

Complexity & Sorting algorithms

Seminar 1.



1. Complexity

- measure of how many operations algorithm will require to proceed an input of size N (or memory)

1. Complexity

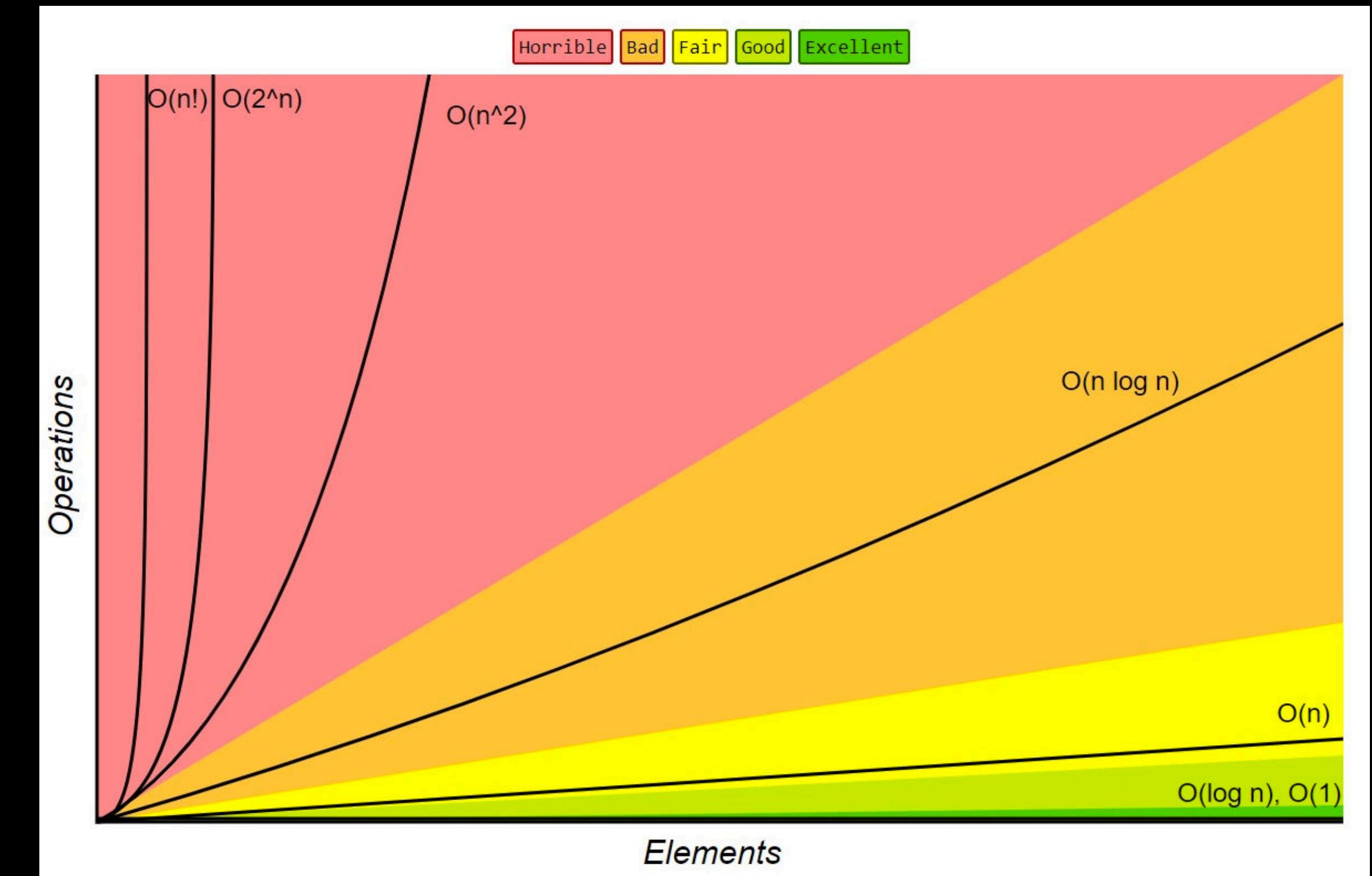
- measure of how many operations algorithm will require to proceed an input of size N (or time)



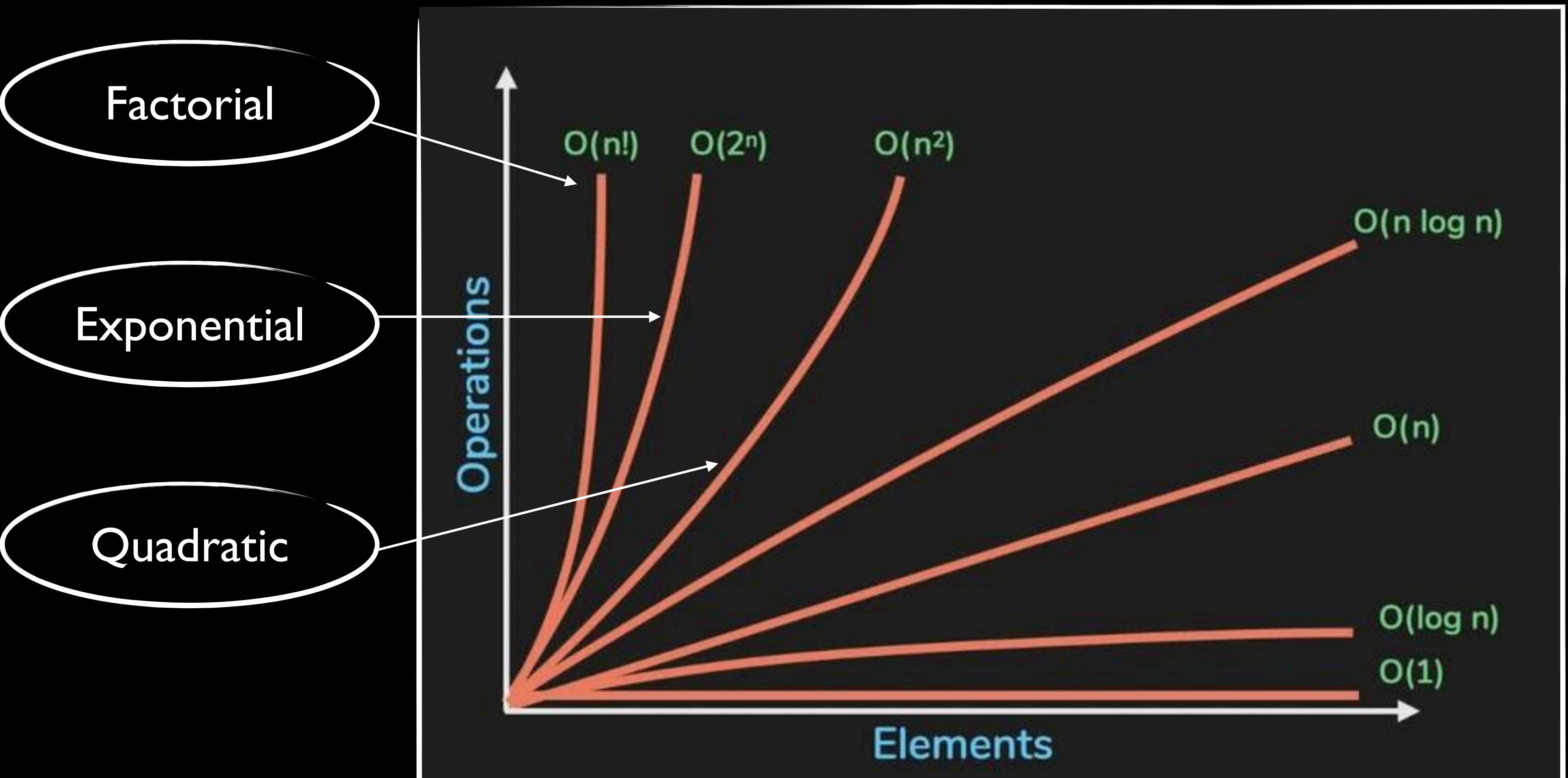
Big-Oh notation

- upper bound of the algorithm's computation time (worst-case)

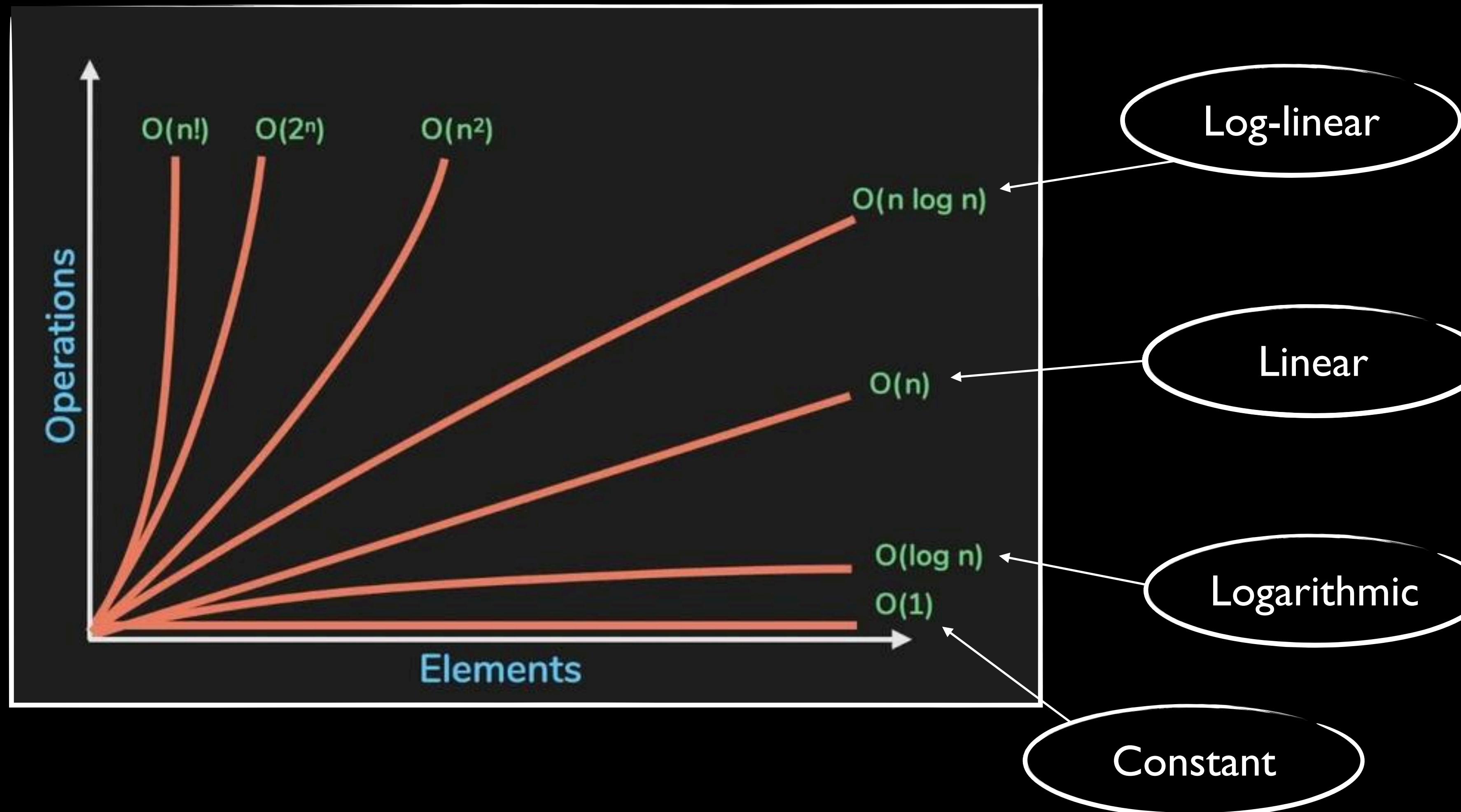
$$f(N) = O(g(N)) \iff f(N) \leq c \cdot g(N) \ \forall N$$



