* Lecture 4 *

→ $\boxed{\ln x^n = n \ln x}$

Examples:-

1) $y = x^x$

→ $\ln y = x \ln x$

$\frac{1}{y} y' = \ln x + \frac{1}{x} \cdot x = \ln x + 1$

$\therefore y' = y(\ln x + 1) = x^x (\ln x + 1)$

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2) $y = (x^2+4)^{3x}$

→ $\ln y = 3x \ln(x^2+4)$

$\frac{1}{y} y' = 3\ln(x^2+4) + 3x \cdot \frac{1}{x^2+4} \cdot 2x$

$\frac{y'}{y} = 3\ln(x^2+4) + \frac{6x^2}{x^2+4}$

$\therefore y' = y\left(3\ln(x^2+4) + \frac{6x^2}{x^2+4}\right) = (x^2+4)^{3x}\left(3\ln(x^2+4) + \frac{6x^2}{x^2+4}\right)$

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3) $y = (2+\sin x)^{\cos x}$

→ $\ln y = \cos x \ln(2+\sin x)$

$\frac{1}{y} y' = -\sin x \ln(2+\sin x) + \cos x \cdot \frac{1}{2+\sin x} \cdot \cos x$

$\frac{y'}{y} = -\sin x \ln(2+\sin x) + \frac{\cos^2 x}{2+\sin x}$

$\therefore y' = y\left(-\sin x \ln(2+\sin x) + \frac{\cos^2 x}{2+\sin x}\right)$

$= (2+\sin x)^{\cos x}\left(-\sin x \ln(2+\sin x) + \frac{\cos^2 x}{2+\sin x}\right)$

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4) $y = (1+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x)$

$\frac{y'}{y} = \frac{-1}{x^2}\ln(1+x) + \frac{1}{1+x}\cdot\frac{1}{x}$

$\therefore y' = y\left(\frac{-1}{x^2}\ln(1+x) + \frac{1}{x(1+x)}\right)$

$= (1+x)^{1/x}\left(\frac{-1}{x^2}\ln(1+x) + \frac{1}{x(1+x)}\right)$

5) $y = (1+\sinh x)^{\cosh x}$

$\ln y = \cosh x \ln(1+\sinh x)$

$\frac{y'}{y} = \sinh x \ln(1+\sinh x) + \cosh x \cdot \frac{1}{1+\sinh x}\cdot\cosh x$

$\therefore y' = y\left(\sinh x \ln(1+\sinh x) + \frac{\cosh^2 x}{1+\sinh x}\right)$

$= (1+\sinh x)^{\cosh x}\left(\sinh x \ln(1+\sinh x) + \frac{\cosh^2 x}{1+\sinh x}\right)$

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* Implict Functions *

1) $x^2 + y^2 = 16$

$2x + 2yy' = 0$, $\therefore 2yy' = -2x$, $\therefore y' = \frac{-x}{y}$

2) $x^3 + y^3 = 4xy$

$3x^2 + 3y^2y' = 4(xy' + y)$

$3y^2y' - 4xy' = 4y - 3x^2$

$y'(3y^2 - 4x) = 4y - 3x^2$, $\therefore y' = \frac{4y - 3x^2}{3y^2 - 4x}$

3) $x\sin y + \cos 3y = \sin 2y$

$\sin y + x\cos y y' - 3\sin 3y y' = 2\cos 2y y'$

$x\cos y y' - 3\sin 3y y' - 2\cos 2y y' = -\sin y$

$y'(x\cos y - 3\sin y - 2\cos 2y) = -\sin y$

$\therefore y' = \dfrac{-\sin y}{x\cos y - 3\sin y - 2\cos 2y}$

H) $\dfrac{(x-2)^2}{9} + \dfrac{(y-3)^2}{16} = 1$

$\rightarrow \dfrac{2}{9}(x-2) + \dfrac{2}{16}(y-3)y' = 0$

$\dfrac{1}{8}y'(y-3) = \dfrac{-2}{9}(x-2)$, $\therefore y' = \dfrac{-16(x-2)}{9(y-3)}$

5) $(x^2+y^2)^2 = 4(x^2-y^2) = 4x^2 - 4y^2$

$\rightarrow 2(x^2+y^2)(2x + 2yy') = 8x - 8yy'$

$4x^3 + 4x^2yy' + 4y^2x + 4y^3y' = 8x - 8yy'$ $\div 4$

$x^3 + x^2yy' + y^2x + y^3y' = 2x - 2yy'$

$x^2yy' + y^3y' + 2yy' = 2x - x^3 - y^2x$

$y'(x^2y + y^3 + 2y) = 2x - x^3 - y^2x$

$\therefore y' = \dfrac{x(2 - x^2 - y^2)}{y(x^2+y^2+2)}$

* Higher derivative *

- Find the Second derivative:-

1) $y = \sin 3x$

$\rightarrow y' = 3\cos 3x$, $y'' = -9\sin 3x$

2) $y = \cos(4x^2)$

$\rightarrow y' = -\sin(4x^2) \cdot 8x = -8x\sin(4x^2)$

, $y'' = -8\sin(4x^2) + \cos(4x^2) \cdot 8x \cdot (-8x)$

$= -8\sin(4x^2) - 64x^2\cos(4x^2)$

3) $y = \tan(3x)$

$y' = 3\sec^2(3x)$

$y'' = 6\sec(3x).\sec(3x)\tan(3x).3$

$= 18\sec^2(3x)\tan(3x)$

4) $y = \sec(2x)$

$y' = \sec(2x)\tan(2x).2$

$= 2\sec(2x)\tan(2x)$

$y'' = 2(2\sec(2x)\tan(2x).\tan(2x) + 2\sec^2(2x).\sec(2x))$

$= 2(2\sec(2x)\tan^2(2x) + 2\sec^3(2x))$

5) $y = \coth(4x)$

$y' = -4\,\mathrm{csch}^2(4x)$

$y'' = -4(2\,\mathrm{csch}(4x).(-\mathrm{csch}4x)\coth(4x)\times 4)$

$= -4(-8\,\mathrm{csch}^2(4x)\coth(4x))$

6) $y = \ln x^2$

$y' = \frac{1}{x^2}.2x = \frac{2}{x}$, $y'' = \frac{-2}{x^2}$

7) $y = e^{x^2}$

$y' = e^{x^2}.2x$, $y'' = 2xe^{x^2}\times 2x + 2e^{x^2} = 4x^2e^{x^2} + 2e^{x^2}$

8) $y = \log_{10}(x^2+1)$

$y' = \frac{1}{(x^2+1)\ln 10}\times 2x = \frac{2x}{(x^2+1)\ln 10}$

$y'' = \frac{2}{\ln 10}\left(\frac{(x^2+1)-2x}{(x^2+1)^2}\right)$

* Try By yourself:

1) $y = 10^{x^2}$

2) $y = \sin^{-1}x$

3) $y = \tan^{-1}x$