Probability and Statistics

Lecture 3

* Expectation «E(x)»:

$$E(x) = \begin{cases} \sum x\,p(x) & , x \rightarrow \text{discrete r.v} \\ \int_{-\infty}^{\infty} x\,f(x)\,dx & , x \rightarrow \text{Continueous r.v} \end{cases}$$

* **Example:** A box contains 10 transistors, 2 of them are defective. A man selected at random one transistor from the box until he obtains a non defective one. Find the expected number of transistors to be chosen.

→ Let x be a random variable showing the number of chosen transistors.

$\overset{8}{G} \quad \overset{2}{D} \rightarrow G$

x	1	2	3
$p(x)$	$p(G) = \frac{8}{10}$	$p(D_1 \cap G_2) = p(D_1).p(G_2 \mid D_1)$ $= \frac{2}{10} \times \frac{8}{9}$	$p(D_1 \cap D_2 \cap G_3)$ $= p(D_1).p(D_2 \mid D_1).p(G_3 \mid D_1 \cap D_2)$ $= \frac{2}{10} \times \frac{1}{9} \times \frac{8}{8}$
$x\,p(x)$	$\frac{8}{10}$	$\frac{32}{90}$	$\frac{6}{90}$

$$\therefore E(x) = \sum x\,p(x) \quad \text{«discrete r.v»}$$

$$= \frac{8}{10} + \frac{32}{90} + \frac{6}{90} = \frac{11}{9}$$

* $\because p(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\therefore p(A \cap B) = p(A \mid B).p(B)$

* Example 2: A man wants to match 3 names with their professional names. He doesn't know the names and he tries to match by guess. Find the expected number of correct matches.

⇒ Let x be a random variable showing the number of correct matches.

$x = 0$ «there is no true matches.»
or $x = 1$ «There is one true match and two false.» "for illustration"
or $x = 3$ « All matches are true.»

* Notice that: we'll not suppose that $x = 2$ because if 2 matches are true, the third one must be true.

x	0	1	3
$p(x)$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{1}{6}$
$xp(x)$	0	$\frac{3}{6}$	$\frac{1}{2}$

$$\therefore E(x) = \sum x p(x)$$
$$= 0 + \frac{3}{6} + \frac{1}{2} = 1$$

* Explanation:

Name	prof
a	A
b	B
d	D

→ $S = \{(aD, bA, dB), (aB, bD, dA), (aA, bD, dB), (aD, bB, dA), (aB, bA, dD), (aA, bB, dD)\}$

, we have 2 probabilities that there is no true matches. « $\frac{2}{6}$ »
, we have 3 probabilities that there is only one true match. « $\frac{3}{6}$ »
, we have only one probability that all matches are true. « $\frac{1}{6}$ ».

Chapter 3: "Some important distributions."

1) Binomial distribution: "Important."

→ There are 3 Conditions must be satisfied to use Binomial dist. :-

1) Random Experiment

P[Success] = p   P[fail] = 1 - p

only 2 states.

* Success means achieving the required at the experiment.

2) The R.E is repeated n independent trials.

3) The probability of success is constant in each trial. , $E(x) = np$

→ Law: $P(x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$

* Example: Tossing a coin 10 times. Find the prob. of appearing head 4 times.

→ $P(x) = {}^{10}C_{4} \cdot \left(\frac{1}{2}\right)^{4} \cdot \left(1 - \frac{1}{2}\right)^{10-4}$

* Example 2: The percentage of defective items in the production of a certain factory is 0.1. A man selected at random 10 items of the production.
Find the probability of getting 2 defective items.
" " " " " at least 2 defective items.
" " " " " 8 good items.

D G

p(D) = 0.1   p(G) = 0.9

→ let x be a random variable showing number of defective items.

here: $x \sim b(n,p)$

→ x submits to binomial conditions at $n=10$, $p=0.1$.

$\therefore p(x=2) = {}^{10}C_2 (0.1)^2 (0.9)^8$ "1st req."

2) $p(x \geq 2) = p(x=2) + p(x=3) + \cdots + p(x=10)$

or $p(x \geq 2) = 1 - p(x^c)$

$= 1 - (p(x=0) + p(x=1))$

3) $p(8 \text{ Good}) = p(2 \text{ def.}) = p(x=2)$

- The expected number of defective items: $E(x) = np = 0.1 \times 10 = 1$