Graph Theory IV

Seminar 15.

Maximum Flow problem

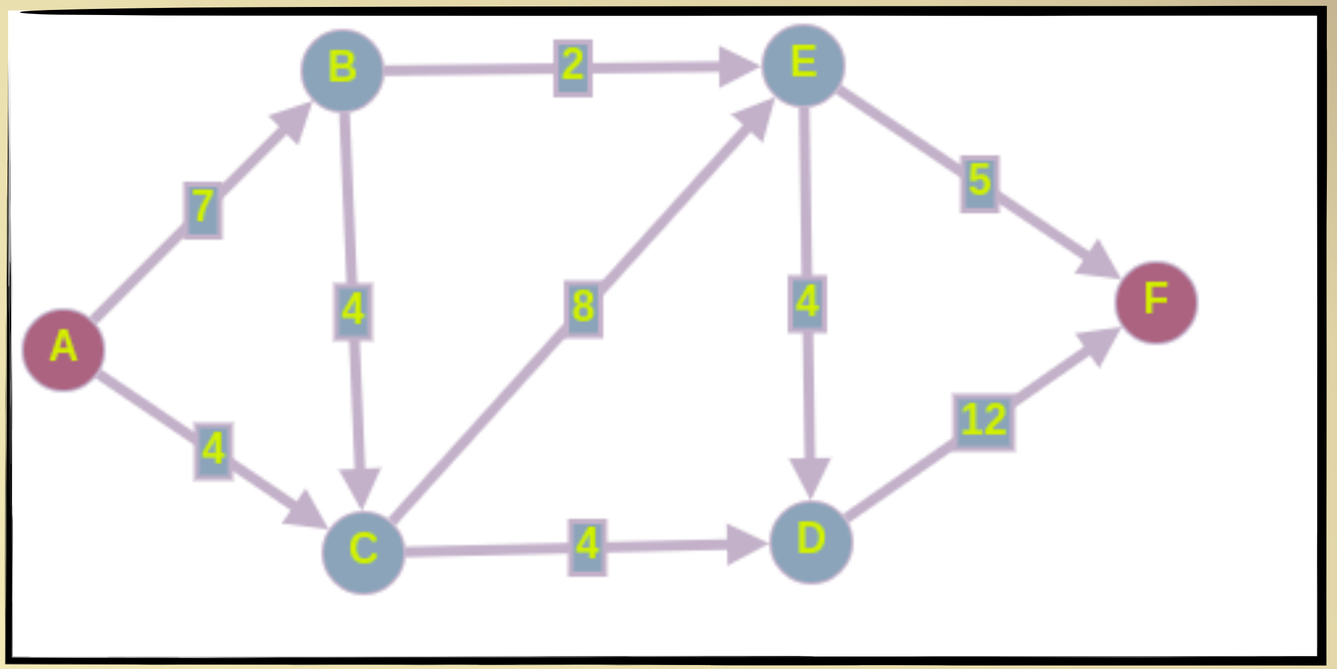
Definitions

- ***Network*** - oriented graph with each edge having its capacity and all edges being called "***inner***" except for the "***source***" with all edges going out and the "***sink***" (vice versa)
- ***Flow*** function follows two rules:
  - Does not go above the capacity of any edge
  - Preserves the value of outer flow in terms of inner flow for each vertex
- ***Cut*** is a set of edges, required for the connectivity of source & sink

Maximum Flow problem

Definitions

- ***Cut*** is a set of edges, required for the connectivity of source & sink, ***s&t*** respectively
- Maximum ***s-t flow*** value equals the minimum capacity of ***s-t cut***



Maximum Flow problem

Ford-Fulkerson algorithm

1. Find ***any*** path from source to sink
2. Send flow of minimum capacity among the found path's edges
3. Take from these edges' capacities the value of sent flow
4. Return to point I until any path exists
5. Resulting flow: sum of all sent flows

