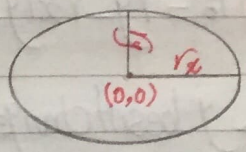
Computer Graphics

* Lecture 3 *

- Ellipse drawing Algorithm:-

Equation of ellipse: $\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$



$\therefore r_y^2 x^2 + r_x^2 y^2 = r_x^2 r_y^2$

$\therefore y = \pm \sqrt{\frac{r_x^2 r_y^2 - r_y^2 x^2}{r_x^2}}$

→ Can't be used in graphics. « decimal results » « too many operations »

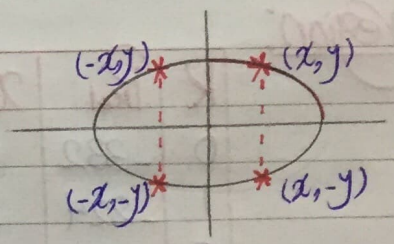
, At Center of ellipse ≠ (0,0) → Equation: $\frac{(x-x_c)^2}{r_x^2} + \frac{(y-y_c)^2}{r_y^2} = 1$

So, we'll use this Algorithm:

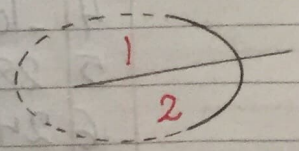
→ $F(x,y) = r_x^2 y^2 + r_y^2 x^2 - r_x^2 r_y^2$

, At $F(x,y)$
- <0 → point inside ellipse.
- $=0$ → point on ellipse.
- >0 → point outside ellipse.

- The ellipse has Similarity property, So we can get 3 quarters from one quarter.



→ we divide it into two regions:



1st region	2nd region
→ x++	→ y--
→ y { → No change, → decrease }	→ x { → No change, → Increase }

* **Rules:-**

1st region

1) Set $(0, r_y)$

2) $P_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$

3) At $P_k < 0$ → y doesn't change. (x_{k+1}, y_k)
→ $P_{k+1} = P_k + 2r_y^2 x_{k+1} + r_y^2$

, At $P_k \geq 0$ → y decreases by 1. $(x_{k+1}, y_k - 1)$
→ $P_{k+1} = P_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$

→ At $2r_y^2 x \geq 2r_x^2 y \Rightarrow$ Stop.

2nd region

1) last point in region 1: (x_0, y_0)

2) $P_0 = r_y^2 (x_0 + \frac{1}{2})^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$

3) At $P_k > 0$ → x doesn't change
→ y--, (x_k, y_{k-1})
→ $P_{k+1} = P_k - 2r_x^2 y_{k+1} + r_x^2$

, At $P_k \leq 0$ → x increases by 1
→ (x_{k+1}, y_{k-1})
→ $P_{k+1} = P_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$

→ At point $(r_x, 0) \Rightarrow$ Stop.

* **Example "midterm":** Draw ellipse of $r_x = 8$, $r_y = 6$

→ **1st region:**

K	P_{k+1}	x_{k+1}	y_{k+1}	$2r_y^2 x_{k+1}$	$-2r_x^2 y_{k+1}$
0	-332	1	6	72	-768
1	-224	2	6	144	-768
2	-44	3	6	216	-768
3	208	4	5	288	-640
4	-108	5	5	360	-640
5	288	6	4	432	-512
6	244	7	3	504	-384

→ 2nd region: Starts from point (7, 3)

K	P_k	x_{k+1}	y_{k+1}	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-23	8	2	576	-256
1	361	8	1	576	-128
2	297	8	0	—	—