

# Trajectory Mining

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# Background and Motivation

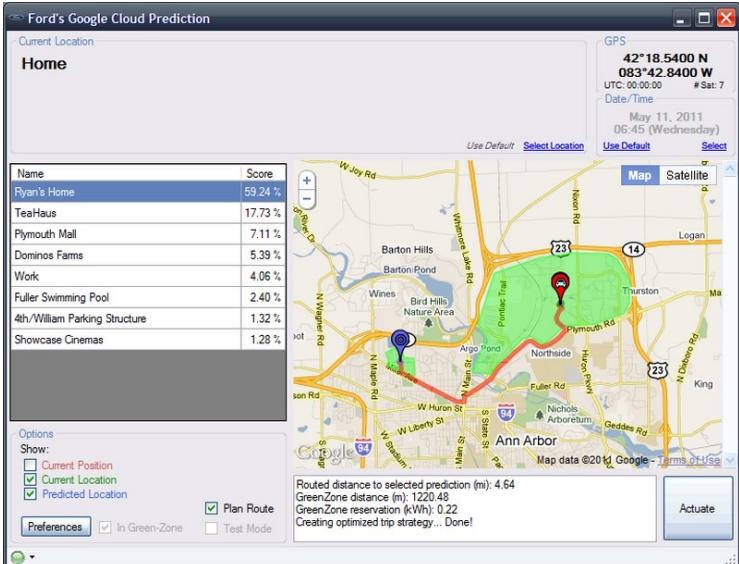
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- Nowadays, many electronic devices are used for real world applications  
*GPS, sensor networks, mobile phone, ..*
- « interesting » patterns for:  
*movement pattern analysis, animal behavior, route planning and vehicle control, location prediction, ...*



# Some Examples

Photo: Ford and Google



Ford with the Google Api Prediction

Photo: cdn.benzinga.com



The world's largest traffic jam in history (China)

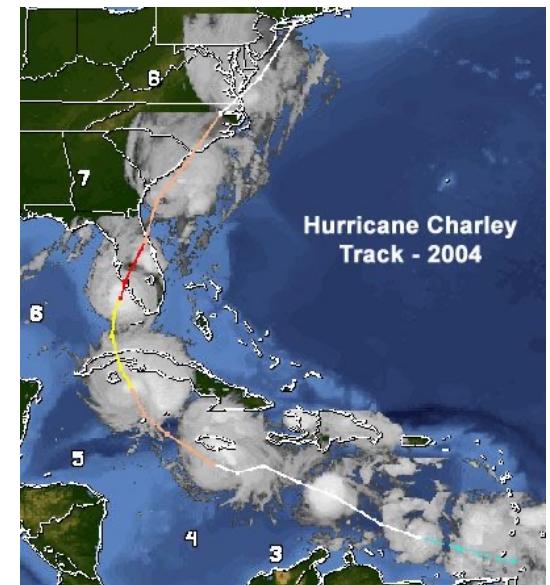
Movebank

MOVEBANK  
For Animal Tracking Data

Home Tracking Data Map Community Help Tools Env-DATA Published Data Search

User login  
Username: \*  
Password: \*

Free, online database of animal tracking data  
Animal migration analysis



Hurricane Trajectory Prediction

# Some Examples

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- Location Prediction

*According to the frequent behavior of the Second Big Data School attendees, the next place, after São Carlos to visit is:*

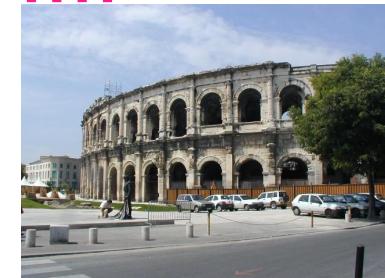
**Montpellier and its Region !!!!**



Antigone



Place de la Comédie



Nîmes



Beaches



Pont du Diable

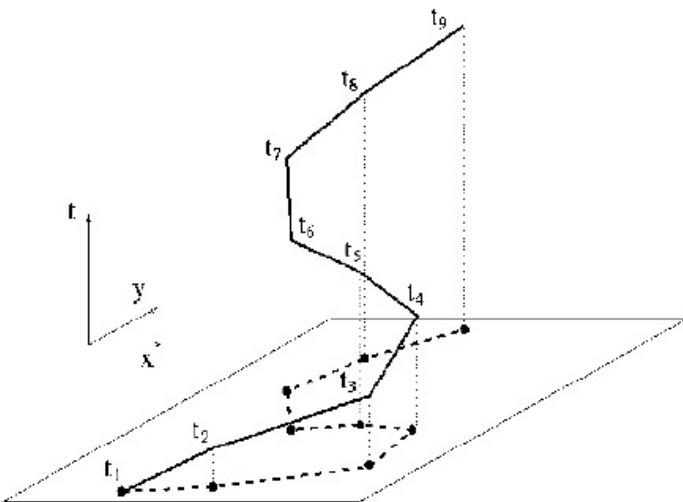


Sète

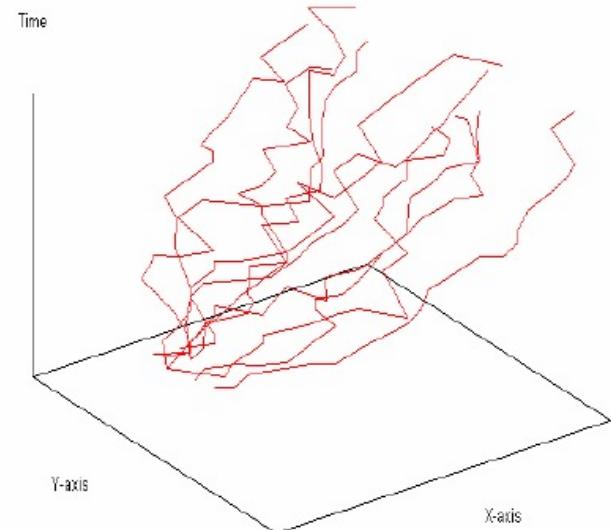


# Spatio-Temporal Data

- Represented as a set of points, located in space and time.
  - $T=(x_1, y_1, t_1), \dots, (x_n, y_n, t_n) \rightarrow$  position in space at time  $t_i$  was  $(x_i, y_i)$ .



Tid	position (x,y)	time (t)
1	48.890018 2.246100	08:25
1	48.890018 2.246100	08:26
...	...	...
1	48.890020 2.246102	08:40
1	48.888880 2.248208	08:41
1	48.885732 2.255031	08:42
...	...	...
1	48.858434 2.336105	09:04
1	48.853611 2.349190	09:05
...	...	...
2	...	...



# Using Ad-hoc Queries

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- Spatial query extensions in GIS applications
  - « *find all the moving objects inside area A between 10:00 am and 2:00 pm* »
  - « *how many cars were driven between Main Square and the Airport on Friday?* »
- Find the best solution by exploring each spatial object at a specific time according to some metric distance measurement



# Capture Collective Behavior

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- Extracting patterns which capture 'group' or 'common' behaviour among moving entities:  
**Trajectories**
- Development of approaches to identify groups of moving objects  
*strong relationship and interaction exist within a defined spatial region during a given time duration*



# Outline

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- A quick reminder on pattern discovery
- The different kinds of patterns from trajectory
- Towards a unified approach for extracting trajectories
- Deal with and without a spatial component?
- A concrete illustration
- Conclusion



# Outline

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- A quick reminder on pattern discovery
  - Association Rules
  - Sequential Patterns
- The different kinds of patterns from trajectory
- Towards a unified approach for extracting trajectories
- Deal with and without a spatial component?
- A concrete illustration
- Conclusion



# Association Rule Mining

- **Items** : A, B, C, D, E, F
- **4 transactions** (subset of items)
  - T1 : {A,D}
- **Support** for an itemset
  - $\text{Supp } \{\{A,D\}\} = 1/4$
  - $\text{Supp } \{\{A,C\}\} = 2/4$
- **Frequent Itemsets** ( $\text{minSupp}=50\%$ )
  - {A,C} is a frequent itemset
- **Rules** : ( $\text{minSupp}$  and  $\text{minConf} = 50\%$ )
  - $A \rightarrow C [50\%, 50\%]$
  - $C \rightarrow A [50\%, 100\%]$

Trans ID	Items
1	A, D
2	A, C
3	A, B, C
4	A, B, E, F

# Association Rule Mining

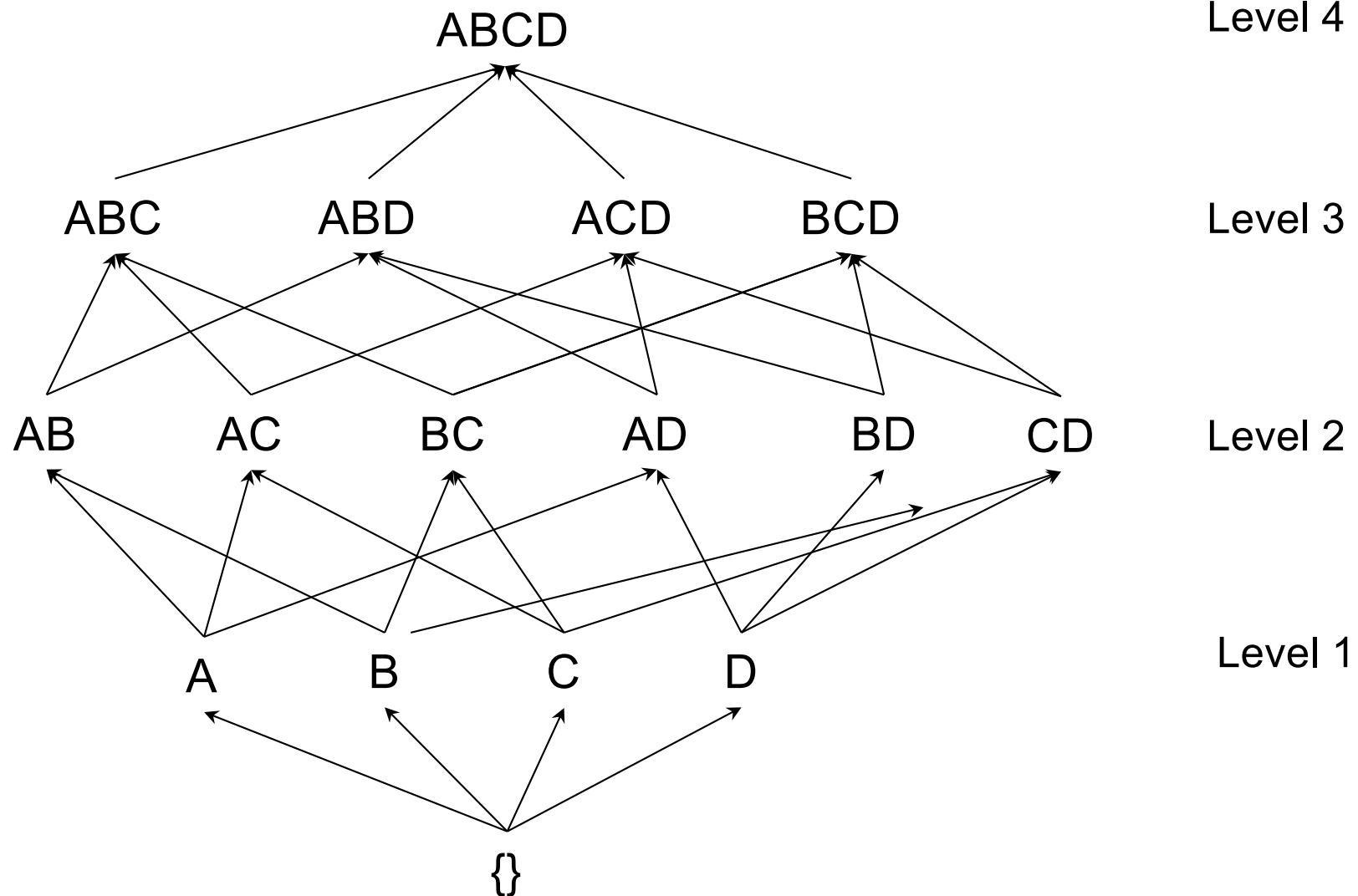
- The corresponding matrix with the frequent itemset {A,C}

	A	B	C	D	E	F
1	1	0	0	1	0	0
2	1	0	1	0	0	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1

Trans ID	Items
1	A, D
2	A, C
3	A, B, C
4	A, B, E, F



# The Search Space



# Sequential Pattern Mining

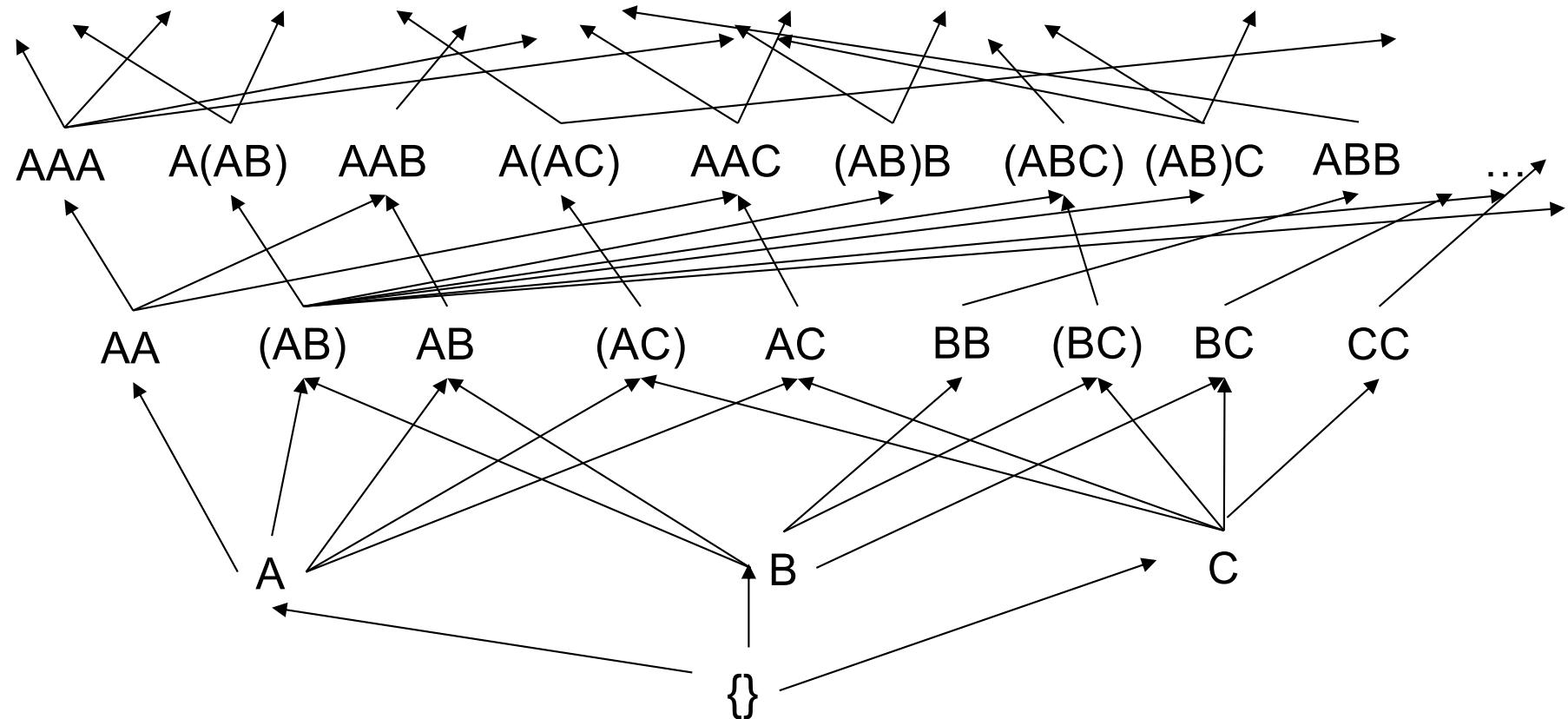
- **Items :** A, B, C, D, E, F
- **Support of a sequence**
  - $\text{Supp}((A)(C))=2/4$
  - $\text{Supp}((A)(B))=2/4$
- **Frequent sequences**  
( $\text{minSupp}=50\%$ ) :
  - (A) (C) and (A) (B) are frequent sequences

Trans ID	Sequences
1	(A, D)
2	(A) (C) (A) (C)
3	(A) (B, C)
4	(A) (E, F) (B)



# The Search Space for SP

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# What kinds of patterns can be mined?

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- $1, \{A, B, C, D\}$ 
  - Association rules
- $1, <(A, t1) (B, C, t2) (D, t3)>$ 
  - Sequences
- What can we do with the spatial dimension?
  - $1, <(x1, y1, t1) (x2, y2, t2) (x3, y3, t3)>$



# Outline

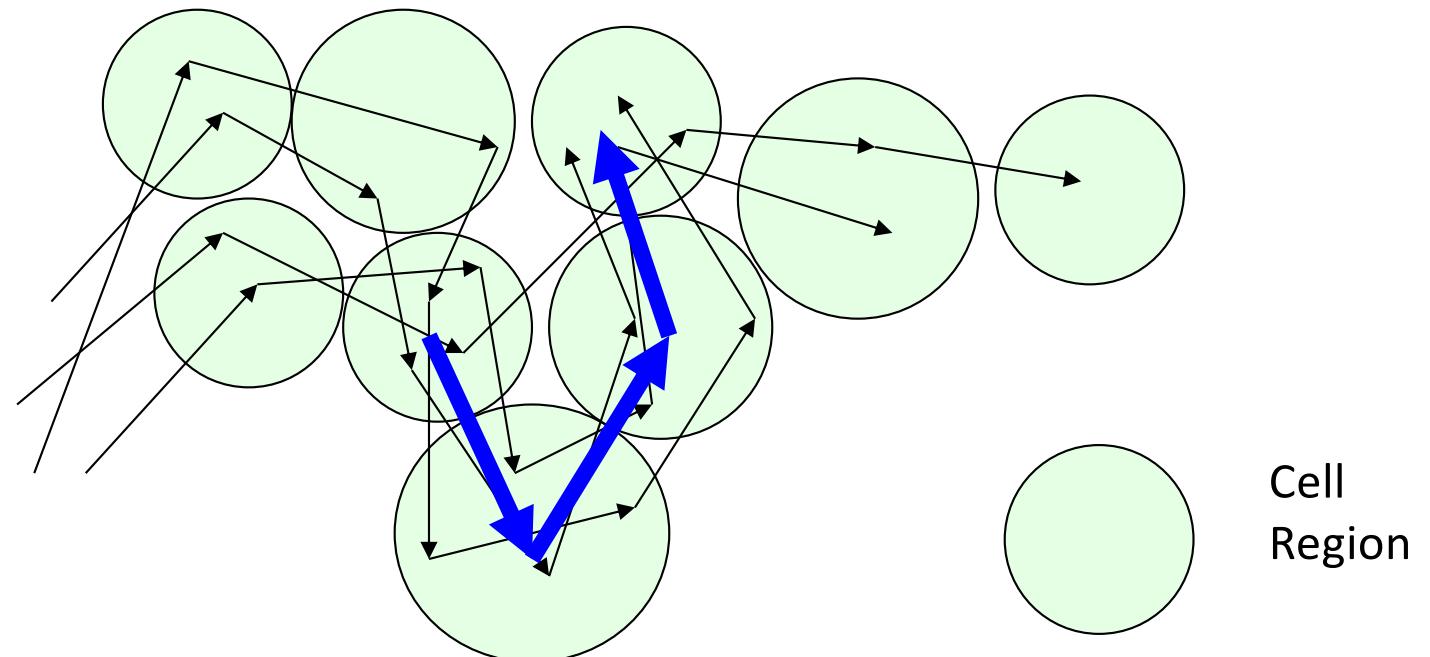
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- A quick reminder on pattern discovery
- **The different kinds of patterns from trajectory**
  - Frequent Patterns
  - Moving Object
- Towards a unified approach for extracting trajectories
- Deal with and without a spatial component?
- A concrete illustration
- Conclusion



# Mining Spatio-Temporal Patterns from Trajectory Data

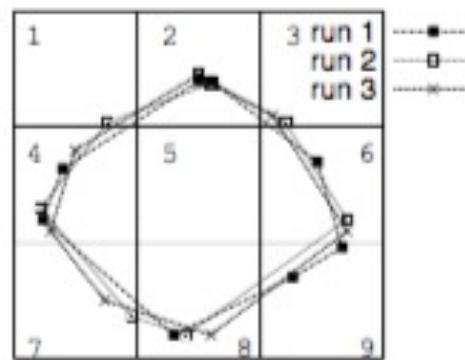
- Frequent Patterns:
  - Frequent followed paths



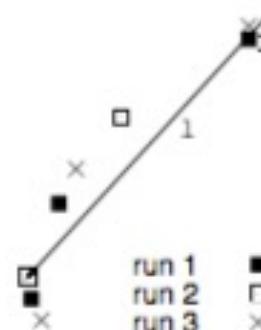
# Frequent Spatio-Temporal Sequence

- One spatio-temporal sequence:

$$S = \langle (x_1, y_1, t_1) (x_2, y_2, t_2) \dots (x_n, y_n, t_n) \rangle$$



(a)



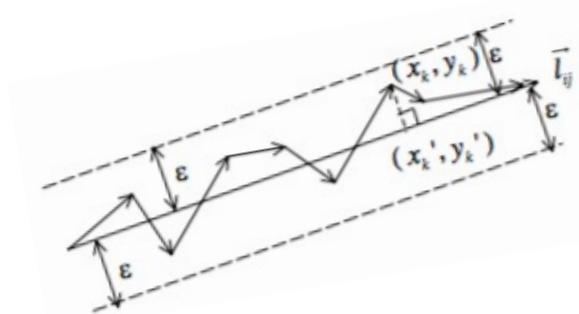
(b)

- Movement patterns that occurs frequently in the sequence: Frequent spatio-temporal sequence

# Frequent Spatio-Temporal Sequence

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- A three step process
- Transform each trajectory in a line with several segments
  - A distance tolerance measure  $\epsilon$  is defined
  - All trajectory points inside this distance can be projected in a representative line segment