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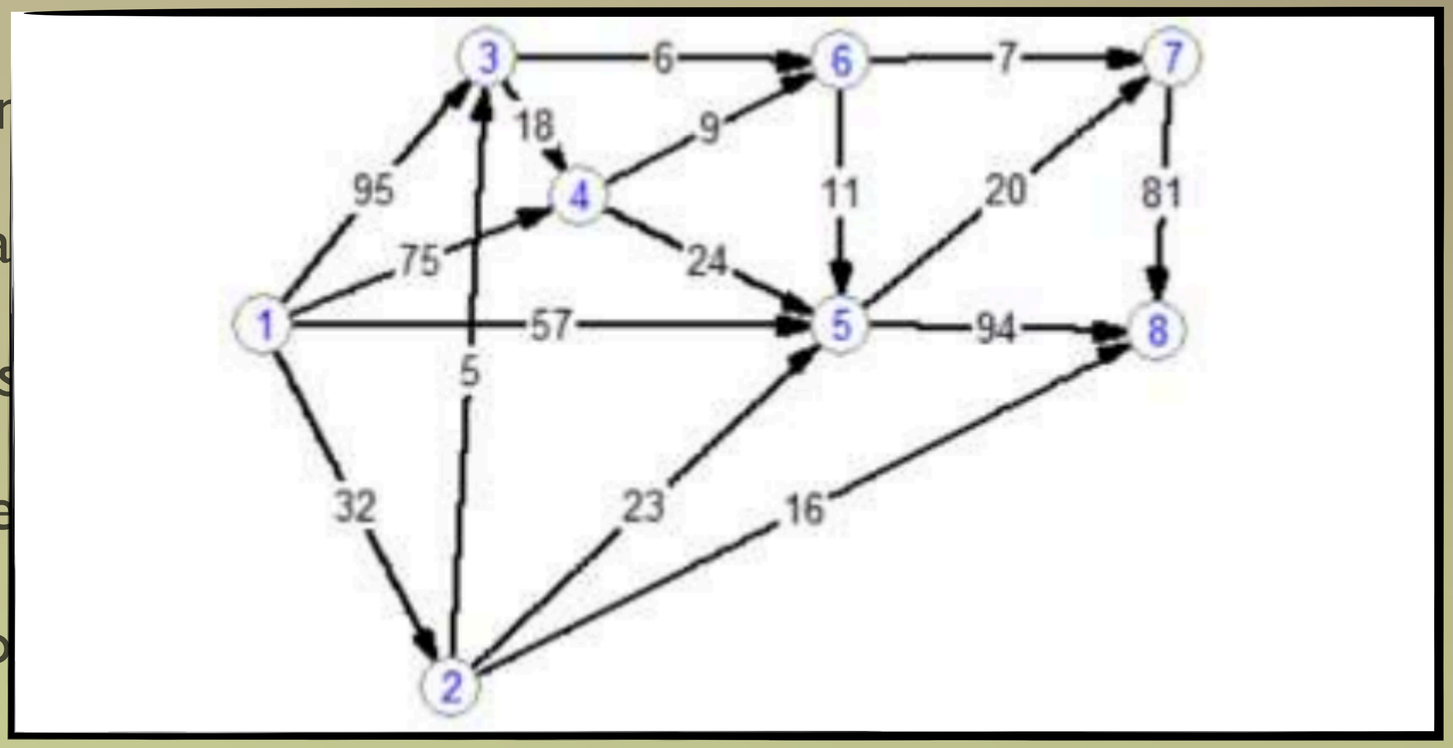
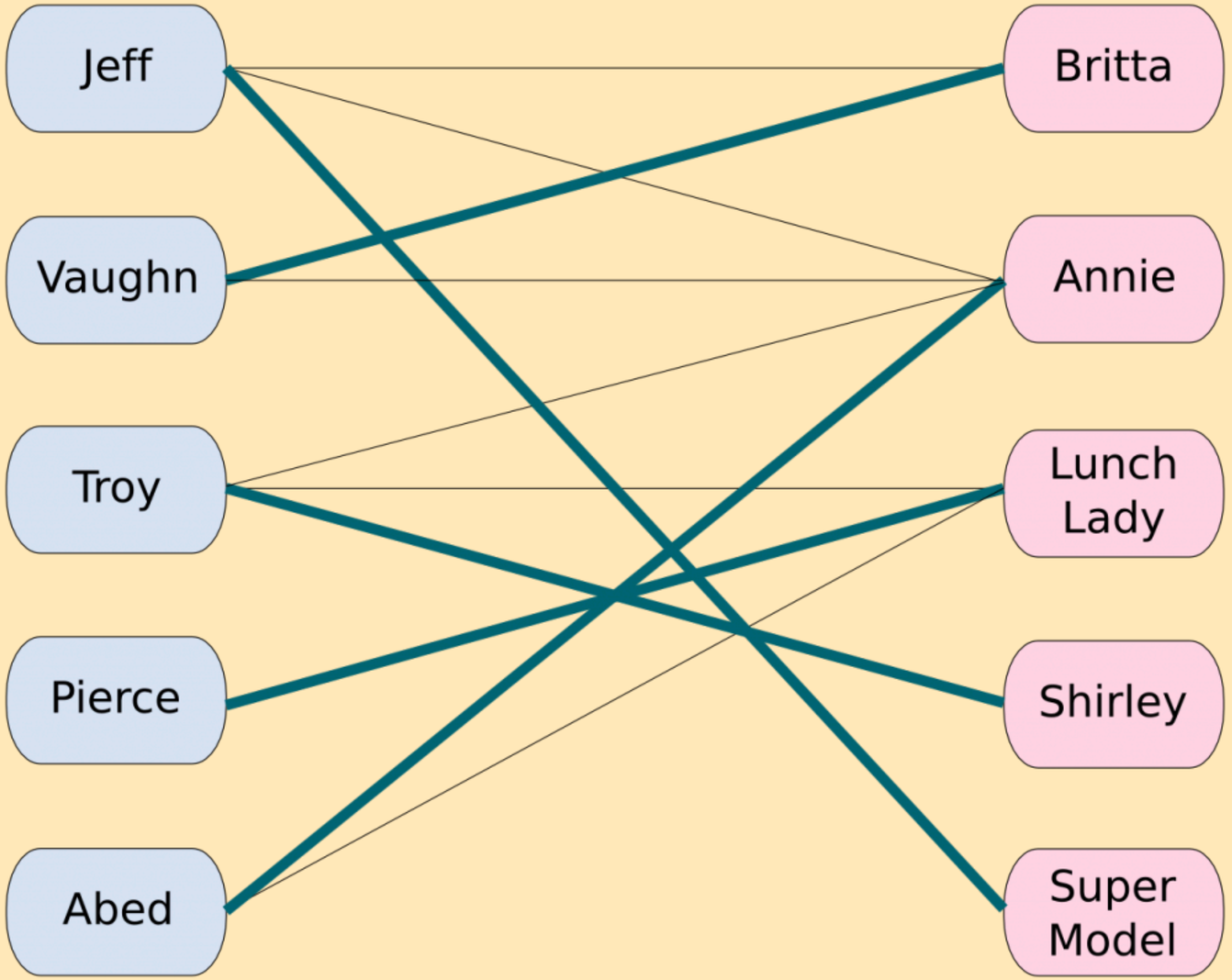


