A (not so) short introduction to Reinforcement Learning

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Disclaimer

▶ Reinforcement Learning (RL) is a huge field, I will focus on the very basics that you need for your exercise

Overview

- 1. What is a MDP?
- 2. The Dimensions of RL
- 3. RL Definitions
- 4. Model Based RL
- 5. Model Free RL
 - 5.1 Q-Learning and TD methods

- ▶ Before I detail anything complicated let us look at the framework of Markov Decision Processes (MDPs) which we will use to describe all RL algorithms
- ▶ The situation an MDP seeks to model:

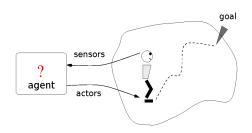
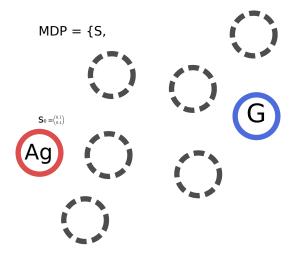
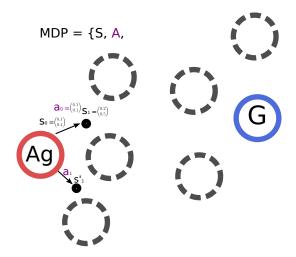
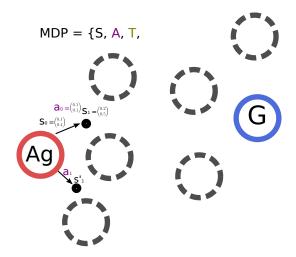
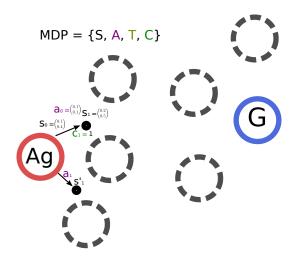


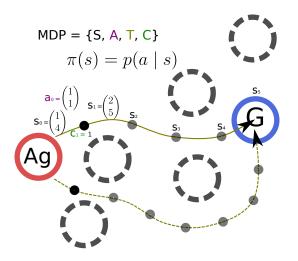
Figure: Taken from RL leacture by Martin Riedmiller





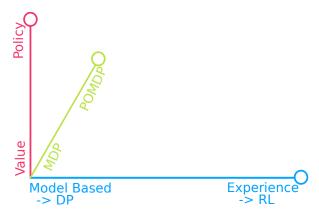






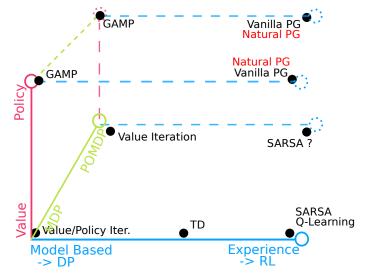
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► A MDP is a 5-tuple

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 - ► Assume the *markov property*: the future depends on the past only through the present state

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 - ► Assume the *markov property*: the future depends on the past only through the present state
 - $\rightarrow p(s_{t+1} \mid s_t, a_t)$ independent of previous states and actions
- ▶ The agents policy is described as: $\pi(s_t, a_t) = p(a_t \mid s_t)$

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- \blacktriangleright We can assign a Value V to each state as follows.
- ▶ Let us assume a finite horizon

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$$V_T^{\pi^*}(s_t) = \mathbb{E}_{\pi^*, P} \{ \sum_{t=0}^{T-1} r_t \mid s_o = s \}$$

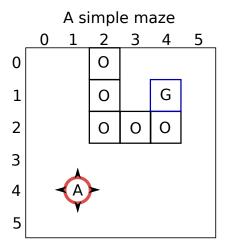
$$= \max_{a} r(s, a) + \sum_{s' \in S} p(s' \mid s, a) V_{T-1}^{\pi^*}(s')$$

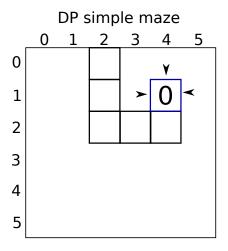
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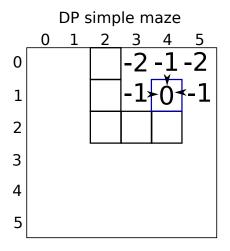
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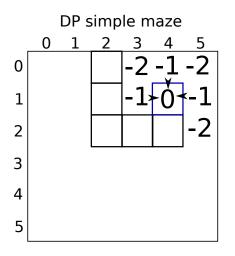
 \rightarrow Set $V_0^{\pi^*}(s') = \max r(s, a)$ and iterate

▶ But is this actually any use? Show me a solvable problem!







