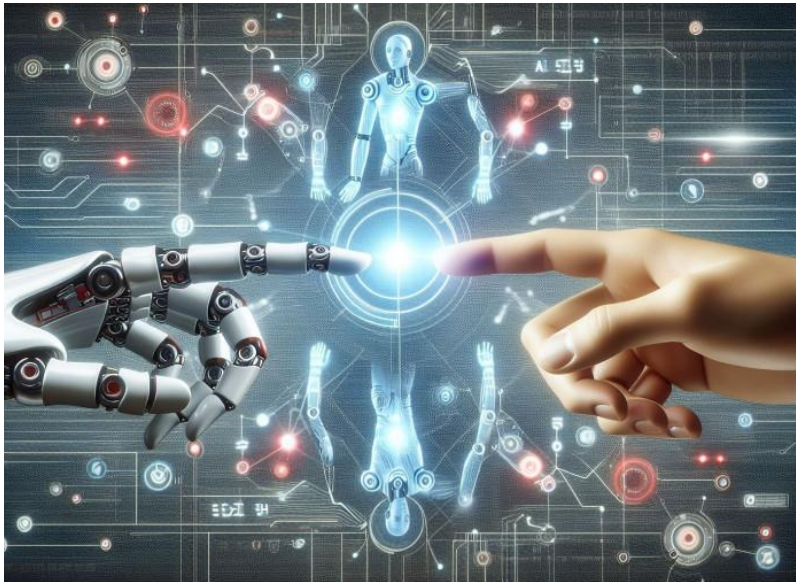
Lecture 5

Expert systems



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Logic Representation Systems

Introduction

❑ What is Logic?

Logic is the study of correct reasoning.

It provides a formal framework to:

- ❑ **Represent** knowledge and statements precisely.
- ❑ **Infer** new information from existing facts.

Topics:

- ✓ **Propositional logic**
- ✓ **Syntax and semantics of PL**
- ✓ **First-order logic**
- ✓ **Syntax and semantics of FOL**

Introduction

❑ Two Pillars of any Formal Language

For any logical system, we must define **two key** :

1- Syntax

- The "**grammar**" or set of rules for constructing valid sentences.
- Deals with structure. Is the sentence well-formed?

Example in English: "Cat the sat mat on" is syntactically invalid.

Introduction

- **Two Pillars of any Formal Language**

2- Semantics

- The "**meaning**" of the sentences.
- **Deals with truth.** What does the sentence mean, and is it true or false?

Example in English: "The cat sat on the mat" has a clear meaning and can be true or false in the world.

1- Propositional Logic (PL)

- **The simplest form of logic.**

Syntax

The syntax defines the **legal symbols** and how to combine them into **well-formed formulas (WFFs)**.

1. Symbols:

- **Propositional Symbols:** P, Q, R, ... (Atomic facts)
  - Examples:
    - P: "It is sunny."
    - Q: "The door is closed."
    - R: "The temperature is greater than 25°C."

1- Propositional Logic (PL)

- **Logical Connectives:**
  - Sentences are combined by connectives:
    - $\wedge$...and [conjunction]
    - $\vee$...or [disjunction]
    - $\Rightarrow$...implies [implication / conditional]
    - $\Leftrightarrow$..is equivalent [biconditional]
    - $\neg$ ...not [negation]

- **Parentheses:** (,) for grouping.

1- Propositional Logic (PL)

2. Rules for Well-Formed Formulas (WFFs):

- Any propositional symbol is a WFF.
- If φ is a WFF, then $\neg\varphi$ is a WFF.
- If φ and ψ are WFFs, then $(\varphi \land \psi)$, $(\varphi \lor \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$ are WFFs

- Examples:

- $(P \land Q)$ → **Valid WFF**
- $P \land Q \rightarrow$ → **Invalid WFF**
- $\neg(P \rightarrow (Q \lor R))$ → **Valid WFF**

1- Propositional Logic (PL)

Semantics

- Semantics gives **meaning** to the symbols by defining their **truth values**.
- **Truth Tables for Logical Connectives**

P	Q	¬P	P ∧ Q	P ∨ Q	P → Q	P ⇔ Q
F	F	T	F	F	T	T
F	T	T	F	T	T	F
T	F	F	F	T	F	F
T	T	F	T	T	T	T

Limitations of Propositional Logic

Problem: PL cannot look inside propositions.

Example:
We want to represent: **"All humans are mortal. Socrates is a human. Therefore, Socrates is mortal."**

In PL, we could try:
- P: "All humans are mortal."
- Q: "Socrates is a human."
- R: "Socrates is mortal."

- The logical form becomes: $(P \land Q) \rightarrow R$. This is **not a tautology** (always true). It's just a statement that could be false.

1- Propositional Logic (PL)

Problems

1- Which of the following is NOT a well-formed formula (WFF) in Propositional Logic?

a) $(P \land Q) \rightarrow R$

b) $P \rightarrow\rightarrow Q$

c) $\neg(P \lor Q)$

d) P

1- Propositional Logic (PL)

Problems

2- According to the truth table for implication ($P \rightarrow Q$), when is the statement false?

a) When P is true and Q is true.

b) When P is false and Q is true.

c) When P is true and Q is false.

d) When P is false and Q is false.

1- Propositional Logic (PL)

Problems

3- What is the correct translation of "If it is raining, then the ground is wet" into PL?

a) Raining ∧ Wet

b) Raining ∨ Wet

c) $\boxed{\text{Raining} \rightarrow \text{Wet}}$

d) Wet → Raining

2- First-Order Logic (FOL)

A more expressive logic.

- It can express statements about **objects**, their **properties**, and **their relationships**.

Key New Concepts:

- **Constants:** Refer to specific objects (e.g., Socrates, Alice).