Matching

Definitions



Matching
Definitions

- ***Matching*** is a set of edges without common vertices. Vertex is ***saturated*** if it acquires a single edge from the matching subgraph
- ***Matching number*** is a quantity of edges in matching
    - Maximum matching — largest possible number of edges
    - Maximal matching — not a subset of any other matching

Matching
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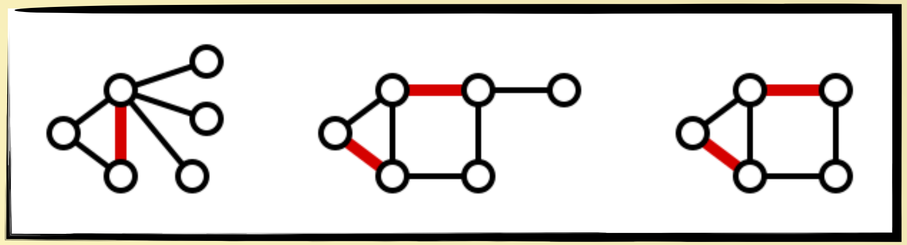

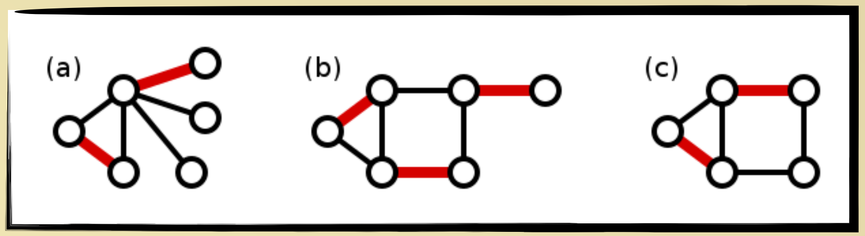

Matching
Definitions

- ***Bipartite graph*** - vertices can be divided into two disjoint & independent sets, and every edge connects two vertices from different sets
- ***Chain -*** a path of ***k*** edges without repeating ones
    - Alternating Path - edges belong to or don’t belong to the matching by turns
    - Augmenting Path - Alternating path, whose first & last vertices are not saturated

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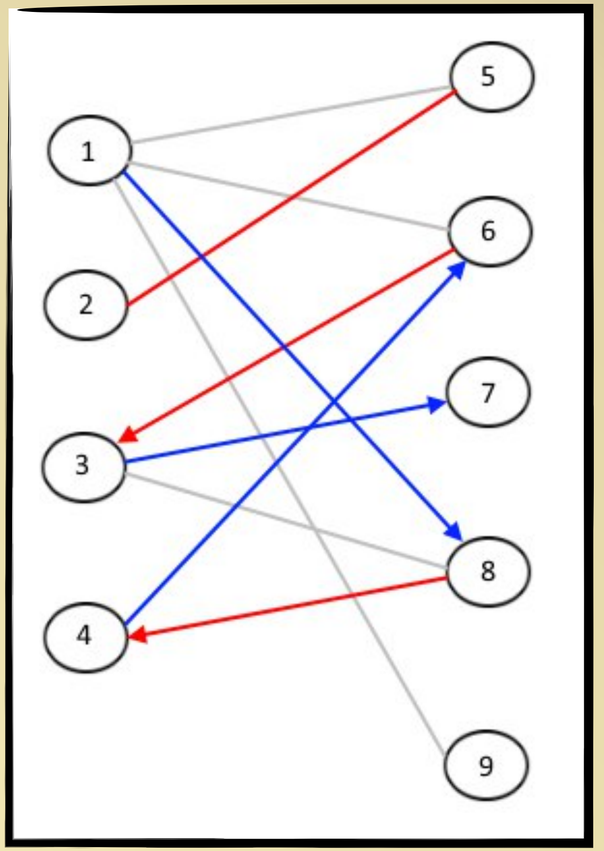

Matching
Hungarian Method

1. Find Augmenting Path

2. Complete ***reversing***: remove all edges within the path from matching and add others to it

3. Repeat the first step until possible