Recurrent Neural Networks II

Deep Dive: Long Short-Term Memory (LSTM)

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Agenda

The Problem with Standard RNNs

Short-Term Memory

Standard Recurrent Networks (RNNs) suffer from the Vanishing Gradient Problem.

- As the gap between relevant information and the current prediction grows, RNNs lose the ability to learn connections.
- They have difficulty retaining information over long sequences.

The LSTM Solution

Proposed by Hochreiter & Schmidhuber (1997).

- Explicitly designed to avoid the long-term dependency problem.
- Remembering information for long periods is their default behavior, not something they struggle to learn.

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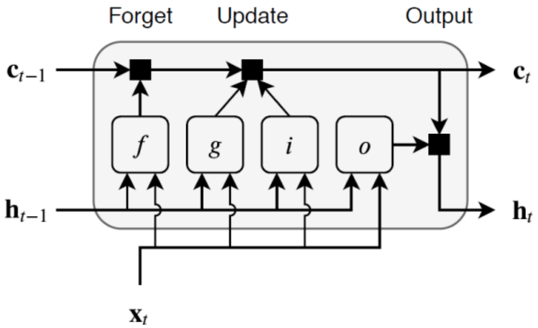
The Core Structure

Unlike the repeating module in a standard RNN (a single tanh layer), the LSTM contains four interacting layers (gates).



Key Concepts: States and Inputs

Looking at the diagram, we identify the primary vectors:

1. **Input Vector ($\mathbf{x}_t$):** The new information entering the network at time step t.
2. **Previous Hidden State ($\mathbf{h}_{t-1}$):** The output of the LSTM from the previous step (Short-term memory).
3. **Cell State ($\mathbf{c}_t$):** The internal memory "highway". It runs straight down the entire chain with only minor linear interactions, allowing gradients to flow unchanged (solving vanishing gradient).

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Understanding the Symbols

The figure introduces four distinct blocks (gates). Each block represents a Neural Network layer with its own weights (W) and bias (b).

Symbol	Name	Activation	Role
f	**Forget Gate**	Sigmoid (σ)	Decides what to delete.
g	**Update Gate**	Tanh	Creates new candidates.
i	**Input Gate**	Sigmoid (σ)	Decides importance of g.
o	**Output Gate**	Sigmoid (σ)	Filters the output $\mathbf{h}_t$.

Step 1: The Forget Gate (f)

Decision: "What do we throw away?"

The gate looks at $\mathbf{h}_{t-1}$ and $\mathbf{x}_t$, and outputs a number between 0 and 1 for each number in the cell state $\mathbf{c}_{t-1}$.

$$\mathbf{f}_t = \sigma(W_f \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + b_f) \tag{1}$$

- **1:** "Keep this completely."
- **0:** "Get rid of this completely."

Symbol in Diagram:

$$\boxed{f}$$

$$\uparrow$$

Control of History

Step 2: The Input & Update Gates (i, g)

Decision: "What new info do we store?"

This step has two parts appearing in the diagram as i and g:

1. Input Gate (i):

$$\mathbf{i}_t = \sigma(W_i \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + b_i) \tag{2}$$

Decides *which values* we'll update.

2. Candidate Update (g):

$$\mathbf{g}_t = \tanh(W_g \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + b_g) \tag{3}$$

Creates a vector of *new candidate values* to be added to the state.

Step 3: Updating the Cell State ($\mathbf{c}_t$)

The Core Operation

We now update the old cell state, $\mathbf{c}_{t-1}$, into the new cell state $\mathbf{c}_t$.

In the diagram, observe the "Merge" line at the top:

$$\mathbf{c}_t = \left(\underbrace{\mathbf{f}_t \odot \mathbf{c}_{t-1}}_{\text{Forget old memory}} \right) + \left(\underbrace{\mathbf{i}_t \odot \mathbf{g}_t}_{\text{Add new scaled info}} \right) \tag{4}$$

Note: The symbol $\odot$ denotes element-wise multiplication (Hadamard product), represented by the black squares in the diagram.

Step 4: The Output Gate (o)

Finally, we need to decide what we're going to output. This output will be based on our cell state, but a filtered version.

1. Calculate Gate Activation:

$$\mathbf{o}_t = \sigma(W_o \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + b_o) \tag{5}$$

2. Calculate Hidden State: Push cell state through tanh (to push values between -1 and 1) and multiply by output gate.

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{c}_t) \tag{6}$$

Symbol in Diagram:

$\boxed{O} \rightarrow$ Output

Outputs both $\mathbf{h}_t$ (for prediction) and passes it to the next step.

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Parameter Complexity

Why does an LSTM have so many learnable parameters in MATLAB?

The Factor of 4

Since an LSTM has 4 separate "Neural Networks" inside (f, i, g, o), the total number of weights is:

$$\text{NumParameters} \approx 4 \times [(n \times m) + (n^2) + n] \tag{7}$$

Where:

- n: Number of Hidden Units.
- m: Input dimension size.

This explains why Deep Network Analyzer shows "Total learnables: 68.1k" for a relatively small network.

