Particle Swarm Optimization Algorithm

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Contenido

What is PSO?

- PSO is a population-based stochastic optimization algorithm inspired by social behavior of bird flocking.
- Each individual (called a particle) represents a potential solution.
- Particles adjust their positions in the search space by learning from their own experience and that of their neighbors.
- Developed originally to simulate the choreography of birds.

Particle Representation and Movement

- A swarm is a collection of particles.
- Each particle has a position x_i and velocity v_i.
- Position update:

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

Velocity Update (gbest model)

$$v_{ij}(t+1) = v_{ij}(t) + c_1 r_1 (y_{ij}(t) - x_{ij}(t)) + c_2 r_2 (\hat{y}_j(t) - x_{ij}(t))$$

- $y_{ij}(t)$: personal best
- $\hat{y}_j(t)$: global best
- r_1, r_2: random numbers in $[0, 1]$

Global vs. Local Best

- **gbest:** global best PSO, fast convergence, less diversity
- **lbest:** local best PSO, slower convergence, more robust
- Neighborhoods overlap to share information

Inertia Weight in PSO

- Introduced by Shi and Eberhart to control the balance between exploration and exploitation.
- Modifies the velocity equation:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_1(y_{ij}(t) - x_{ij}(t)) + c_2 r_2(\hat{y}_j(t) - x_{ij}(t))$$

- w is the inertia weight. High w encourages exploration; low w promotes exploitation.

Effects and Tuning of Inertia Weight

- If $w \geq 1$: velocity may diverge, particles can overshoot.
- If $w < 1$: velocities decay over time, enabling convergence.
- Typically $w \in [0{,}4, 0{,}9]$.
- Must be chosen in conjunction with c_1, c_2 to avoid divergence:

$$w > \frac{1}{2}(c_1 + c_2) - 1$$

Dynamic Inertia Weight Strategies

- **Linear Decrease**:

$$w(t) = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{t_{\max}} \right) t$$

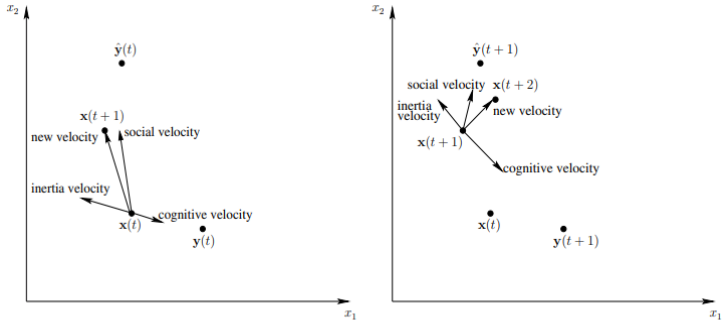
- **Nonlinear Decay** (e.g., exponential):

$$w(t) = \alpha w(t-1) \quad \text{with } \alpha \in (0,1)$$

- **Fuzzy Adaptive** and **Stochastic** adjustments also explored.
- Goal: allow wide exploration initially, focus on refinement in later iterations.

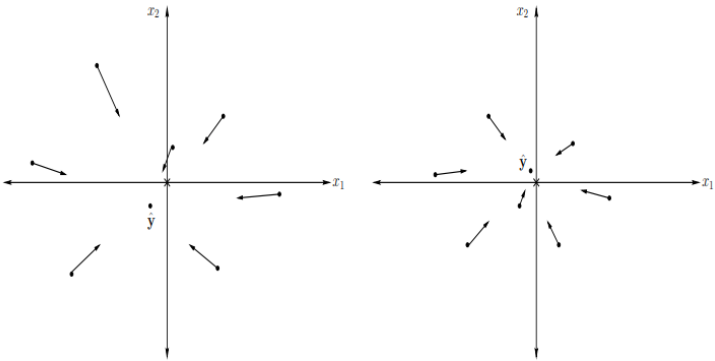
Velocity Components

- **Inertia:** memory of previous direction
- **Cognitive:** tendency to return to personal best
- **Social:** tendency to follow best in neighborhood



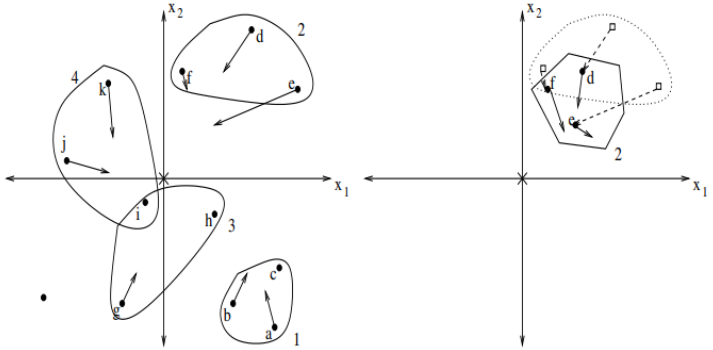
Geometric Interpretation-gbest

- Motion guided by vector components
- Balance between exploration and exploitation