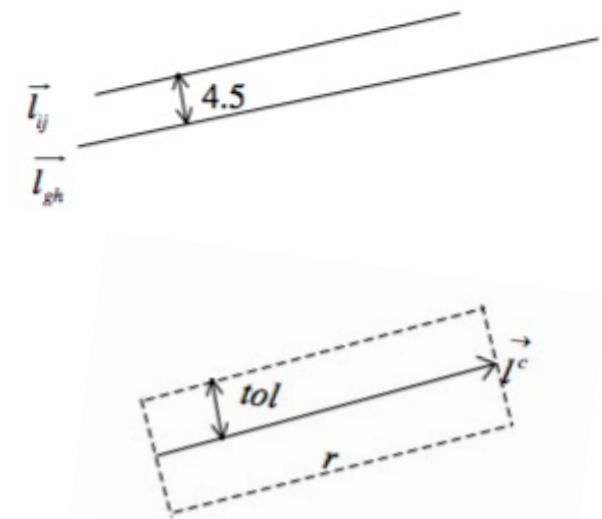


# Frequent Spatio-Temporal Sequence

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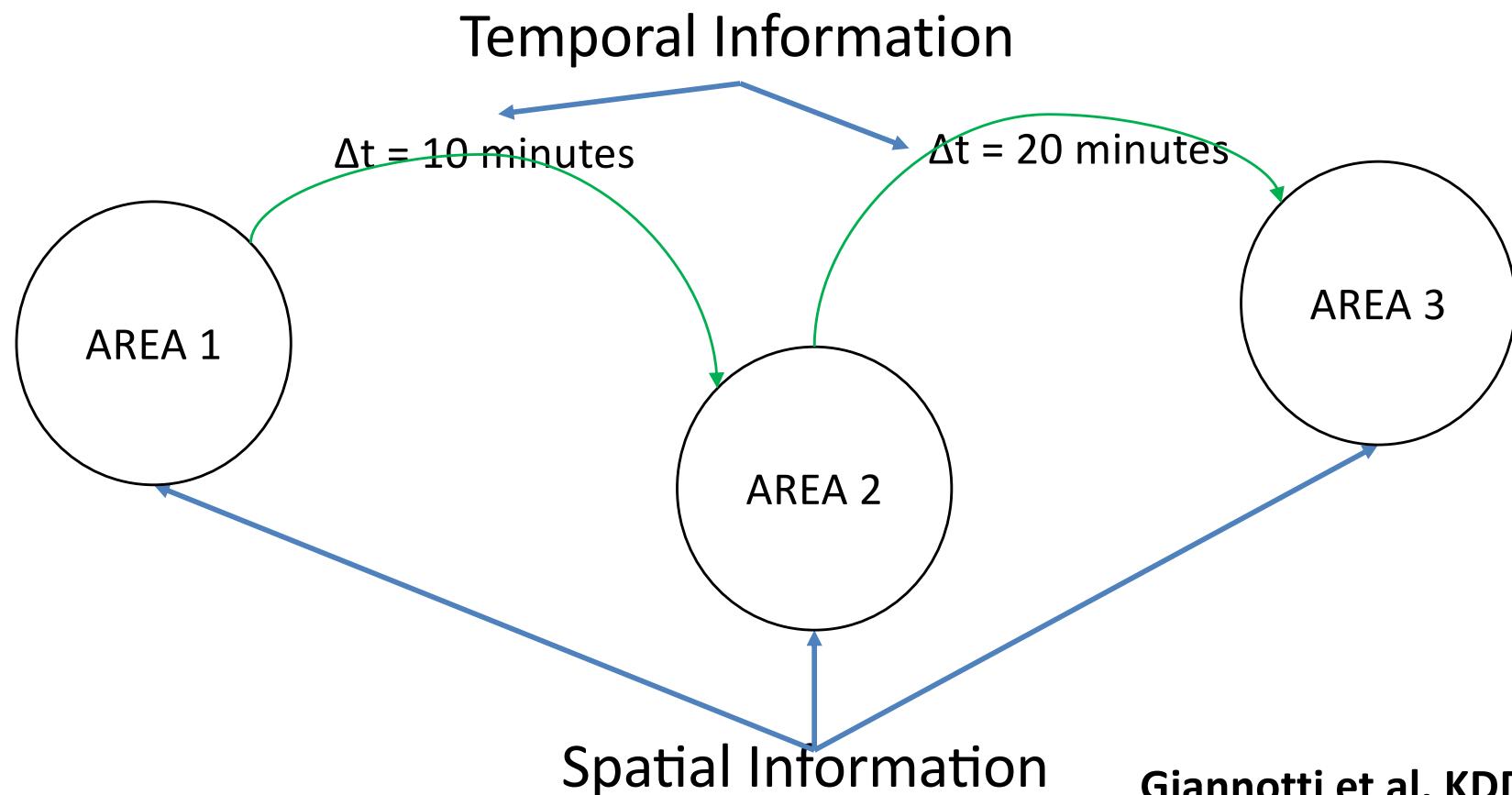
- Group together similar segments and define a region
  - The similarity is based on the angle and the spatial length of the segment
  - A line segment with same angle and length is close to another one if their distance is lower than a distance threshold  $d$
  - A medium segment is created, i.e. a region
- Compute frequent sequences of regions (minSupp)

$r1-r2-r3-r1-r2 \Rightarrow r1, r2, r1-r2$   
(minSupp=2)



# Trajectory Pattern Mining

- A trajectory pattern describes the movements of objects both in space and in time



# Trajectory (T-) Patterns

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- A *Trajectory Pattern (T-pattern)* is a couple  $(s, \alpha)$ :
  - $s = \langle (x_0, y_0), \dots, (x_k, y_k) \rangle$  is a sequence of  $k+1$  locations
  - $\alpha = \langle \alpha_1, \dots, \alpha_k \rangle$  are the transition times

$$(x_0, y_0) \xrightarrow{\alpha_1} (x_1, y_1) \xrightarrow{\alpha_2} \dots \xrightarrow{\alpha_k} (x_k, y_k)$$

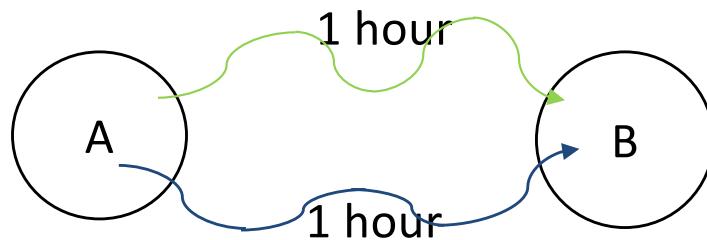
- A T-pattern  $T_p$  occurs in a trajectory if the trajectory contains a subsequence  $S$  such that:
  - Each  $(x_i, y_i)$  in  $T_p$  matches a point  $(x_{i'}, y_{i'})$  in  $S$ , and the transition times in  $T_p$  are similar to those in  $S$



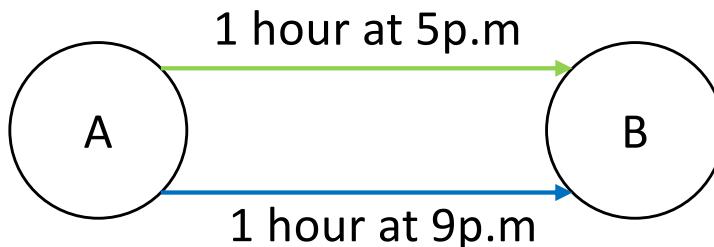
# Characteristics of Trajectory-Patterns

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- Routes between two consecutive regions are not relevant

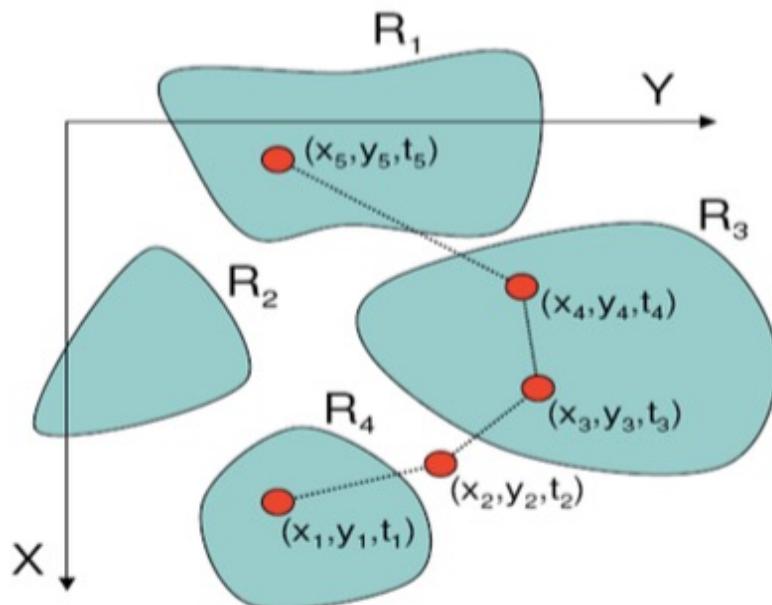


- Absolute times are not relevant



# Trajectory-Pattern Mining

- A preprocessing step is performed :
  - Convert each trajectory to a sequence, i.e., by converting a location  $(x, y)$  into a region



$S = \langle (x_1, y_1, t_1), \dots, (x_5, y_5, t_5) \rangle$

↓

$\langle (R_4, t_1), (R_3, t_3), (R_3, t_4), (R_1, t_5) \rangle$

# A Neighborhood Function

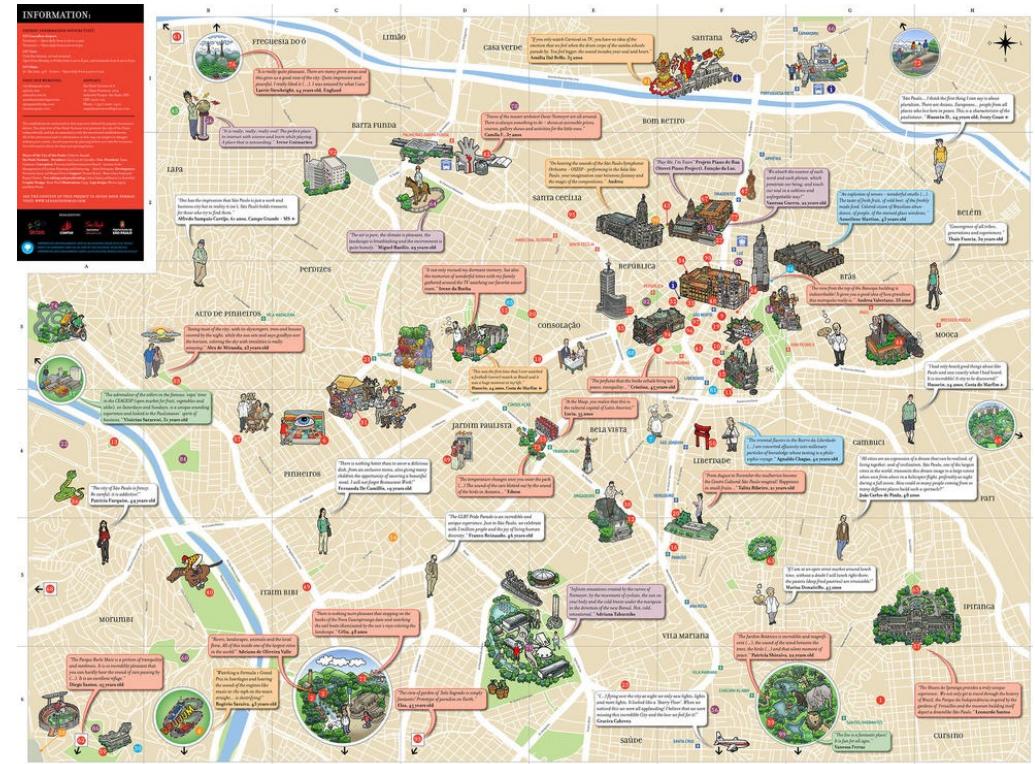
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- A preprocessing step
- Neighborhood Function
  - Calculates spatial containment of regions
  - Input point to find enclosing Region of Interest
  - Defines the necessary proximity to fall into a region
  - Parameters:  $e$  – radius or necessary proximity of points
- Translate each set of points into regions
- Timestamp is selected from when the trajectory first entered the region



# Using prior Knowledge

- Static Region of Interest
  - Initially receives set of R disjoint spatial regions
  - R regions are predefined based on prior knowledge
  - Each represents relevant place for processing



# Popular Regions

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- Data sets often do not possess predetermined regions
- Regions based on criteria of density of the trajectories
  - A grid  $G$  of  $n \times m$  cells
  - A density Threshold  $d$
  - Each cell with density  $G(i,j)$
- Each region forms rectangular region
- Dense cells always contained in some region
- All regions have average density above  $d$
- All regions cannot expand without their average density decreasing below  $d$



# Grid Density Preparation

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- Split space into  $n \times m$  grid with small cells
- Increment cells where trajectory passes
- Neighborhood Function  $NR()$  determines which surrounding cells
- Regression - increment continuously along trajectory

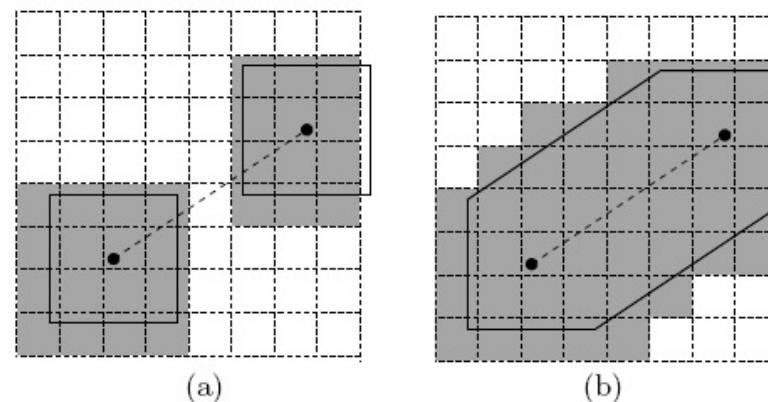


Figure 2: Density with and without regression

# Popular Regions Algorithm

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- Iteratively consider each dense cell
- For each:
  - Expands in all four directions
  - Select expansion that maximizes density
  - Repeat until expansion would decrease below density threshold



# Trajectory-Pattern Mining

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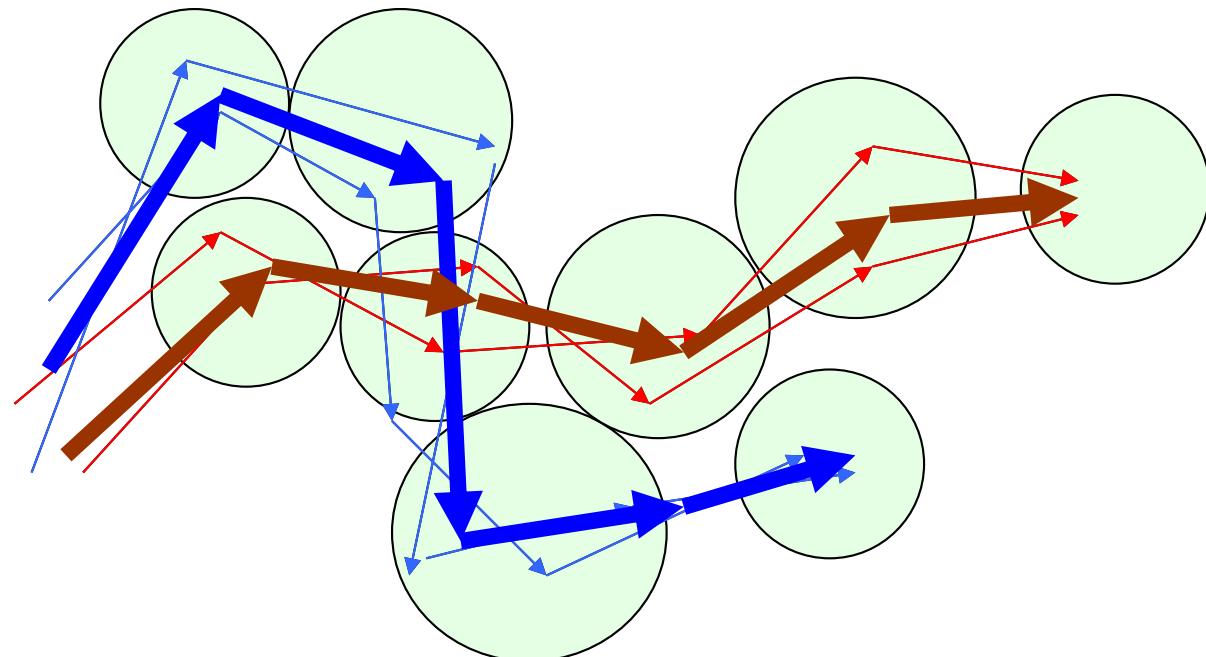
- After the preprocessing
- Execute a TAS (temporally annotated sequence) algorithm
  - A TAS is a sequential pattern annotated with typical transition times between its elements
  - The algorithm of TAS mining is an extension of PrefixSpan



# Mining Spatio-Temporal Patterns from Trajectory Data

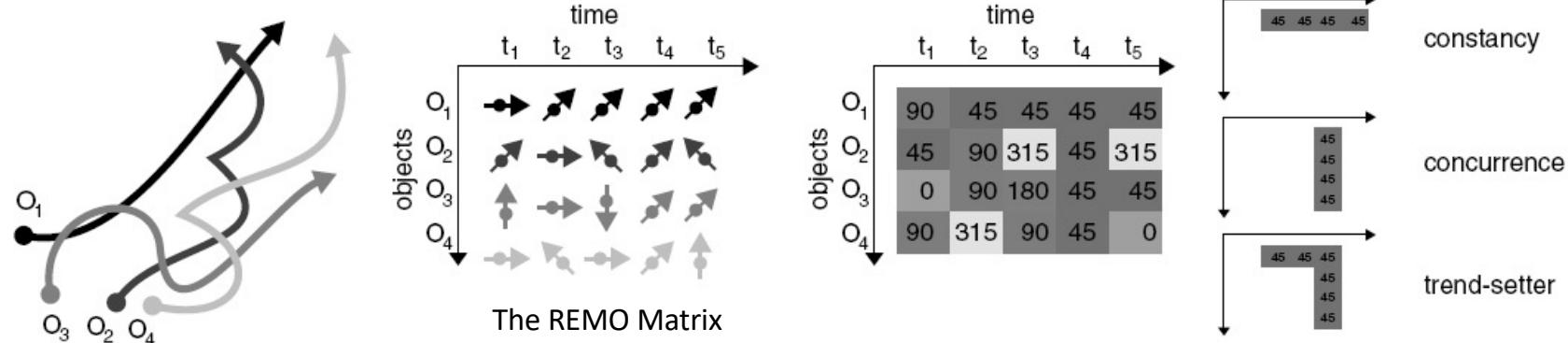
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- Clustering:
  - Group together similar trajectories
  - For each group produce a summary



# Relative Motion Patterns

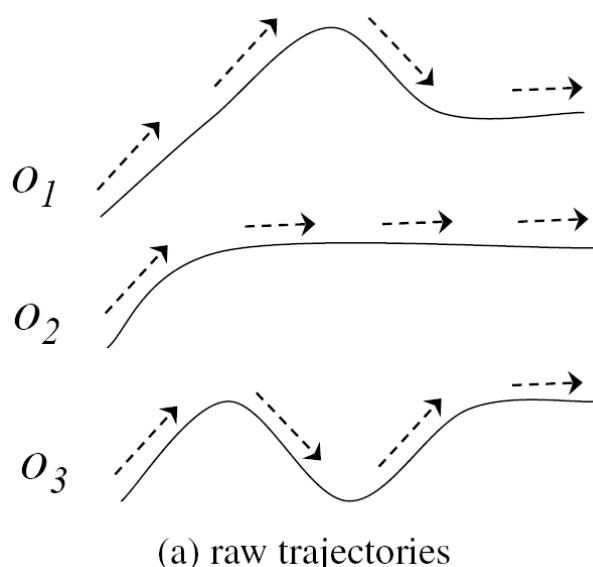
- Identify similar movements in a collection of moving-object trajectories
- Transform raw trajectories into motion attributes (speed, motion azimuth)



Laube, P. et al. GIScience, 2002  
Laube, P. et al., IJGIS, 2005<sup>32</sup>

# Basic Motion Patterns

- Describing motion events, disregarding absolute positions
  - **Constance**: a sequence of equal motion attributes for consecutive times
  - **Concurrence**: the incidence of multiple objects with the same motion attributes
  - **Trendsetter**: a certain motion pattern that is shared by a set of other objects in the future. E.g., “constance” + “concurrence.”



	$t_1$	$t_2$	$t_3$	$t_4$
$O_1$	↗	↗	↘	→
$O_2$	↗	→	→	→
$O_3$	↗	↘	↗	→

*constance*

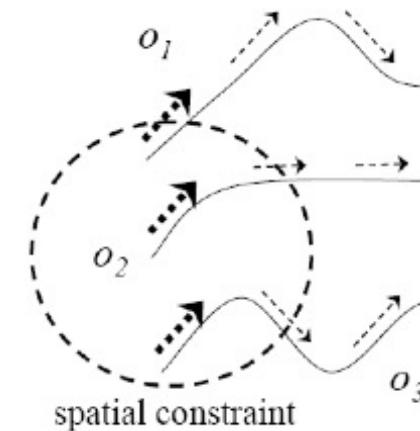
*concurrence*

*trendsetter*

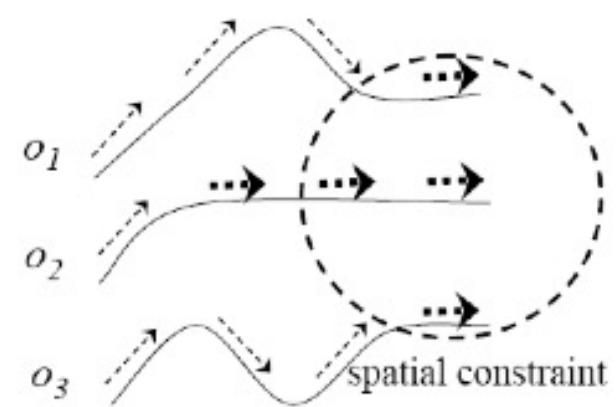
# Relative Motion Patterns

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- *Flock* ( $m > 1, r > 0$ ): At least  $m$  entities are within a circular region of radius  $r$  and they move in the same direction (“constance” + a spatial constraint)



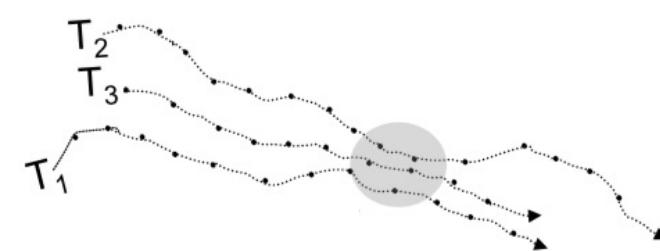
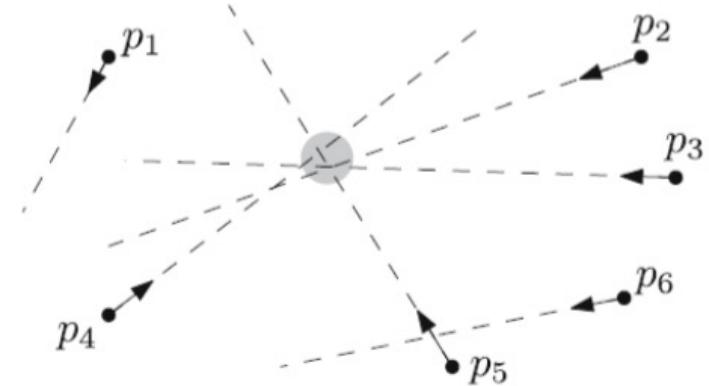
- *Leadership* ( $m > 1, r > 0, s > 0$ ) At least  $m$  entities are within a circular region of radius  $r$ , they move in the same direction, and at least one of the entities was already heading in this direction for at least  $s$  time steps



# Relative Motion Patterns

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- *Convergence* ( $m > 1, r > 0$ ) At least  $m$  entities will pass through the same circular region of radius  $r$  (assuming they keep their direction)
- *Encounter* ( $m > 1, r > 0$ ) At least  $m$  entities will be simultaneously inside the same circular region of radius  $r$  (assuming they keep their speed and direction)



# Disc-Based Trajectory Patterns

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- Circular spatial constraint is considered (Disc-Based)
- Basic relative motion patterns are no longer considered
- Integration of time constraints in pattern

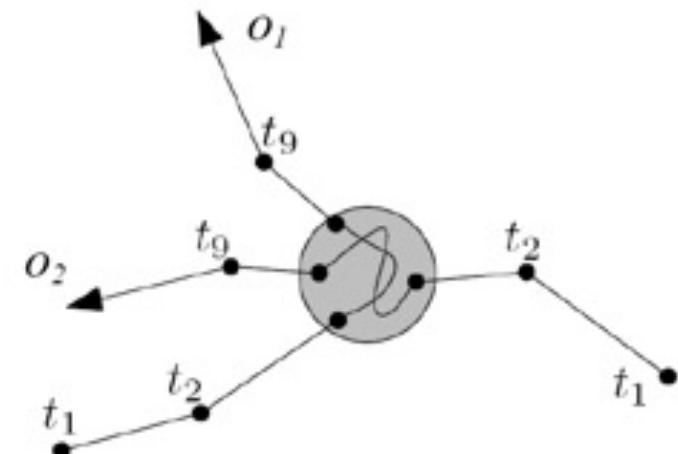
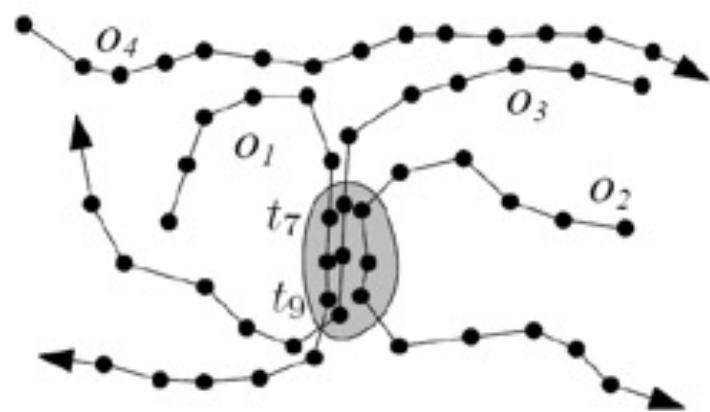


