



حركة الشبيبة الفتحاوية

جامعة كلية الهندسة وتكنولوجيا المعلوما

تلخيص الطالب

هلا الفالوجي

حركة الشبيبة الفتحاوية
جامعة الأزهر



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Discrete Structures

(Lecture 1)

chapter(1) The Foundations: Logic & proofs
proposition is a declarative sentence that
is either true or false.

① $0+0=0$ is the capital of Canada.
② $2+1=3$ is not the capital of Canada.
(propositions).

③ Sit down!

④ $x+1=2$

] not propositions?

Constructing propositions:

variables: P, q, r, s

true: T false: F

- Negation \neg (نفي) $\neg P$ not (\bar{P})
- Conjunction \wedge (رابط) $P \wedge q$ and
- Disjunction \vee (أو) $P \vee q$ inclusive or
- Implication \rightarrow (يمplies)
- Biconditional \leftrightarrow (إذا وفقط إذا)

(Lecture 2)

Inclusive or: $P \vee q$ either P or q or both must be true

Exclusive or: one of p or q is true, not both

$P \oplus q$

Implication:

الصواب \rightarrow إذا $P \rightarrow q$ فالـ

$$P \rightarrow q \Rightarrow (\text{False})$$

$$\begin{array}{c} T \\ P \rightarrow q \end{array} \quad (\text{If } q \text{ is } F \text{ and } P \text{ is } T) \quad P \text{ is true} \rightarrow b \rightarrow q$$

* if P , then q

* P implies q

* if P , q

* only if q ($q \rightarrow P$)

* q unless $\neg P$

* q when P

* q if P

$q \rightarrow P$

* q whenever P

* P is sufficient for q

* q follows from P

* q is necessary for P

a necessary condition for P is q .

a sufficient condition for q is P .

* Sufficient Condition: ($= G \wedge b \rightarrow q$)

q if P = $G \wedge b \rightarrow q$ $P \wedge \neg \sigma \rightarrow q$

q \rightarrow q \rightarrow P if P is b

(q follows from P), (P) \rightarrow q \rightarrow b

* necessary condition: (σ is b)

$P \rightarrow q$ if $\neg q$ \rightarrow $\neg P$ \rightarrow $\neg q$

($\neg q$ is a condition that is necessary for P)

It rains when the sky is cloudy \rightarrow (If p, q)

$$\frac{(q)}{(P \rightarrow q) \text{ (if } p, q\text{)}} \quad (P) = q \text{ when } p$$

(q) allows \rightarrow $((P) \rightarrow q, \text{ if } p)$

$(q) \rightarrow P = q \text{ when } p$

$(P) \rightarrow q = q \text{ when } p$

عندما ينعد المطر \rightarrow يكون السماء صافية (عندما ينعد المطر)

$(P) \rightarrow q = q \text{ when } p$

P are needed for q ellid

$q \rightarrow (P \rightarrow q)$

Converse: $q \rightarrow P$

contrapositive: $\neg q \rightarrow \neg P$

Inverse: $\neg P \rightarrow \neg q$

$P \rightarrow q \leftarrow$
 $q \rightarrow P \leftarrow$ p is sufficient for q
p is necessary for q

Biconditional: $P \leftrightarrow q$

- p is necessary and sufficient for q.
- if p then q, and conversely.
- p iff q.

if $1+1=2$, then $2+2=5$ (F)
True False

if $1+1=3$, then $2+2=4$ (T)

if $1+1=3$, then $2+2=5$ (T)

If monkeys can fly, then $1+1=3$ (T)

Truth table → column doesn't ale, no sum
الكلام الباقي بـ

(lecture 4) $P \vee q \rightarrow \neg r$

$(P \vee q) \rightarrow \neg r$ ~~is valid~~

~~and~~ $\underline{P \vee q \rightarrow \neg r}$

$(P \rightarrow q)$ is equivalent to $(\neg q \rightarrow \neg P)$

Two propositions are equivalent if they have the same truth value.