1. Complexity



1. Complexity

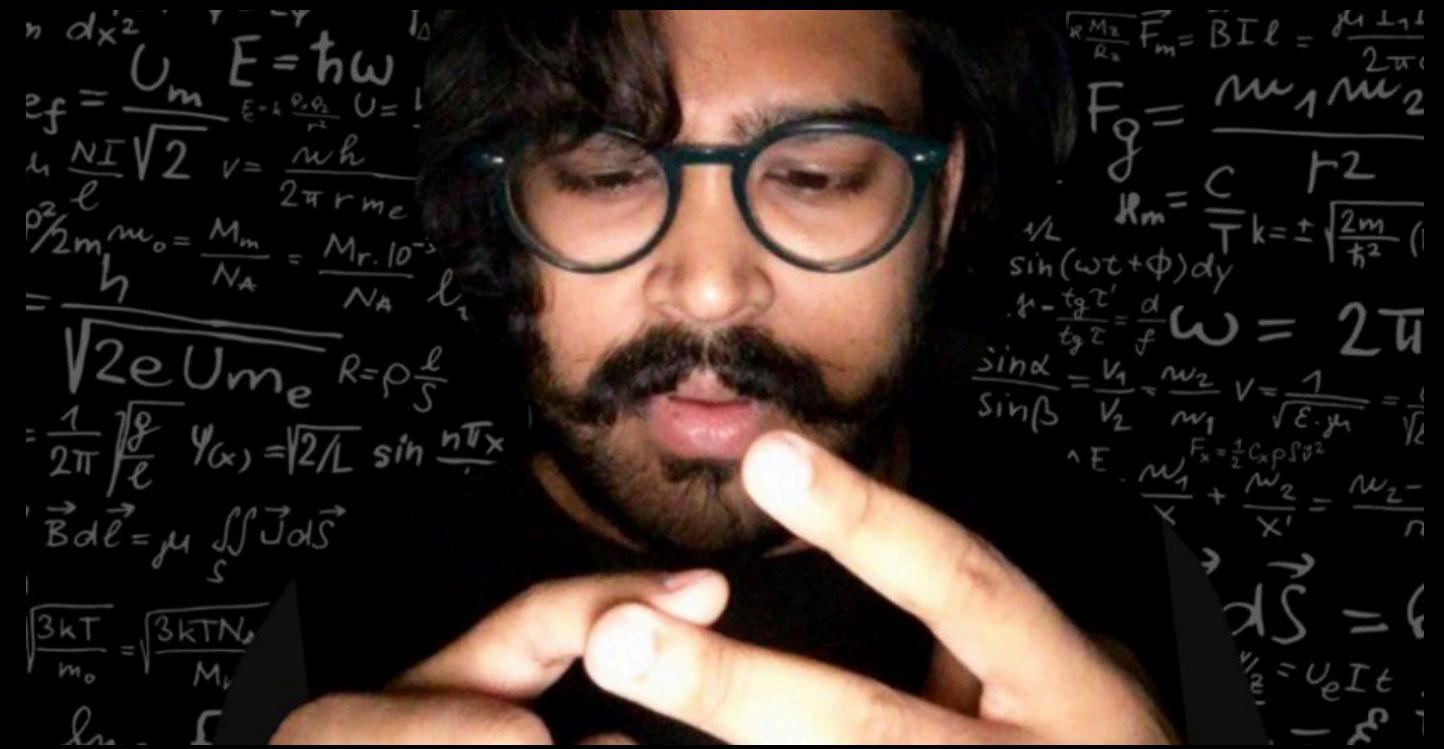


1. Complexity

Practical tasks

Complexity of discovering:

1. if value t is in array A of length N
2. if value t is in array A or array B , each of size N
3. if array A of length N contains duplicates
4. *if $f(n) = a_k n^k + \dots + a_1 n + a_0$, then $f(n) = O(n^k)$
5. two matrix of size $N \times N$ multiplication
6. all subsets of array A of size N
7. break password of size N , consisting only of digits
8. empty recursion of depth N , called twice each time

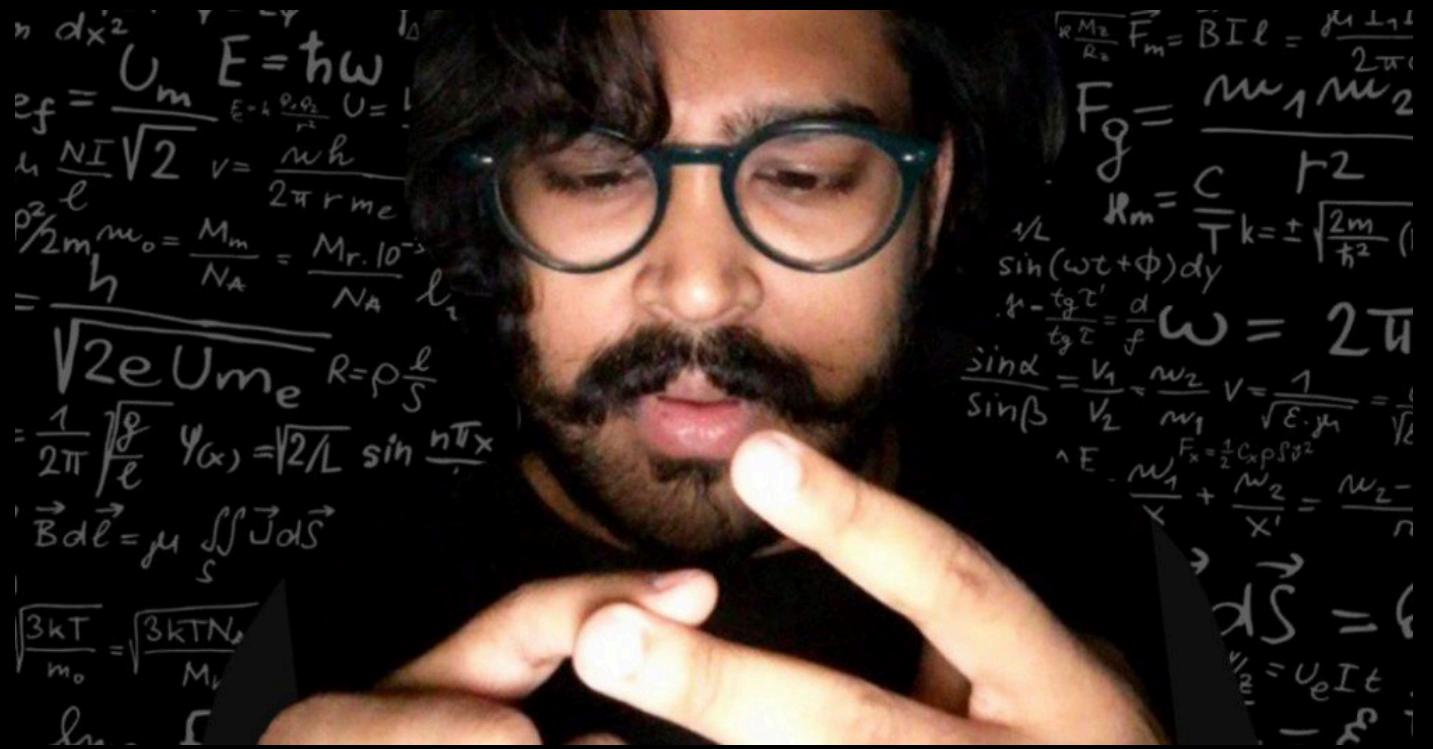


1. Complexity

Practical tasks

Complexity of discovering:

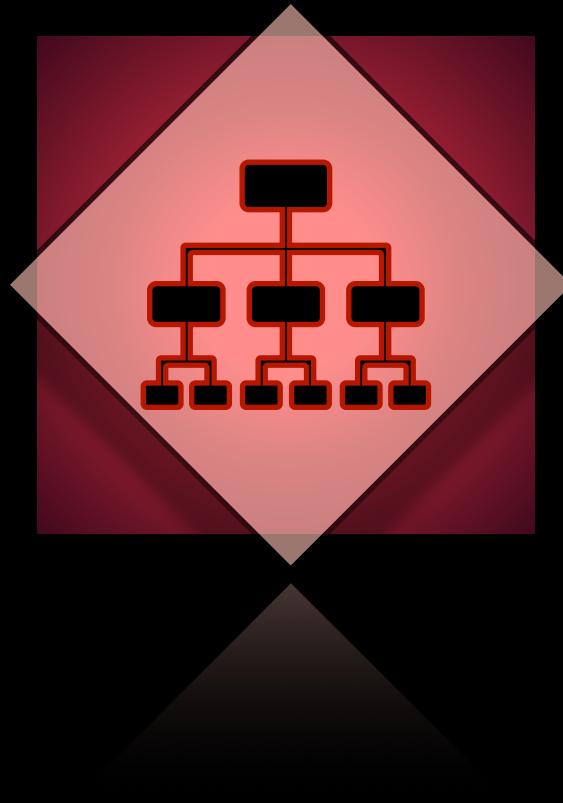
1. if value t is in array A of length N **$O(n)$**
2. if value t is in array A or array B , each of size N **$O(n)$**
3. if array A of length N contains duplicates **$O(n^2)$**
4. *if $f(n) = a_k n^k + \dots + a_1 n + a_0$, then $f(n) = O(n^k)$
5. two matrix of size $N \times N$ multiplication **$O(n^3)$**
6. all subsets of array A of size N **$O(2^n)$**
7. break password of size N , consisting only of digits **$O(10^n)$**
8. empty recursion of depth N , called twice each time **$O(2^n)$**



