

$$x_{10} = 1.4146$$

$$f(1.4146) = 0.001170$$

2] بفرض $y = f(x)$ والـ صيغة التكرارية التالية

x	0.00	0.05	0.10	0.15	0.20	0.25
$y = f(x)$	0.0000	0.10017	0.20134	0.30452	0.41075	0.52110

اوجد $f'(x)$ و $f''(x)$ عند $x=0$ و $x=0.1$

3.01

$$h = 0.05$$

x	$y = f(x)$	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
0.00	0.0000					
0.05	0.10017	0.10017				
0.10	0.20134	0.10117	10^{-3}			
0.15	0.30452	0.10318	2.01×10^{-3}	1.01×10^{-3}		
0.20	0.41075	0.10623	3.05×10^{-3}	1.04×10^{-3}	3×10^{-5}	
0.25	0.52110	0.11035	4.12×10^{-3}	1.07×10^{-3}	3×10^{-5}	0

$$\text{عند } x=0 \Rightarrow \alpha=0$$

$$\begin{aligned} \therefore f'(x) &= \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \frac{1}{5} \Delta^5 y_0 - \dots \right] \\ &= \frac{1}{0.05} \left[0.10017 - \frac{1}{2} \times 10^{-3} + \frac{1}{3} \times 1.01 \times 10^{-3} - \frac{1}{4} \times 3 \times 10^{-5} \right] \\ &= 1.999 \end{aligned}$$

$$\begin{aligned} f''(x_0) &= \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \frac{5}{6} \Delta^5 y_0 \right] \\ &= \frac{1}{(0.05)^2} \left[10^{-3} - 1.01 \times 10^{-3} + \frac{11}{12} \times 3 \times 10^{-5} \right] \\ &= 7 \times 10^{-3} \end{aligned}$$

$$\text{عند } x=0.1 \quad \alpha = \frac{x - x_0}{h} = \frac{0.1 - 0}{0.05} = 2$$

$$\begin{aligned} f'(0.1) &= \frac{1}{h} \left[\Delta y_0 + \left(\alpha - \frac{1}{2} \right) \Delta^2 y_0 \right. \\ &\quad \left. + \frac{1}{6} (3\alpha^2 - 6\alpha + 2) \Delta^3 y_0 \right. \\ &\quad \left. + \frac{1}{12} (2\alpha^3 - 9\alpha^2 + 11\alpha - 3) \Delta^4 y_0 + \dots \right] \end{aligned}$$

$$\begin{aligned} f'(0.1) &= \frac{1}{0.05} \left[0.10318 + \left(2 - \frac{1}{2} \right) 3.05 \times 10^{-3} \right. \\ &\quad \left. + \frac{1}{6} (3(2)^2 - 6(2) + 2) 1.07 \times 10^{-3} \right] \\ &= \dots \end{aligned}$$

$$\begin{aligned} f''(0.1) &= \frac{1}{h^2} \left[\Delta^2 y_0 + (\alpha - 1) \Delta^3 y_0 + \dots \right] \\ &= \frac{1}{(0.05)^2} \left[3.05 \times 10^{-3} + 1.07 \times 10^{-3} \right] \\ &= \dots \end{aligned}$$

المسألة الأولى

المطلوب: إيجاد $f'(4)$ و $f''(4)$ و $f'''(4)$ عند النقطة $x=4$

x	4	6	8	10
$y=f(x)$	1	3	8	20

أوجد $f'(4)$ و $f''(4)$ و $f'''(4)$ عند النقطة $x=4$

الحل

نكتب جدول الفروق المتقدمة

x	$y=f(x)$	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
4	1	2		
6	3	5	3	
8	8	12	7	4
10	20			

$$\because x = x_0 \Rightarrow \alpha = 0 \quad \& \quad h = 2$$

$$f'(x_0) = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \dots \right]$$

$$f'(4) = \frac{1}{2} \left[2 - \frac{1}{2} \cdot 3 + \frac{1}{3} \cdot 4 \right] =$$

$$= \frac{1}{2} \left[2 - \frac{3}{2} + \frac{4}{3} \right] = 0.917$$

$$f''(4) = \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 \right] = \frac{1}{4} [3 - 4] = -\frac{1}{4}$$

$$f'''(4) = \frac{1}{h^3} [\Delta^3 y_0] = \frac{1}{8} \cdot 4 = \frac{1}{2}$$