* Lecture u8" * * Important USes: - $1 \quad \sqrt{\alpha^2 - \chi^2} \quad \Rightarrow \quad \chi = a \sin \theta$ $2\sqrt{\alpha^2+\alpha^2}$ $\chi = atan\theta$ $3 - \sqrt{2} - \alpha^2 \rightarrow \alpha = \alpha \sec \theta$ * You must know that: $Cos^2 d + Sin^2 \alpha = 1$ 1 + Tan'x = Sec'x . Examples. 1) $\int d^2 x^2 dx$ = $\int d^2 - (c^2 \sin^2 \theta) dx$ $\int dx = a \cos \theta d\theta$ Ja2/11_Sin20 da = af Cost (a Cost) do = a2 f Cos2 0 de $GS^2\theta = \frac{1}{2}(1+GS2\theta)$ $=\frac{\alpha^2}{3}J(1+\cos 2\theta)d\theta$ $= \alpha^2 (\theta + \sin 2\theta) + C$

2) $\int \sqrt{9} \cdot 2^2 d\alpha = \int \sqrt{9} \cdot (9\sin^2\theta) d\alpha$ $d\alpha = 3\sin\theta$ $d\alpha = 3\int \sqrt{1-\sin^2\theta} d\alpha$ = 3 / GSO (3GSO) do $=\frac{9}{2}\int (1+6s2\theta) d\theta$ $=\frac{9}{2}\left(\theta+\frac{\sin 2\theta}{2}\right)+C$ 3) $\int a^2 + \alpha^2 d\alpha = \int a^2 + a^2 t a^3 \theta d\alpha$, $\int a = a t a n \theta$ = $a \int \int 1 + t a n^2 \theta d\alpha$ = a J Seco (a Seco) do $= a^2 \int Sec^3 \theta \ d\theta$ *Note. Jsec o in example 6 4) $\int \frac{d\alpha}{\sqrt{16+2^2}} = I$ $\int \frac{d\alpha}{\sqrt{16+2^2$ $I = \int \frac{4Sec^2\theta \, d\theta}{4Sec\theta} = \int \frac{Sec\theta \, d\theta}{4Sec\theta} \times \frac{Sec\theta + tan\theta}{Sec\theta + tan\theta}$ $= \int \frac{Sec\theta + Sec\theta \, tan\theta}{Sec\theta + tan\theta} \, d\theta$ $= ln(Sec\theta + tan\theta) + C$

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 $I = \ln(\sqrt{\frac{\chi^{2}+16}{H}} + \frac{\chi}{H}) + C$ $Cos\theta = \frac{H}{\sqrt{\chi^{2}+16}}$ $Another Solution: - \int \frac{d\chi}{\sqrt{16+\chi^{2}}} = Sinh' \frac{\chi}{H} + C$ 5) 1 12- a2 da = 1 la2 sec0 - c2 da $dx = asec \theta$ $dx = asec \theta tand d\theta$ $= a \int \sqrt{Sec^2\theta_{-1}} d\alpha$ = a 1 tano (asecotano) do = a2 / tan20 Sec 0 do = a2 ((Sec20_1) Sec0 d0 = a [| Sec 30 d0 - | Sec 0 d0] Note. 1 Secodo in ex. 6 , Jecodo ina 5 6) $\int Sec^3\theta \ d\theta = \int Sec^3\theta Sec^3\theta \ d\theta = I$ $U = Sec\theta$ $du = Sec\theta tan\theta$ $v = tan\theta$ I = Secotand - I Seco tand do = Secotand - I seco (Secol - 1) do = Secotand - Kseco - Seco) do = Sect tand - I Sect do + I sect all

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: I + I = Secotand + ln (seco + tano) : I = 1 [Secotand + ln(Seco+tend)] + C *Definite Integration *

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* $(\alpha^2 + \alpha) d\alpha = \frac{\alpha^3}{3} + \frac{\alpha^2}{2} \Big|_{1}^{2} = (\frac{8}{3} + 2) - (\frac{1}{3} + \frac{1}{2}) = \frac{23}{6}$ $\int_{a}^{a} f(\alpha) d\alpha$ if (fx) is abd if f(x) obx is even. 1) $\int_{0}^{\frac{\pi}{4}} e^{tonx} sec^{2}x dx = e^{tonx} \int_{0}^{\frac{\pi}{4}} = e^{-1}$ 2) $\int_{1}^{2} \frac{\chi}{\sqrt{2}} d\chi = \frac{1}{2} \int_{1}^{2} \frac{2\chi}{\sqrt{2}} d\chi = \left[\frac{\chi^{2} + H}{2} \right]_{1}^{2} = \sqrt{8} \sqrt{5}$ 3) $\int_{0}^{2} \cos x \sin^{2}x \, dx = \frac{\sin^{2}x}{3} \Big|_{0}^{2} = \frac{1}{3}$ 4), $\sqrt{(2e^{2}+3)}dx = e^{2}+3x/, = e^{2}+12 e^{2}-3 = e^{2}-e^{2}+9$ 5) $\sqrt{(2e^{2}+3)}dx = 2\sqrt{(2e^{2}+3)}x$ (as $x dx = (2e^{2}-2e^{2})x$) $\sqrt{(2e^{2}+3)}dx = 2\sqrt{(2e^{2}+3)}x$ (as $x dx = (2e^{2}-2e^{2})x$) $\sqrt{(2e^{2}+3)}dx = 2\sqrt{(2e^{2}+3)}x$ (b) $\sqrt{(2e^{2}+3)}dx = 2\sqrt{(2e^{2}+3)}x$ (c) $\sqrt{(2e^{2}+3)}dx = 2\sqrt{(2e^{2}+3)}x$ (

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