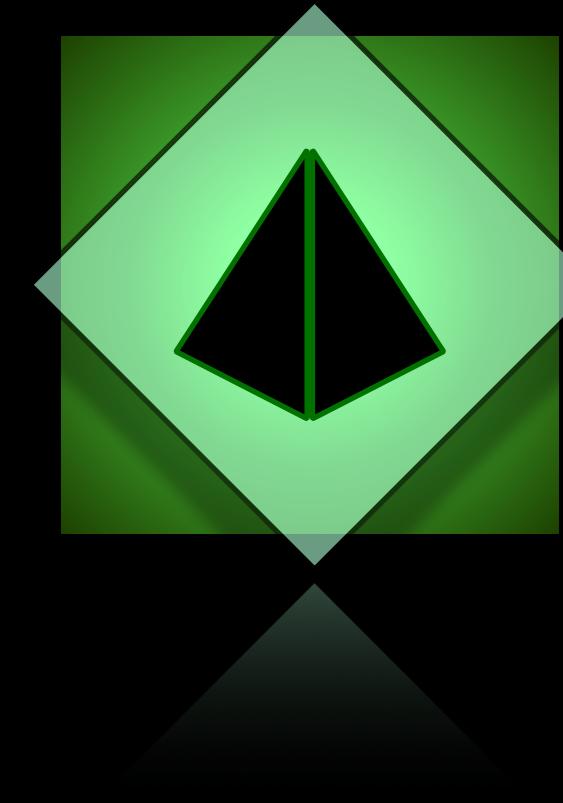
2. Sorting algorithms



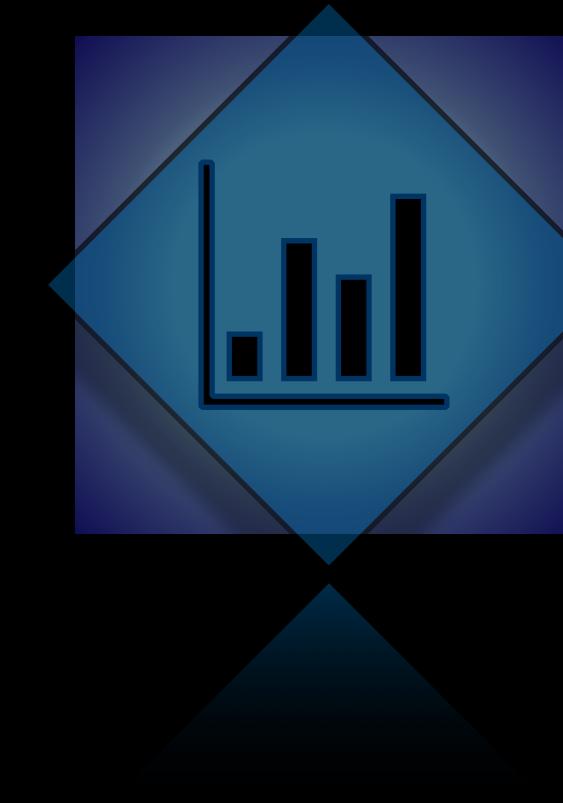
Insertion sort



Merge sort



Heap sort



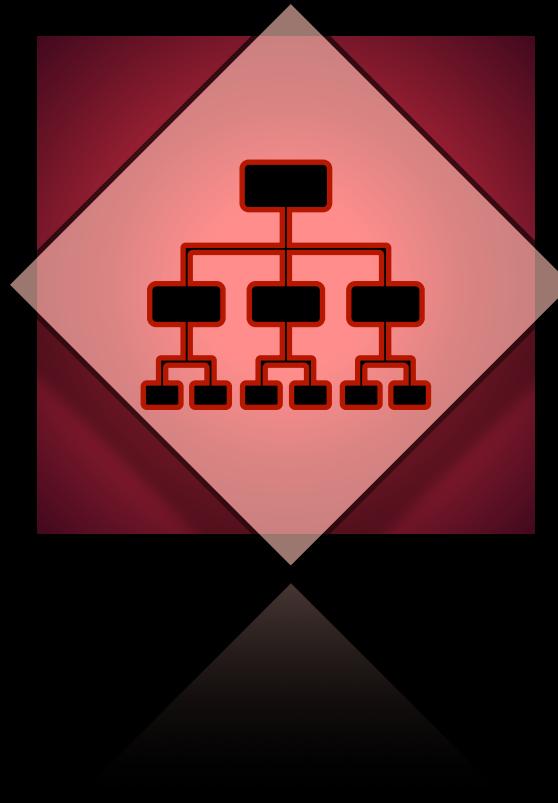
Count sort

2. Sorting algorithms

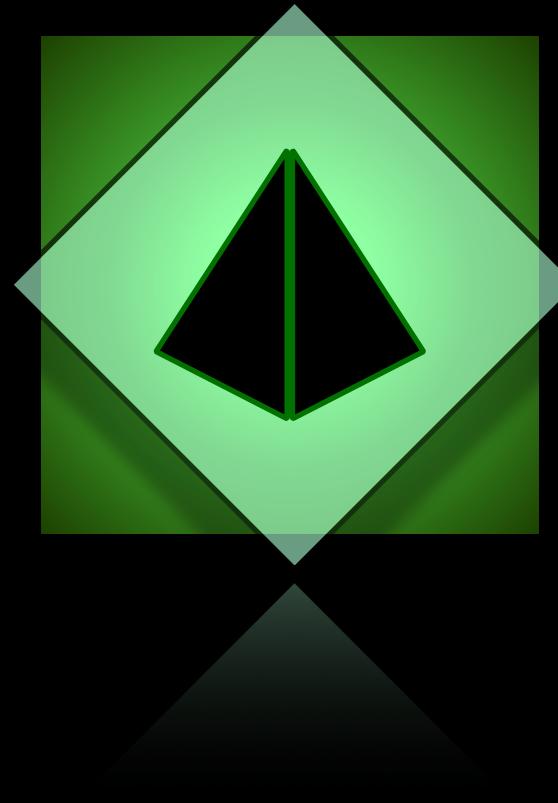


Insertion sort

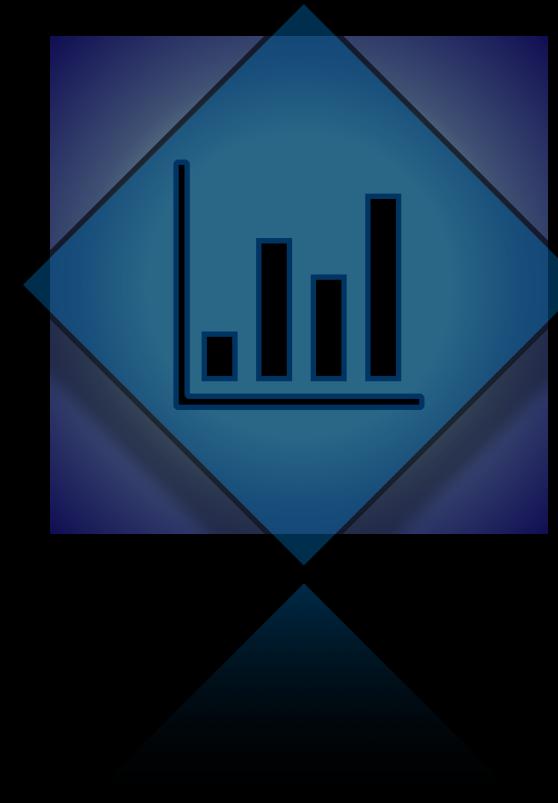
- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region