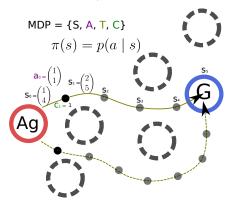


 \rightarrow lather, rinse, repeat

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- ▶ Remember (continuous states/actions, infinite horizon):



▶ Recall the definition of the state value:

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- ▶ Not bounded, we don't want infinite costs
- \rightarrow Solve this via discounting with $\gamma \leq 1$

$$V^{\pi}(s) = \mathbb{E}_{\pi,P} \{ \sum_{t=0}^{\infty} \gamma^t r_t \mid s_o = s \}$$

Model Based RL - DP Problems

▶ Recall the definition of the state value:

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- ▶ Not bounded, we don't want infinite costs
- \rightarrow Solve this via discounting with $\gamma < 1$

MLL

$$V^{\pi}(s) = \mathbb{E}_{\pi,P}\{\sum_{t=0}^{\infty} \gamma^{t} r_{t} \mid s_{o} = s\}$$

▶ Discounting is like assuming a non-zero interest rate — money arriving in the future is worth less than money arriving now.

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- ▶ Solution is to iteratively do DP like updates with discounting

Algorithm 2: Value Iteration

```
repeat
       for s \in S do
2
             update V values using current estimate V_i
3
            V_{i+1}^{\star}(s) \leftarrow \max_{a} \sum_{s' \in S} p(s' \mid s, a) (r(s, a) + \gamma V_{i}^{\star}(s'))
4
5 until converged
6 return V^*
```

Tobias Springenberg MLL

- ► Can we solve for the optimal value function (and hence policy)?
- ▶ Solution is to iteratively do DP like updates with discounting

Algorithm 3: Value Iteration

```
repeat
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            V_{i+1}^{\star}(s) \leftarrow \max_{a} \sum_{s' \in S} p(s' \mid s, a) (r(s, a) + \gamma V_{i}^{\star}(s'))
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5 until converged
6 return V^{\star}
```

 \rightarrow Converges under mild assumptions

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Algorithm 4: Value Iteration

```
repeat
        for s \in S do
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3
            V_{i+1}^{\star}(s) \leftarrow \max_{a} \sum_{s' \in S} p(s' \mid s, a) (r(s, a) + \gamma V_i^{\star}(s'))
4
5 until converged
```

- \rightarrow Converges under mild assumptions
 - \rightarrow However only in the limit of infinite iterations ...

6 return V^{\star}

Model Based RL - Further Problems

- ▶ But what if we don't have a model?
 - → Learn a model (see e.g. PILCO, or our ADPRL paper)

movie1

→ Use model free methods (e.g. Q-learning, SARSA)

Model Based RL - Further Problems

- ▶ But what if we don't have a model?
 - → Learn a model (see e.g. PILCO, or our ADPRL paper)

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- \rightarrow Use model free methods (e.g. Q-learning, SARSA)
- ▶ And what if we have continuous states?
 - → Use function approximation (e.g. a Neural Network)

Model Free RL

▶ But what if we don't have a model?

