Probability and Statistics

Lecture 2

* Hollow Partitions: "Total Probability"

- if $A_1, A_2, \ldots, A_n$ partition to S and B is another event in S, then:

$$P(B) = \sum_{i=1}^{n} P(A_i)\, P(B|A_i)$$

- **Example 1:** In the fourth level of FCAI, The dean of the faculty chose one branch from the four branches: MM, IS, CS, IT and then he chose a student from it. Find the probability that this student is a girl. Notice That MM → 30g, 30b, IS → 20b, 20g, CS → 25b, 25g, IT → 30b, 20g.

⇒ let A_1 be the event of choosing the branch IT.
" A_2 " " " " " " CS.
" A_3 " " " " " " IS.
" A_4 " " " " " " MM.
, let B be the event of choosing a girl.

$$p(A_1) = p(A_2) = p(A_3) = p(A_4) = \frac{1}{4}$$

$$p(B|A_1) = \frac{20}{50},\ p(B|A_2) = \frac{25}{50},\ p(B|A_3) = \frac{20}{40},\ p(B|A_4) = \frac{30}{60}$$

$$\therefore p(B) = \sum_{i=1}^{n} p(A)\,p(B|A)$$

$$= \frac{1}{4}\left(\frac{20}{50} + \frac{25}{50} + \frac{20}{40} + \frac{30}{60}\right)$$

- **Example 2:** A class contains 40 boy and 30 girl. The doctor will choose 2 persons, Find the probability that the second one is a girl.

→ let A be the event that the first chosen one is a boy.
, let B be the event that the second chosen one is a girl.

$\therefore p(B) = p(A)\,p(B|A) + p(A^c)\,p(B|A^c)$

$= \frac{40}{70} \times \frac{30}{69} + \frac{30}{70} \times \frac{29}{69}$

* Bay's Theorem: "Important"

→ If $A_1, \ldots, A_n$ make partition to S, B is another event in S, then:

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(B)}, \quad i = 1, 2, \ldots, n$$

- By Applying Bay's Theorem on Example 1:

→ If we know that the chosen student is a girl, then find the probability that this girl was chosen from CS.

$$\Rightarrow P(A_2|B) = \frac{P(A_2)P(B|A_2)}{P(B)}$$

* Chapter 2: Random Variables, probability functions and Expectations.

* Random Variables: functions defined on the Sample Space.

Example 1: $x: S \rightarrow \mathbb{R}$, S: sample space, $\mathbb{R}$: set of real numbers, x: random variable.

, Let S(two coins) $= \{HH, HT, TH, TT\}$
x: Shows number of heads.

$\Rightarrow x(HH) = 2$, $x(HT) = 1$, $x(TH) = 1$, $x(TT) = 0$
, Range$(x) = \{0, 1, 2\}$

Example 2: Let S(Two dies), x: Sum of the two appeared numbers.
$\Rightarrow$ Range$(x) = \{2, \ldots, 12\}$

* Types of Random Variables: "Discrete r.v , Continueous r.v"

→ discrete r.v: The range of r.v can be counted or numbered.
→ Continueous r.v: The range of r.v can't be counted or numered.

* Note: If Range of r.v = $[0,1]$, it is continueous r.v as the interval can't be counted or numered.

* Probability Functions "$p(x)$": - Functions defined on values of Random Variables.

1) Discrete r.v:-

- Named: probability mass functions.
- Has 2 Conditions: 1) $p(x) \geq 0$, $\forall x \in Range(x)$
2) $\sum p(x) = 1$

2) Continueous r.v:-

- Named: probability density functions. $f(x)$
- Has 2 Conditions: 1) $f(x) \geq 0$, $\forall x \in Range(x)$
2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

- Example: Suppose that S is a sample space of tossing two coins, and x is a r.v showing the number of heads. Find the probability distribution of x.

⇒ $S = \{HH, HT, TH, TT\}$, $Range(x) = \{0, 1, 2\}$

* Law: $p(x) = P\{s \in S : X(s) = x\}$

x	0	1	2
$p(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

$\therefore\ 0 = p\{TT\} = \frac{1}{4}$

, $1 = p\{HT, TH\} = \frac{2}{4}$

, $2 = p\{HH\} = \frac{1}{4}$

* Sum of $p(x)$ must be equal 1.