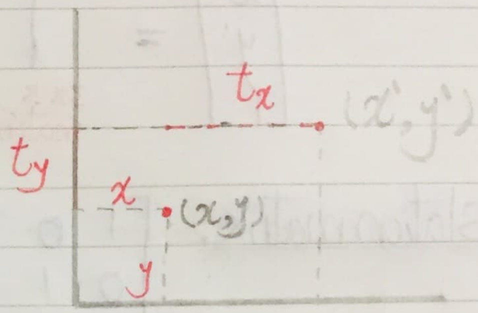
Computer Graphics

Lecture 4

Transformations:-

1) Translation:

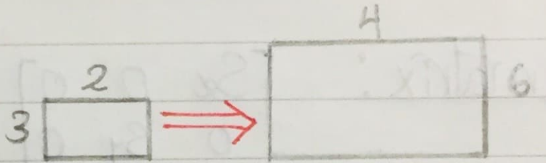


→ we want to move the point (x,y) to position (x',y').

- then:

$$x' = x + t_x$$
$$y' = y + t_y$$

2) Scaling:

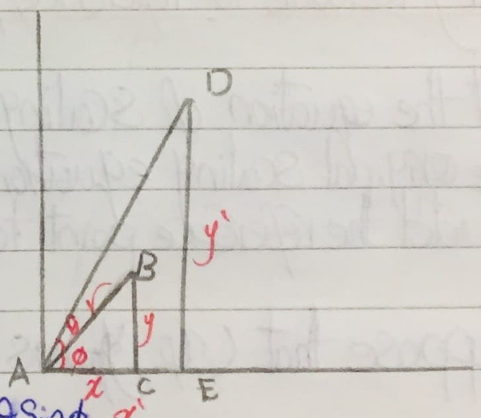


→ we want to scale this shape by scaling value equals 2, then we multiply its height by 2 and its width by 2.

- then:

$$x' = x \times S_x$$
$$y' = y \times S_y$$

3) Rotation:



→ we want to rotate triangle ABC by angle θ.

In $\triangle$ ABC: → $\cos\phi = \frac{x}{r}$ → $x = r\cos\phi$

→ $\sin\phi = \frac{y}{r}$ → $y = r\sin\phi$

In $\triangle$ ADE: → $x' = r\cos(\theta + \phi) = r\cos\theta\cos\phi - r\sin\theta\sin\phi$

→ $y' = r\sin(\theta + \phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$\because r\cos\phi = x$, $r\sin\phi = y$

- then: → $x' = x\cos\theta - y\sin\theta$

→ $y' = x\sin\theta + y\cos\theta$

* The previous operations are So difficult in Graphics, So We will use " Homogeneous matrix / System" To make all transformations by one method.

→ For all transformations:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underset{\substack{3\times 3 \\ \text{matrix}}}{\overline{T}} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

→ Translation matrix: $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

→ Scaling matrix: $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ Rotation matrix: $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* Scaling Relative to reference/pivot/Fixed point:

- To get the equation of Scaling relative to reference point, We remove x and y from the orginal Scaling equation and put $(x - x_f)$ and $(y - y_f)$ instead of them then add the reference point to the equation.

→ Suppose that (x_f, y_f) is the reference point.

$$\therefore x' = x_f + S_x(x - x_f) = x_f(1 - S_x) + S_x x$$

$$, y' = y_f + S_y(y - y_f) = y_f(1 - S_y) + S_y y$$

* Rotation Relative to reference point:

$$\rightarrow x' = x_f + (x - x_f)\cos\theta - (y - y_f)\sin\theta$$
$$= x\cos\theta - y\sin\theta + y_f\sin\theta + x_f - x_f\cos\theta$$

$$\rightarrow y' = y_f + (x - x_f)\sin\theta + (y - y_f)\cos\theta$$
$$= x\sin\theta + y\cos\theta + y_f - x_f\sin\theta - y_f\cos\theta$$

* Reflection:

- relative to x-axis: $x' = x$, $y' = -y$
- relative to y-axis: $x' = -x$, $y' = y$

- reflection matrix:

$$ref_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad ref_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Shearing:

- x-direction shearing: $x' = x + sh_x y$, $y' = y$
- y-direction shearing: $x' = x$, $y' = y + sh_y x$

- Shearing matrix:

$$sh_x = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad sh_y = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- with reference point: $\begin{bmatrix} 1 & sh_x & -y_f \times sh_x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -x_f \times sh_y \\ 0 & 0 & 1 \end{bmatrix}$

* Translation Inverse:

$$\begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

* Scaling Inverse:

$$\begin{bmatrix} 1/s_x & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Rotation Inverse:

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