Gudmundsson et al. GIS'06,  
Benkert et al. SAC'07  
Vieira, M. Et al. SIGSPATIAL'09

# Disc-Based Trajectory Patterns

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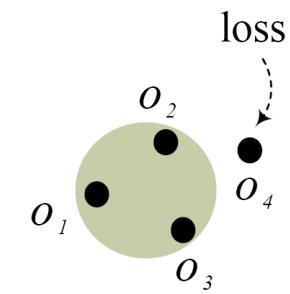
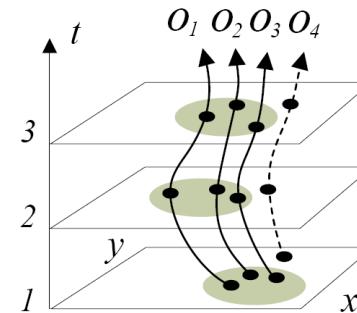
- *Flock* ( $m, k, r$ ): a group of at least  $m$  objects that move together for at least  $k$  consecutive time points, while staying within a disc with radius  $r$
- *Meet* ( $m, k, r$ ): a group of at least  $m$  objects that stay together in a stationary disc with radius  $r$  for at least  $k$  consecutive time points



# Density-Based Trajectory Patterns

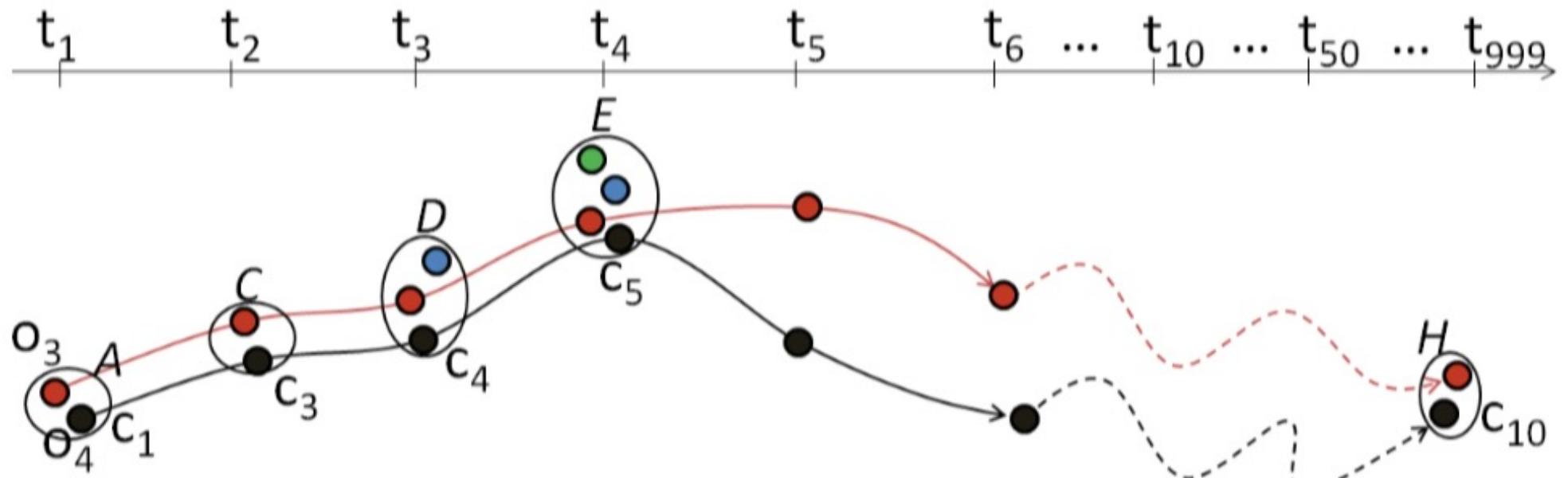
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- The selection of a proper disc size  $r$  is difficult
  - A large  $r$  may capture objects that are intuitively not in the same group
  - A small  $r$  may miss some objects that are intuitively in the same group
- *lossy-flock* problem
- Employ density concepts

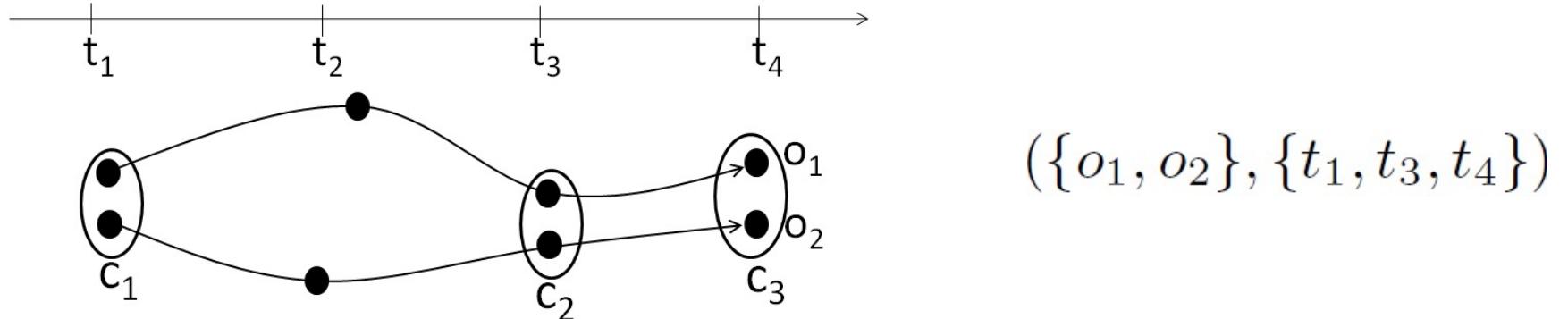


# Swarm

- Swarm : At least  $\mathcal{E}$  objects move together (density-connected objects) during  $min_t$  timestamps



# Swarm – Closed Swarm

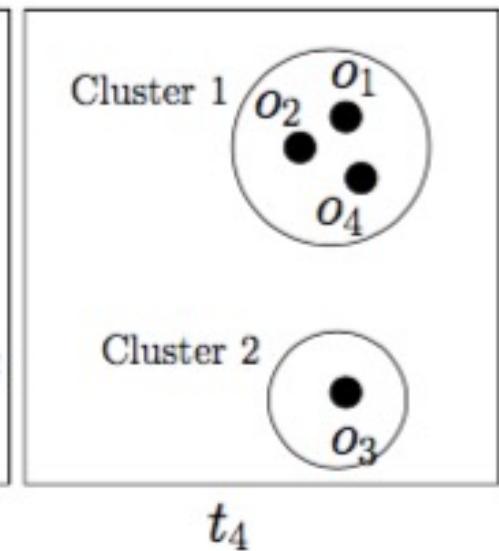
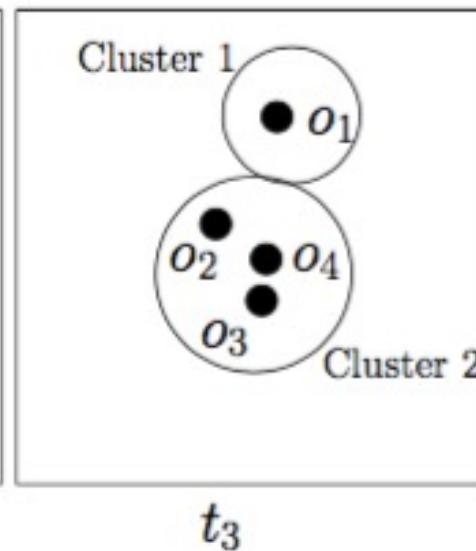
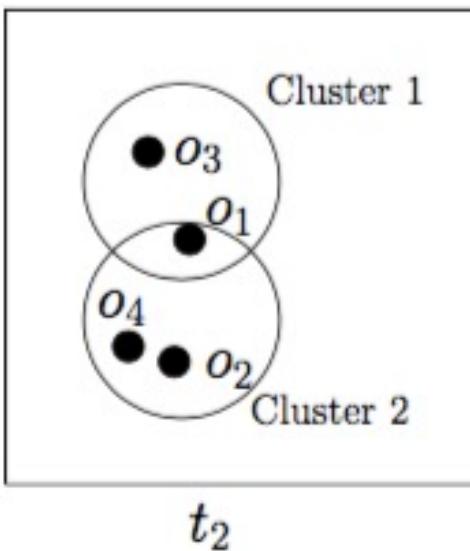
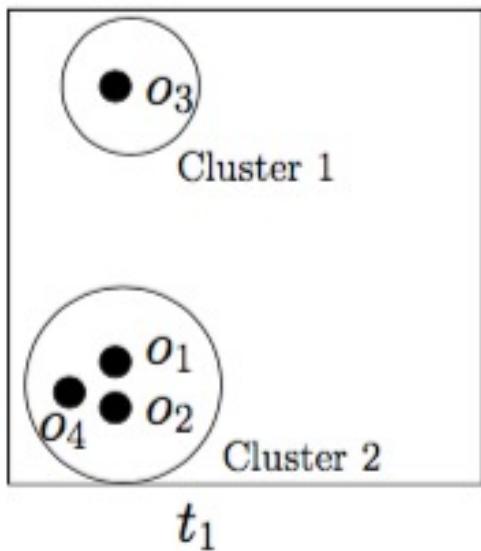


DEFINITION 1. *Swarm* [6]. A pair  $(O, T)$  is a swarm if:

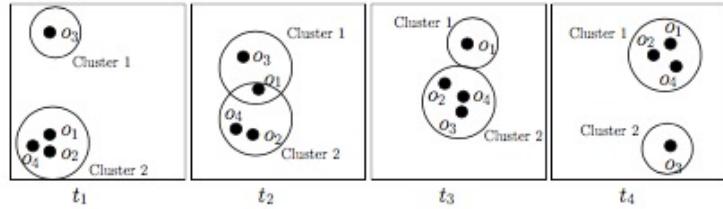
- $$\left\{ \begin{array}{l} (1) : \forall t_{a_i} \in T, \exists c \text{ s.t. } O \subseteq c, \text{ } c \text{ is a cluster.} \\ \text{There is at least one cluster containing} \\ \text{all the objects in } O \text{ at each timestamp in } T. \\ (2) : |O| \geq \varepsilon. \\ \text{There must be at least } \varepsilon \text{ objects.} \\ (3) : |T| \geq \min_t. \\ \text{There must be at least } \min_t \text{ timestamps.} \end{array} \right.$$

# An illustrative Example

- How many Swarms?



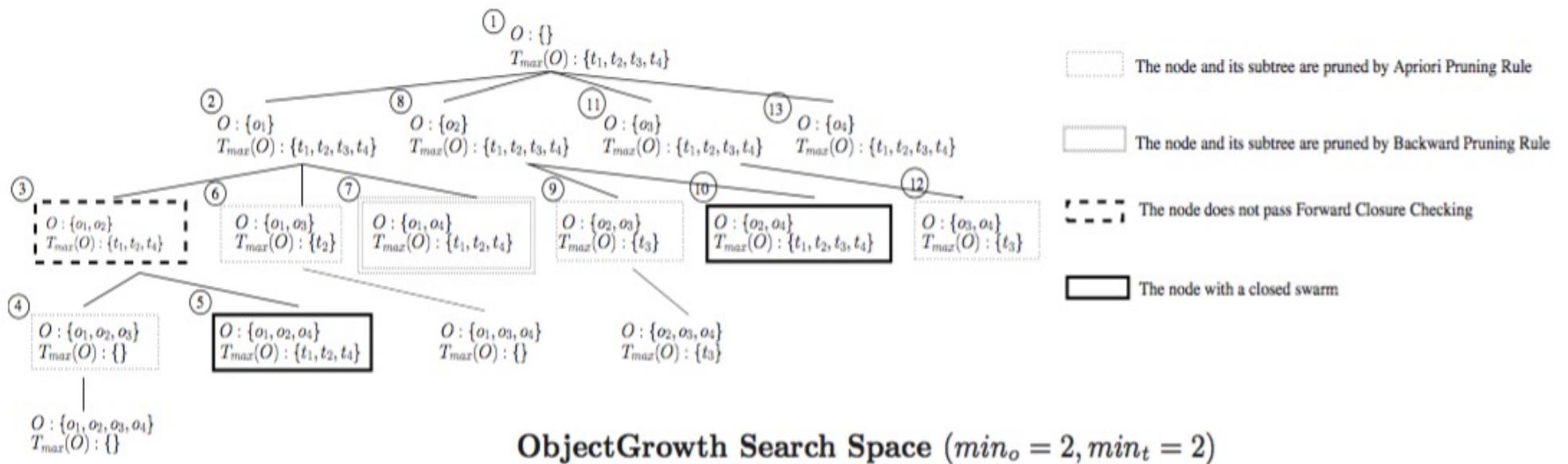
# The Algorithm



2 closed swarms:  $(\{o_2, o_4\}, \{t_1, t_2, t_3, t_4\})$   
 $(\{o_1, o_2, o_4\}, \{t_1, t_2, t_4\})$

A Depth First Search:  
Get the maximal Timeset

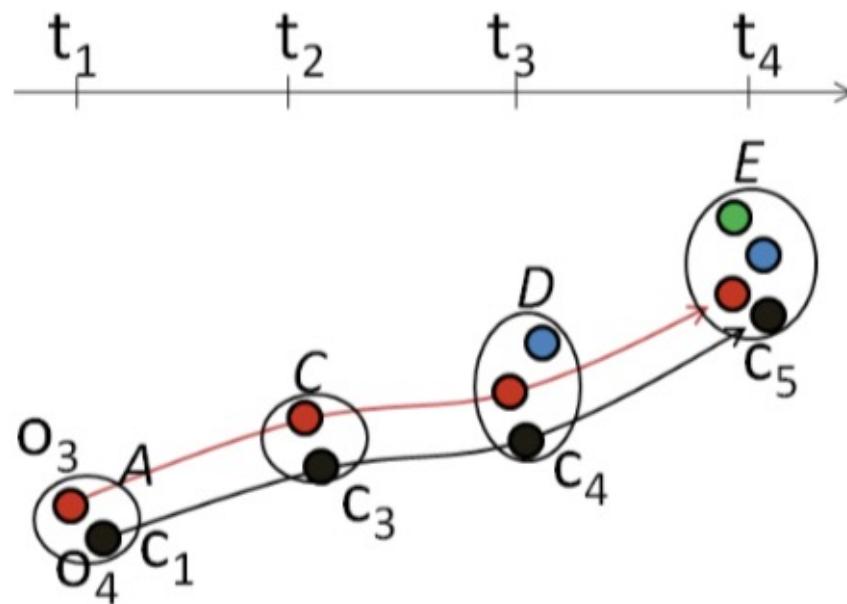
- Pruning Rules:
  - A Priori Pruning
  - Backward Pruning



# Convoy

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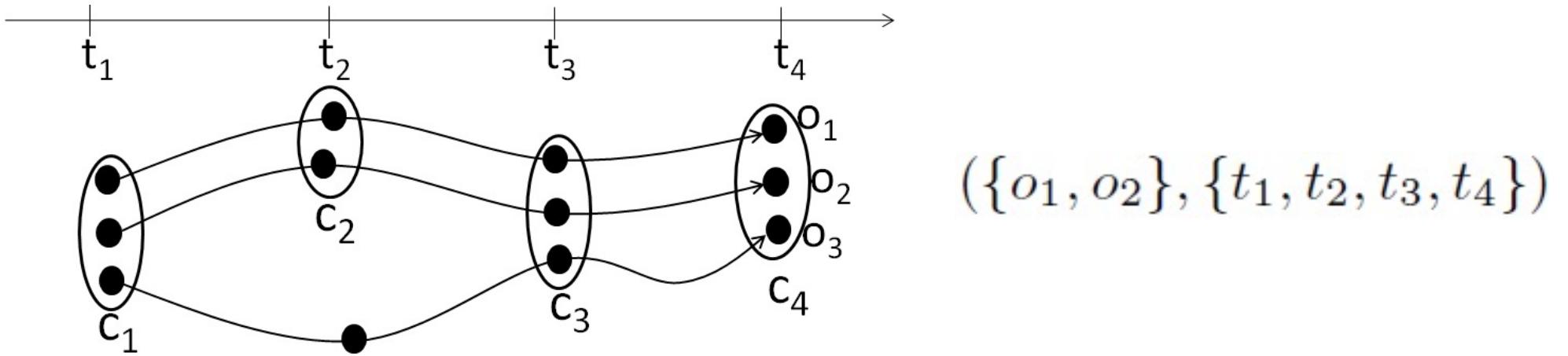
- Convoy: At least  $\mathcal{E}$  objects move together (density-connected objects) during  $\min_t$  consecutive timestamps



Jeung et al. ICDE'08 & VLDB'08

# Convoy

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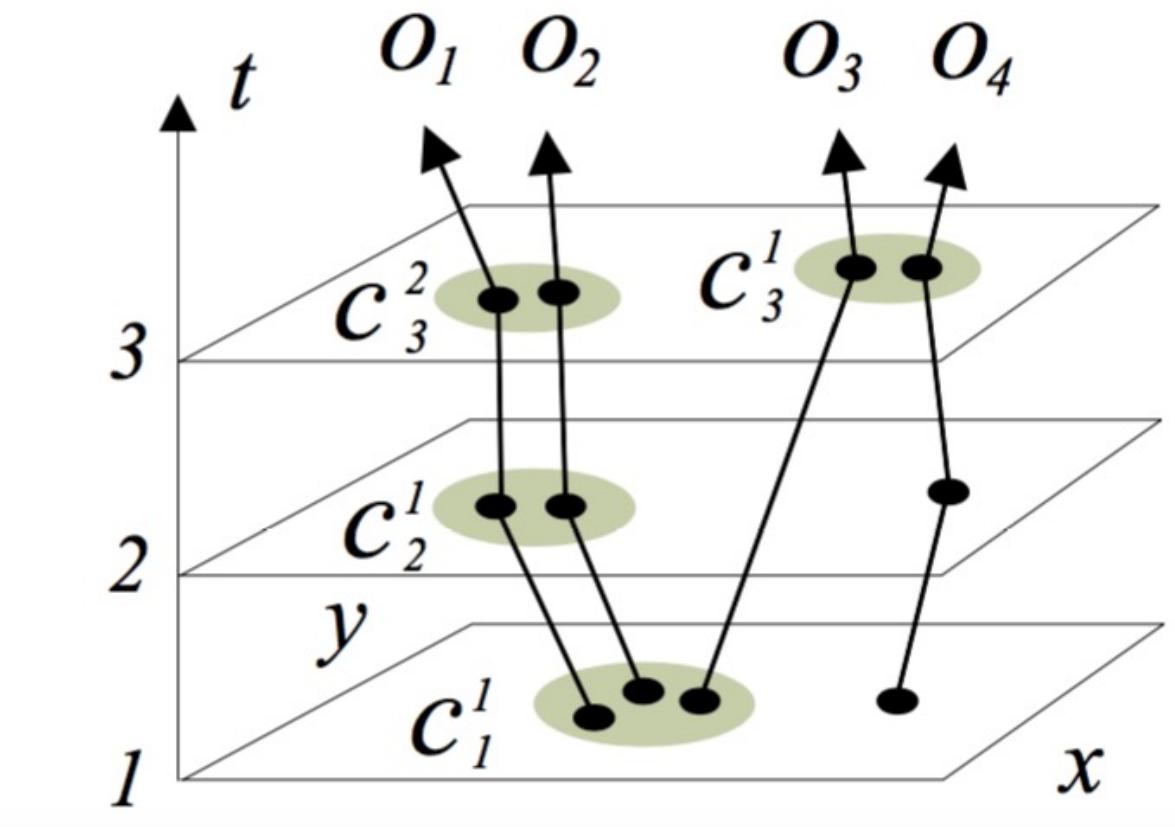
DEFINITION 3. *Convoy [3]. A pair  $(O, T)$ , is a convoy if:*

- $$\left\{ \begin{array}{l} (1) : (O, T) \text{ is a swarm.} \\ (2) : \forall i, 1 \leq i < |T|, t_{a_i}, t_{a_{i+1}} \text{ are consecutive.} \end{array} \right.$$

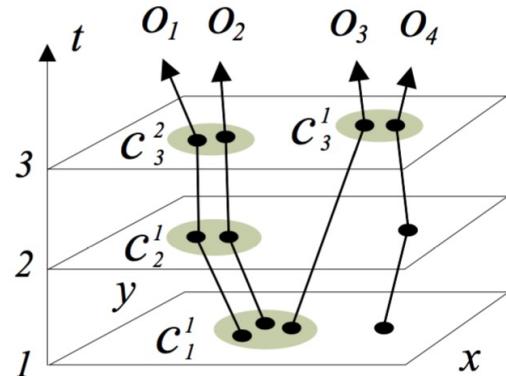
# An illustrative Example

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- How many Convoys?



