"Maths III"

Lecture 1

* Chapter 1: Partial derivative:-

- Partial derivative is used when the function is in 2 variables or more.

$$Z = f(x,y)$$

$$\rightarrow \frac{\partial z}{\partial x} = Z_x \quad , \quad \frac{\partial z}{\partial y} = Z_y$$

- To find $\frac{\partial f}{\partial x}$, we regard y as a constant and differentiate with respect to x.

* Examples *

1) $f(x,y) = x^3y^2 - e^{xy}$

$\rightarrow f_x = 3x^2y^2 - ye^{xy} \quad , \quad f_y = 2x^3y - xe^{xy}$

2) $f(x,y) = \ln(x^2+y^2-xy)$

$\rightarrow f_x = \frac{2x-y}{x^2+y^2-xy} \quad , \quad f_y = \frac{2y-x}{x^2+y^2-xy}$

3) $f(x,y,z) = e^{x-2y+3z}$

$\rightarrow f_x = e^{x-2y+3z} \quad , \quad f_y = -2e^{x-2y+3z} \quad , \quad f_z = 3e^{x-2y+3z}$

* Higher order partial derivatives *

$$\rightarrow \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = f_{xx} \quad , \quad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = f_{yy}$$

$$\rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = f_{yx} \quad , \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = f_{xy}$$

* Examples *

1) $f(x,y) = x\cos y + y e^{x}$, find $f_{xx}, f_{yy}, f_{xy}, f_{yx}$.

$\rightarrow f_x = \cos y + y e^{x}$, $f_y = -x\sin y + e^{x}$

. $f_{xx} = \frac{\partial}{\partial x}(f_x) = y e^{x}$, $f_{yy} = -x\cos y$

, $f_{xy} = \frac{\partial}{\partial y}(f_x) = -\sin y + y e^{x}$, $f_{yx} = \frac{\partial}{\partial x}(f_y) = -\sin y + e^{x}$

2) if $f(x,y) = x^2 \ln y + \sin(xy)$, Verify that: $f_{xy} = f_{yx}$

$\rightarrow f_x = 2x\ln y + y\cos(x,y)$, $f_y = \frac{x^2}{y} + x\cos(xy)$

$f_{xy} = \frac{\partial}{\partial y}(fx) = \frac{2x}{y} + \cos(xy) - xy\sin(xy) \rightarrow (1)$

$f_{yx} = \frac{\partial}{\partial x}(f_y) = \frac{2x}{y} + \cos(xy) - xy\sin(xy) \rightarrow (2)$

from (1), (2) ∴ $f_{xy} = f_{yx}$

3) if $u(x,y) = e^{x}\sin y$, prove that : $u_{xx} + u_{yy} = 0$

$\rightarrow u_x = e^{x}\sin y$, $u_y = e^{x}\cos y$

$u_{xx} = e^{x}\sin y \rightarrow (1)$, $u_{yy} = -e^{x}\sin y \rightarrow (2)$

from (1), (2) ∴ $u_{xx} + u_{yy} = 0$

---

* Chain Rule *

* $z = f(x,y)$, $x = x(t)$, $y = y(t)$

$$\rightarrow \frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

* $W = f(x,y,z)$, $x = x(t)$, $y = y(t)$, $z = z(t)$

$$\rightarrow \frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

$* \; z = f(x,y) \;,\; x = x(u,v) \;,\; y = y(u,v)$

$$\rightarrow \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$, \; \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

«3»