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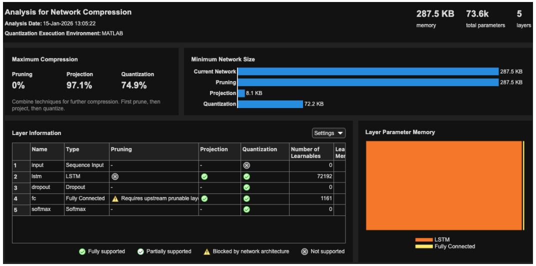
Validating Complexity with Network Analyzer



Analysis for Network Compression
Analysis Date: 15-Jan-2026 13:05:22
Quantization Execution Environment: MATLAB

287.5 KB memory · 73.6k total parameters · 5 layers

Maximum Compression

Pruning	Projection	Quantization
0%	97.1%	74.9%

Combine techniques for further compression. First prune, then project, then quantize.

Minimum Network Size

Current Network	287.5 KB
Pruning	287.5 KB
Projection	8.1 KB
Quantization	72.2 KB

Layer Information

	Name	Type	Pruning	Projection	Quantization	Number of Learnables	Lea Mer
1	input	Sequence Input	-	-	⊗	0	
2	lstm	LSTM	⊗	✓	✓	72192	
3	dropout	Dropout	-	-	✓	0	
4	fc	Fully Connected	⚠ Requires upstream prunable lay	✓	✓	1161	
5	softmax	Softmax	-	-	✓	0	

✓ Fully supported ✓ Partially supported ⚠ Blocked by network architecture ⊗ Not supported

Figure 2: Deep Network Designer Report: 73.3k Learnables.

Applying the Formula

In the provided screenshot: Input $(m) = 12$, Hidden $(n) = 128$.

LSTM Parameters Calculation:

$$P \approx 4 \times ((12 \times 128) + (128^2) + 128)$$

$$P \approx 4 \times (1,536 + 16,384 + 128)$$

$$P \approx 72,192 \text{ weights}$$

Final Count: Adding the Fully Connected

The Softmax Layer

From Logits to Probabilities

In the architecture diagram, the LSTM feeds a Fully Connected (FC) layer, which outputs raw scores called **logits**. These can be negative or unbounded (e.g., $2.5, -0.1, 5.0$).

The **Softmax Layer** squashes these values to form a valid probability distribution:

$$P(y = j \mid \mathbf{z}) = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}} \tag{8}$$

- **Rule 1:** All outputs sum to exactly 1 (100%).
- **Rule 2:** Competitive nature (if Class A goes up, B and C must go down).
- **Usage:** Essential for Multi-class Classification loss calculation.

Initialization Strategies

Choosing the right initialization is critical for convergence in LSTMs. MATLAB offers several options in the dropdown menu:

- **Glorot (Xavier):** The default for LSTMs.
  - optimized for symmetric activations like **Tanh** and **Sigmoid** (the internal gates of LSTM).
  - Keeps variance constant across layers.
- **He:**
  - Optimized for **ReLU** layers. usually avoided for the internal gates of RNNs but used in the FC layers.
- **Orthogonal:**
  - Specifically useful for RNNs/LSTMs to prevent vanishing gradients during very long sequences.

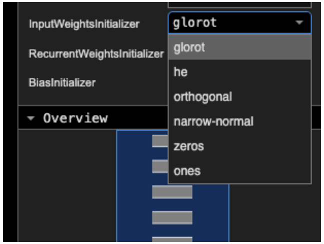


Figure 3: Weight Initializers in MATLAB.

Deployment & Compression

Resource Constrains

LSTMs are memory intensive. The analyzer shows **287.5 KB** for a small network, but deep LSTMs can grow to Gigabytes.

MATLAB's Compression Tools (Pruning, Quantization) help deploy to embedded systems:

Method	**Concept**	**Trade-off**
Pruning	Removing "weak" connections	Can reduce accuracy if aggressive.
Quantization	Float32 $\rightarrow$ Int8	4x Smaller, faster inference.
Projection	Compressing feature maps	Requires retraining.

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Real-World Applications

LSTMs are the state-of-the-art for sequence problems (before Transformers took over NLP):

1. **Time Series Classification:** Activity recognition (Acc/Gyro), ECG arrhythmia detection.
2. **Forecasting:** Stock prediction, energy load demand.
3. **Anomaly Detection:** Predictive maintenance in machinery (detecting patterns that deviate from normal history).
4. **Control Systems:** Modeling dynamic systems where state depends on deep history.

MATLAB Implementation

Constructing the network discussed in theory:

Listing 1: Defining LSTM Architecture

```
% Sequence Input: 3 Channels (sensors)
inputSize = 3;
numHiddenUnits = 128; % Generates 128x4 internal weights
numClasses = 4;

layers = [ ...
    sequenceInputLayer(inputSize)
    lstmLayer(numHiddenUnits, 'OutputMode', 'last') % Use 'sequence' for
        seq2seq
    dropoutLayer(0.5)
    fullyConnectedLayer(numClasses)
    softmaxLayer
    classificationLayer];
```

Conclusion

- **Structure:** LSTMs separate the memory stream ($\mathbf{c}_t$) from the processing stream ($\mathbf{h}_t$).
- **Gates:** Through mechanisms f, g, i, o, the network learns purely by gradient descent what to remember and what to ignore.
- **Result:** They solve the vanishing gradient problem, enabling the modeling of complex temporal dependencies over thousands of time steps.