# The Algorithm



Applying DBSCAN at each timestamp

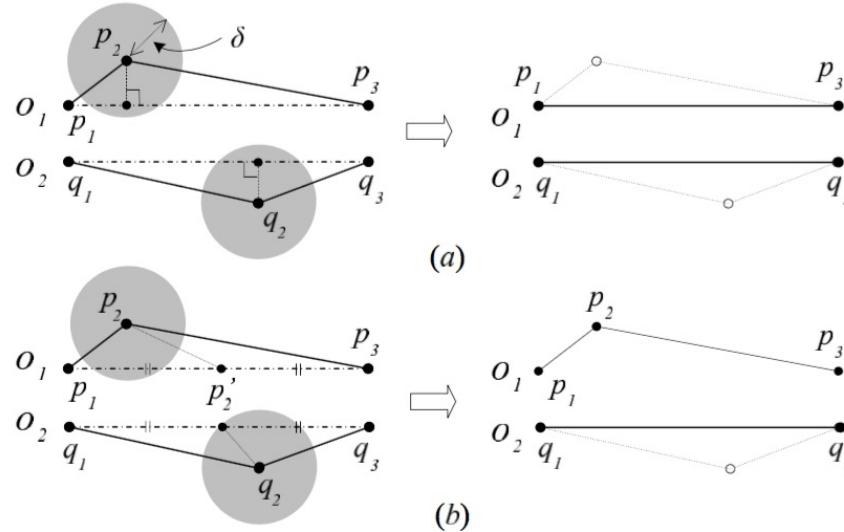
1 convoy:  $(\{o_2, o_3\}, \{t_1, t_3\})$

Timestamp	Clusters	Candidate set $V$
$t_1$	$c_1^1$	$v_1 = c_1^1$
$t_2$	$c_2^1$	$v_1 = c_1^1 \cap c_2^1$
$t_3$	$c_3^1, c_3^2$	$v_1 = c_1^1 \cap c_2^1 \cap c_3^2, v_2 = c_3^1$

$$\begin{aligned}v_1 &= \{o_1, o_2, o_3\} \\v_1 &= \{o_1, o_2\} \\v_1 &= \{o_1, o_2\}\end{aligned}$$

# Algorithm Improvements

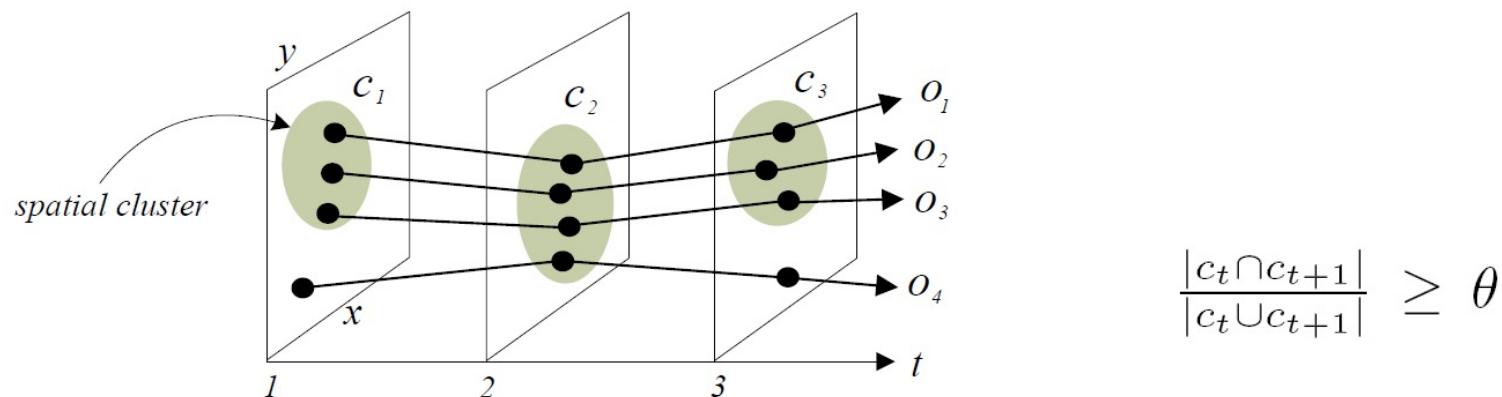
- Reducing the search space:
  - Simplifying the original trajectories



- Apply DBSCAN at each timestamp: CUTS, CUTS+

# Moving Clusters

- Moving Cluster : At least  $\varepsilon$  **common** objects move together (density-connected objects) during  $min_t$  consecutive timestamps



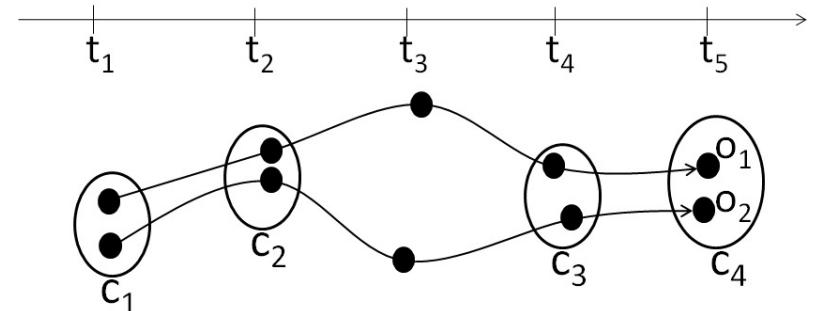
- Convoys + additional condition that they need to share some objects between two consecutive timestamps

# Group Pattern

- Combination of both convoy and closed swarm
  - A closed swarm of disjointed convoys

$$T_S = \{(\{o_1, o_2\}, \{t_1, t_2\}), (\{o_1, o_2\}, \{t_4, t_5\})\}$$

$(\{o_1, o_2\}, T_S)$  is a closed swarm of convoys



**Definition 4.** *Group Pattern*<sup>21</sup>. Given a set of objects  $O$ , a minimum weight threshold  $min_{wei}$ , a set of disjointed convoys  $T_S = \{s_1, s_2, \dots, s_n\}$ , a minimum number of convoys  $min_c$ .  $(O, T_S)$  is a group pattern if:

$$\begin{cases} (1) : (O, T_S) \text{ is a closed swarm w.r.t } min_c. \\ (2) : \frac{\sum_{i=1}^{|T_S|} |s_i|}{|T_{DB}|} \geq min_{wei}. \end{cases}$$

Note that  $min_c$  is only applied for  $T_S$  (i.e.  $|T_S| \geq min_c$ ).

Wang, DKE 2006

# Periodic Patterns

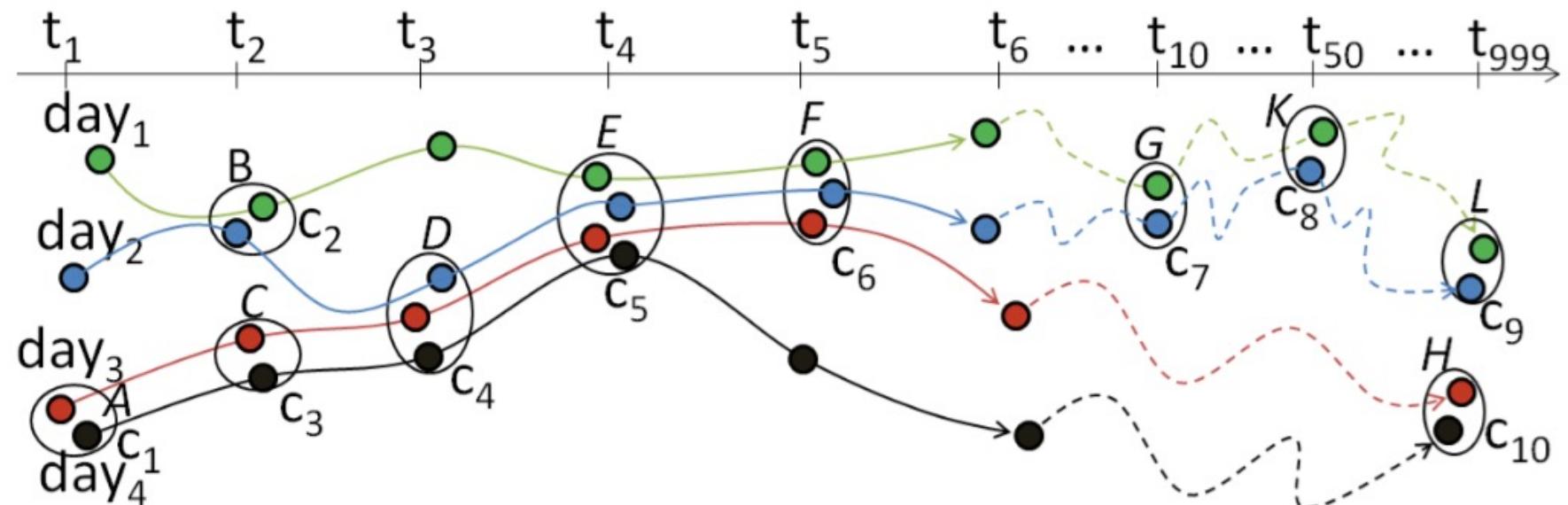
---

- An object follows the same routes (approximately) over regular time intervals
  - People wake up at the same time and generally follow the same route to their work everyday
- Given an object's trajectory including  $N$  time-points,  $T_P$  which is the number of timestamps that a pattern may re-appear.
- The object's trajectory is decomposed into  $N / T_P$  sub-trajectories
- $T_P$  is data dependent
  - $T_P = \text{'a day'}$  in traffic control applications since many vehicles have daily patterns
  - $T_P = \text{'a year'}$  for annual animal migration patterns



# Periodic Patterns

- A periodic pattern is a closed swarm discovery from  $N/T_P$  sub-trajectories.



4 daily sub-trajectories

We extract potential periodic patterns such as  $\{c_1, c_3, c_4, c_5\}$ ,  $\{c_2, c_5, c_6\}$

# Outline

---

- A quick reminder on pattern discovery
- The different kinds of patterns from trajectory
- **Towards a unified approach for extracting trajectories**
  - Cluster Matrix
  - Properties
  - GetMove
  - How to Manage Blocks
  - Lord of the Rings
  - Gradual Spatio-Temporal Rules
  - The top-k informative Patterns
- Deal with and without a spatial component?
- An illustration
- Conclusion



# Motivations

---

- Clustering:
  - Flock, convoy, moving cluster, swarm, closed swarm, k-Stars, ....
- Different approaches to mine them:
  - CuTS\*, ObjectGrowth, CMC, Vg-Growth, ...
  - Complexity, computation cost, time consuming!!!
- *How about unifying algorithm?*
- What happen when new data arriving?
  - *How about incremental algorithm?*



# The main intuition

---

- Very efficient approaches have been defined for extracting a set of association rules
  - Apriori, Closed, Derivable, ...

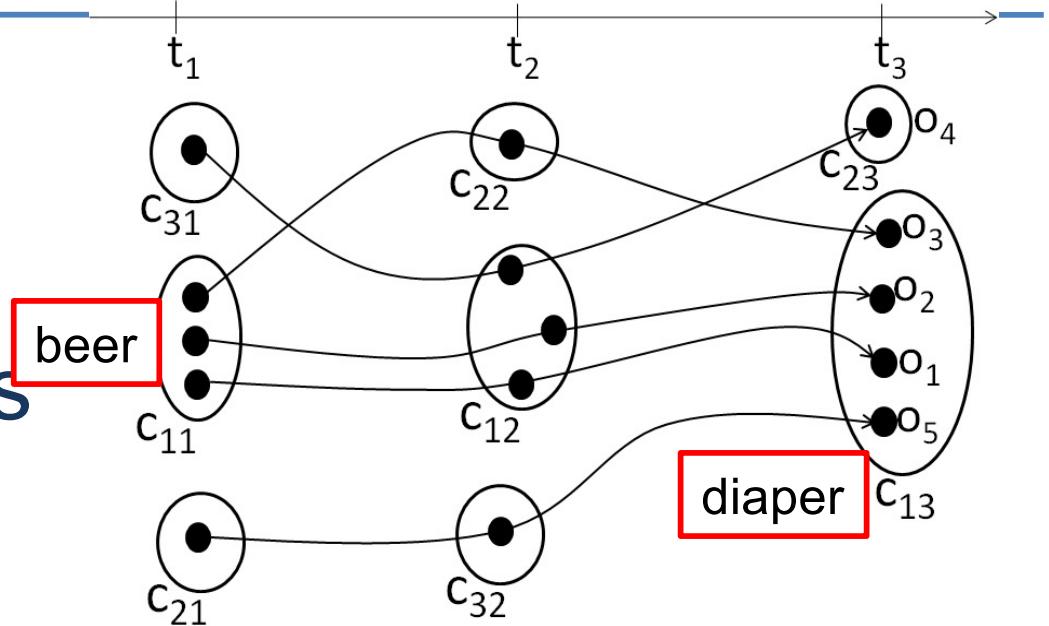
	A	B	C	D	E	F
1	1	0	0	1	0	0
2	1	0	1	0	0	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1

- *Is it possible to adapt a same kind of approach for extracting trajectories?*



# Cluster Matrix

- Patterns:
  - Evolution of clusters
- Objects: transactions
- Clusters: items



Cluster Matrix

$T_{DB}$		$t_1$		$t_2$			$t_3$		
Clusters	$C_{DB}$	beer	$c_{21}$	$c_{31}$	$c_{12}$	$c_{22}$	$c_{32}$	diaper	$c_{23}$
$O_{DB}$	$o_1$	1			1			1	
	$o_2$	1			1			1	
	$o_3$	1				1		1	
	$o_4$			1	1				1
	$o_5$		1				1	1	

# Frequent Closed Itemset from Cluster Matrix

Frequent Closed Itemset  $\Upsilon = \{c_{t_{a_1}}, c_{t_{a_2}}, \dots, c_{t_{a_p}}\}$

Life Time  $T_\Upsilon = \{t_{a_1}, t_{a_2}, \dots, t_{a_p}\}$

Set of Objects  $O(\Upsilon) = \bigcap_{i=1}^p c_{t_{a_i}}$  Support  $\sigma(\Upsilon)$

$\Upsilon = \{c_{11}, c_{13}\}$  ( $O(\Upsilon) = \{o_1, o_2, o_3\}$ ,  $T_\Upsilon = \{t_1, t_3\}$ )

$\sigma(\Upsilon) = 3$

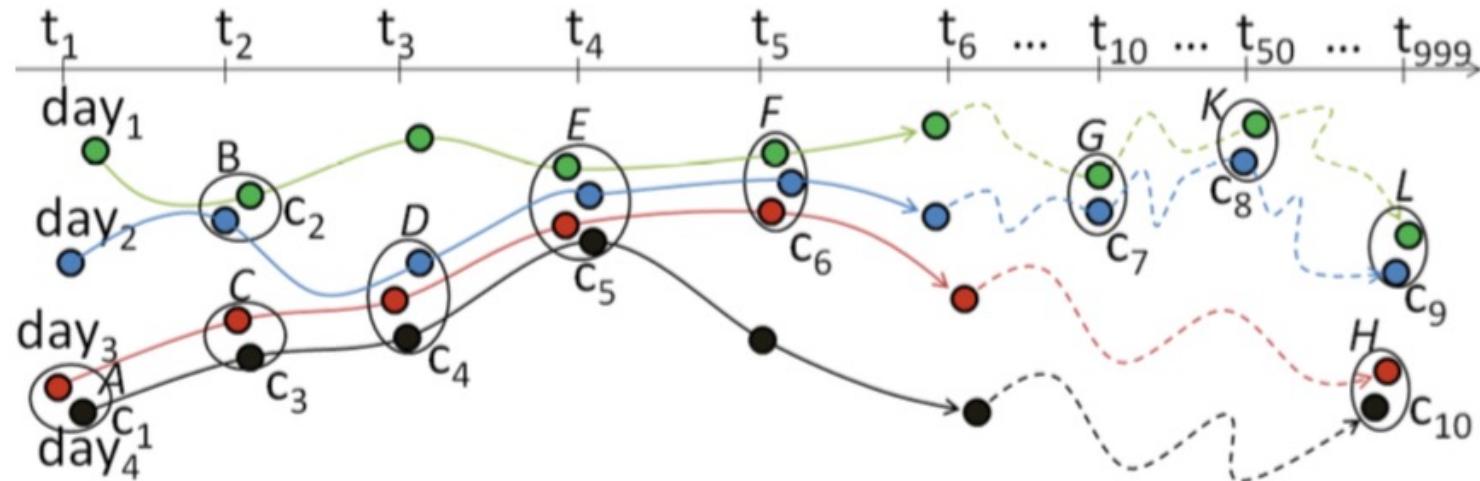
$|\Upsilon| = 2$

Cluster Matrix

		$T_{DB}$		$t_1$		$t_2$		$t_3$	
Clusters $C_{DB}$		$c_{11}$	$c_{21}$	$c_{31}$	$c_{12}$	$c_{22}$	$c_{32}$	$c_{13}$	$c_{23}$
$O_{DB}$	$o_1$	1			1			1	
	$o_2$	1			1			1	
	$o_3$	1				1		1	
	$o_4$			1	1				1
	$o_5$		1				1	1	56



# Periodic Cluster Matrix



$T_{db}$	$t_1$	$t_2$		$t_3$	$t_4$	$t_5$	$t_{10}$	$t_{50}$	$t_{999}$	
Clusters $C_{db}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$ST_{db}$	$st_1$		1			1	1	1	1	
	$st_2$		1		1	1	1	1	1	
	$st_3$	1		1	1	1				1
	$st_4$	1		1	1	1				1

# The main intuition (following...)

---

- We are now able to extract itemsets corresponding to a set of clusters occurring over time
- Not trajectories yet!
- *What about properties on Itemsets?*



# Property for Swarm

---

DEFINITION 1. *Swarm* [6]. A pair  $(O, T)$  is a swarm if:

- $$\left\{ \begin{array}{l} (1) : \forall t_{a_i} \in T, \exists c \text{ s.t. } O \subseteq c, \text{ } c \text{ is a cluster.} \\ \quad \text{There is at least one cluster containing} \\ \quad \text{all the objects in } O \text{ at each timestamp in } T. \\ (2) : |O| \geq \varepsilon. \\ \quad \text{There must be at least } \varepsilon \text{ objects.} \\ (3) : |T| \geq min_t. \\ \quad \text{There must be at least } min_t \text{ timestamps.} \end{array} \right.$$

PROPERTY 1. *Swarm*. Given a frequent itemset  $\Upsilon = \{c_{t_{a_1}}, c_{t_{a_2}}, \dots, c_{t_{a_p}}\}$ .  $(O(\Upsilon), T_\Upsilon)$  is a swarm if, and only if:

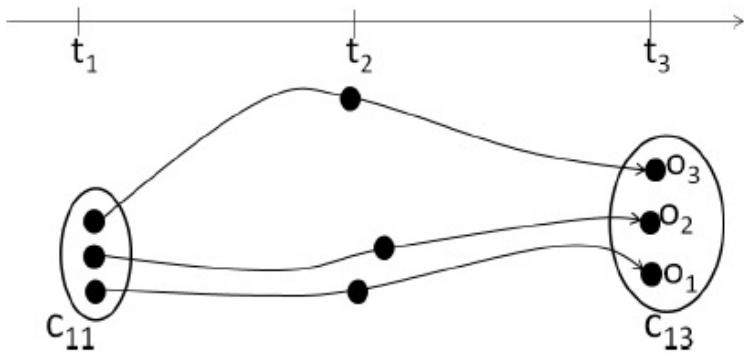
$$\left\{ \begin{array}{l} (1) : \sigma(\Upsilon) \geq \varepsilon \\ (2) : |\Upsilon| \geq min_t \end{array} \right.$$



# Swarm

PROPERTY 1. *Swarm.* Given a frequent itemset  $\Upsilon = \{c_{t_{a_1}}, c_{t_{a_2}}, \dots, c_{t_{a_p}}\}$ .  $(O(\Upsilon), T_\Upsilon)$  is a *swarm* if, and only if:

$$\left\{ \begin{array}{l} (1) : \sigma(\Upsilon) \geq \varepsilon \\ (2) : |\Upsilon| \geq min_t \end{array} \right.$$



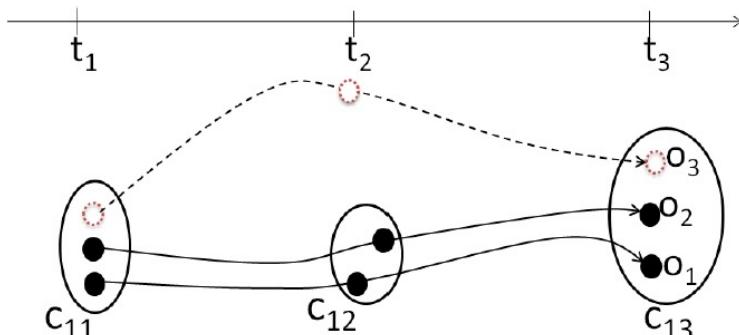
Cluster Matrix									
T <sub>DB</sub>		t <sub>1</sub>			t <sub>2</sub>			t <sub>3</sub>	
Clusters C <sub>DB</sub>	O <sub>DB</sub>	c <sub>11</sub>	c <sub>21</sub>	c <sub>31</sub>	c <sub>12</sub>	c <sub>22</sub>	c <sub>32</sub>	c <sub>13</sub>	c <sub>23</sub>
O <sub>DB</sub>	o <sub>1</sub>	1			1			1	
	o <sub>2</sub>	1			1			1	
	o <sub>3</sub>	1				1		1	
	o <sub>4</sub>			1	1				1
	o <sub>5</sub>		1				1	1	

$\Upsilon = \{c_{11}, c_{13}\}$  ( $O(\Upsilon) = \{o_1, o_2, o_3\}$ ,  $T_\Upsilon = \{t_1, t_3\}$ )  
 $\sigma(\Upsilon) = 3 > \varepsilon$  and  $|\Upsilon| = 2 \geq min_t$

# Convoy

PROPERTY 3. *Convoy.* Given a frequent itemset  $\Upsilon = \{c_{t_{a_1}}, c_{t_{a_2}}, \dots, c_{t_{a_p}}\}$ .  $(O(\Upsilon), T_\Upsilon)$  is a *convoy* if and only if:

- $\left\{ \begin{array}{l} (1) : (O(\Upsilon), T_\Upsilon) \text{ is a swarm.} \\ (2) : \forall j, 1 \leq j < p : t_{a_j}, t_{a_{j+1}} \text{ are consecutive.} \end{array} \right.$



		Cluster Matrix								
		T <sub>DB</sub>		t <sub>1</sub>		t <sub>2</sub>		t <sub>3</sub>		
		Clusters C <sub>DB</sub>	c <sub>11</sub>	c <sub>21</sub>	c <sub>31</sub>	c <sub>12</sub>	c <sub>22</sub>	c <sub>32</sub>	c <sub>13</sub>	c <sub>23</sub>
O <sub>DB</sub>	o <sub>1</sub>		1				1			1
	o <sub>2</sub>		1				1			1
	o <sub>3</sub>		1					1		1
	o <sub>4</sub>				1	1				1
	o <sub>5</sub>			1				1	1	

$$\begin{aligned} \Upsilon &= \{c_{11}, c_{12}, c_{13}\} \\ (O(\Upsilon) &= \{o_1, o_2\}, T_\Upsilon = \{t_1, t_2, t_3\}) \end{aligned}$$

# Properties

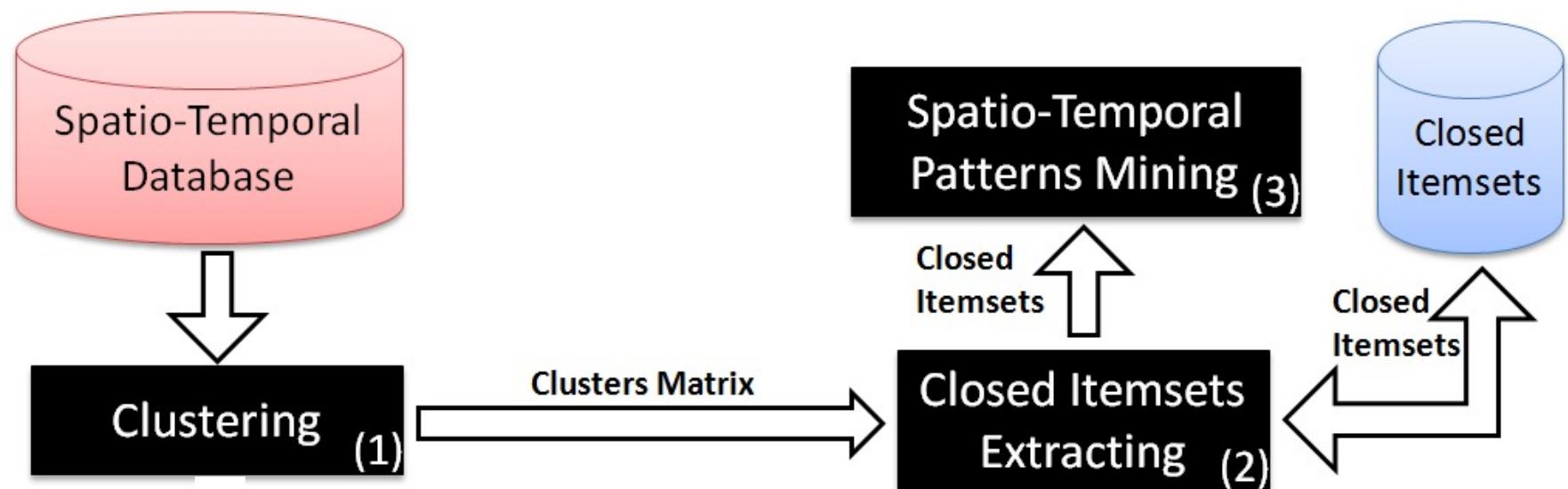
---

- In the same way it is possible to define properties for:  
*Swarm, Closed Swarm, Convoy, Moving Cluster, Periodic Pattern, ...*
- We are now able to extract trajectories!