Algorithm 5: Value Iteration

MLL

- 1 repeat
- for $s \in S$ do
- ${f a}$ update V values using current estimate V_i

4
$$V_{i+1}^{\star}(s) \leftarrow \max_{a} \sum_{s \in S} p(s' \mid s, a) \quad (r(s, a) + \gamma V_i^{\star}(s'))$$

- 5 until converged
- 6 return V^*

▶ Somewhat Model Free $RL \rightarrow use sampling$

Algorithm 6: TD learning

1 repeat

choose
$$a = \arg\max_{a} r(s, a) + \sum_{s'} p(s' \mid s, a) V(s')$$

- **3** record transition (s, a, r, s')
- 4 update V using sample (td) error $\delta_t = r(s, a) + \gamma V_i(s') V_i(s)$

$$V_{i+1}(s) \leftarrow V_i(s) + \alpha \delta_t$$

- 6 until converged
- 7 return V

▶ Almost there!

MLL

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- ▶ Need a way to query the value of actions without using the model
- ▶ Introducing the Q-function

$$Q^{\pi}(s, a) = r(s, a) + \sum_{s'} p(s' \mid s, a) V^{\pi}(s')$$

Algorithm 7: Q-learning

1 repeat

choose
$$a = \arg\max_{a} \quad Q(s, a)$$
record transition (s, a, r, s')
update Q using sample Q -error
$$\delta_t = r(s, a) + \gamma \max_{a'} Q_i(s', a') - Q_i(s, a)$$

$$Q_{i+1}(s, a) \leftarrow Q_i(s, a) + \alpha \delta_t$$

- 6 until converged
- 7 return Q

Algorithm 8: Q-learning

MLL

1 repeat

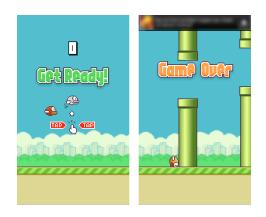
choose
$$a = \begin{cases} \arg\max_{a} & Q(s,a) & \text{with probability } \epsilon \\ a \in A & \text{else choose randomly} \end{cases}$$
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- 6 until converged
- 7 return Q

- → Remarkably Q-learning converges
- ▶ Okay, we don't need a model anymore, let's see some more applications!

▶ Finally solving some important problems!



- ➤ Solving FlappyBird with a simple table based Q-learning approach (see http://sarvagyavaish.github.io/FlappyBirdRL/)
- ▶ 2-dimensional State-space S: (x, y) pixel distance to tube discretized in 4 pixel steps



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$$r(s, \mathbf{a}) = \begin{cases} 1 & \text{if alive} \\ -1000 & \text{otherwise} \end{cases} \tag{1}$$

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$$r(s, \mathbf{a}) = \begin{cases} 1 & \text{if alive} \\ -1000 & \text{otherwise} \end{cases} \tag{1}$$

- ► Solved using on-line Q-learning (training took about 6-7 hours)
- Show VIDEO

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 - → Curse of dimensionality I: table is not going to work for high dimensions
 - \rightarrow We have only considered discrete states, next: function approximation
 - → Curse of dimensionality II: Efficient Exploration is key! Even with only 2 state-dimensions + 2 actions FlappyBird took hours ...

Modern approaches / Function approximation

► To (partially) solve the curse of dimensionalty one can try to use function approximation

- ➤ To (partially) solve the curse of dimensionalty one can try to use function approximation
- → Idea: replace Q-table by a function approximator
 - use Q-error $\delta_t = r(s, a) + \gamma \max_a Q_i(s', a) Q_i(s, a)$ as the gradient

Algorithm 9: Approximate Q-learning

1 repeat

choose
$$a = \begin{cases} \arg \max_{a} & Q(s, a) \\ a \in A \end{cases}$$
 with probability ϵ else choose randomly
record transition (s, a, r, s')
update Q using sample Q -error

MLL

update Q using sample Q-error

$$\delta_t = r(s, a) + \gamma \max_{a'} \quad Q_i(s', a') - \quad Q_i(s, a)$$
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- until converged
- return Q

Algorithm 10: Approximate Q-learning

1 repeat

choose
$$a = \begin{cases} \arg\max_{a} & Q_W(s, a) \\ a \in A \end{cases}$$
 with probability ϵ else choose randomly record transition (s, a, r, s') update Q_W using sample Q-error
$$\delta_t = r(s, a) + \gamma \max_{a'} & Q_W(s', a') - Q_W(s, a)$$

$$Q_{i+1}(s, a) \leftarrow Q_i(s, a) + \alpha \delta_t$$

6 until converged

MLL

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Algorithm 11: Approximate Q-learning

