Phonons

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Exercises

(24.1) :

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\begin{aligned}
\Theta_D &= \frac{\hbar\omega_D}{k_B} \Rightarrow \omega_D = 100k_B/\hbar \\
\omega_D &= (\frac{6N\pi^2 v_s}{V})^{1/3} \\
v_s &= \omega_D^3 \frac{V}{6N\pi^2} \\
\Longrightarrow v_s &= \frac{0.3k_B}{6N\pi^2\hbar}10^3
\end{aligned}
$$

$$
\begin{aligned}
\omega &= (4K/m)^{1/2}|sin(qa/2)| \Rightarrow \omega_{Max} = 2(K/m)^{1/2} \\
v_s &= a(K/m)^{1/2} \Rightarrow (K/m)^{1/2} = \frac{v_s}{a} \\
\Longrightarrow \omega_{Max} &= 2\frac{v_s}{a}
\end{aligned}
$$

(24.2) :

$$
\begin{aligned}
U &= \frac{9}{8}N\hbar\omega_D + \frac{9RT}{x_D^3}\int_0^{x_D}\frac{x^3dx}{e^x-1} \\
&= \frac{9}{8}N\hbar\omega_D + \frac{9RT}{(\hbar\beta\omega_D)^3}\int_0^{x_D}\frac{(\hbar\beta\omega)^3 d(\hbar\beta\omega)}{e^{(\hbar\beta\omega)}-1} \\
&= \frac{9}{8}N\hbar\omega_D + \frac{9RT(\hbar\beta)^4}{(\hbar\beta\omega_D)^3}\int_0^{x_D}\frac{(\omega)^3 d(\omega)}{e^{(\hbar\beta\omega)}-1} \\
&= \frac{9}{8}N\hbar\omega_D + \frac{9RT(\hbar\beta)}{\omega^3}\int_0^{x_D}\frac{(\omega)^3 d(\omega)}{e^{(\hbar\beta\omega)}-1}
\end{aligned}
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(24.3):

$$
\begin{aligned}
n_{(\omega)}d\omega &= \frac{g_{(\omega)}d\omega}{e^{\beta\hbar\omega}-1} = \frac{3V\omega^2}{2\pi^2 v^3(e^{\beta\hbar\omega}-1)} \\
u &= \int_0^{\omega_D} \hbar\omega n_{(\omega)} d\omega = 3V\hbar/2\pi^2 v^3 \int 0^{\omega_D} \frac{\omega^2 d\omega}{e^{\beta\hbar\omega}-1} \\
&= \frac{3}{2}\frac{V k_B^4 T^4}{v^3\hbar^3\pi^2}\int \frac{x^3 dx}{e^x-1} \\
if \rightarrow x << \frac{\hbar\omega}{k_B} \Rightarrow u &= \frac{3}{2}\frac{V k_B^4 T^4}{v^3\hbar^3\pi^2}\int_0^\infty \frac{x^3 dx}{e^x-1} \\
&= \frac{V k_B^4 T^4}{10 v^3\hbar^3} \\
\Longrightarrow C_v &= \frac{\partial u}{\partial T} \propto T^d =! : |
\end{aligned}
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(24.4)

$$
\begin{aligned}
\omega &= (4K/m)^{1/2}|sin(qa/2)| \\
g_{(q)}dq &= g_{(\omega)}d\omega \Rightarrow g_{(\omega)} = \frac{g_{(q)}}{d\omega/dq} \\
\frac{d\omega}{dq} &= (4K/m)^{1/2}(a/2)cos(qa/2) \\
\omega^2 &= (4K/m)sin^2(qa/2) = \frac{4K}{m}(1-cos^2(qa/2)) \\
\Rightarrow cos(qa/2) &= (1-\frac{m\omega^2}{4K})^(1/2) \\
g_{(q)}dq &= \frac{2dq}{2\pi/l} = \frac{ldq}{\pi} \\
\Longrightarrow g_\omega &= \frac{2}{\pi a}[\omega^2 - 4K/m]^{1/2}
\end{aligned}
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