# The Main Process

---



# GeT\_Move – Pruning Rule

---

- Spatio-temporal patterns:
  - Clusters (items) must belong to different timestamps
  - Items which form FCIs must be in different timestamps
- FCIs include more than 1 item in the same timestamp will be discarded

Cluster Matrix

$T_{DB}$		$t_1$		$t_2$		$t_3$	
Clusters $C_{DB}$		$c_{11}$	$c_{21}$	$c_{31}$	$c_{12}$	$c_{22}$	$c_{32}$
$O_{DB}$	$o_1$	1			1		1
	$o_2$	1			1		1
	$o_3$	1				1	1
	$o_4$			1	1		1
	$o_5$		1			1	1



# GeT\_Move

---

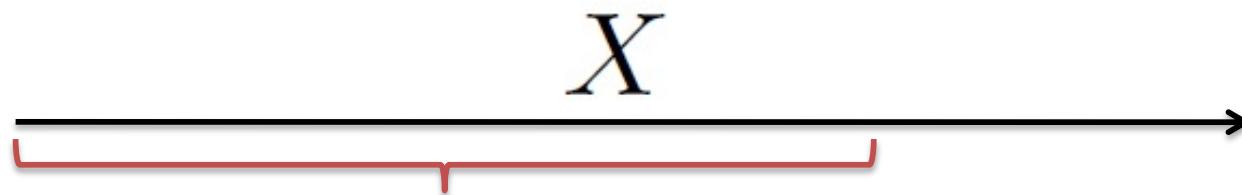
- Frequent closed itemset mining algorithm to extract FCIs:  
LCM Algorithm (Linear time Closed itemset Miner)
- Extract patterns from FCIs

$$\overbrace{|X| \geq min_t}^X : \text{Closed Swarm}$$


# GeT\_Move

---

- Frequent closed itemset mining algorithm to extract FCIs:  
LCM Algorithm (Linear time Closed itemset Miner)
- Extract patterns from FCIs



*Consecutiveness, object correctness, min<sub>t</sub>, Convoy*

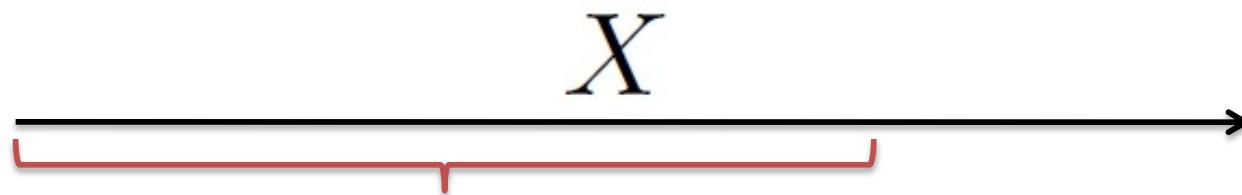
Set of disjointed convoys is a **Group Pattern**



# GeT\_Move

---

- Frequent closed itemset mining algorithm to extract FCIs:  
LCM Algorithm (Linear time Closed itemset Miner)
- Extract patterns from FCIs



*Consecutiveness, integrity proportion*  
Moving Cluster



# Two remaining problems

---

- Association rule mining algorithms have some problems when dealing with a huge number of items, i.e. long transactions!
- *What happens with new data?*

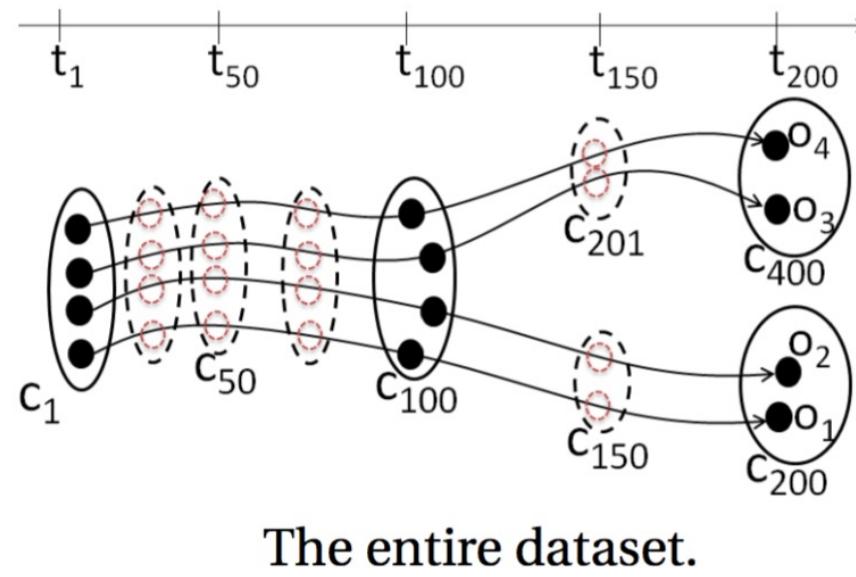


# Towards an Incremental Approach

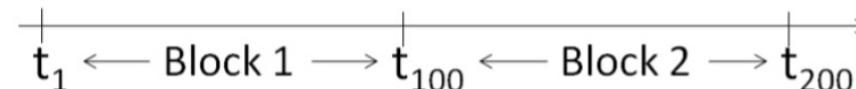
- Main idea: “Compress the dataset”
  - Shorten the transactions by splitting the trajectory matrix into short intervals called Blocks
  - Apply FCI mining on blocks (local FCI)

*Maximal size of  
Itemset on the dataset=200*

*Maximal size of itemset  
in the dataset=100*



The entire dataset.



# Block

## Cluster Matrix

$t_1 - t_{100}$	$t_{101} - t_{200}$	$t_{201} - t_{300}$

block<sub>1</sub>

FCI

Set of frequent itemsets

block<sub>2</sub>

FCI

Set of frequent itemsets

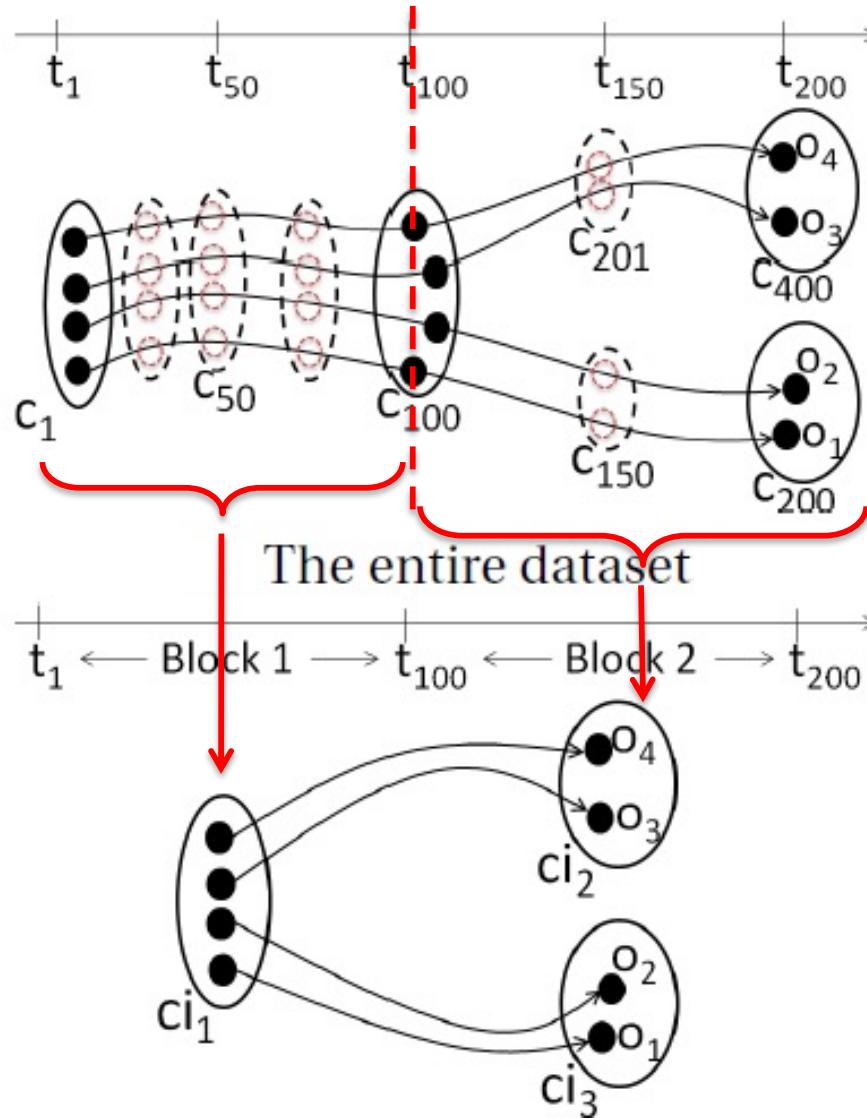
block<sub>3</sub>

FCI

Set of frequent itemsets

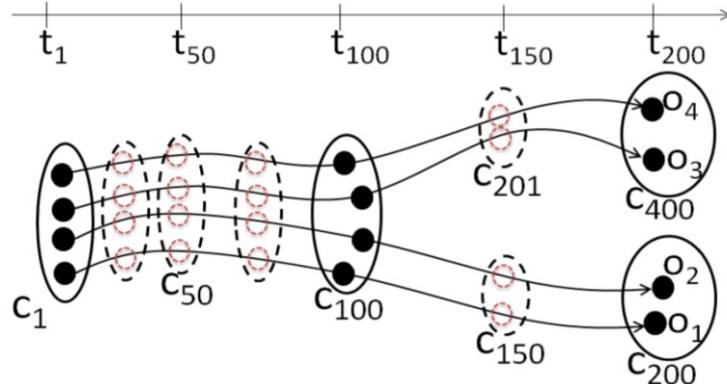


# Incremental GeT\_Move

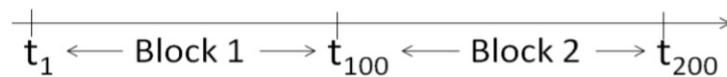


Data after applying frequent closed itemsets mining  
on Blocks

# Closed Itemset Matrix

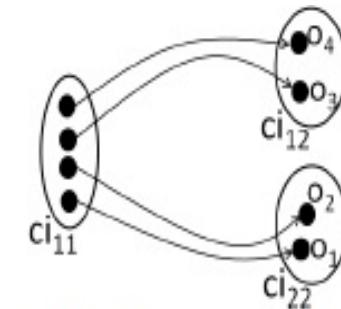


The entire dataset.



Closed Itemset Matrix.

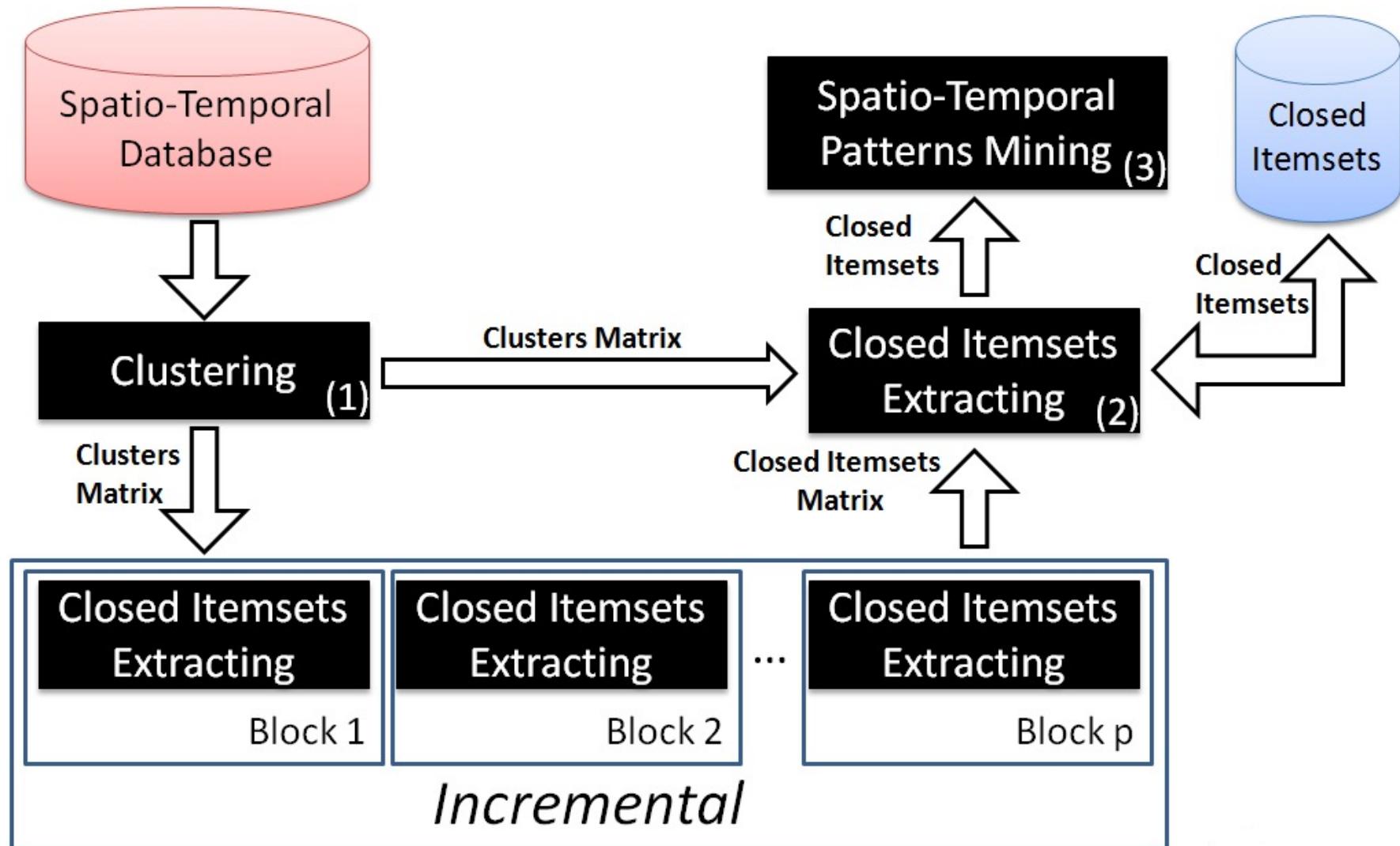
Block B		$b_1$	$b_2$
Frequent Closed Itemsets CI		$ci_1$	$ci_2$
$O_{db}$	$o_1$	1	1
	$o_2$	1	1
	$o_3$	1	1
	$o_4$	1	1



Data after applying frequent closed itemsets mining on Blocks.

**Definition 8.** *Closed Itemset Matrix (CIM).* Closed itemset matrix is a cluster matrix with some differences as follows: 1) Timestamp t now becomes a block b, 2) Item c is a frequent closed itemset ci.

# The Main Process



# Experimental Results

- Datasets:

	#objects	#timestamps
<b>Swainsoni</b>	43	4,425
<b>Buffalo</b>	165	3,000
<b>Synthetic*</b>	500	10,000
<b>Synthetic 2</b>	50,000	10,000

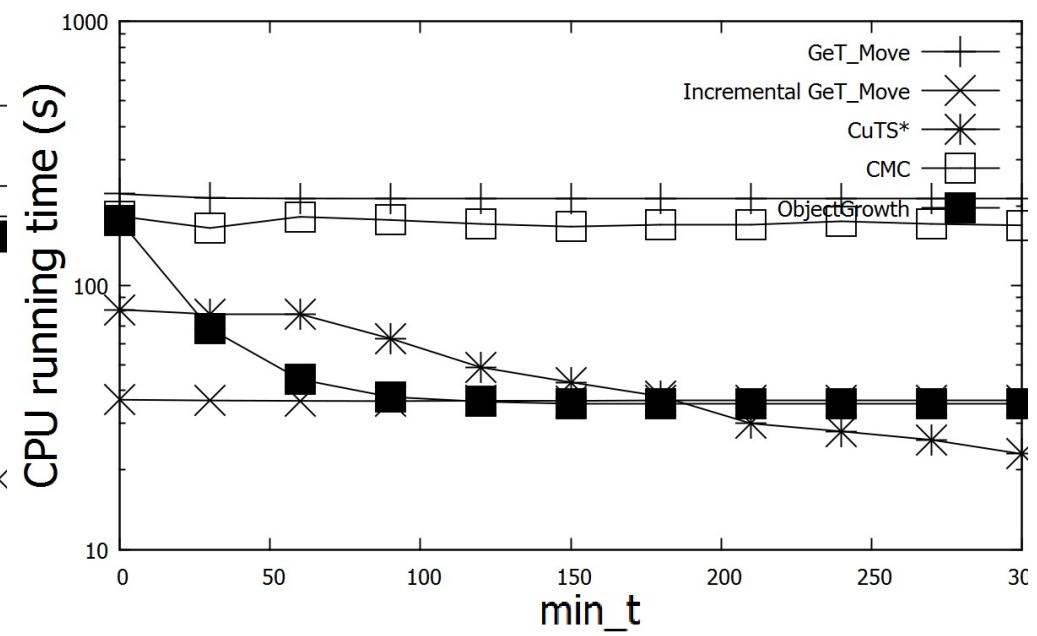
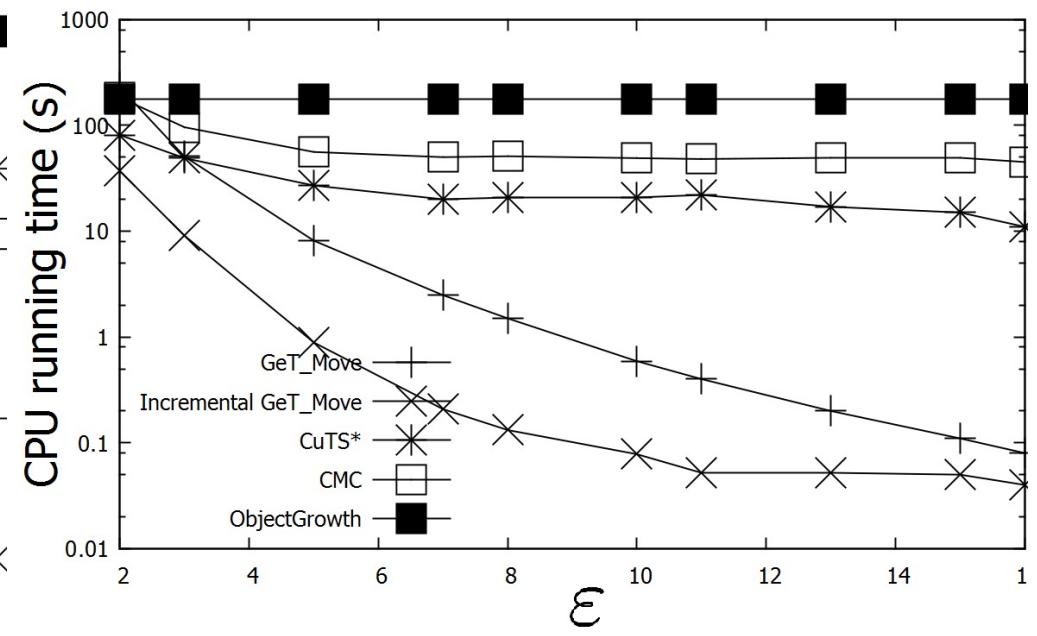
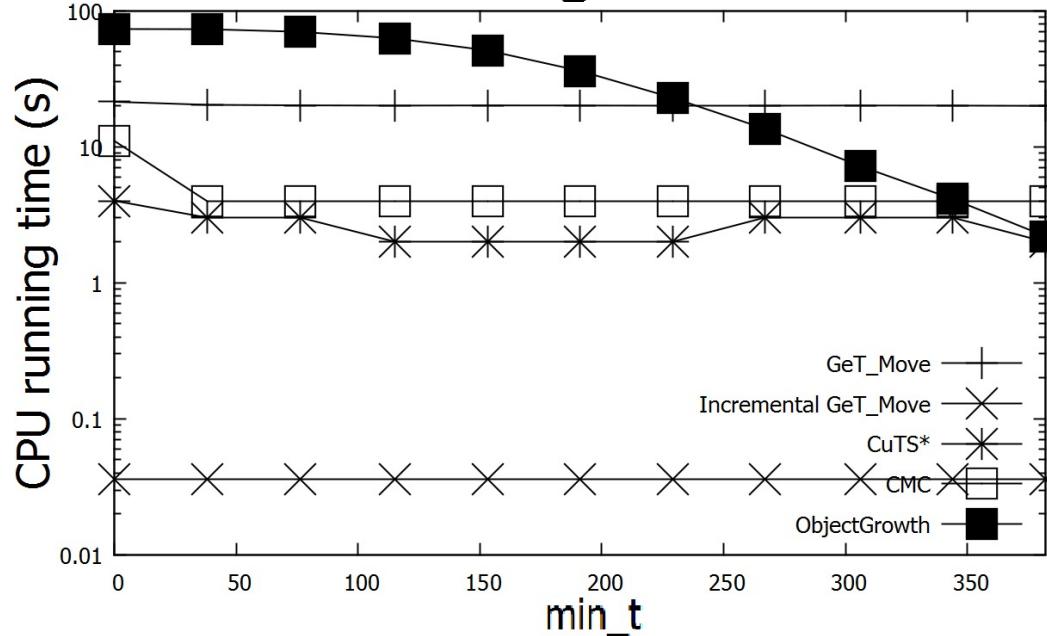
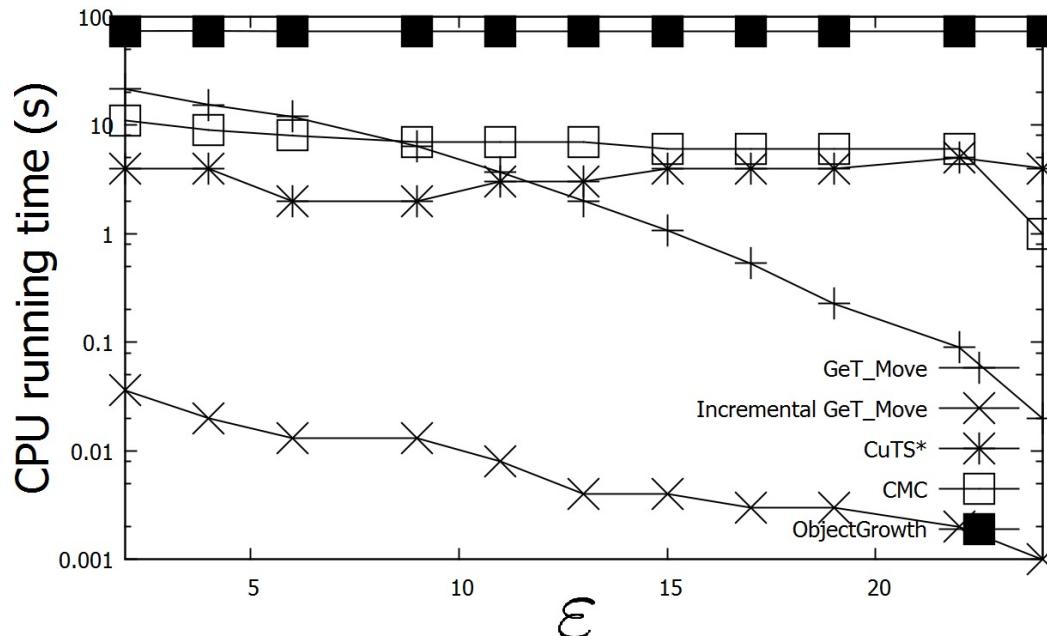
- Competitive algorithms:

	<b>CMC</b>	<b>CuTS*</b>	<b>ObjectGrowth</b>	<b>Vg-Growth</b>	<b><i>Incremental GeT_Move</i></b>
<b>Convoys</b>	X	X			X
<b>Closed Swarms</b>			X		X
<b>Group Patterns</b>				X	X