Geometric Interpretation-lbest

- Motion guided by vector components
- Balance between exploration and exploitation



Key Parameters

- Swarm size n_s
- Cognitive and social coefficients c_1, c_2
- Inertia weight w
- Maximum velocity v_{max}

Termination Criteria

- Maximum number of iterations
- Acceptable fitness threshold
- No improvement over iterations
- Normalized swarm radius
- Objective function slope ~ 0

Common Variants

- Inertia weight PSO
- Constriction factor PSO
- Barebones PSO
- Fully Informed PSO (FIPS)
- Charged PSO

Hybrid PSO Models

- PSO with mutation (Gaussian, Cauchy)
- PSO with DE (Differential Evolution) crossover
- PSO + local search (GCPSO)
- PSO with genetic selection pressure

Multi-Swarm and Niching PSOs

- Multi-swarm: cooperative or competitive subgroups
- Predator-prey PSO
- Self-organizing and adaptive networks
- Split dimension optimization

PSO in Matlab

- `particleswarm` is a solver in MATLAB's Global Optimization Toolbox.
- It implements Particle Swarm Optimization (PSO) for solving bound-constrained optimization problems.
- Useful for problems where the objective function is non-differentiable, noisy, or has multiple local minima.

Basic Syntax

- `x = particleswarm(fun, nvars)`
  Minimizes the objective function `fun` with `nvars` variables.
- `x = particleswarm(fun, nvars, lb, ub)`
  Adds lower (`lb`) and upper (`ub`) bounds to the variables.
- `x = particleswarm(fun, nvars, lb, ub, options)`
  Allows customization through the `options` structure.

Key Parameters (1/2)

- **SwarmSize**
  Number of particles in the swarm. Default is 100.
- **MaxIterations**
  Maximum number of iterations allowed. Default is 100.
- **FunctionTolerance**
  Termination tolerance on the function value. Default is 1e−6.
- **InertiaRange**
  Range for inertia weight. Controls exploration vs. exploitation.

Key Parameters (2/2)

- **SelfAdjustmentWeight**
  Weight for the particle's own best position influence.
- **SocialAdjustmentWeight**
  Weight for the swarm's best position influence.
- **HybridFcn**
  Function handle for a hybrid function to refine the solution.
- **Display**
  Level of display output: `'off'`, `'iter'`, or `'final'`.

Velocity Update Equation

$$v = W \cdot v + y_1 \cdot u_1 \cdot (p - x) + y_2 \cdot u_2 \cdot (g - x)$$

- v: Current velocity
- W: Inertia weight
- y_1: SelfAdjustmentWeight
- y_2: SocialAdjustmentWeight
- u_1, u_2: Random vectors in [0,1]
- p: Particle's best-known position
- g: Global best-known position

Termination Criteria

- Maximum number of iterations reached.
- Change in the best function value is less than `FunctionTolerance`.
- No improvement in the best function value for `MaxStallIterations`.

PSO Parameters: Exploration vs. Exploitation

Parameter	Typical Range	Exploration	Exploitation
w	0.4 – 0.9	High w (e.g., 0.9)	Low w (e.g., 0.4)
c_1	1.5 – 2.5	High c_1	Low c_1
c_2	1.5 – 2.5	Low c_2	High c_2

Cuadro 1: Impact of PSO parameters on exploration and exploitation

PSO Parameters: Exploration vs. Exploitation

- **Inertia Weight (w)**: Controls the momentum of particles. Higher values encourage global exploration, while lower values promote local exploitation.
- **Cognitive Coefficient (c_1)**: Represents the particle's tendency to return to its own best position. Higher values increase individual exploration.
- **Social Coefficient (c_2)**: Represents the particle's tendency to move towards the swarm's best position. Higher values increase convergence and exploitation.

Best Practices

- Start with default parameters; adjust based on problem complexity.
- Use bounds (`lb`, `ub`) to confine the search space.
- Consider using a hybrid function for fine-tuning the solution.
- Monitor convergence using the `Display` option.

Conclusion

- PSO is an efficient population-based optimization algorithm
- Many variants have improved its robustness and flexibility
- Understanding social structure and velocity dynamics is key
- Suitable for continuous, discrete, and dynamic environments

References

- MATLAB Documentation: particleswarm
- MATLAB Documentation: Particle Swarm Options
- Engelbrecht, A. P. (2007). *Computational Intelligence: An Introduction*. John Wiley & Sons