Merge sort



Heap sort



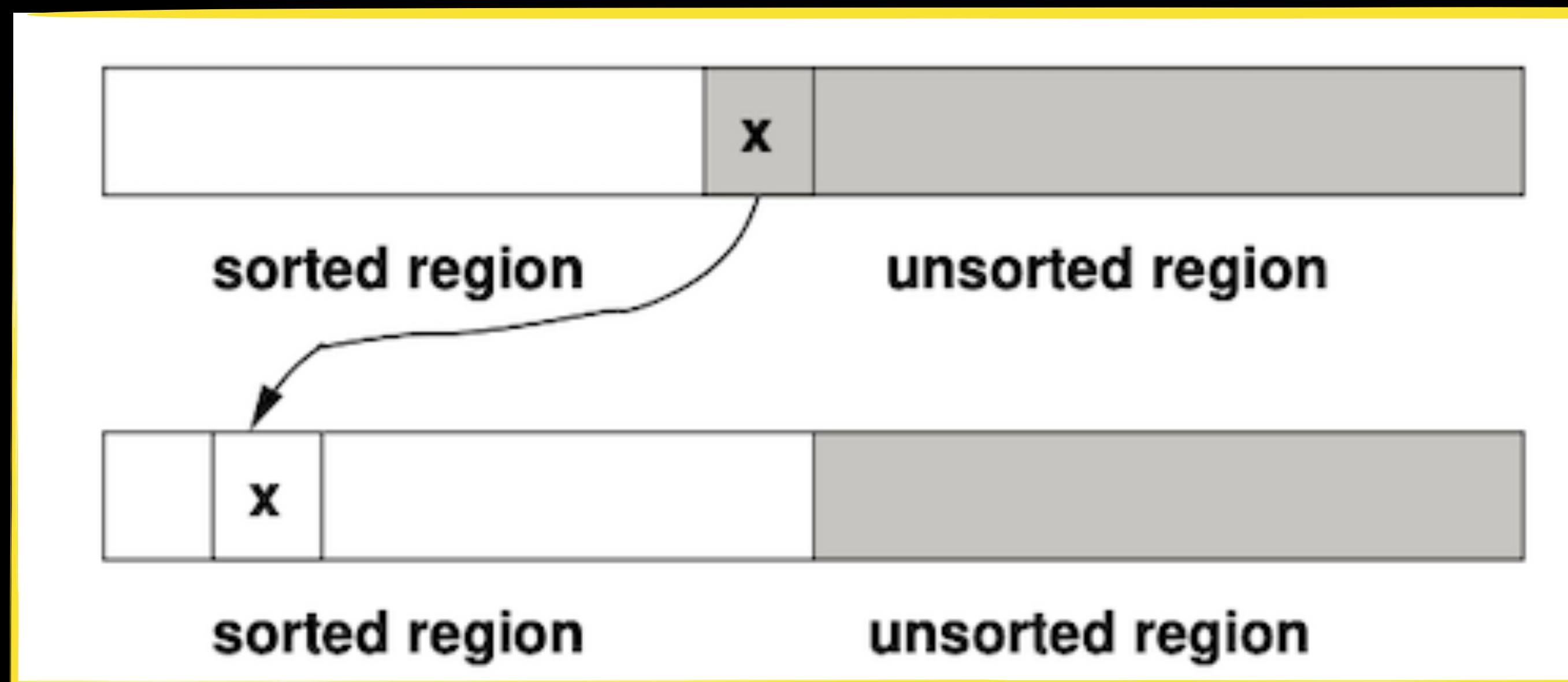
Count sort

2. Sorting algorithms



Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region

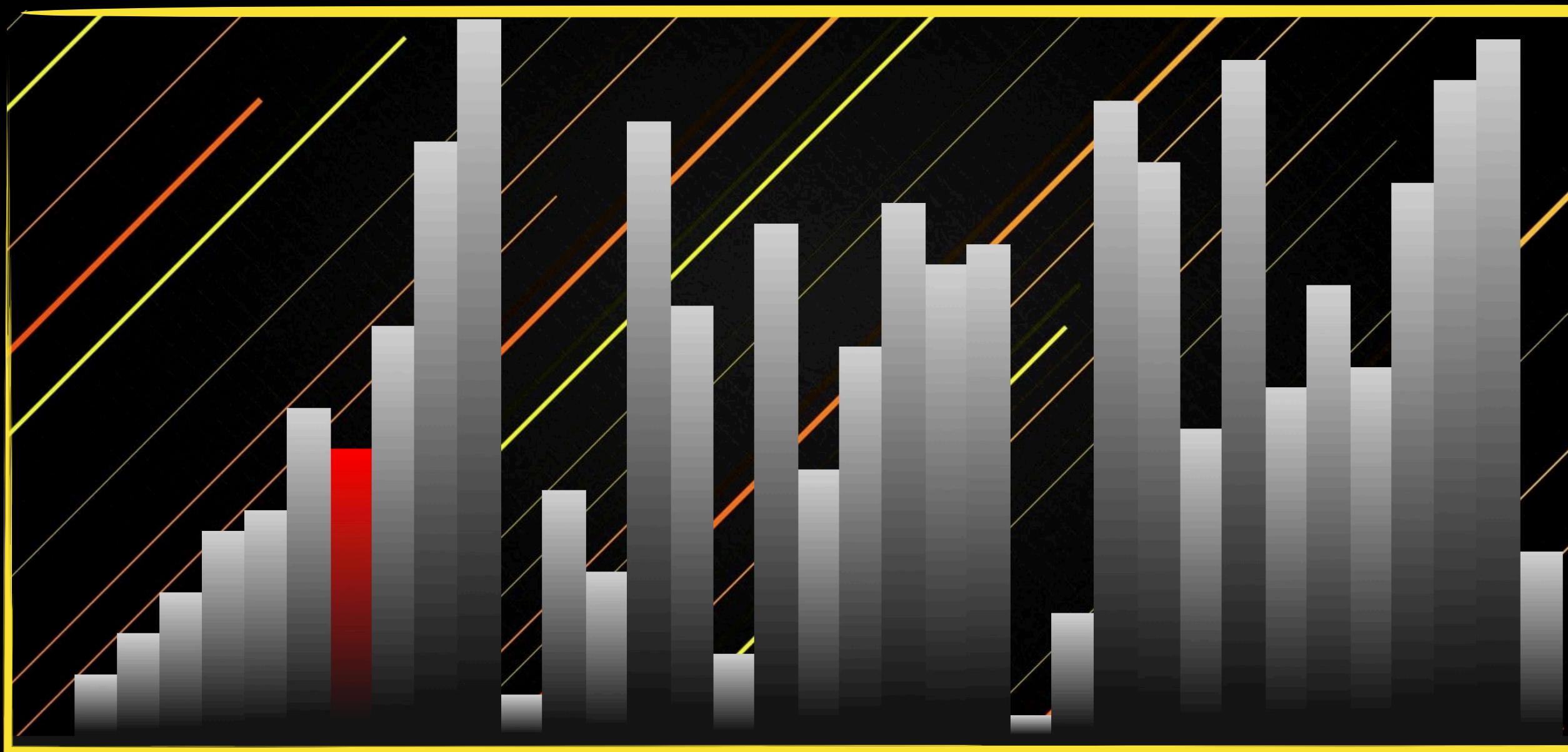


2. Sorting algorithms



Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region



2. Sorting algorithms



Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region

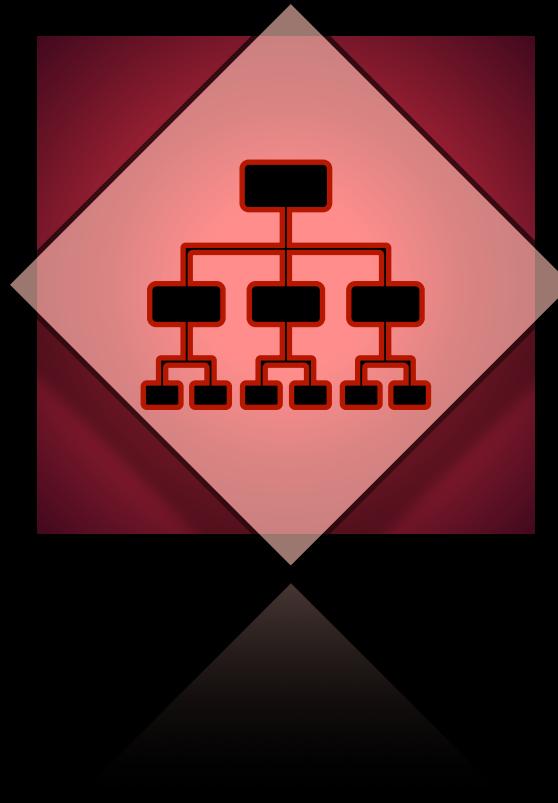


2. Sorting algorithms



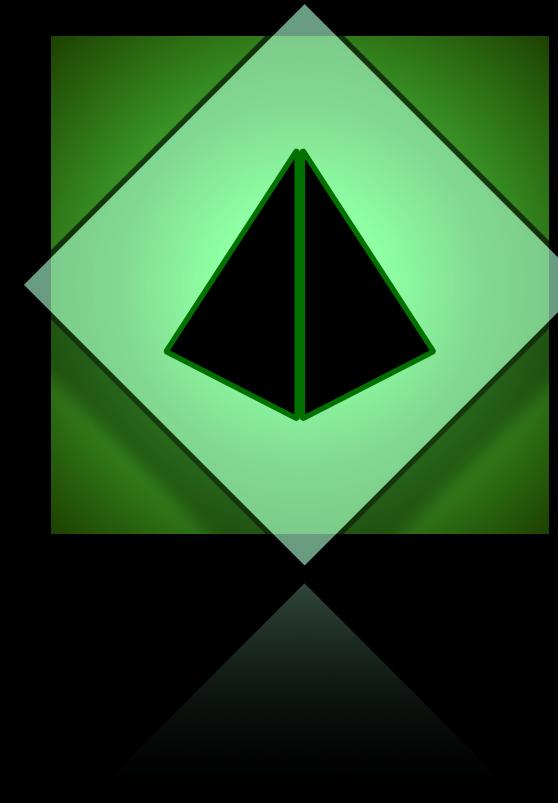
Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region