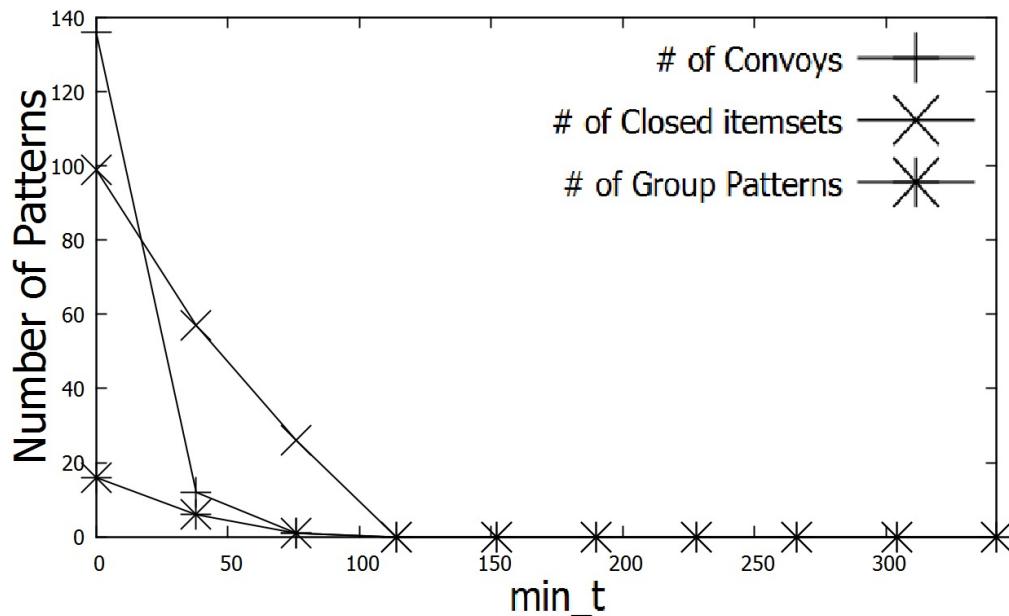


\* <http://iapg.jade-hs.de/personen/brinkhoff/generator/>

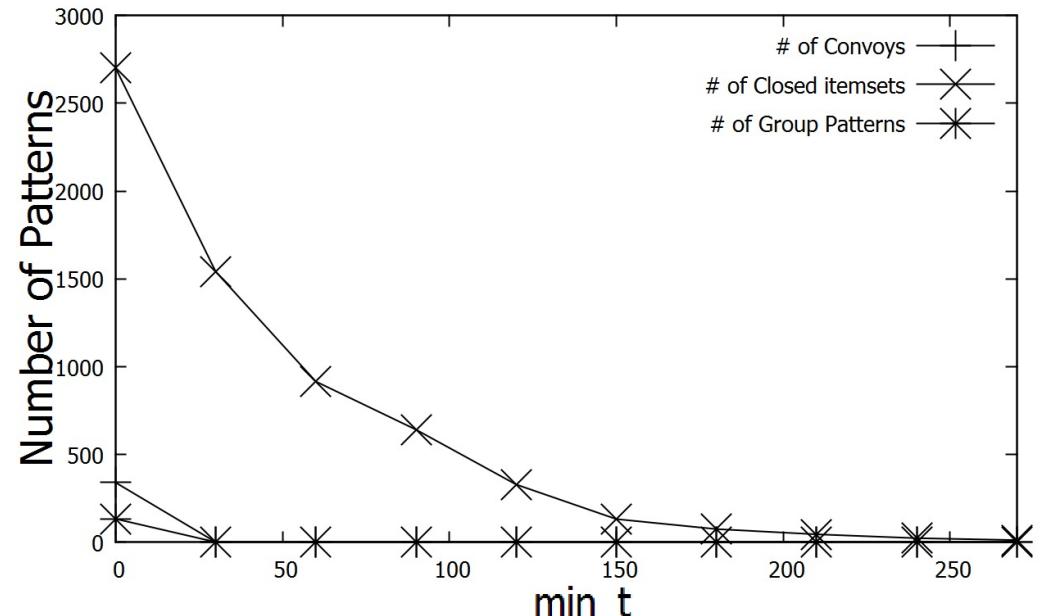
# Swainsoni - Buffalo



# Swainsoni - Buffalo

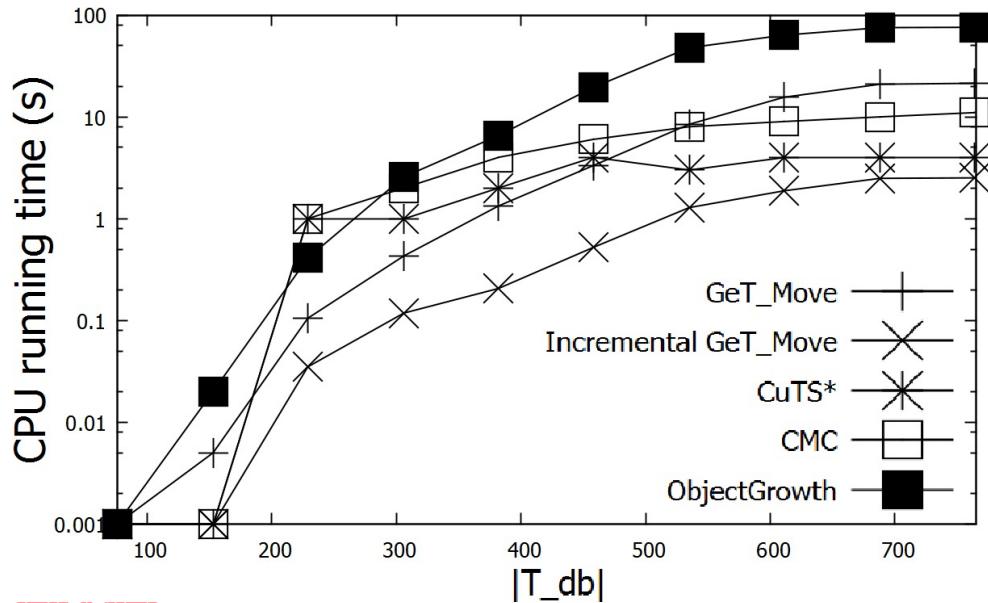
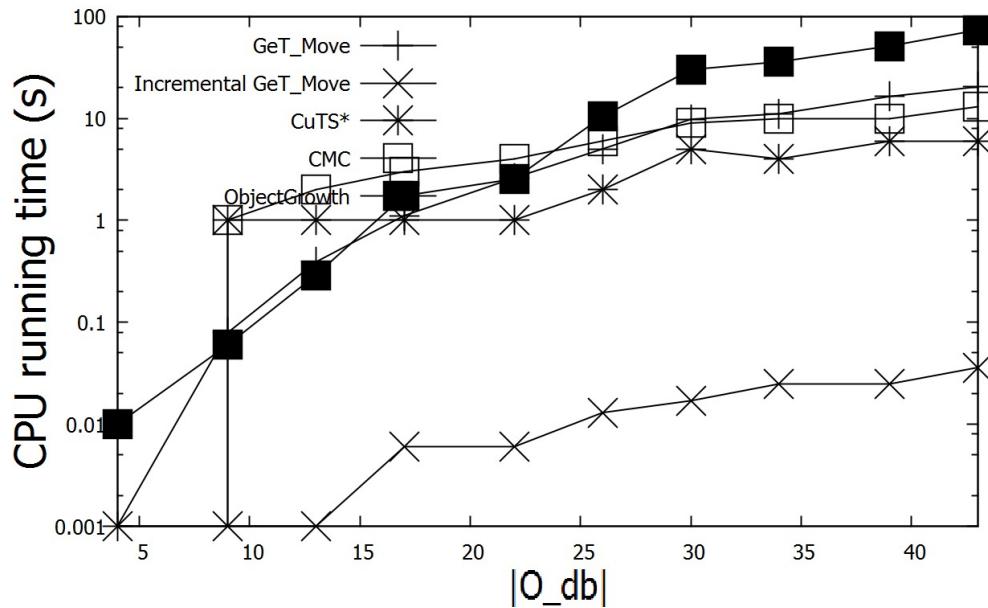


Swainsoni

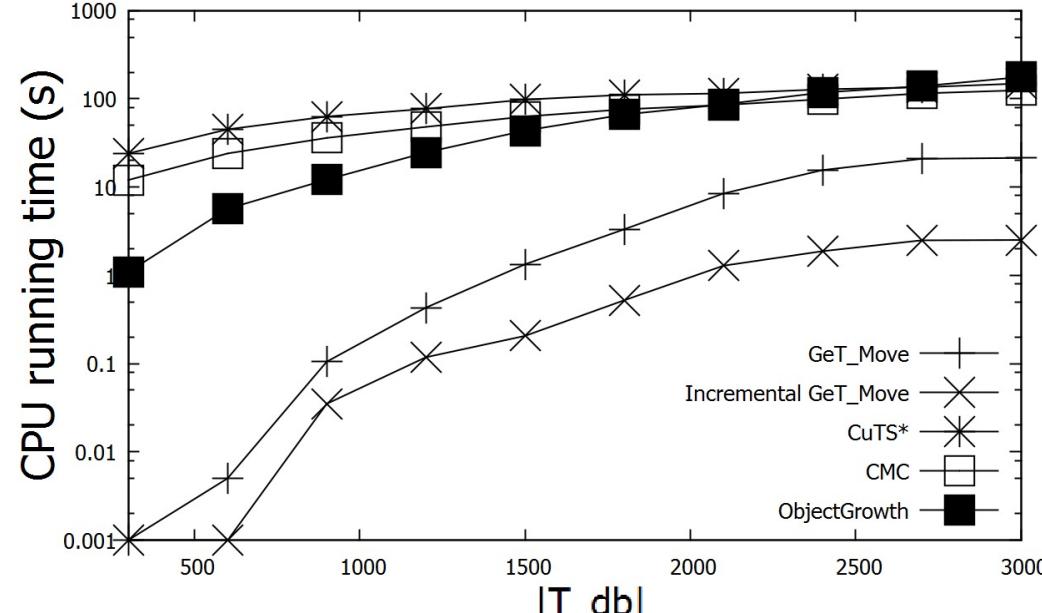
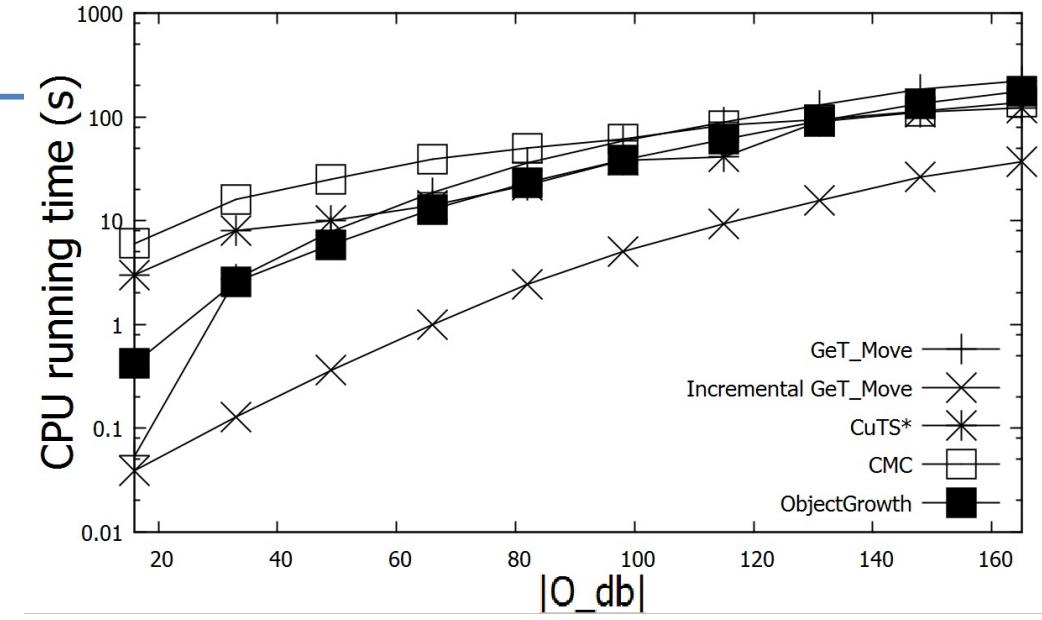


Buffalo

# Swainsoni - Buffalo

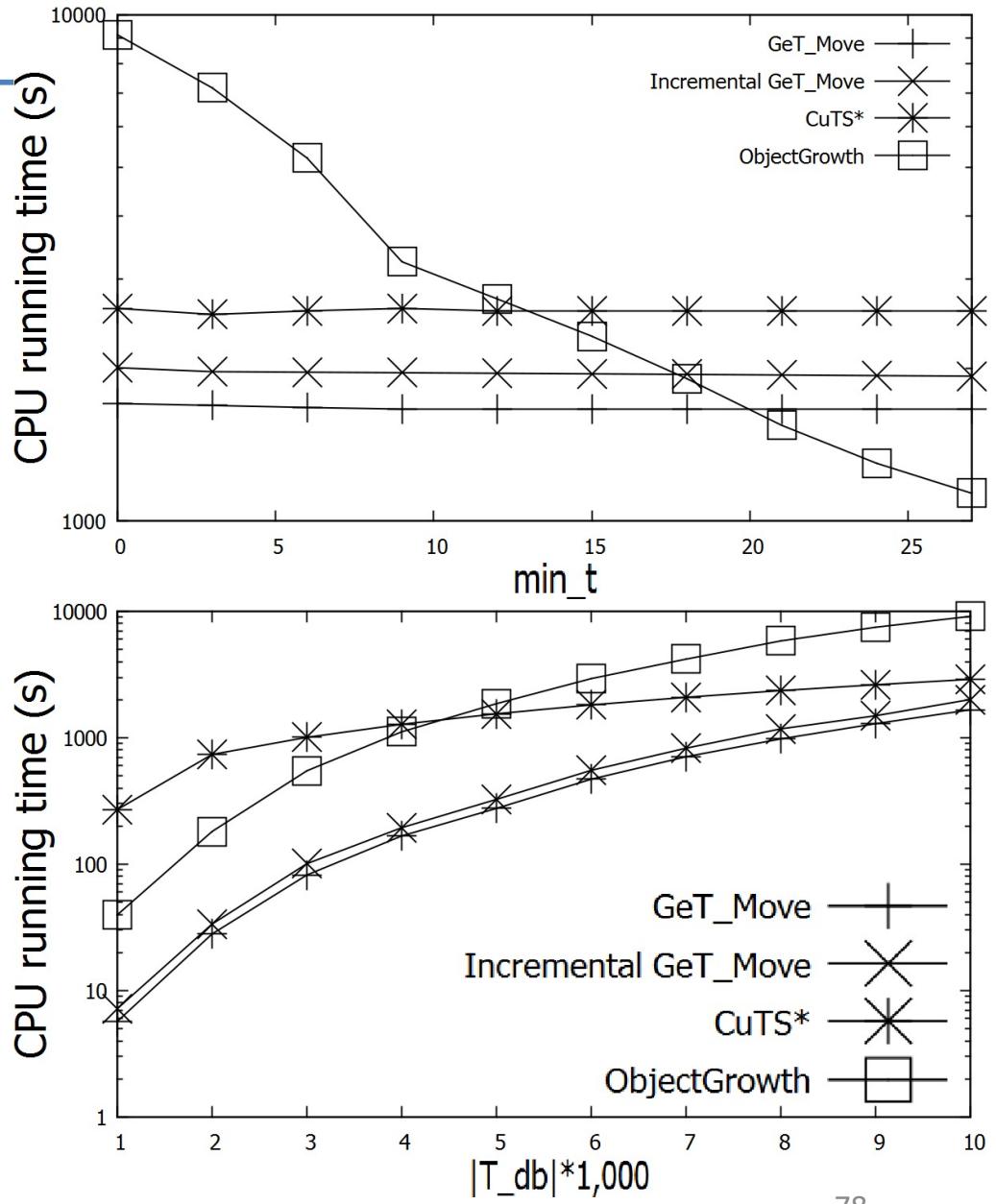
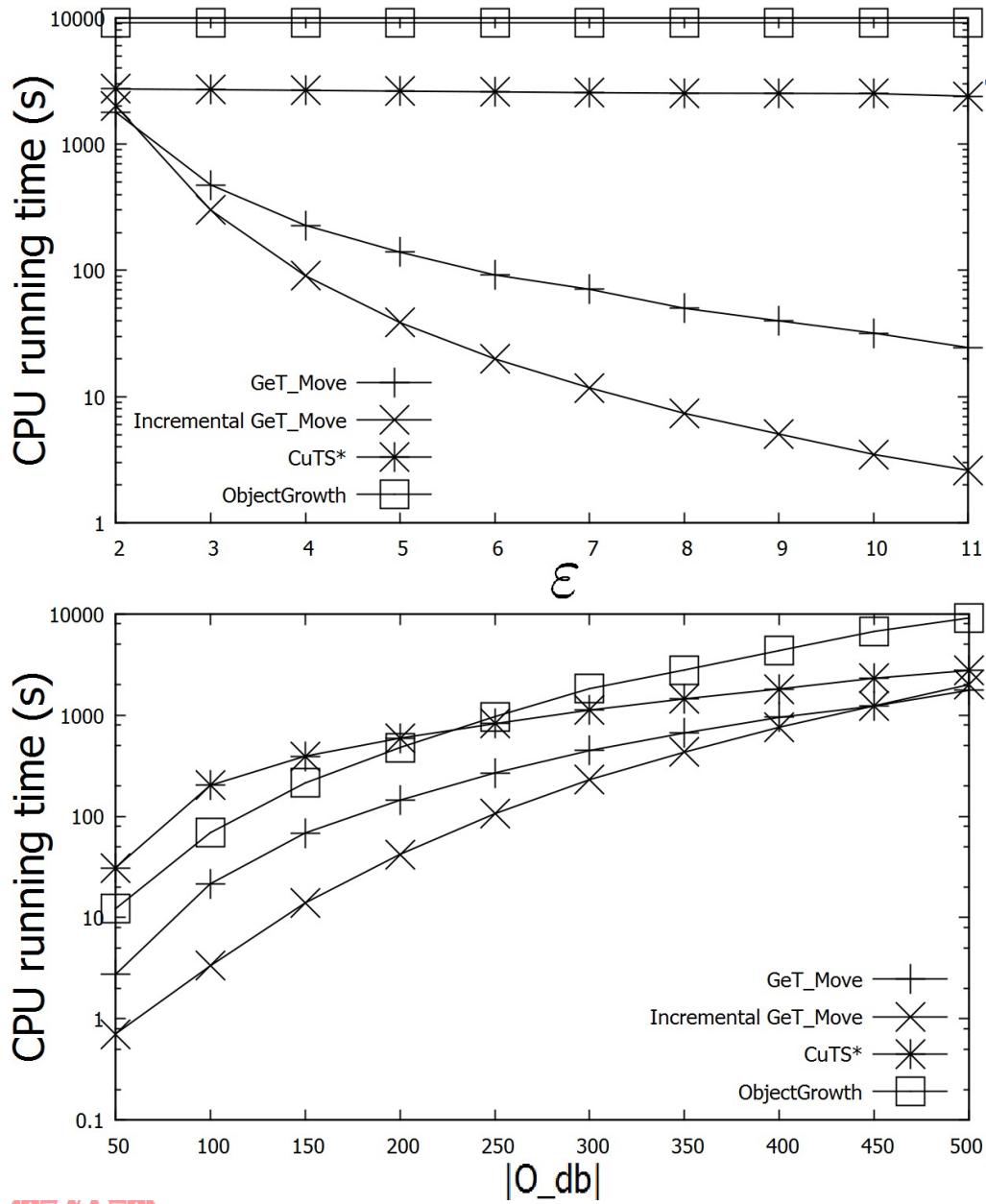


Swainsoni



Buffalo

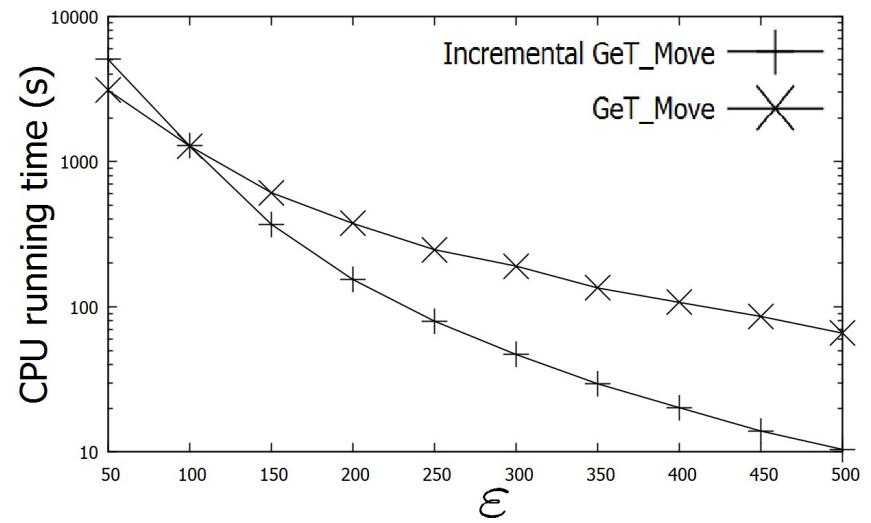
# Synthetic Data



# Scalability

---

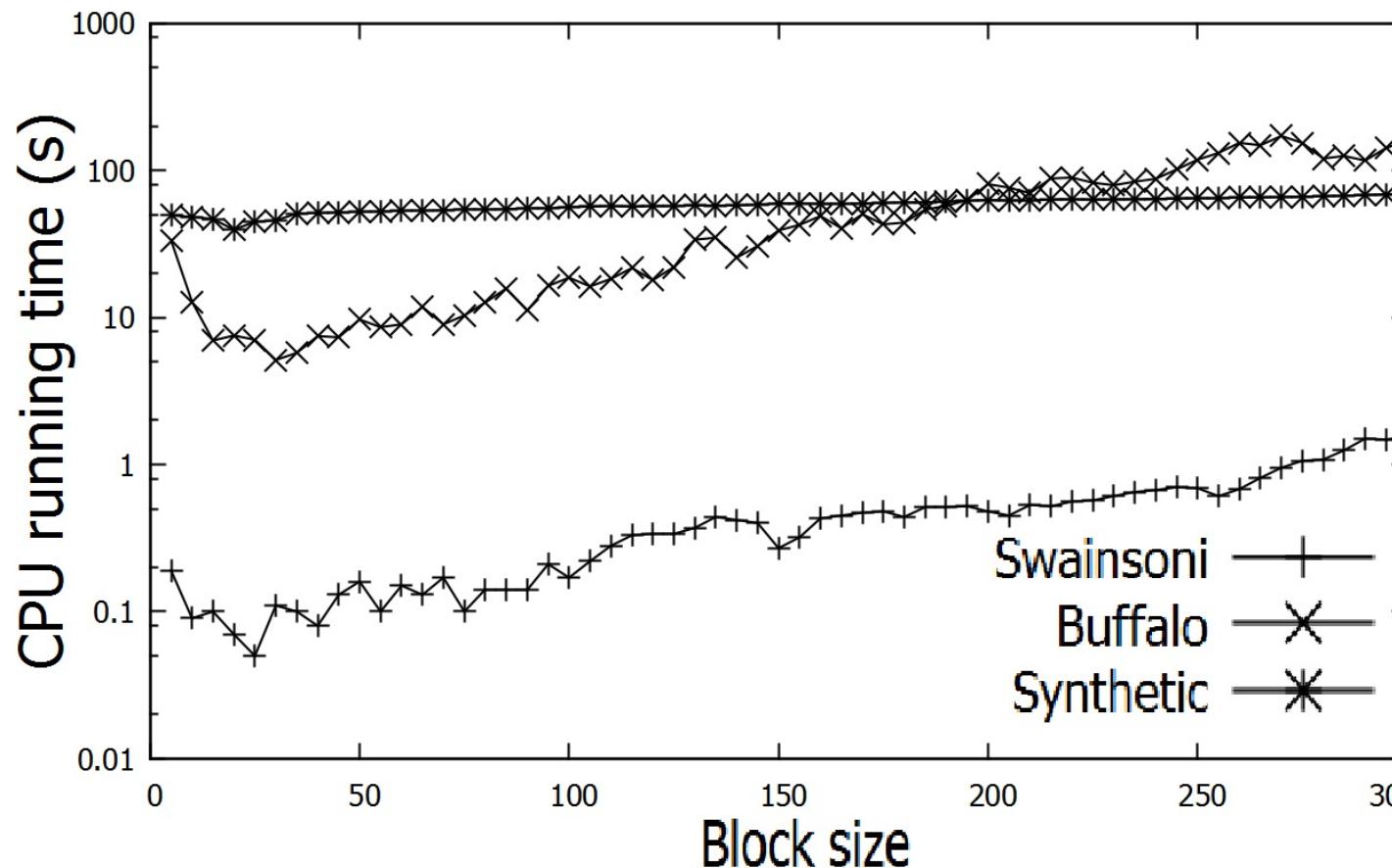
- Synthetic data:
  - 50,000 objects in 10,000 timestamps
  - 500 million locations in total
  - CMC and CuTS\* stop due to a lack of memory capacity after processing 300 million locations
  - ObjectGrowth cannot provide the results after 1day running



# Block Size

---

- Different block sizes:
  - Range 20-30.