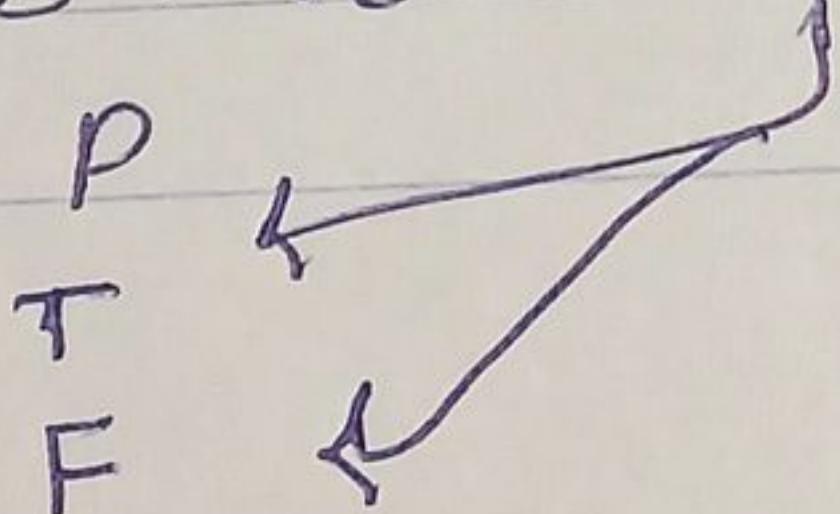
number of rows in a truth table: with n variables ? 2^n n: number of True or False propositional variables.

Lectures 5:

Tautology: proposition always (T)
 $P \vee \neg P$ (T)

Contradiction: proposition always (F)
 $P \wedge \neg P$

Contingency: proposition neither tautology nor contradiction such as P



* Two compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology

$$\boxed{P \equiv q}$$

is equivalent to

$$P \leftrightarrow q$$

T	\rightarrow	لما $q \rightarrow p$ لوكه
T		فهي دالة (T)
T		غير دالة (F)

$q \equiv p$, tautology \therefore

* Two compound propositions P and q are equivalent if and only if the columns in a truth table giving their truth values

$$\boxed{\neg P \vee q}$$

T

$$\boxed{P \rightarrow q}$$

T

$$(\neg P \vee q) \equiv (P \rightarrow q)$$

F

F

T

T

T

T

Dal Morgan's laws:

$$\neg(P \wedge q) \equiv \neg P \vee \neg q$$

$$\neg(P \vee q) \equiv \neg P \wedge \neg q$$

key logical Equivalences

- Identity Laws: $P \wedge T \equiv P$, $P \vee F \equiv P$
 $\begin{array}{cc} P & T \\ T & T \\ F & F \end{array} \quad \begin{array}{c} P \wedge T \\ T = P \\ F \end{array} \quad \left\{ \begin{array}{l} \{T\} \\ \{F\} \end{array} \right\} \quad \begin{array}{c} P \vee F \\ F = P \\ F \end{array} \quad \left\{ \begin{array}{l} \{P\} \\ \{F\} \end{array} \right\}$
- Domination Laws: $P \vee T \equiv T$, $P \wedge F \equiv F$
 $\begin{array}{cc} T & T \\ F & F \end{array} \quad \begin{array}{c} P \vee T \\ T = T \\ F \end{array} \quad \left\{ \begin{array}{l} \{T\} \\ \{F\} \end{array} \right\} \quad \begin{array}{c} P \wedge F \\ F = F \\ T \end{array} \quad \left\{ \begin{array}{l} \{F\} \\ \{T\} \end{array} \right\}$
- Idempotent Laws: $P \vee P \equiv P$, $P \wedge P \equiv P$
 $\begin{array}{cc} P & P \\ P & P \end{array} \quad \begin{array}{c} P \vee P \\ P = P \\ P \end{array} \quad \left\{ \begin{array}{l} \{P\} \\ \{P\} \end{array} \right\} \quad \begin{array}{c} P \wedge P \\ P = P \\ P \end{array} \quad \left\{ \begin{array}{l} \{P\} \\ \{P\} \end{array} \right\}$
- Double Negation Laws: $\neg(\neg p) \equiv p$
- Negation Laws: $P \vee \neg p \equiv T$, $P \wedge \neg p \equiv F$

lecture "6"

- Commutative laws: $P \vee q \equiv q \vee P$ $P \wedge q \equiv q \wedge P$
- Associative laws: $(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$
 $\leftarrow \text{左} \rightarrow \text{右}$ $(P \vee q) \vee r \equiv P \vee (q \vee r)$
- Distributive laws: $(P \vee (q \wedge r)) \equiv (P \vee q) \wedge (P \vee r)$
 $\text{左} \equiv \text{右}$ $(P \wedge (q \vee r)) \equiv (P \wedge q) \vee (P \wedge r)$
- Absorptive laws: $P \vee (P \wedge q) \equiv P$
 $\cdot \text{左} \equiv \text{右}$ $P \wedge (P \vee q) \equiv P$
 $P \vee P \wedge P \vee q$

lecture "7"

n is even if $n = 2k$

n is odd if $n = 2k+1$

P	q	$P \wedge q$	$P \vee (P \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

* absorptive laws

$$\neg(P \leftrightarrow q) \equiv P \leftrightarrow \neg q$$