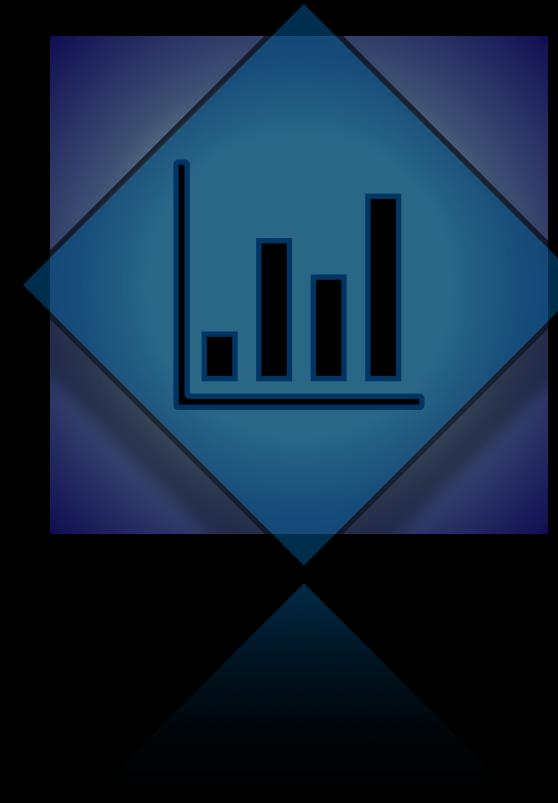


Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively

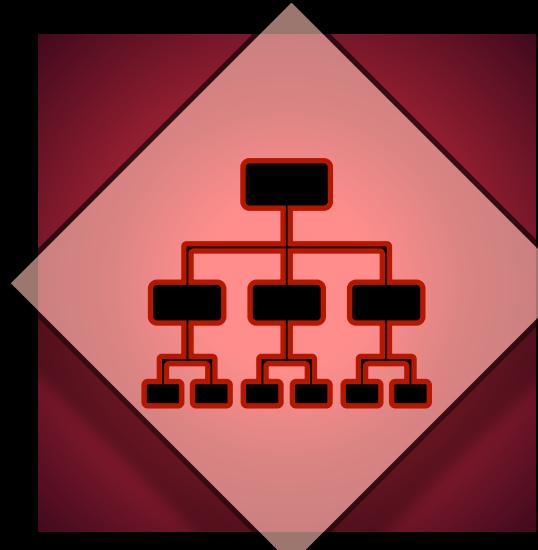


Heap sort



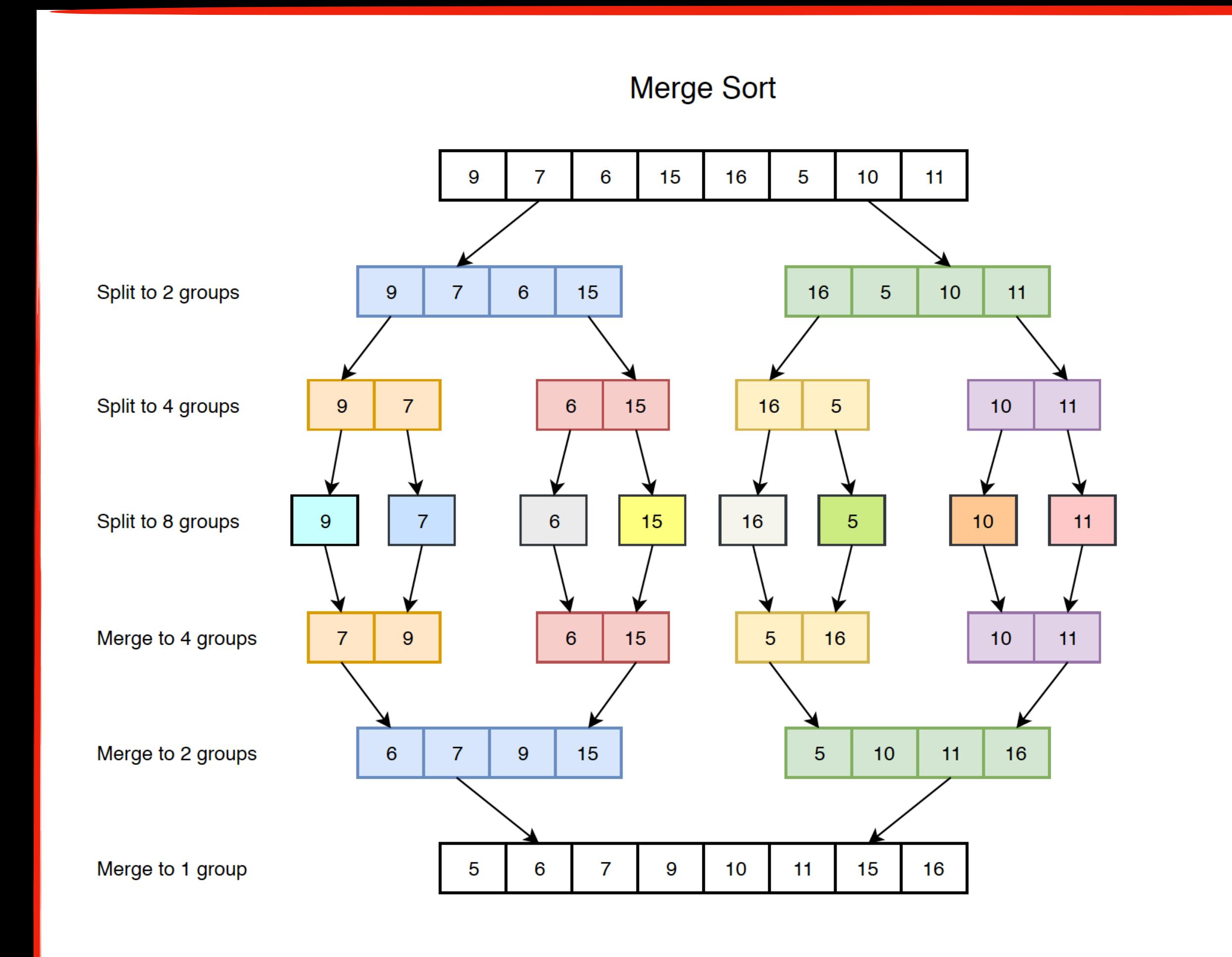
Count sort

2. Sorting algorithms

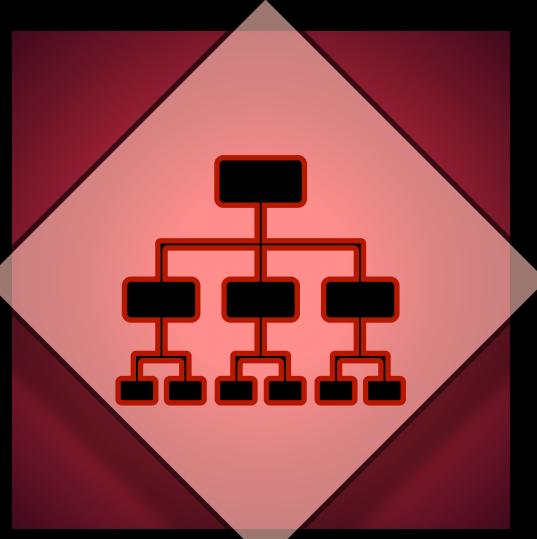


Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively

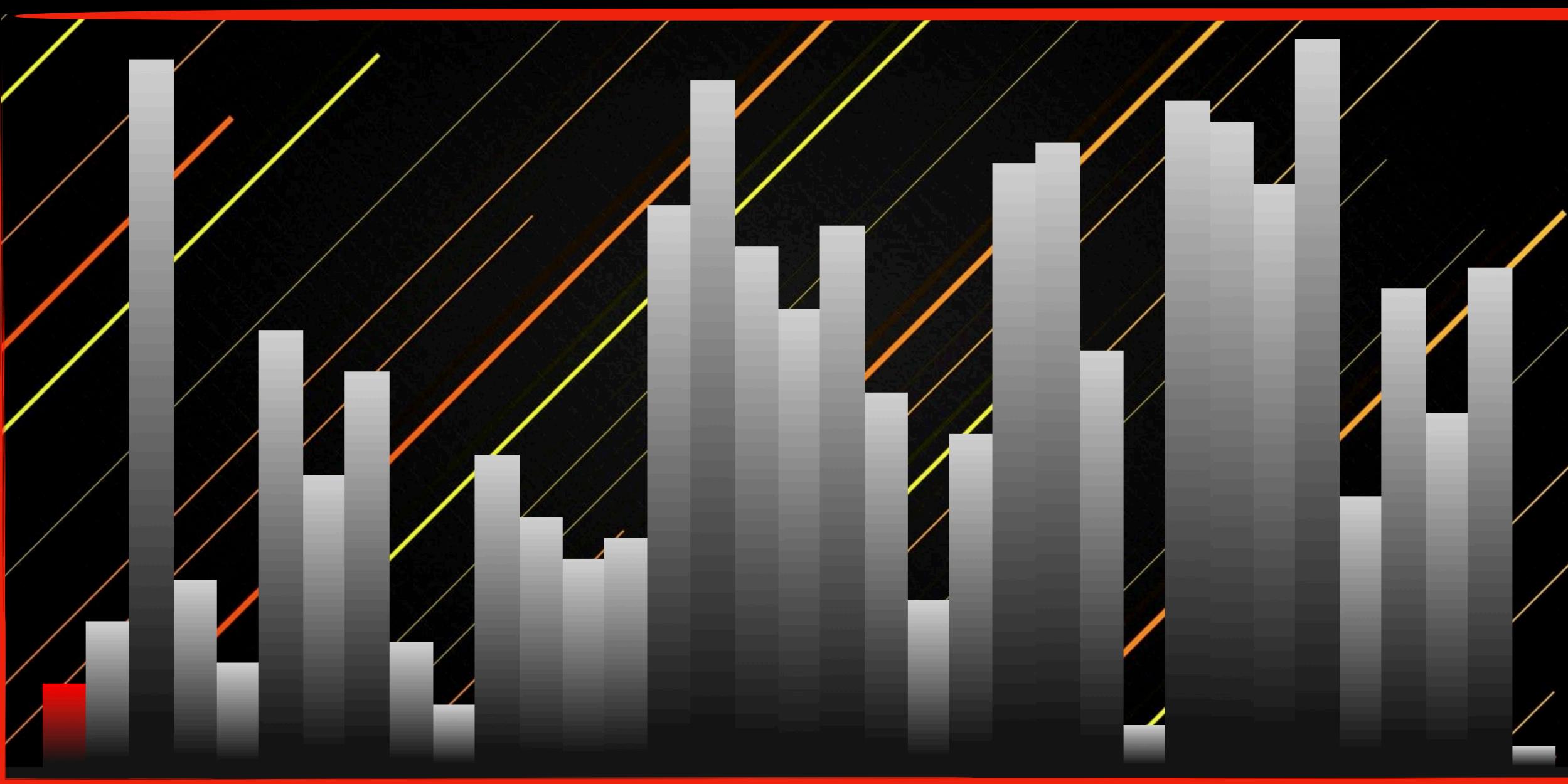


2. Sorting algorithms

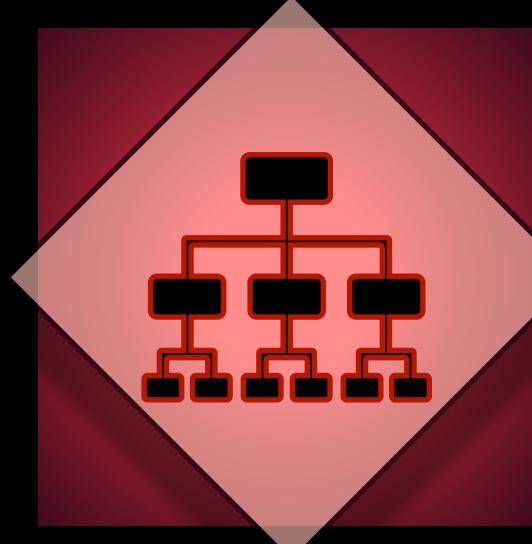


Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively

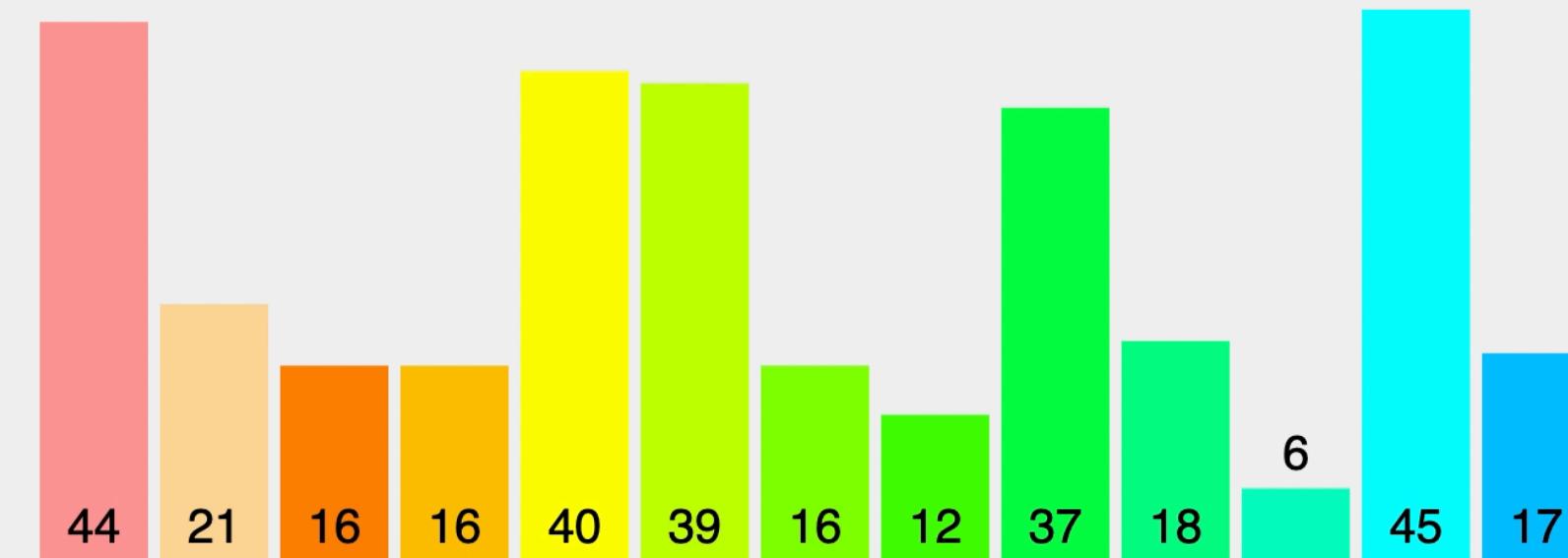


2. Sorting algorithms



Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively



Merge Sort

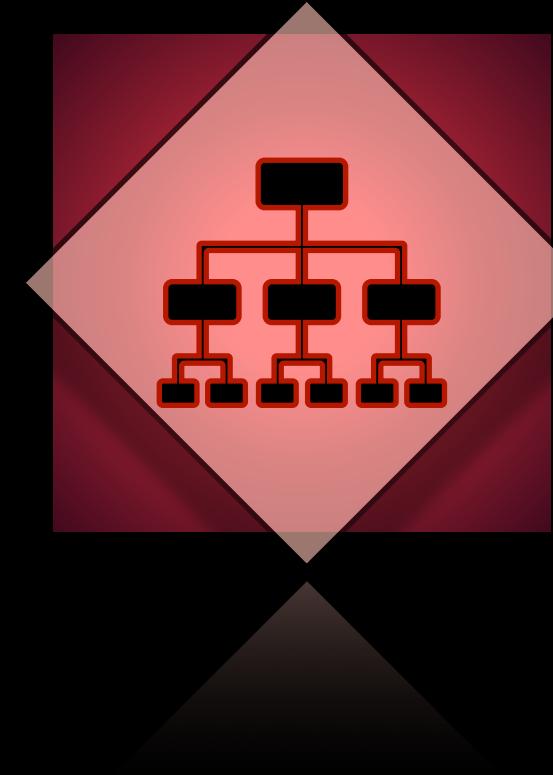
```
Merge partitions [44] (index 0 to 0) and [21] (index 1 to 1).
split each element into partitions of size 1
recursively merge adjacent partitions
for i = leftPartIdx to rightPartIdx
    if leftPartHeadValue <= rightPartHeadValue
        copy leftPartHeadValue
    else: copy rightPartHeadValue; Increase InvIdx
copy elements back to original array
```

2. Sorting algorithms



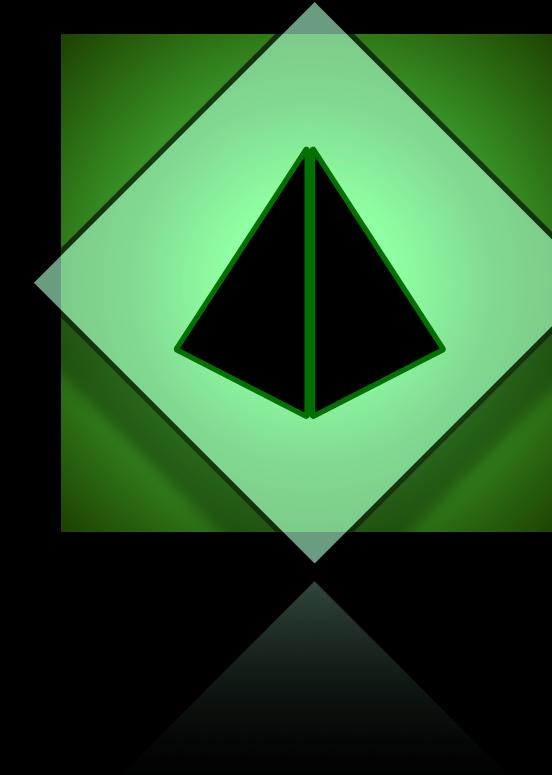
Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region



Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively



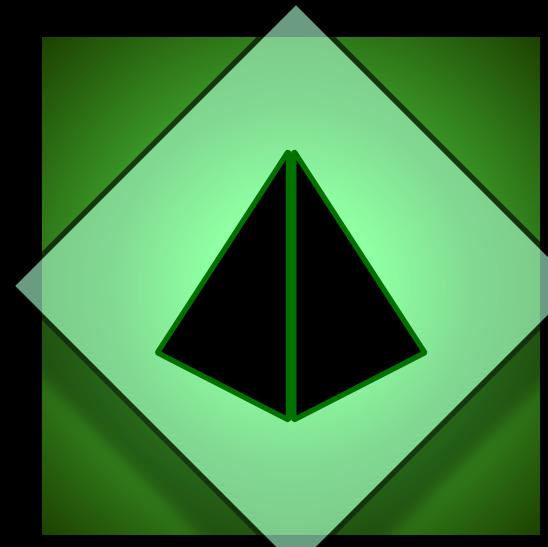
Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root



