أوجد قيمة كثيرة الحدود $P_6(x) = x^6 - 2x^3 + x^2 + 5$

في النقطة $x=2$ بطريقة هورنر وأوجد المشتقات عند $x=2$

الحل

	1	0	0	-2	1	0	5
$x=2$		2	4	8	12	26	52
	1	2	4	6	13	26	57

$$\boxed{P_6(2) = 57}$$

$$q_1(x) = x^5 + 2x^4 + 4x^3 + 6x^2 + 13x + 26$$

	1	2	4	6	13	26
$x=2$		2	8	24	60	146
	1	4	12	30	73	172

$$q_1(3) = 172 = \frac{P_6'(2)}{1!}$$

$$P_6'(2) = 172$$

$$q_2(x) = x^4 + 4x^3 + 12x^2 + 30x + 73$$

	1	4	12	30	73
$x=2$		2	12	48	156
	1	6	24	78	229

$$q_2(3) = 229 = \frac{P_6''(2)}{2!}$$

$$P_6''(2) = 458$$

$$q_3(x) = x^3 + 6x^2 + 24x + 78$$

	1	6	24	78
$x=2$		2	16	80
	1	8	40	158

$$q_3(2) = 158 = \frac{P_6^{(3)}(2)}{3!}$$

$$P_6^{(3)}(2) = 948$$

$$q_4(x) = x^2 + 8x + 40$$

	1	8	40
$x=2$		2	20
	1	10	60

$$q_4(2) = 60 = \frac{P_6^{(4)}(2)}{4!}$$

$$P_6^{(4)}(2) = 1440$$

$q_5(x) = x + 10$

$$\begin{array}{r|rr} & 1 & 10 \\ x=2 & & 2 \\ \hline & 1 & 12 \end{array}$$

$q_5(2) = 12 = \dfrac{P_6^{(5)}(2)}{5!}$

$P_6^{(5)}(2) = 1440$

$q_6(x) = 1$

$$\begin{array}{r|r} & 1 \\ x=2 & \\ \hline & 1 \end{array}$$

$q_6(2) = 1 = \dfrac{P_6^{(6)}(2)}{6!}$

$P_6^{(6)}(2) = 720$

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(26) بتطبيق طريقة هورنر عين قيمة الحدودية التالية $P_4(x) = x^4 - 2x^3 + 3x^2 + 4x - 1$ عند النقطة $x=2$ ثم عين المشتقات المتتالية لهذه الحدودية عند النقطة $x=2$

الحل

$$\begin{array}{r|rrrrr} & 1 & -2 & 3 & 4 & -1 \\ x=2 & & 2 & 0 & 6 & 20 \\ \hline & 1 & \cancel{0} & 3 & 10 & \boxed{19} \end{array}$$

يكون صفر ← $\cancel{x^3}$

$P_4(2) = 19$ صحيح

$q_1(x) = x^3 + 3x + 10$

$$\begin{array}{r|rrrr} & 1 & 0 & 3 & 10 \\ x=2 & & 2 & 4 & 14 \\ \hline & 1 & 2 & 7 & 24 \end{array}$$

$\dfrac{P_4'(2)}{1!} = q_1(2) = 24$

$q_2(x) = x^2 + 2x + 7$

$$\begin{array}{r|rrr} & 1 & 2 & 7 \\ x=2 & & 2 & 8 \\ \hline & 1 & 4 & 15 \end{array}$$

$q_2(2) = 15 = \dfrac{P_4''(2)}{2!}$

$P_4''(2) = 30$

$q_3(x) = x + 4$

$$\begin{array}{r|rr} & 1 & 4 \\ x=2 & & 2 \\ \hline & 1 & 6 \end{array}$$

الجزء الثاني

$q_3(2) = \dfrac{P_4'''(2)}{3!} = 16$

$P_4'''(2) = 96$

$q_4(x) = 1$

$$\begin{array}{r|r} & 1 \\ x=2 & \\ \hline & 1 \end{array}$$

$q_4(2) = 1 = \dfrac{P_4''''(2)}{4!}$

$P_4^{(4)}(2) = 24$

| 0.567 | 0.567142... |

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5) أوجد حل جملة المعادلتين بطريقة نيوتن حيث

$f(x,y) = y - e^{x} + 2 = 0$

$g(x,y) = y - \ln(x+2) = 0$

$(x_0, y_0) = (1, 1)$

الحل

$$x_{n+1} = x_n - \frac{1}{J}\begin{vmatrix} f & f_y \\ g & g_y \end{vmatrix}$$

$$y_{n+1} = y_n - \frac{1}{J}\begin{vmatrix} f_x & f \\ g_x & g \end{vmatrix}$$

$$J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix} = \begin{vmatrix} -e^{x} & 1 \\ \frac{-1}{x+2} & 1 \end{vmatrix} = -e^{x} + \frac{1}{x+2}$$

$f_x = -e^{x}$ $\quad f_y = 1$

$g_x = \frac{-1}{x+2}$ $\quad g_y = 1$

$J(x_0, y_0) \Leftarrow J = -2.3849$

$$x_1 = 1 + \frac{1}{2.3849}\begin{vmatrix} 0.2817 & 1 \\ -0.0986 & 1 \end{vmatrix} = 1.1595$$

$$y_1 = 1 + \frac{1}{2.3849}\begin{vmatrix} -2.7183 & 0.2817 \\ -0.3333 & -0.0986 \end{vmatrix} = 1.1518$$

$J(x_1, y_1)$ $\quad J = -2.8718$

$$x_2 = 1.1595 + \frac{1}{2.8718}\begin{vmatrix} -0.0365 & 1 \\ 0.0014 & 1 \end{vmatrix}$$

$x_2 = 1.1463$

$$y_2 = 1.1518 + \frac{1}{2.8718}\begin{vmatrix} -3.1883 & -0.0365 \\ -0.3165 & 0.0014 \end{vmatrix}$$

$y_2 = 1.1462$

$J = -2.8287$

$$x_3 = 1.1463 + \frac{1}{2.8287}\begin{vmatrix} -0.00033 & 1 \\ -0.00003 & 1 \end{vmatrix}$$

$x_3 = 1.1462$

$$y_3 = 1.1462 + \frac{1}{2.8287}\begin{vmatrix} -3.1465 & -0.00033 \\ -0.3178 & -0.00003 \end{vmatrix}$$

$y_3 = 1.1462$

الجذر هو

$x_3 = 1.462$

$y_3 = 1.462$