# Effectiveness



One of discovered closed swarm

One of discovered convoy



# Effectiveness



One of discovered group pattern

# Towards a Parameter Free

---

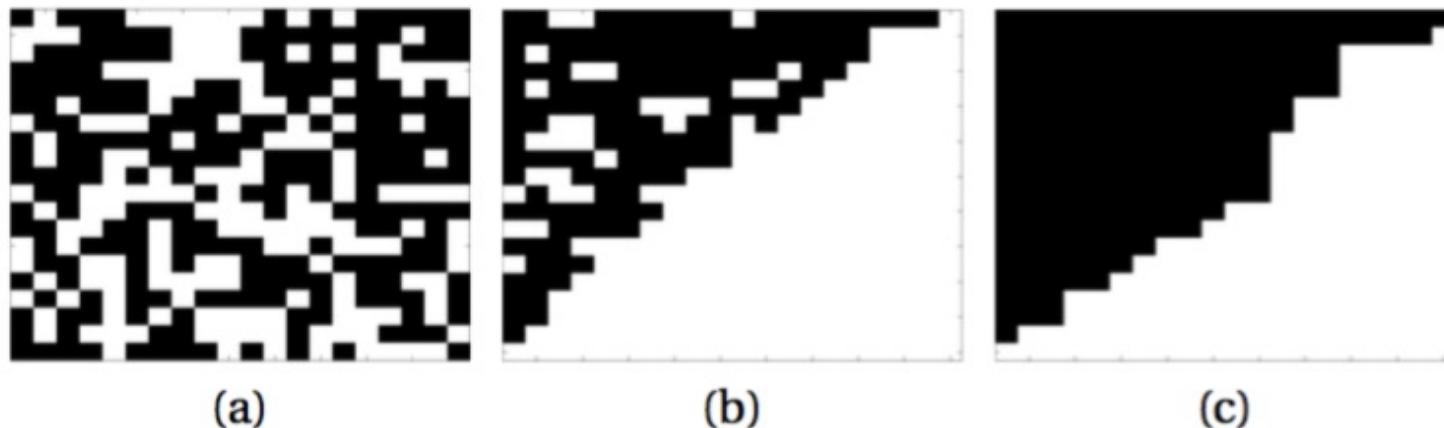
- How to specify the optimal size of blocks?



# Reordering the dataset

---

- LCM is very efficient for dense data
- Reorganizing the data

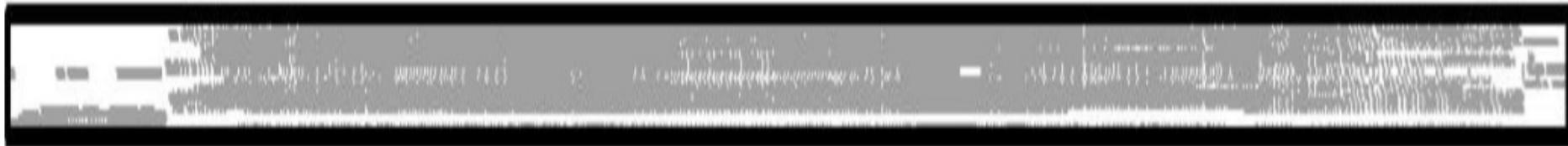


Examples of non-nested , almost nested, fully nested datasets [37]. Black = 1, white = 0. (a) Original, (b) Almost nested, (c) Fully nested.

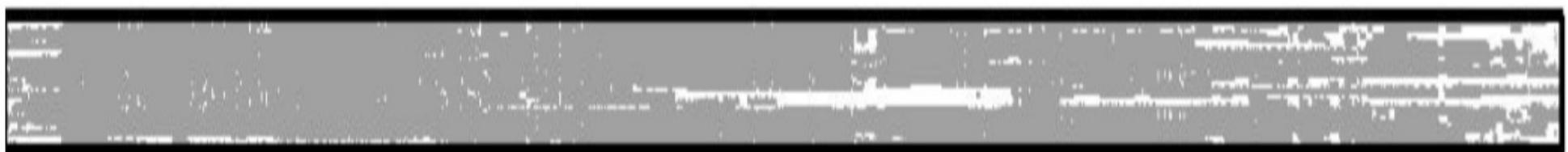
H. Mannila, E. Terzi. *Nestedness and Segmented Nestedness*. In KDD'07

# The Original Distributions

---



Original Swainsoni cluster matrix.



Original Buffalo cluster matrix.



Original Synthetic cluster matrix.

# An Example of Nested

---



(a) Original Swainsoni cluster matrix.



(b) Nested Swainsoni cluster matrix.



(c) Original Buffalo cluster matrix.



(d) Nested Buffalo cluster matrix.



(e) Original Synthetic cluster matrix.



(f) Nested Synthetic cluster matrix.

Dataset	Matrix fill	#Nested blocks	avg.length
Swainsoni	17.8%	102	4.52
Buffalo	7.2%	602	2.894
Synthetic	0.1%	8	2.00

# Reordering the dataset

---

- New definition of cluster matrix

**Definition 9.** Fully nested cluster matrix (resp. block). An  $n \times m$  0-1 block  $b$  is fully nested if for any two column  $r_i$  and  $r_{i+1}$  ( $r_i, r_{i+1} \in b$ ) we have  $r_i \cap r_{i+1} = r_{i+1}$ .



# The Algorithm

They share the same objects. It is a block.

They do not share the same objects. They are stored in a separate place that will be considered at the end

---

### Algorithm 3: Fully Nested Block Partition

---

**Input** : a nested cluster matrix  $CM_N$

**Output**: a set of blocks  $B$

```
1 begin
2    $B := \emptyset; NestedB := \emptyset; SpareB := \emptyset;$ 
3   foreach item  $i \in CM_N$  do
4     if  $i \cap (i+1) = (i+1)$  then
5        $NestedB := NestedB \cup i;$ 
6     else
7        $NestedB := NestedB \cup i;$ 
8     if  $|NestedB| \leq 1$  then
9        $SpareB.push\_all(NestedB);$ 
10       $NestedB := \emptyset$ 
11    else
12       $B := B \cup NestedB;$ 
13       $NestedB := \emptyset$ 
14  return  $B := B \cup SpareB;$ 
15 where the purpose  $SpareB.push\_all(NestedB)$  function is to put
NestedB to SpareB.
```

---



# The Global Approach

---

- *Apply the nested and segment nested Greedy algorithm*
- Apply the partition algorithm to get blocks
- Apply incremental mining with FCI



# Experimental Results

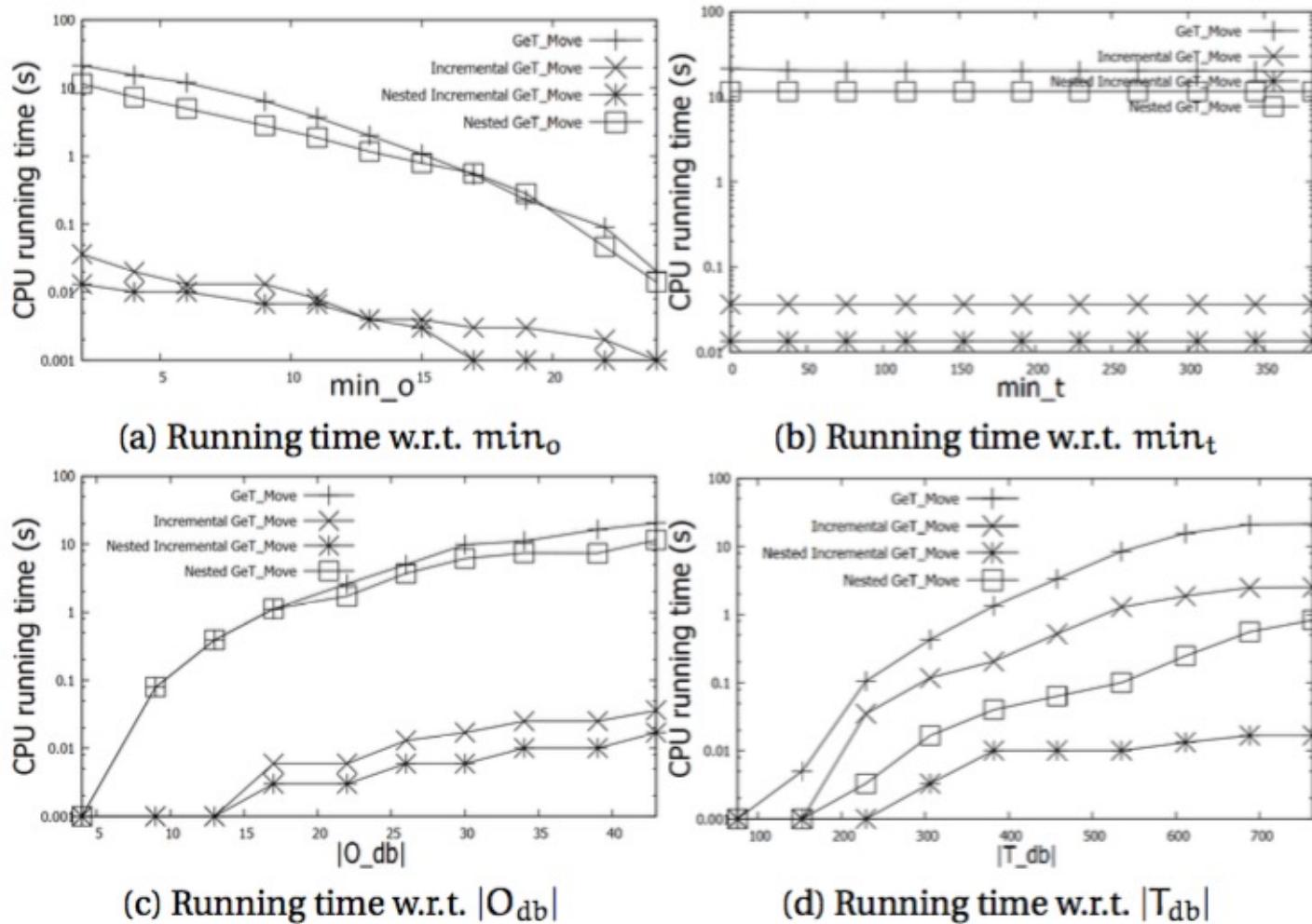


Figure 3.17: Running time on Swainsoni dataset.

# Experimental Results

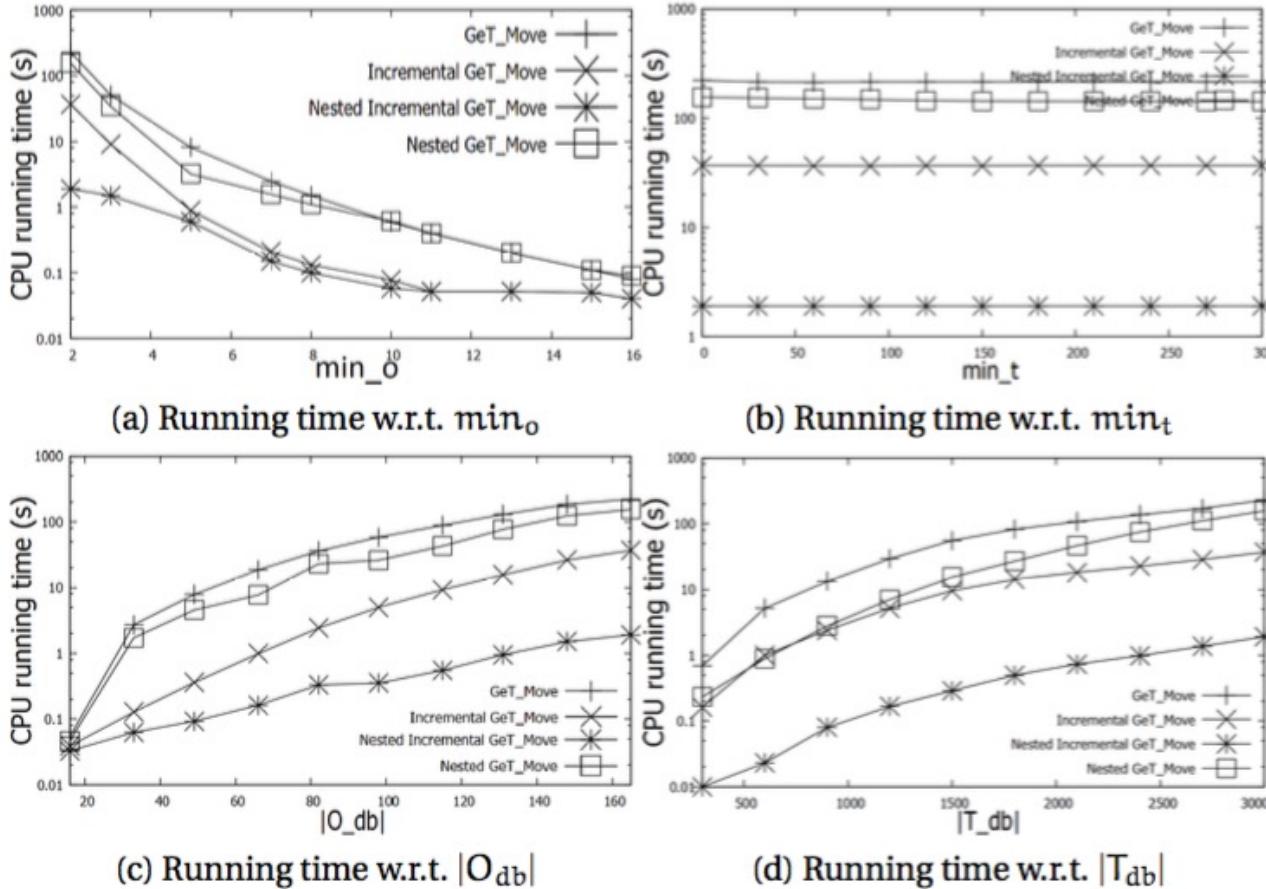
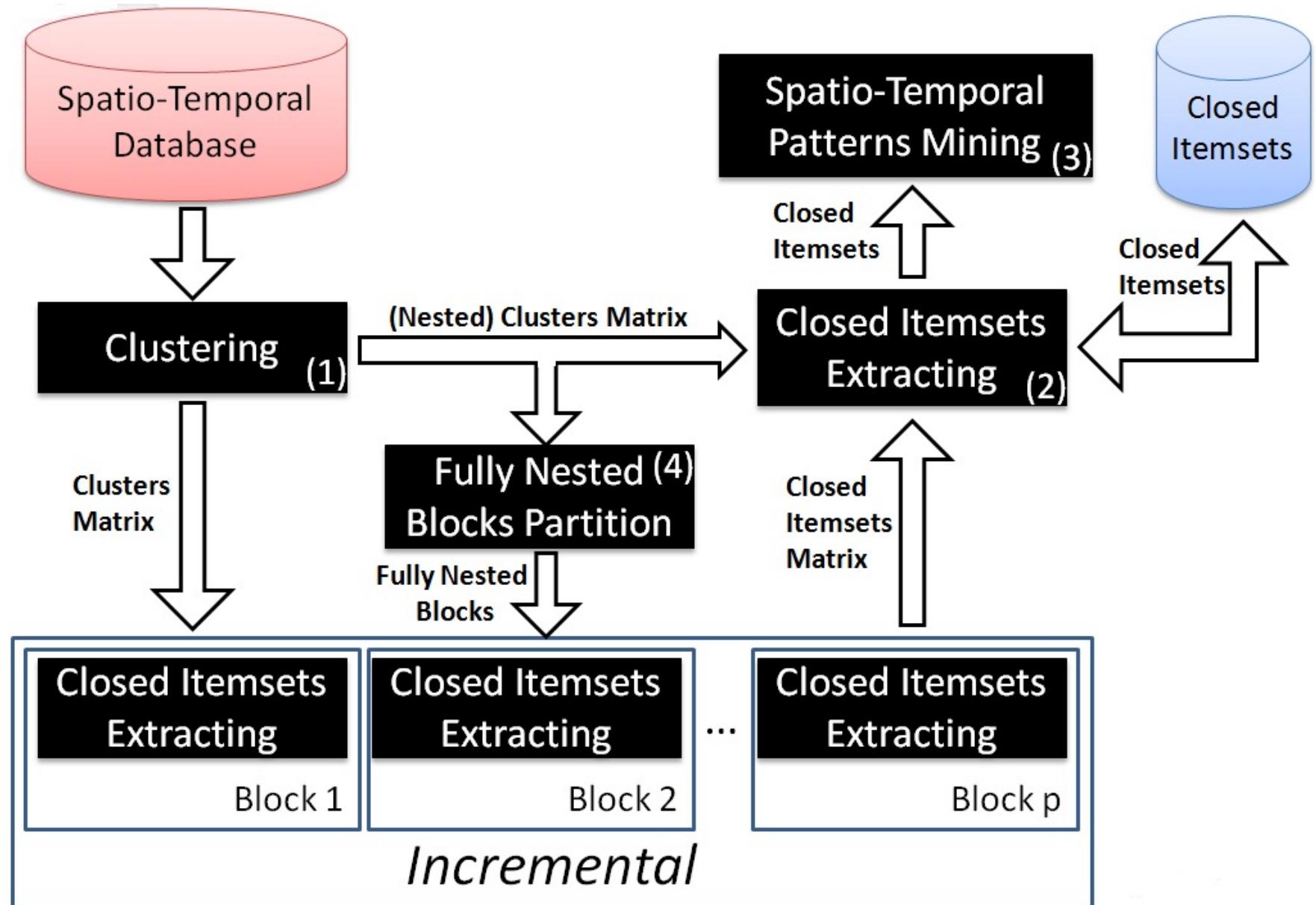
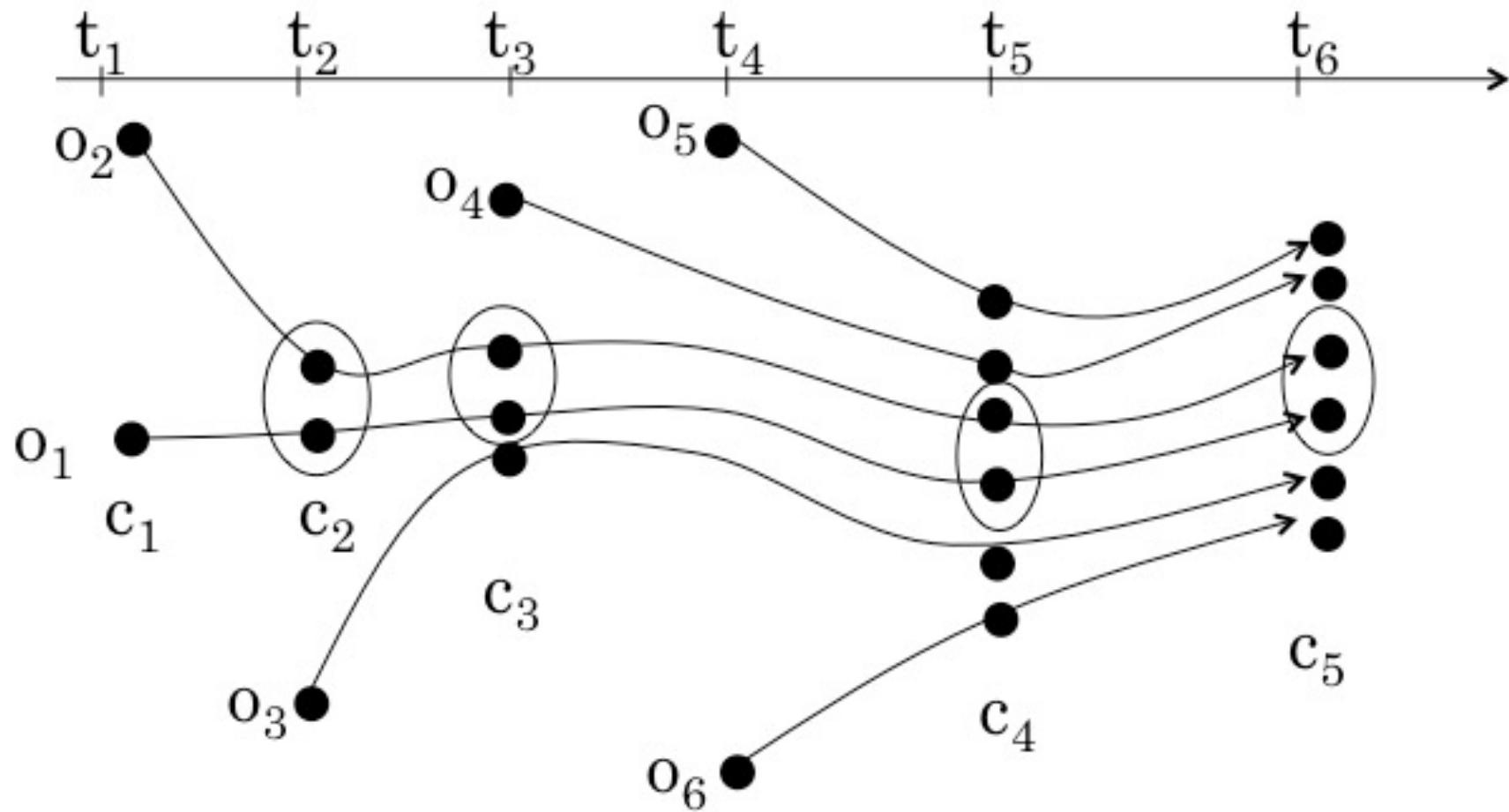


Figure 3.18: Running time on Buffalo dataset.

# The Full Process



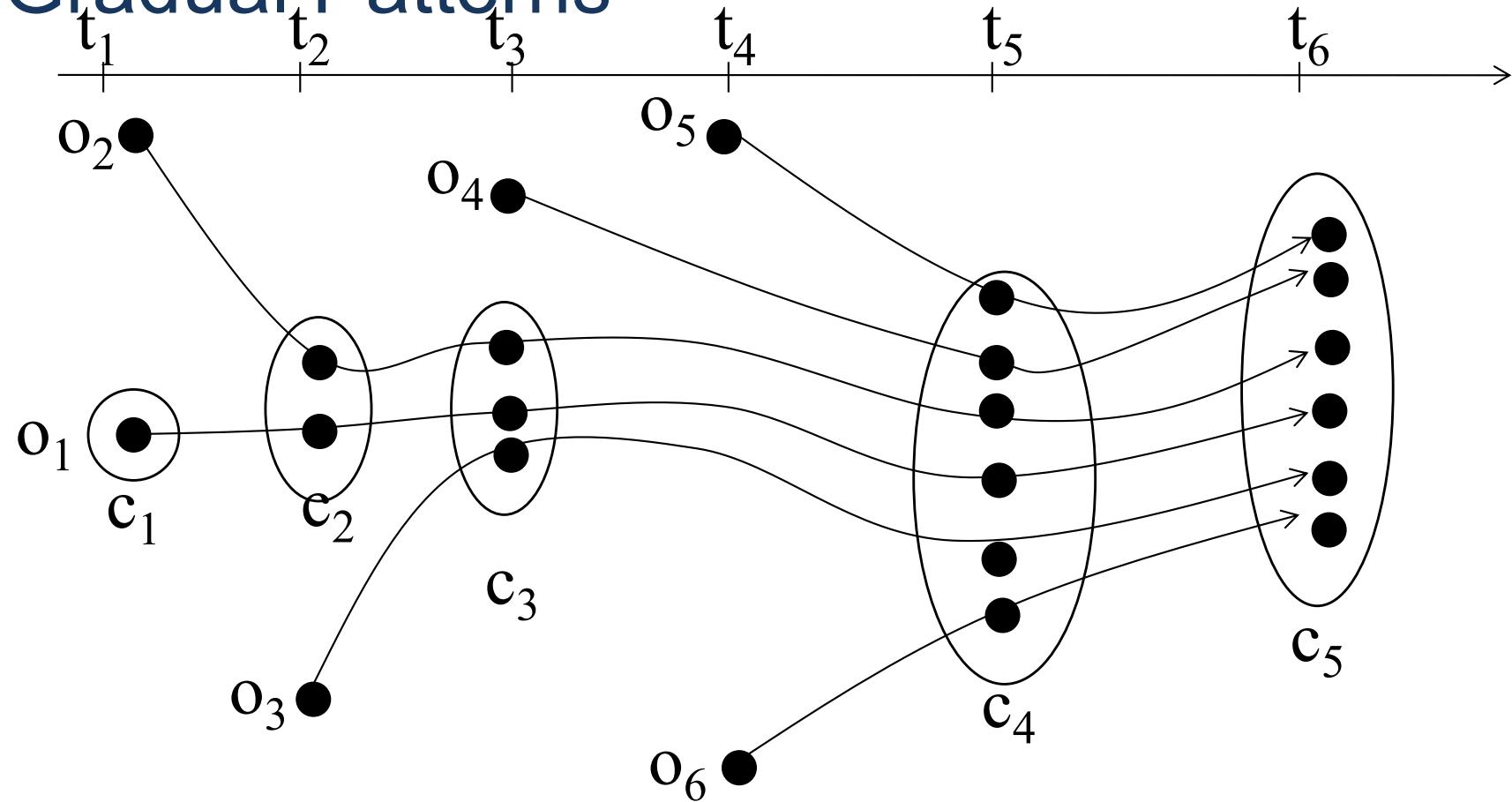
# What's about lord of the Rings?