1 repeat

4

chose
$$a^* = \begin{cases} \max_a & Q_W(s, a) & \text{with probability } \epsilon \\ a \in A & \text{else chose randomly} \end{cases}$$

 \mathbf{a} record transition (s, a, r, s')

update Q_W using sample Q-error as derivative

$$\delta_t = r(s, a) + \gamma \max_{a'} Q_W(s', a') - Q_W(s, a)$$

$$W \leftarrow W + \alpha \left(\delta_t \frac{\partial Q_W(s, a)}{\partial W} \right)$$

- 6 until converged
- 7 return Q_W

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- → Problem: Non-stationarity/Floating targets? If Q changes the function we minimize changes
- \rightarrow Lots of exciting ongoing work: LSTD / NFQ / DQN etc.

Next exercise

NOTE: An exercise sheet is on the course webpage and in the git repository!

1) By hand, on a piece of paper solve the following discretized maze via Q-learning



- 2) Implement a simple neural Q-learning agent using a simple map in the same environment you already know
- ▶ API slightly changed so that you get a reward in each step
- ▶ hand in report showing the solution to 1) and explain what you did for 2)
- ► Exercise sheet and update code available Monday 19.12.16 deadline for exercise 23.01.17
- ▶ Bonus points for trying some more advanced ideas (e.g. target network, experience replay as in DQN)

Algorithm 12: Smoothed Approximate Q-learning (DQN)

1 repeat

chose
$$a^* = \begin{cases} \max_a & Q_W(s, a) & \text{with probability } \epsilon \\ a \in A & \text{else chose randomly} \end{cases}$$

- record transition (s, a, r, s')3
- update Q_W using sample Q-error as derivative 4

$$\delta_t = r(s, a) + \gamma \max_{a'} \quad Q_W(s', a') - Q_W(s, a)$$

$$W \leftarrow W - \alpha \left(\delta_t \frac{\partial Q_W(s, a)}{\partial W} \right)$$

$$\mathbf{5} \quad W \leftarrow W - \alpha \left(\delta_t \frac{\partial Q_W(s, a)}{\partial W} \right)$$

- 6 until converged
- 7 return Q_W

Algorithm 13: Smoothed Approximate Q-learning (DQN)

$$\begin{array}{ll} \mathbf{1} & T = \{\} \\ \mathbf{2} & \mathbf{repeat} \\ \mathbf{3} & \mathrm{chose} \ a^\star = \begin{cases} \max_a Q_W(s,a) & \mathrm{with \ probability} \ \epsilon \\ a \in A & \mathrm{else \ chose \ randomly} \end{cases} \\ \mathbf{4} & \mathrm{record \ transition} & T = T \cup \{(s,a,r,s')\} \\ \mathbf{5} & \mathrm{update} \ Q_W \ \mathrm{using \ sample} \ \mathrm{Q}\text{-error \ as \ derivative} \\ \mathbf{6} & \mathbf{for} \quad s \in [1,K] \quad \mathbf{do} \\ \mathbf{7} & \left[\begin{array}{c} (s,a,r,s') \sim T \\ \delta = \delta + r(s,a) + \gamma \max_{a'} Q_W(s',a') - Q_W(s,a) \\ \mathbf{9} & W \leftarrow W + \alpha \left(\delta \frac{\partial Q_W(s,a)}{\partial W} \right) \\ \end{cases} \\ \mathbf{9} & W \leftarrow W + \alpha \left(\delta \frac{\partial Q_W(s,a)}{\partial W} \right) \end{aligned}$$

- 10 until converged
- 11 return Q_W

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 - → Although Q-update looks like gradient we have "moving targets" (only partially addressed)
 - → Convergence can be slow
 - → Reiterate: Efficient Exploration is key!
 - → What about continuous actions?
 - → When are we really going to have a fully observed state?