- **Variables:** Placeholders for objects (e.g., x, y).

2- First-Order Logic (FOL)

•**Predicates:** Represent properties or relations (e.g., Human(x), Mortal(x), Loves(x, y)).

•**Functions:** Map objects to objects (e.g., motherOf(x)).

Quantifiers:

•∀ (For all): Universal Quantifier.

•∃ (There exists): Existential Quantifier

2- First-Order Logic (FOL)

Syntax

•The syntax defines how to build **terms** and **formulas**.

1. Terms:

•Constants (a, b, socrates),

• Variables (x, y),

•Functions applied to terms (fatherOf(john)).

2. Atomic Formulas:

•A Predicate applied to terms:

• Human(socrates),

•Loves(john, mary),

•GreaterThan(5, 2).

3. Well-Formed Formulas (WFFs):

- Any atomic formula is a WFF.
- If φ and ψ are WFFs, then $\neg\varphi$, $(\varphi \wedge \psi)$, etc., are WFFs.
- If φ is a WFF and x is a variable, then $\forall x\ \varphi$ and $\exists x\ \varphi$ are WFFs

Examples of Valid FOL WFFs:

- $\forall x\ (Human(x) \rightarrow Mortal(x))$
- $\exists y\ (Dog(y) \wedge Loves(john, y))$
- $\neg(P(a) \vee \forall x\ Q(x))$

2- First-Order Logic (FOL)

Element	Example	Meaning
Constant	1, 2, A, John, Mumbai, cat, ...	Values that can not be changed
Variables	x, y, z, a, b, ...	Can take up any value and can also change
Predicates	Brother, Father, >, ...	Defines a relationship between its input terms
Function	sqrt, LeftLegOf, ...	Computes a defined relation of input term
Connectives	$\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\Leftrightarrow$	Used to form complex sentences using atomic sentences
Equality	==	Relational operator that checks equality
Quantifier	$\forall$, $\exists$	Imposes a quantity on the respective variable

2- First-Order Logic (FOL)

Semantics

Semantics for FOL is more complex. It requires a **model** (M) to assign meaning.

A Model M = (D, I) consists of:

1. **Domain (D):** A non-empty set of objects (e.g., D = {socrates, plato, a_dog}).

2. **Interpretation (I):** A mapping that assigns:
   - To each constant, an object in D: I(socrates) = socrates
   - To each predicate, a relation in D: I(Human) = {socrates, plato}, I(Mortal) = {socrates, plato, a_dog}

2- First-Order Logic (FOL)

- To each function, a function on D: I(motherOf) is a function from D to D.

Truth is evaluated with respect to a model:

- Human(socrates) is **True** in M because socrates is in the set I(Human).

- $\forall x$ (Human(x) $\rightarrow$ Mortal(x)) is **True** in M because for every object in D that is in I(Human), it is also in I(Mortal).

2- First-Order Logic (FOL)

The Socrates Argument in FOL

Let's represent the original argument correctly.

Knowledge Base (FOL Formulas):

1. $\forall x\ (Human(x) \rightarrow Mortal(x))$ // "For all x, if x is human, then x is mortal."
2. Human(Socrates) // "Socrates is human."

Query:

- Mortal(Socrates) // "Is Socrates mortal?"

2- First-Order Logic (FOL)

Feature	Propositional Logic (PL)	First-Order Logic (FOL)
Basic Elements	Propositions (atomic)	Objects, Relations/Functions
Quantifiers	No	Yes (∀, ∃)
Variables	No	Yes
Expressiveness	Low	High
Example	`P ∧ Q → R`	`∀x (Cat(x) → Animal(x))`
Good for...	Boolean circuits, simple facts	Representing real-world knowledge

2- First-Order Logic (FOL)

Problems

1- Which of the following is a valid FOL term?

a) $\forall x\ P(x)$

b) Human(Socrates)

c) motherOf(John)

d) $P \rightarrow Q$

(A function applied to a constant is a term)

2- First-Order Logic (FOL)

Problems

2- What is the meaning of the quantifier ∃x in First-Order Logic?

a) For all x...

b) There exists an x such that...

c) For every x...

d) It is false that for all x...

2- First-Order Logic (FOL)

Problems

3- Which FOL formula correctly represents "Every dog has a tail"?

a) $\exists x\ (Dog(x) \rightarrow HasTail(x))$

b) $\forall x\ (Dog(x) \land HasTail(x))$

c) $\forall x\ (Dog(x) \rightarrow HasTail(x))$

d) $\exists x\ (Dog(x) \land HasTail(x))$

(For all x, if x is a dog, then x has a tail)

2- First-Order Logic (FOL)

Problems

4- In FOL, what does a predicate typically represent?

a) A specific object.

b) A property or a relationship between objects.

c) A logical connective.

d) A function that maps to truth values.

2- First-Order Logic (FOL)

Problems

5- In a model M, the Interpretation (I) assigns:

a) Truth values to all formulas.

b) Objects in the domain to constants, predicates, and functions.

c) Variables to quantifiers.

d) Well-formed formulas to the knowledge base.

2- First-Order Logic (FOL)

Problems

6- True / $\boxed{\text{False:}}$ In FOL, ∀x ∃y Loves(x, y) means that everyone loves one specific person.

It means that for every person x, there exists some person y (who could be different for each x) that x loves..

7- $\boxed{\text{True}}$ / False: The formula P(socrates) is a valid atomic formula in FOL if P is a predicate and socrates is a constant.

2- First-Order Logic (FOL)

Problems

8- True / False: The FOL formula ∀x Human(x) would be true in a model where the domain contains only cats.

It would be false, as the objects in the domain (cats) are not in the interpretation of the predicate Human.