3) \int e^{\sin x} \cos x dx = e^{\sin x} + C$$

$$4) \int e^{x^2} x dx = \frac{1}{2} \int 2x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$$

$$5) \int \frac{x}{x^2+5} dx = \frac{1}{2} \int \frac{2x}{x^2+5} dx = \frac{1}{2} \ln(x^2+5) + C$$

$$* \int \sin x dx = -\cos x + C$$

$$* \int \sec^2 x dx = \tan x + C$$

$$* \int \csc x \cot x dx = -\csc x + C$$

$$* \int \cos x dx = \sin x + C$$

$$* \int \sec x \tan x dx = \sec x + C$$

$$* \int \csc^2 x dx = -\cot x + C$$

Examples

$$1) \int \tan x dx = -\int \frac{\sin x}{\cos x} dx = -\ln |\cos x| + C = \ln |\sec x| + C \\ = \ln \frac{1}{\cos x} + C$$

$$2) \int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln \sin x + C$$

$$3) \int \sec x \, dx = \int \frac{\sec^2 x + \tan x \sec x}{\sec x + \tan x} \, dx \quad \left(\times \frac{\sec x + \tan x}{\sec x + \tan x} \right)$$

$$= \ln(\sec x + \tan x) + C$$

$$4) \int \csc x \, dx = - \int \frac{-(\csc^2 x + \csc x \cot x)}{\csc x + \cot x} \, dx \quad \left(\times \frac{\csc x + \cot x}{\csc x + \cot x} \right)$$

$$= -\ln(\csc x + \cot x) + C$$

$$5) \int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \tan x - x + C \quad \text{« } 1 + \tan^2 x = \sec^2 x \text{ »}$$

$$6) \int \sin^2 x \, dx = \frac{1}{2} \int (1 - \cos 2x) \, dx \quad \text{« } \cos 2x = 1 - 2\sin^2 x \text{ »}$$

$$= \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + C$$

$$7) \int \cos^2 x \, dx = \frac{1}{2} \int (1 + \cos 2x) \, dx \quad \text{« } \cos 2x = 2\cos^2 x - 1 \text{ »}$$

$$= \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) + C$$

* Various Examples *

$$1) \int \frac{x}{\sqrt{5-x^2}} \, dx = \int x(5-x^2)^{-\frac{1}{2}} \, dx = -\frac{1}{2} \int -2x(5-x^2)^{-\frac{1}{2}} \, dx$$

$$= -\frac{1}{2} \times \frac{(5-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$2) \int x(x^2+3)^7 \, dx = \frac{1}{2} \int 2x(x^2+3)^7 \, dx = \frac{1}{2} \times \frac{(x^2+3)^8}{8} + C$$

$$3) \int (\tan x + 7)^3 \sec^2 x \, dx = \frac{(\tan x + 7)^4}{4} + C$$

$$4) \int e^{\ln x} \left(\frac{1}{x} \right) \, dx = e^{\ln x} + C$$

$$5) \int \frac{\sin x + \cos x}{-\cos x + \sin x} dx = \ln(-\cos x + \sin x) + C$$

$$6) \int e^{1+\sin x} \cos x dx = e^{1+\sin x} + C$$

$$7) \int e^{\sin x + \tan x} (\sec^2 x + \cos x) dx = e^{\sin x + \tan x} + C$$

$$8) \int \frac{x^2}{(5-x^3)^4} dx = \frac{-1}{3} \int \frac{-3x^2}{(5-x^3)^4} dx = \frac{-1}{3} \int (5-x^3)^{-4} \times (-3x^2) dx$$

$$= \frac{-1}{3} \left(\frac{(5-x^3)^{-3}}{-3} \right) + C$$

$$9) \int \frac{dx}{1-\sin^2 x} = \int \frac{dx}{\cos^2 x} = \int \sec^2 x dx = \tan x + C$$

$$10) \int \frac{x}{(3-x^2)^2} dx = \frac{-1}{2} \int \frac{-2x}{(3-x^2)^2} dx = \frac{-1}{2} \int -2x(3-x^2)^{-2} dx$$

$$= \frac{-1}{2} \left(\frac{(3-x^2)^{-1}}{-1} \right) + C$$
