→ Computer Graphics ←

* Lecture 2 *

Algorithms of drawing Circles:-

1) mathematically:

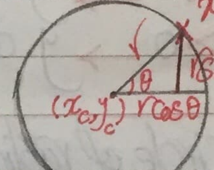
$$\rightarrow x^2 + y^2 = r^2$$

$$\text{, at any point: } (x - x_c)^2 + (y - y_c)^2 = r^2$$

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$

- So difficult and slow. « decimal results. »

2) Trigonometric Functions:



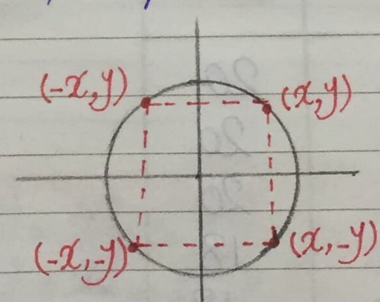
→ Set θ from 0.1 to 2π

$\therefore$ point x: $(r\cos\theta, r\sin\theta)$

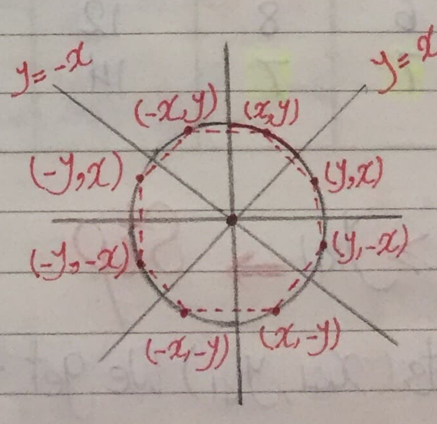
- So difficult in graphics because of decimal results of trigonometric functions.

3) Functions of the Circle:

1- The four quarters are similar.



2- We can get 7 quarters from one quarter.



Midpoint Algorithm:

→ $x^2 + y^2 = r^2$

$F(x,y) = x^2 + y^2 - r^2$

If $F(x,y) < 0$ → point inside Circle.
, If $F(x,y) = 0$ → point on Circle.
, If $F(x,y) > 0$ → point outside Circle.

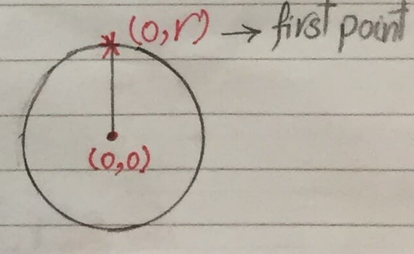
* **Rules:-**

1) $P_0 = 1 - r$
2) If $P_k < 0$ → y doesn't change, $P_{k+1} = P_k + 2x_{k+1} + 1$
, If $P_k \geq 0$ → y decreases by 1, $P_{k+1} = P_k + 2x_{k+1} - 2y_{k+1} + 1$

* **Example:** Draw a circle of radius 10 and its center is the origin point.

→ $P_0 = 1 - r = 1 - 10 = -9$

K	P_k	x_{k+1}	y_{k+1}	$2x_{k+1}$	$2y_{k+1}$
0	-9	1	10	2	20
1	-6	2	10	4	20
2	-1	3	10	6	20
3	6	4	9	8	18
4	-3	5	9	10	18
5	8	6	8	12	16
6	5	7	7	14	14



* At $x_{k+1} \geq y_{k+1}$ ⇒ Stop.

- From the points (x_{k+1}, y_{k+1}) we get the first quarter of the circle.

- To get the other 7 quarters, Follow this rule:-

$+x$	$+y$
$-x$	$+y$
$+x$	$-y$
$-x$	$-y$
$+y$	$+x$
$-y$	$+x$
$+y$	$-x$
$-y$	$-x$

* Example: at the point (3, 10)

$+3$	$+10$
-3	$+10$
$+3$	-10
-3	-10
$+10$	$+3$
-10	$+3$
$+10$	-3
-10	-3

* Proof of the Rules:

$$F(x,y) = x^2 + y^2 - r^2$$

$$F(x_k, y_k) = x_k^2 + y_k^2 - r^2$$

$$F(x_{k+1}, y_k - \tfrac{1}{2}) = x_{k+1}^2 + (y_k - \tfrac{1}{2})^2 - r^2 \Rightarrow P_k$$

, at P_{k+1}:

$$F(x_{k+2}, y_{k+1} - \tfrac{1}{2}) = x_{k+2}^2 + (y_{k+1} - \tfrac{1}{2})^2 - r^2$$

By Subtracting the 2 equations:

$$P_{k+1} = P_k + 2x_k + 3 + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k)$$

$$= P_k + 2(x_k+1) + 1 + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k)$$

$x_{k+1}^2 = (x_k+1)^2$

$x_{k+2}^2 = (x_k+2)^2$

→ If $P_k < 0$ → y doesn't change

$\therefore y_{k+1} = y_k$

$\therefore P_{k+1} = P_k + 2x_{k+1} + 1$

→ If $P_k > 0$ → y decreases by 1

$\therefore y_{k+1} = y_{k-1}$

$\therefore P_{k+1} = P_k + 2x_{k+1} - 2y_k + 2 + 1$

$= P_k + 2x_{k+1} - 2(y_k - 1) + 1$

$= P_k + 2x_{k+1} - 2y_{k+1} + 1$