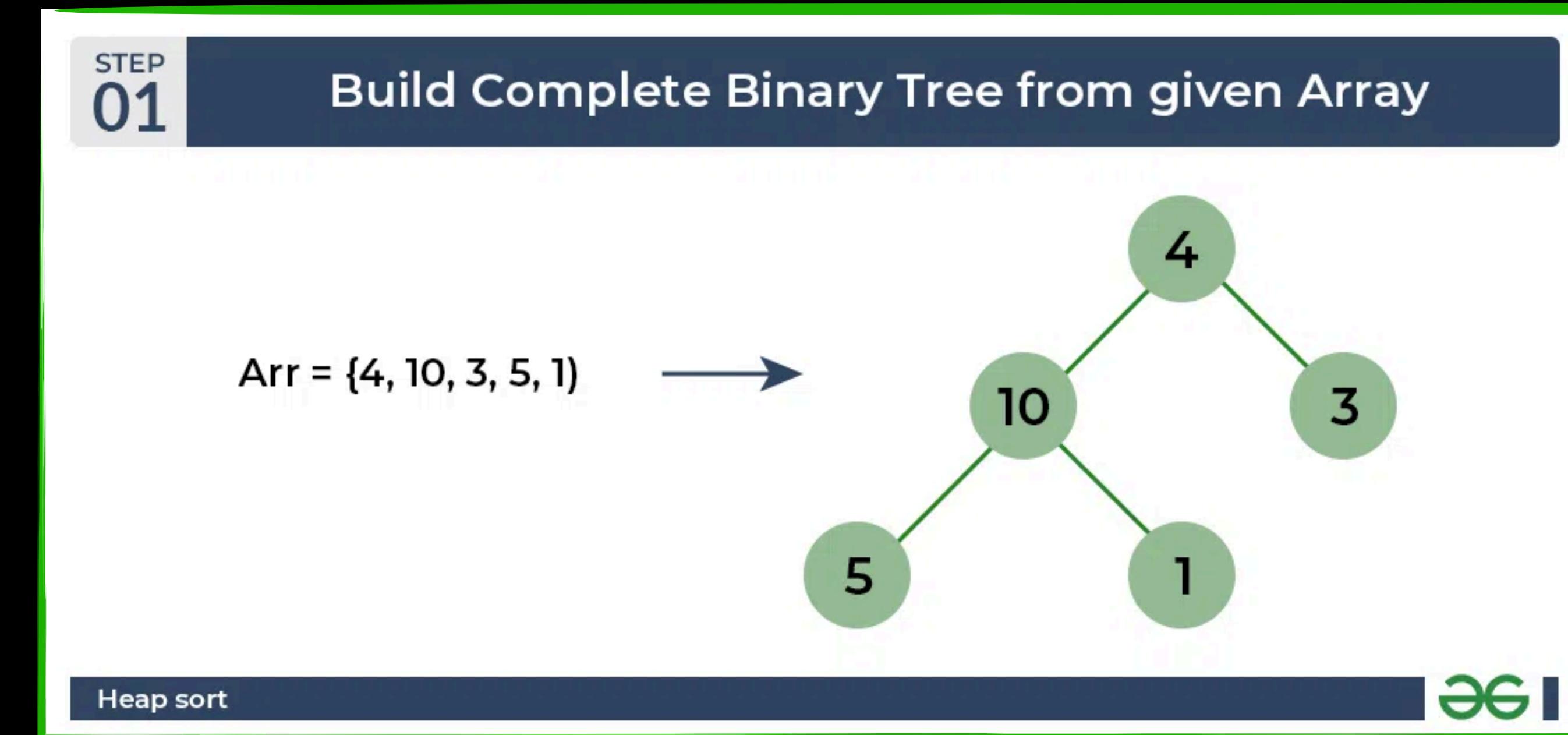
Count sort

2. Sorting algorithms



Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root

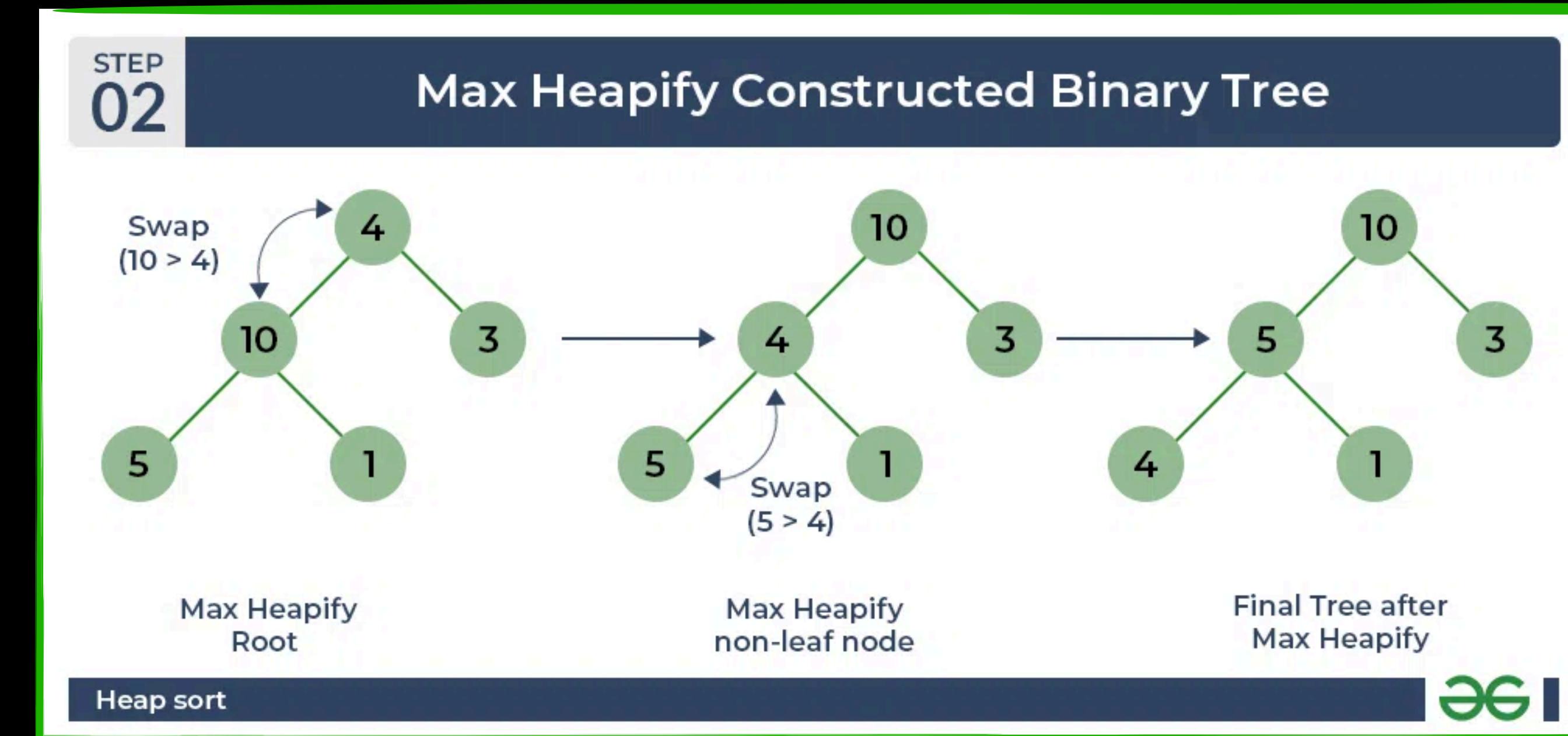


2. Sorting algorithms



Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root

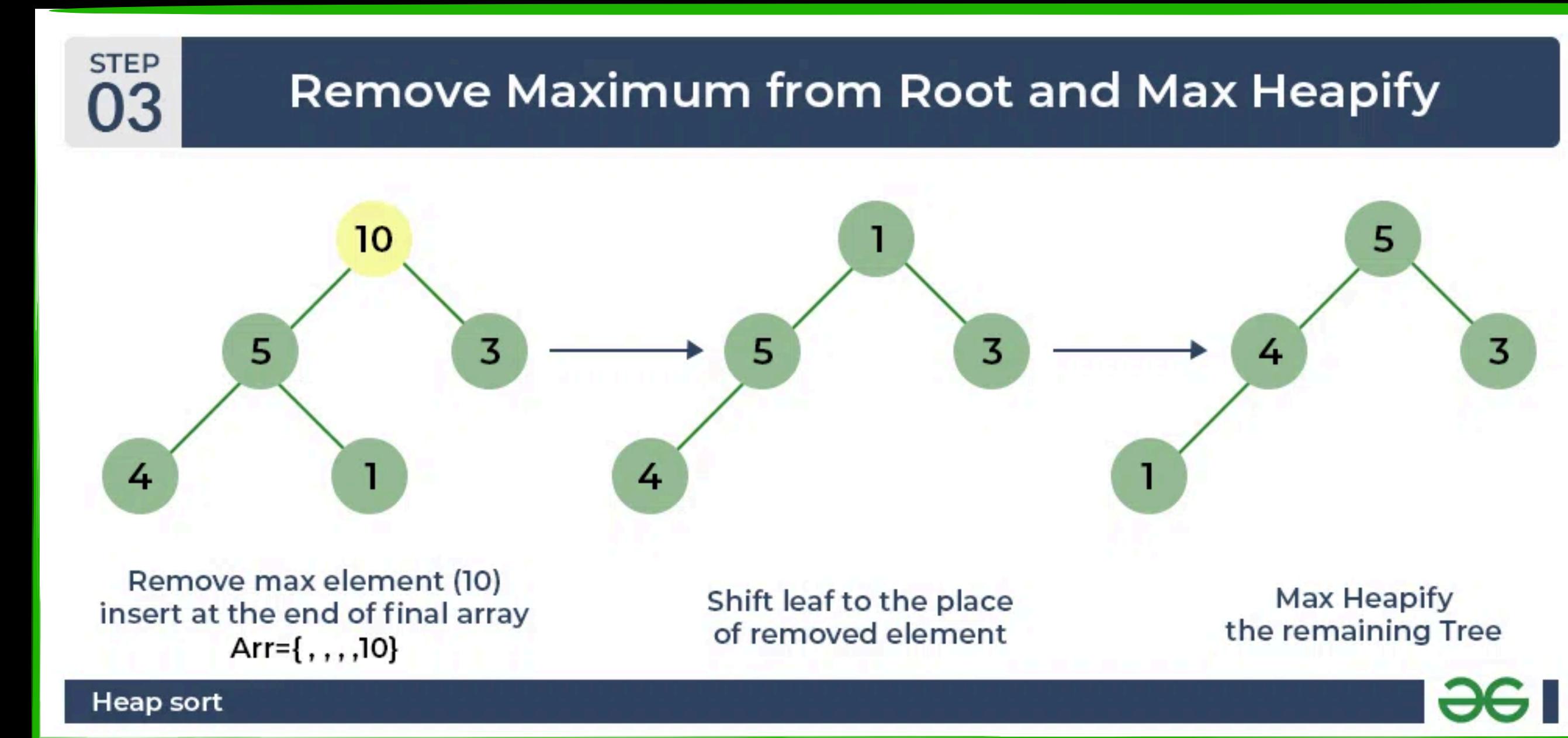


2. Sorting algorithms



Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root

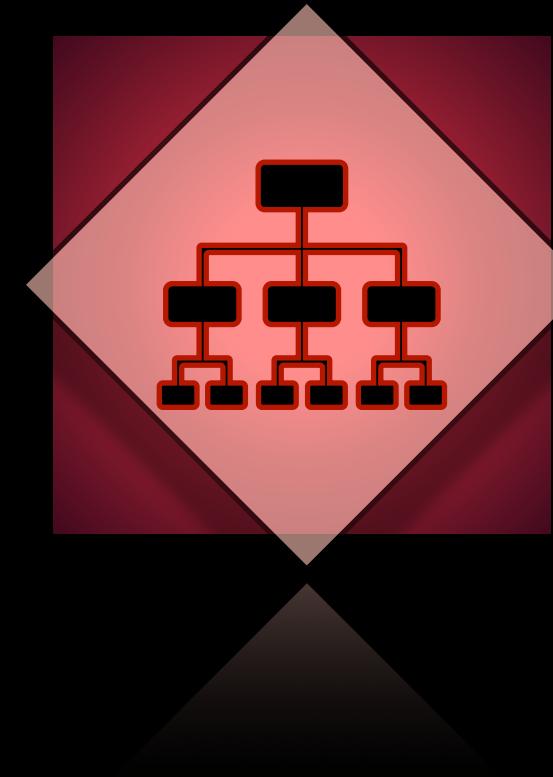


2. Sorting algorithms



Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region



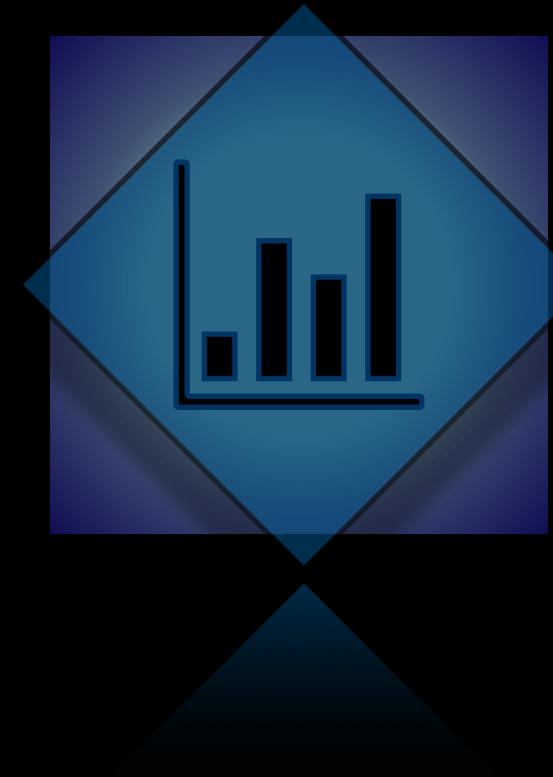
Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively



Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root



Count sort

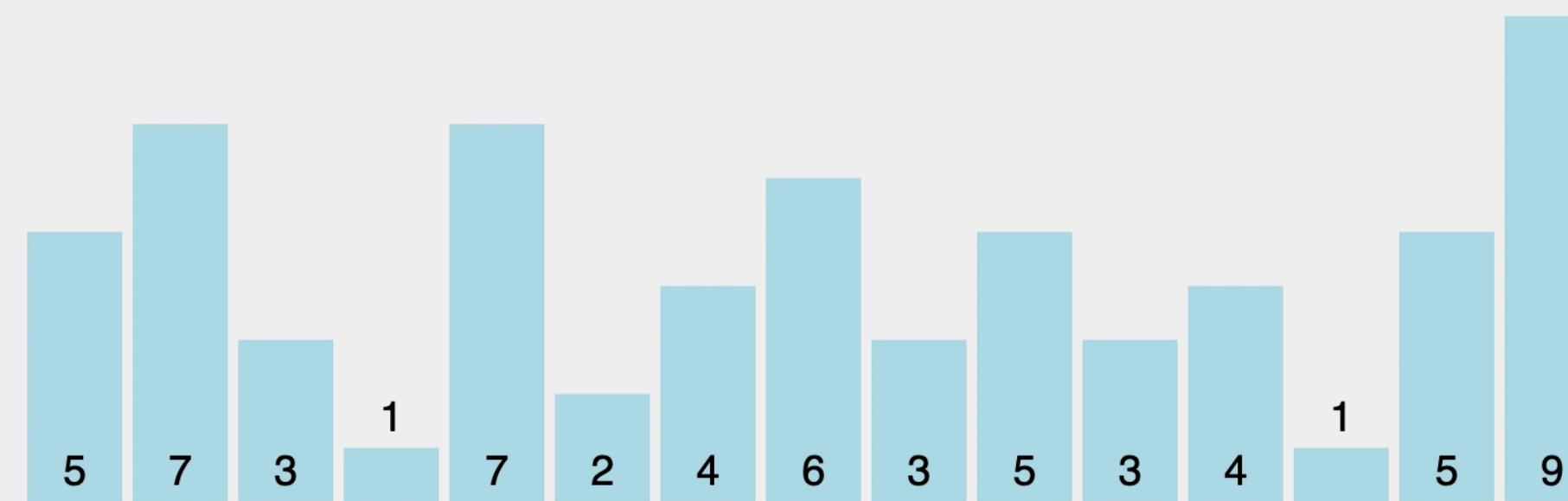
- make bins in range from 0 to K
- group all the elements into bins
- find each bin's index

2. Sorting algorithms



Count sort

- make bins in range from 0 to K
- group all the elements into bins
- find each bin's index



Counting Sort

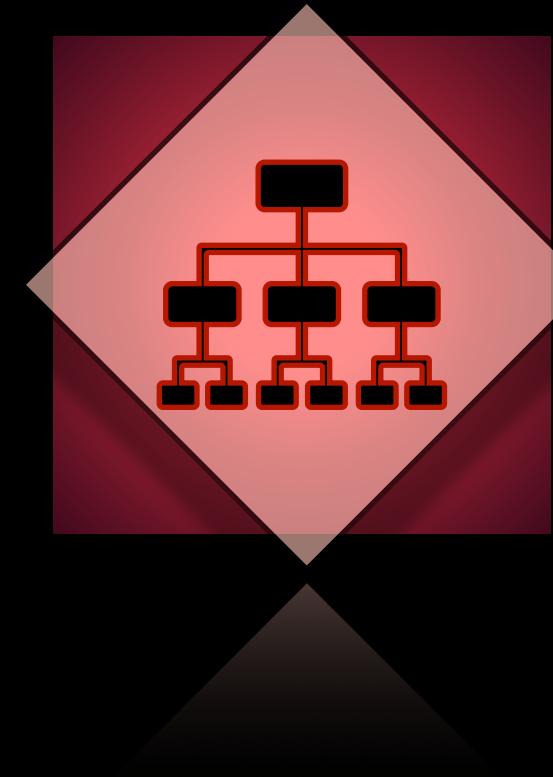
```
create key (counting) array
for each element in list
    increase the respective counter by 1
for each counter, starting from smallest key
    while counter is non-zero
        restore element to list
        decrease counter by 1
```

2. Sorting algorithms



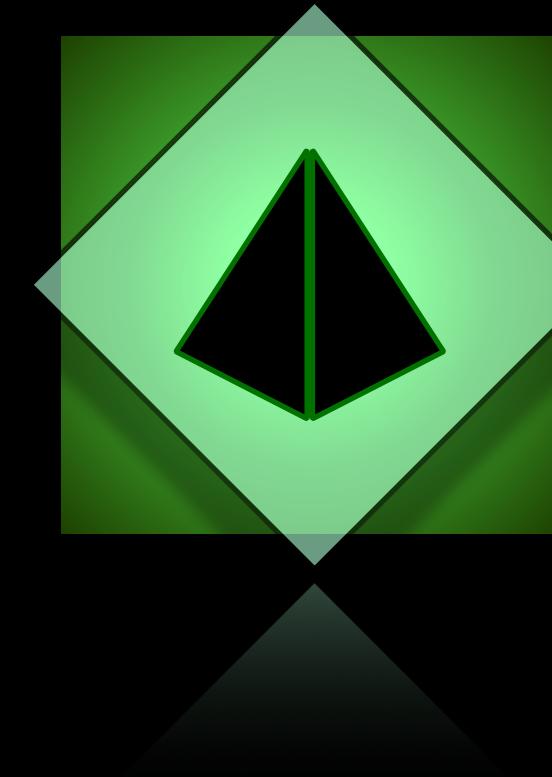
Insertion sort

- divide array into sorted & unsorted regions
- take next lowest-rank object
- find its proper position in sorted region



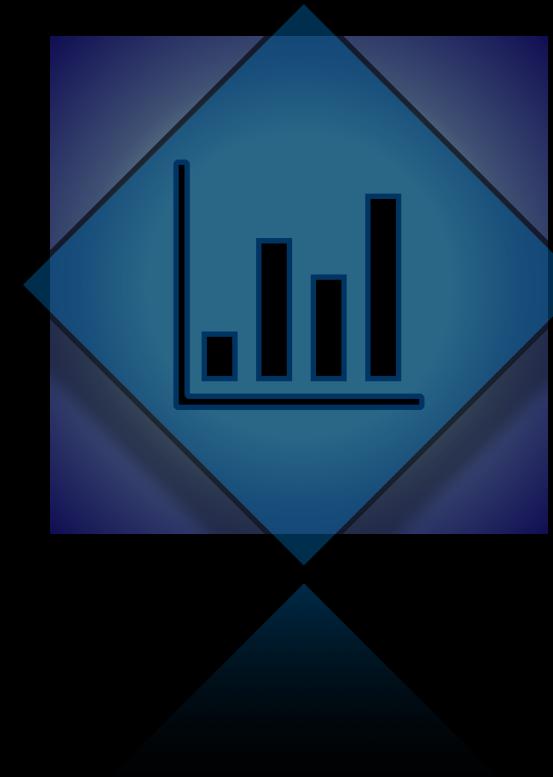
Merge sort

- divide-and-conquer technique
- separate each object into particular bin
- join & sort bins iteratively



Heap sort

- build binary tree from the array
- make each node greater than child
- remove maximum from root



Count sort

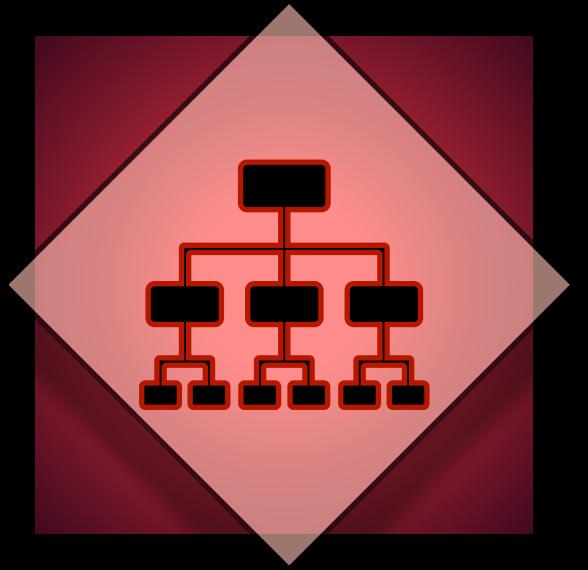
- make bins in range from 0 to K
- group all the elements into bins
- find each bin's index

