Discrete Structures

* Lecture 2 *

* Revision on Last lecture: -

- if there are 40 students use hp laptop, 60 use Dell, 45 use Toshiba, 20 use dell and hp, 25 use hp and toshiba, 30 use dell and toshiba, 12 use the three types and number of all students is 170. How many student doesn't use any of these three types? How many students use dell or toshiba or only both of them?

→ $|hp| = 40$, $|d| = 60$, $|T| = 45$, $|d \cap hp| = 20$, $|hp \cap T| = 25$, $|d \cap T| = 30$, $|hp \cap d \cap T| = 12$, $U = 170$

$|d \cup hp \cup T| = |hp| + |T| + |D| - |d \cap hp| - |hp \cap T| - |d \cap T| + |hp \cap d \cap T|$

$= 40 + 45 + 60 - 20 - 25 - 30 + 12 = 82$

$\therefore |\overline{d \cup hp \cup T}| = |U| - |d \cup hp \cup T| = 170 - 82 = 88$

$|d \cup \overline{T}| = |d| + |\overline{T}| - |d \cap T| = 60 + 45 - 30 = 75$

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* Sequence *

→ S: 1, 2, 4, 6, 10, 11 order and repitition are important.
B: 2, 1, 4, 6, 10, 11 ≠ S

→ A: 0, 1, 1, 0, 0, 1, 1, 1
Set corresponding to sequence is {0, 1}

Formula of Sequence

- explicit functions
- Recursive functions

1) Explicit Functions:-

* depends on the index "n" of the element

Ex: $S(n) = 2^n$ $\quad 0 \le n \le 10$

→ $S(0) = 2^0 = 1$ → first element
$S(1) = 2^1 = 2$ → second element
$S(2) = 2^2 = 4$ → third element
∴ S: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.

Ex₂: $B(n) = n + 3$ $\quad 3 \le n \le 7$

→ $B(3) = 6$ → first element
$B(4) = 7$ → second element
∴ B: 6, 7, 8, 9, 10

Ex₃: $M(n) = \frac{1}{n}$ $\quad 1 \le n \le 5$

→ $M(1) = 1$ → first element
$M(2) = 1/2$ → second element
$M(3) = 1/3$ → third element
∴ M: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

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2) Recursive Functions:-

* depends on the previous elements.

For example:
$5! = 5 \times 4! = 120$
$4! = 4 \times 3! = 24$
$3! = 3 \times 2! = 6$
$2! = 2 \times 1! = 2$
$1! = 1$ → base case

Recursive formula using Functions :-

$F(1) = 1$, $F(n) = n * F(n-1)$

→ $F(4) = 4F(3) = 24$
$F(3) = 3F(2) = 6$
$F(2) = 2F(1) = 2$
$F(1) = 1$ → base case

* Example: S: 1, 1, 2, 3, 5, 8, ...

→ $S(1) = 1$, $S(2) = 1$ → 2 base cases
$S(n) = S(n-1) + S(n-2)$
$S(7) = S(6) + S(5) = 13$
$S(6) = S(5) + S(4) = 8$
$S(5) = S(4) + S(3) = 5$
$S(4) = S(3) + S(2) = 3$
$S(3) = S(2) + S(1) = 2$

* Ex_2: S: 3, 8, 13, 18, 23, ...

→ $S(1) = 3$, $S(2) = 8$ → 2 base cases
$\therefore S(n) = S(n-1) + 5$
explicit: $S(n) = 3 + 5n$

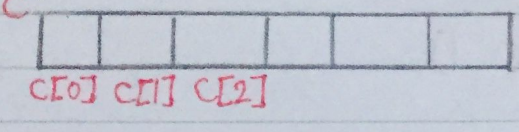
* Ex_3: S: 5, 10, 20, 40, 80

→ $S(1) = 5$, $\therefore S(n) = 2S(n-1)$

* Ex_4: S: 87, 82, 77, 72, 67

→ $S(n) = 87 - 5n$ $\quad 0 \le n \le 4$
or $S(n) = 92 - 5n$ $\quad 1 \le n \le 5$

* Arrays: Sequence of positions.



* Characteristic Functions:-

if $A=\{a,b,c,d\}$, $B=\{a,g\}$, $U=\{a,b,c,d,e,f,g\}$

$$\therefore F_A(x)=\begin{cases}0 & x\notin A\\ 1 & x\in A\end{cases}$$

	a	b	c	d	e	f	g
f_A	1	1	1	1	0	0	0

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	a	b	c	d	e	f	g
f_B	1	0	0	0	0	0	1

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* Example: $A=\{a,b,c,d\}$, $B=\{c,d,e,f\}$

	a	b	c	d	e	f	g
f_u	1	1	1	1	1	1	1
f_A	1	1	1	1	0	0	0
f_B	0	0	1	1	1	1	0
$f_{A\cap B}$	0	0	1	1	0	0	0
$f_{A\cup B}$	1	1	1	1	1	1	0
f_{A-B}	1	1	0	0	0	0	0
$f_{A\oplus B}$	1	1	0	0	1	1	0

- $F_{A\cap B} = F_A F_B$
- $F_{A\cup B} = F_A + F_B - F_A F_B$
- $F_{A-B} = F_A - F_A F_B$
- $F_{A\oplus B} = F_A + F_B - 2F_A F_B$
