# * Lecture 3 *

1) $u = \sin^{-1} x$ $\qquad -1 < x < 1$

$\rightarrow \; x = \sin u$

$1 = \cos u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{1}{\cos u}$

$\dfrac{du}{dx} = \dfrac{1}{\sqrt{1 - \sin^2 u}} = \dfrac{1}{\sqrt{1 - x^2}}$

$\therefore \boxed{\dfrac{d}{dx} \sin^{-1} = \dfrac{1}{\sqrt{1 - x^2}}}$

$* \; \cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos x = \sqrt{1 - \sin^2 x}$

2) $u = \cos^{-1} x$

$\rightarrow \; x = \cos u$

$1 = -\sin u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{-1}{\sin u}$

$\dfrac{du}{dx} = \dfrac{-1}{\sqrt{1 - \cos^2 u}} = \dfrac{-1}{\sqrt{1 - x^2}}$

$\therefore \boxed{\dfrac{d}{dx} \cos^{-1} x = \dfrac{-1}{\sqrt{1 - x^2}}}$

3) $u = \tan^{-1} x$

$\rightarrow \; x = \tan u$

$1 = \sec^2 u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{1}{\sec^2 u}$

$\dfrac{du}{dx} = \dfrac{1}{1 + \tan^2 u} = \dfrac{1}{1 + x^2}$

$\therefore \boxed{\dfrac{d}{dx} \tan^{-1} x = \dfrac{1}{1 + x^2}}$

$* \; \sec^2 x = 1 + \tan^2 x$

4) $u = \sec^{-1} x$

$\rightarrow \quad x = \sec u$

$1 = \sec u \tan u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{1}{\sec u \tan u} = \dfrac{1}{|x|\sqrt{\sec^2 u - 1}} = \dfrac{1}{|x|\sqrt{x^2-1}}$

$* \tan^2 x = \sec^2 x - 1$

$\tan x = \sqrt{\sec^2 x - 1}$

$\therefore \boxed{\dfrac{d}{dx} \sec^{-1} x = \dfrac{1}{|x|\sqrt{x^2-1}}}$

5) $u = \cot^{-1} x$

$\rightarrow \quad x = \cot x$

$1 = -\csc^2 u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{-1}{\csc^2 u} = \dfrac{-1}{1+\cot^2 u} = \dfrac{-1}{1+x^2}$

$* \csc^2 x = 1 + \cot^2 x$

$\therefore \boxed{\dfrac{d}{dx} \cot^{-1} x = \dfrac{-1}{1+x^2}}$

6) $u = \csc^{-1} x$

$\rightarrow \quad x = \csc u$

$1 = -\csc u \cot u \dfrac{du}{dx}$

$\therefore \dfrac{du}{dx} = \dfrac{-1}{\csc u \cot u} = \dfrac{-1}{|x|\sqrt{x^2-1}}$

$\therefore \boxed{\dfrac{d}{dx} \csc^{-1} x = \dfrac{-1}{|x|\sqrt{x^2-1}}}$

* $\frac{d}{dx} \ln x = \frac{1}{x}$
* $\frac{d}{dx} b^x = b^x \ln b$

* $\frac{d}{dx} e^x = e^x$
* $\log_b x = \frac{1}{x \ln b}$

# Examples

1) $y = \log_{10}(x^2+1)$

$\rightarrow y' = \frac{1}{(x^2+1)\ln 10} \cdot 2x = \frac{2x}{(x^2+1)\ln 10}$

2) $y = e^{x^2+1}$

$\rightarrow y' = e^{x^2+1} \cdot 2x$

3) $y = 10^{(x^3+2x+1)}$

$\rightarrow y' = 10^{(x^3+2x+1)} \cdot \ln 10 \cdot 3x^2+2$

4) $y = 3^{\sin x}$

$\rightarrow y' = 3^{\sin x} \cdot \ln 3 \cos x$

5) $y = \ln x^2+1$

$\rightarrow y' = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

6) $y = \ln\left(\frac{1-x}{1+x}\right)$

$\rightarrow y' = \frac{1+x}{1-x} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2}$

Another Solution: $y = \ln(1-x) - \ln(1+x)$

$\rightarrow y' = \frac{-1}{1-x} - \frac{1}{1+x}$

* $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

* $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

* $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^{x} + e^{-x}}$

* $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

* $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$

* $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^{x} - e^{-x}}$

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1) $y = \sinh x$

$\rightarrow y = \dfrac{e^{x} - e^{-x}}{2}, \quad \dfrac{dy}{dx} = \dfrac{1}{2}(e^{x} + e^{-x}) = \cosh x$

2) $y = \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$\rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}(e^{x} - e^{-x}) = \sinh x$

3) $y = \tanh x$

$\rightarrow y = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}, \quad \dfrac{dy}{dx} = \dfrac{(e^{x} + e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^2}$

$$= \frac{(e^{x} + e^{-x})^2 - (e^{x} - e^{-x})^2}{(e^{x} + e^{-x})^2}$$

$$= 1 - \frac{(e^{x} - e^{-x})^2}{(e^{x} + e^{-x})^2} = 1 - \tanh^2 x = \operatorname{sech}^2 x$$

* $1 - \tanh^2 x = \operatorname{sech}^2 x$

Similarly:

* $\dfrac{d}{dx} \tanh x = \operatorname{sech}^2 x$
* $\dfrac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$
* $\dfrac{d}{dx} \coth x = -\operatorname{csch}^2 x$
* $\dfrac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

* $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$
* $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$
* $\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$

## * Examples *

1) $y = \sin^{-1} \frac{x}{2}$

$\rightarrow y' = \frac{1}{\sqrt{1-\frac{x^2}{4}}} \times \frac{1}{2}$

2) $y = \cos^{-1} e^{x}$

$\rightarrow y' = \frac{-1}{\sqrt{1-(e^{x})^2}} \times e^{x}$