2. Sorting algorithms

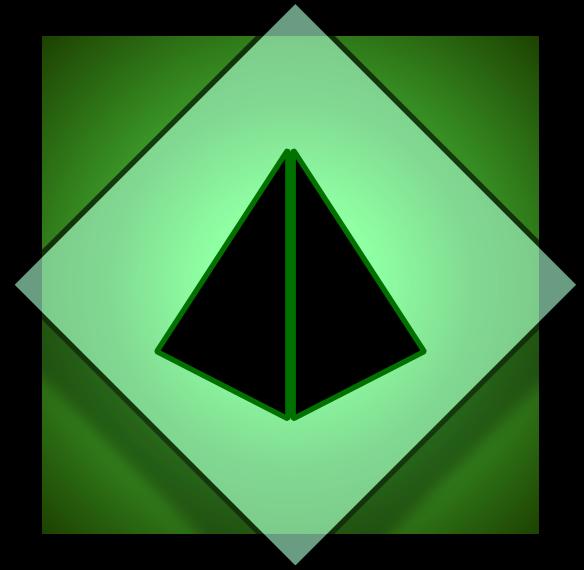
Properties



Insertion sort



Merge sort



Heap sort



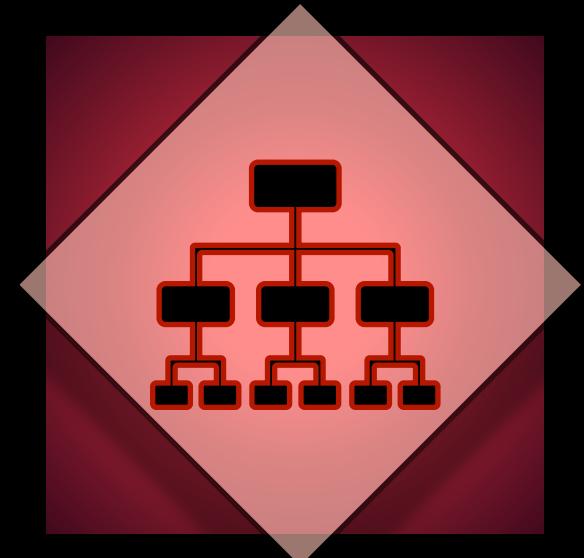
Count sort

2. Sorting algorithms

Properties



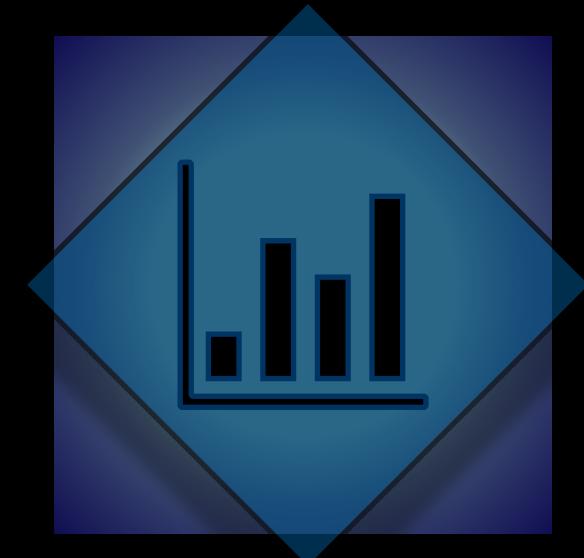
Insertion sort



Merge sort



Heap sort



Count sort

Inplace

Stability

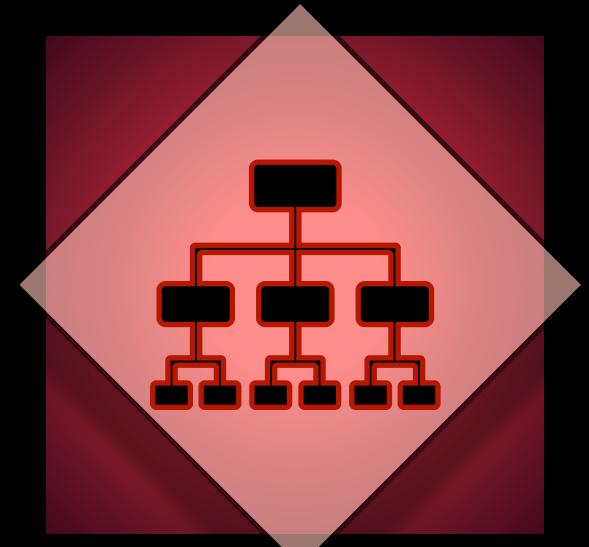
Comparison-based

2. Sorting algorithms

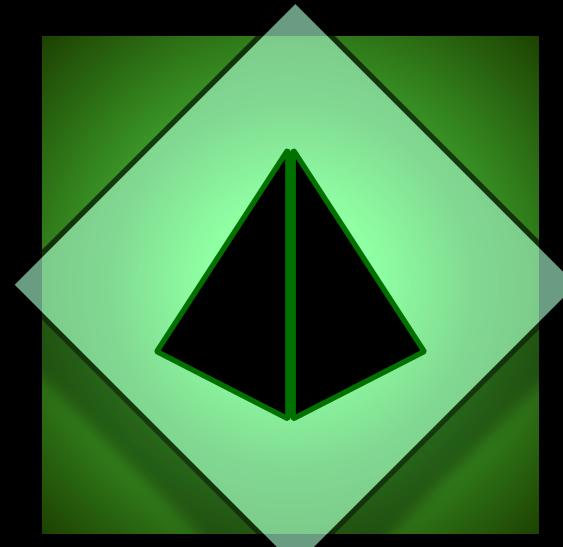
Properties



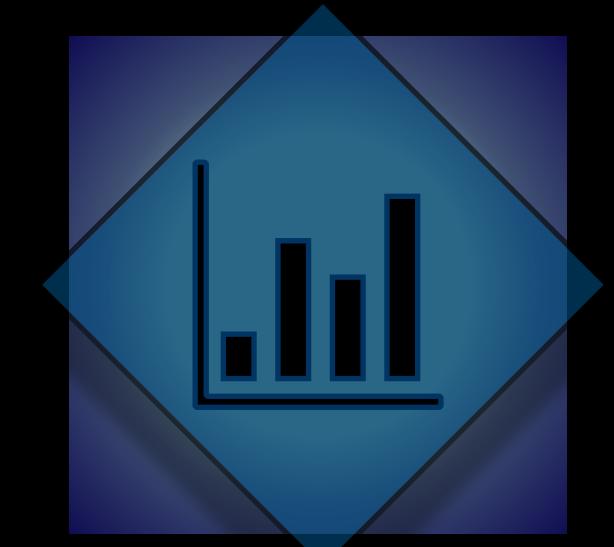
Insertion sort



Merge sort



Heap sort

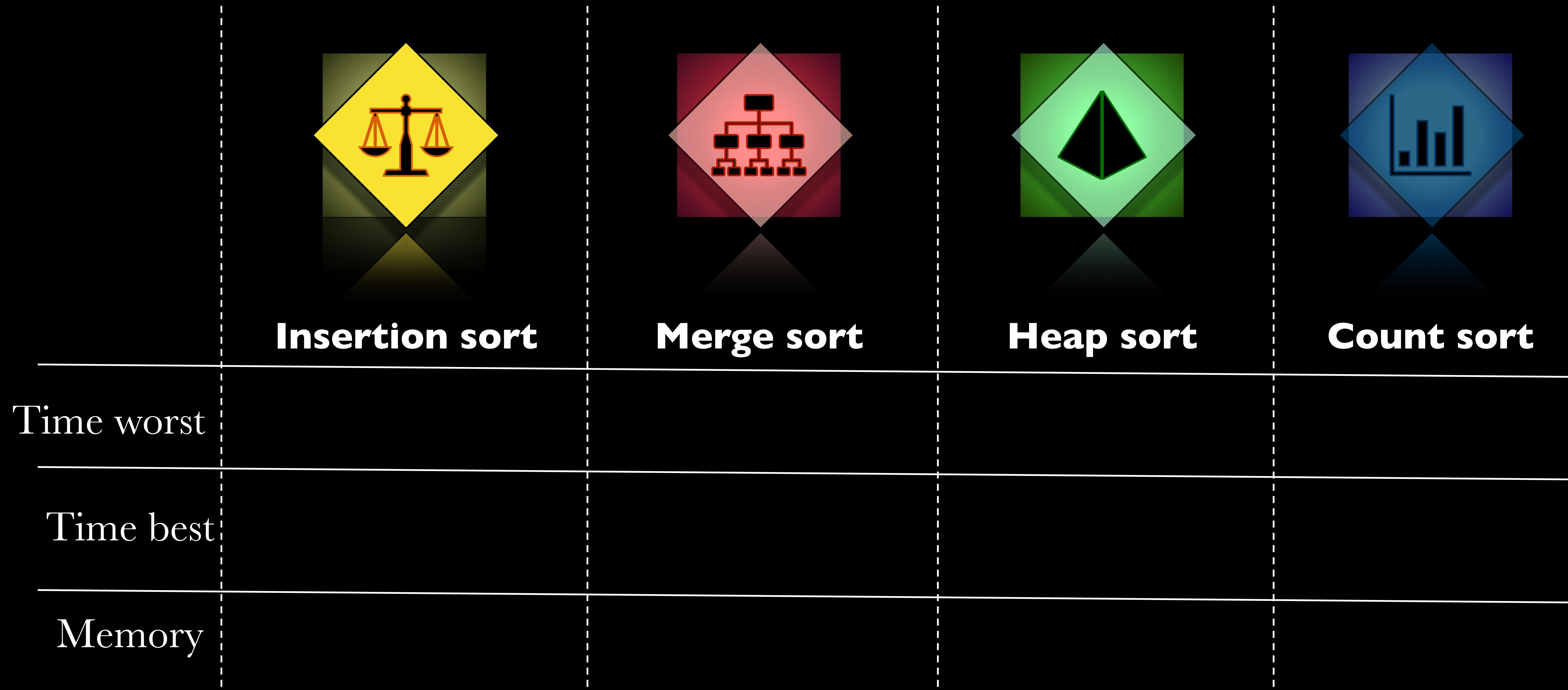


Count sort

	Insertion sort	Merge sort	Heap sort	Count sort
Inplace	YES	NO	YES	NO
Stability	YES	YES	NO	YES
Comparison-based	YES	YES	YES	NO

2. Sorting algorithms

Complexity

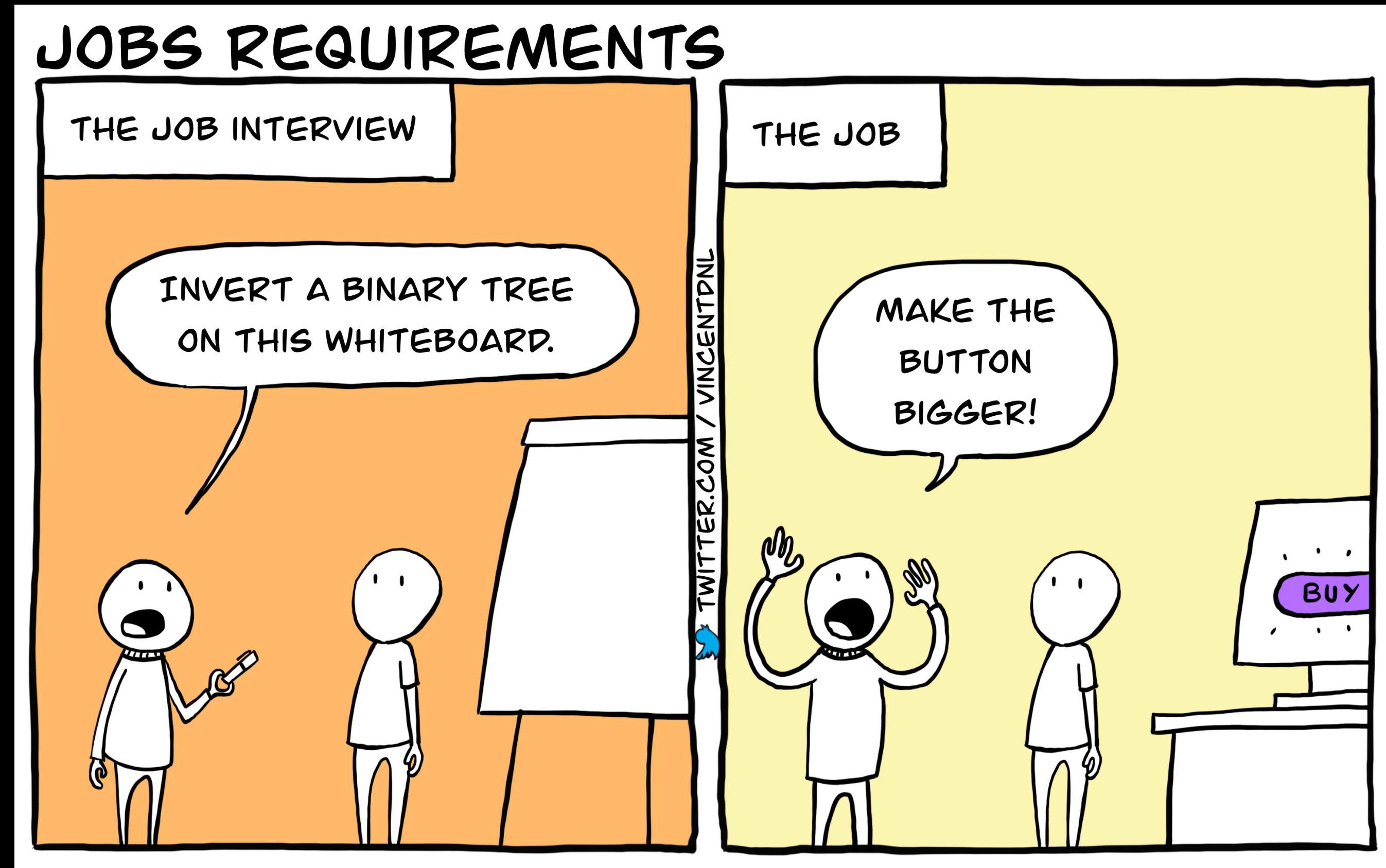


2. Sorting algorithms

Complexity

	Insertion sort	Merge sort	Heap sort	Count sort
Time worst	$O(n^2)$	$O(n \log n)$	$O(n \log n)$	$O(n+k)$
Time best	$\Omega(n)$	$\Omega(n \log n)$	$\Omega(n \log n)$	$\Omega(n+k)$
Memory	$O(1)$	$O(n)$	$O(1)$	$O(k)$

3. Coding



3. Coding

Insertion sort

```
for i = 1 to n-1  
    j = i  
    while j > 0 AND A[j] < A[j-1]  
        swap(A[j],A[j+1])  
        j = j-1
```

3. Coding

Heap sort

```
Heapify (A, i)
    lft <- Left(i)
    rt <- Right(i)
    if lft <= heap-size[A] and A[lft] > A[i]
        then largest <- lft
        else largest <- i
    if rt <= heap-size[A] and A[rt] > A[largest]
        then largest <- rt
    if largest != i
        then Swap( A, i, largest );
        Heapify (A, largest);
```