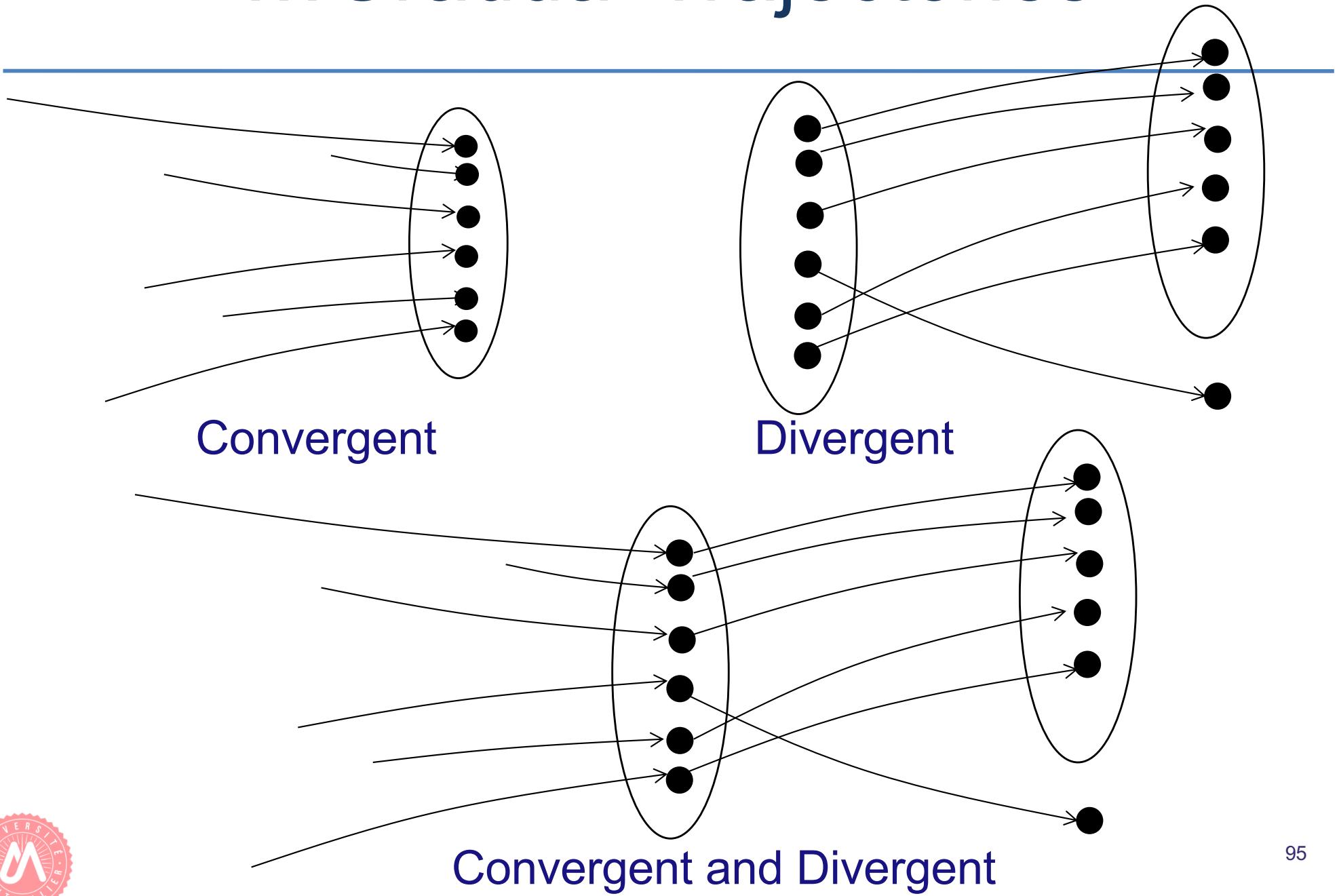
# What's about lord of the Rings?



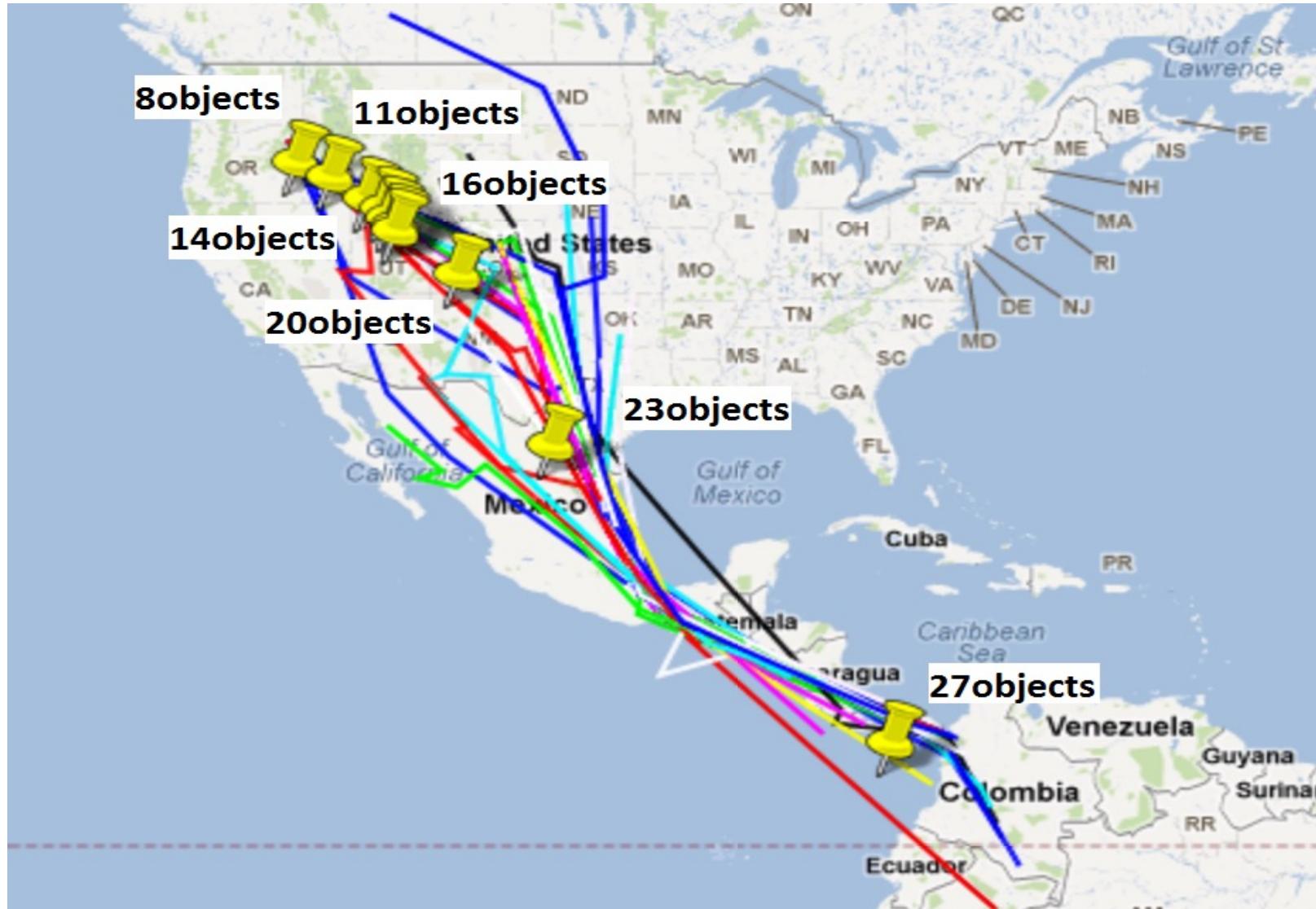
- **Gradual Patterns**



# ...Gradual Trajectories

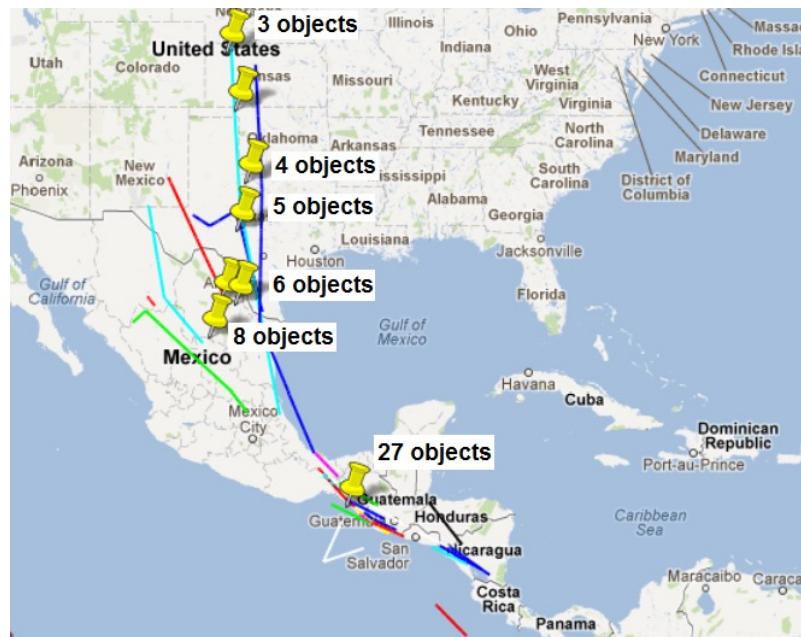


# More seriously



# Pattern Definition

- The objects still remain in the next cluster
- The number of objects is increasing (resp. decreasing)
- At least a number of certain timestamps



# rGpattern

---

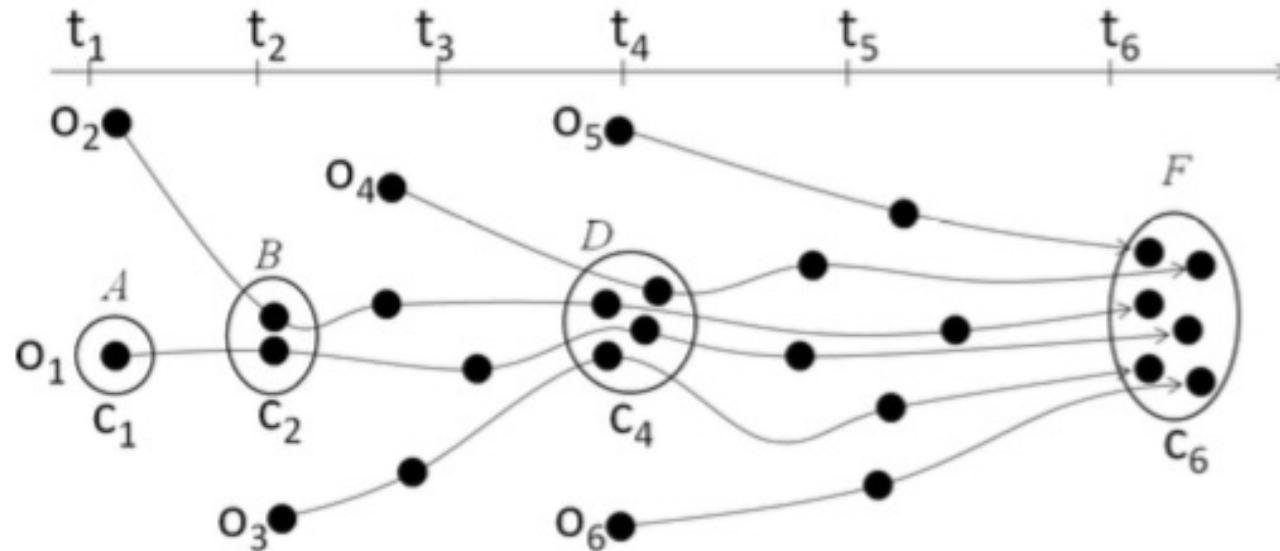
**Definition 17.** *rGpattern.* Given a list of clusters  $C^* = \{c_1, \dots, c_n\}$  and a minimum threshold  $\min_t$ .  $C^*$  is a rGpattern if:

$$C^* = C^{\geq} : \begin{cases} (1) : |C^*| \geq \min_t, \\ \forall i \in \{1, \dots, n-1\}, \\ (2) : c_i \subseteq c_{i+1}. \\ (3) : |c_n| > |c_1|. \end{cases}$$

$$C^* = C^{\leq} : \begin{cases} (1) : |C^*| \geq \min_t, \\ \forall i \in \{1, \dots, n-1\}, \\ (2) : c_i \supseteq c_{i+1}. \\ (3) : |c_n| < |c_1|. \end{cases}$$



# Example



$C_1^{\geq} = \{c_1, c_2, c_4\}$  is a rGpattern

$|C_1^{\geq}| \geq \min_t, c_1 \subset c_2 \subset c_4$  and  $|c_4| = 4 > |c_1| = 1$

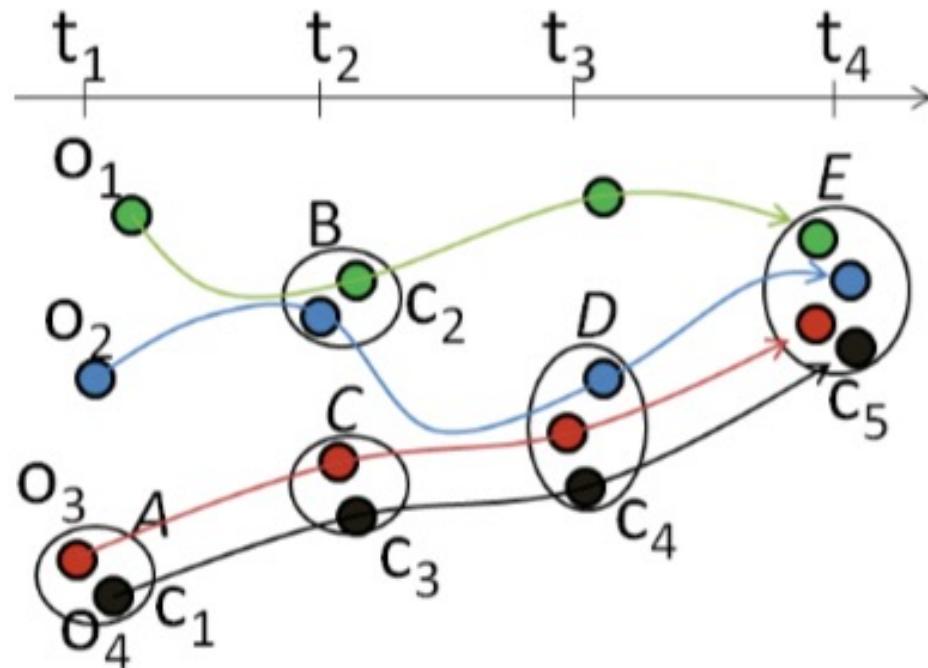
$C_1^{\geq} = \{c_1, c_2, c_4\}$ ,  $C_2^{\geq} = \{c_1, c_2, c_6\}$ ,  $C_3^{\geq} = \{c_2, c_4, c_6\}$  and  $C_4^{\geq} = \{c_1, c_2, c_4, c_6\}$

**Definition 18.** *Maximal rGpattern.* Given a rGpattern  $C^* = \{c_1, \dots, c_n\}$ .  $C^*$  is maximal if  $\exists C'^*, C^* \subset C'^* \text{ and } C'^* \text{ is a rGpattern}$ .

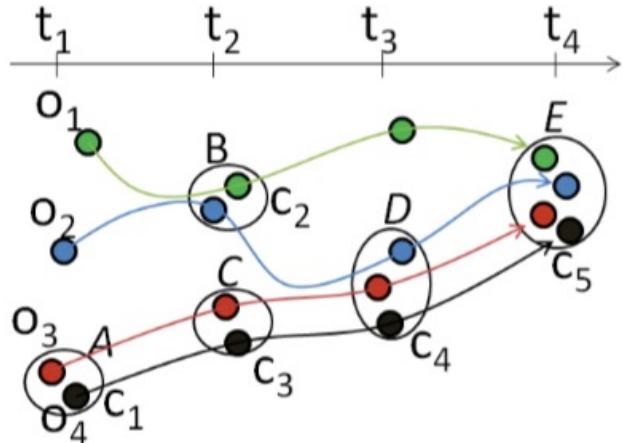
# An Illustrative Example

---

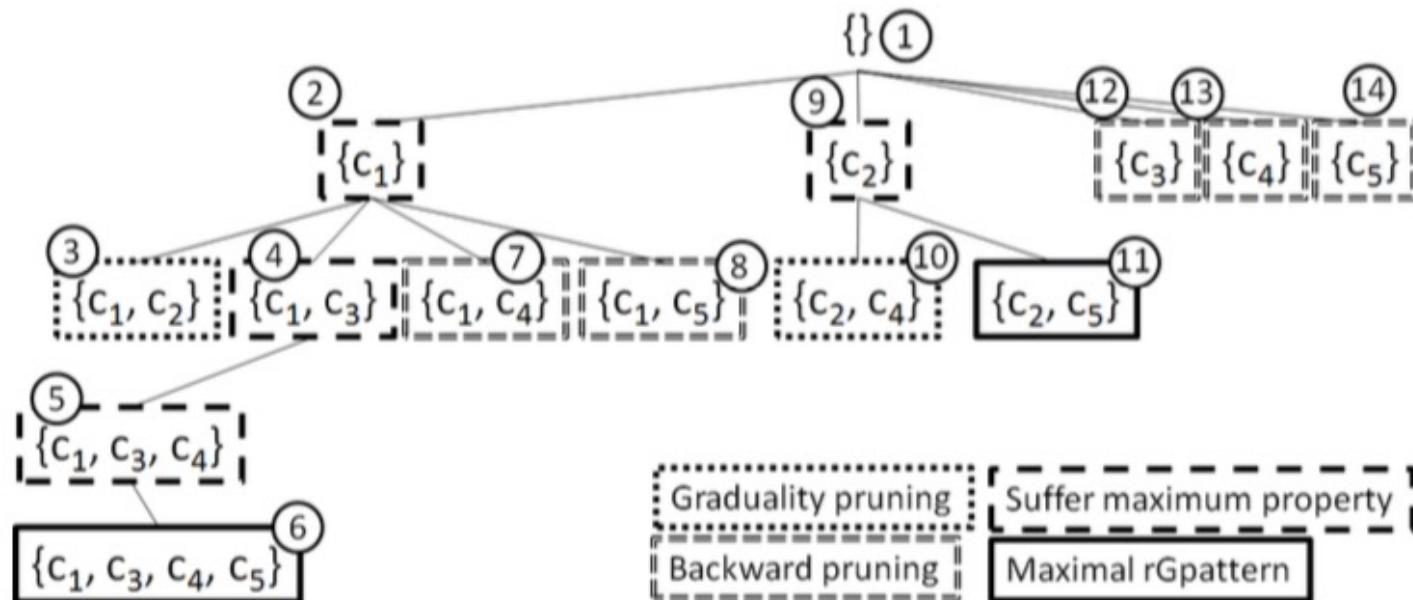
- How many maximal rGpatterns?



# The Algorithm

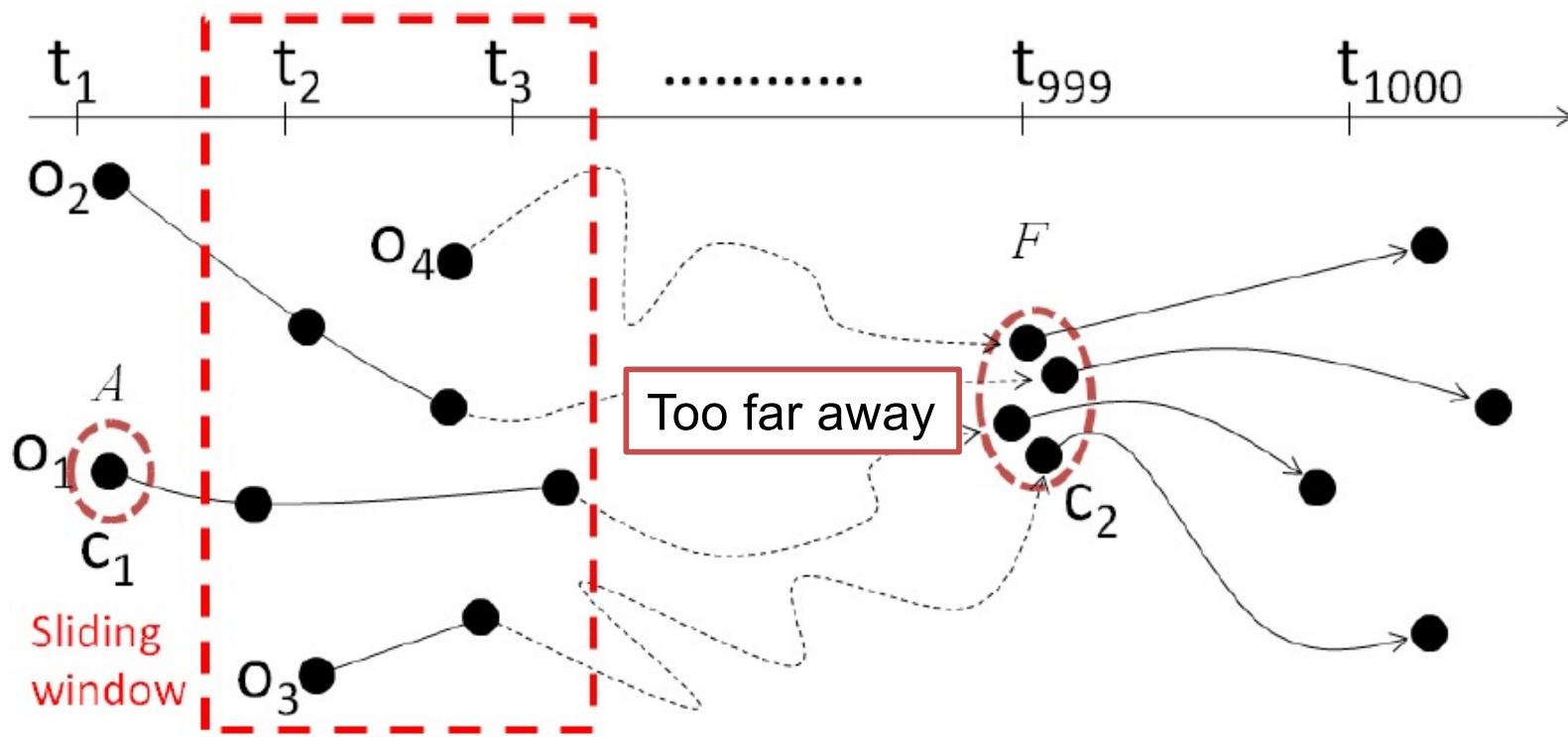


A Depth First Search:  
- 2 Pruning Rules (Graduality, Backward)



# Time Relaxed Gradual Trajectories

- Timestamps can be:
  - non-consecutive
  - within a sliding time window



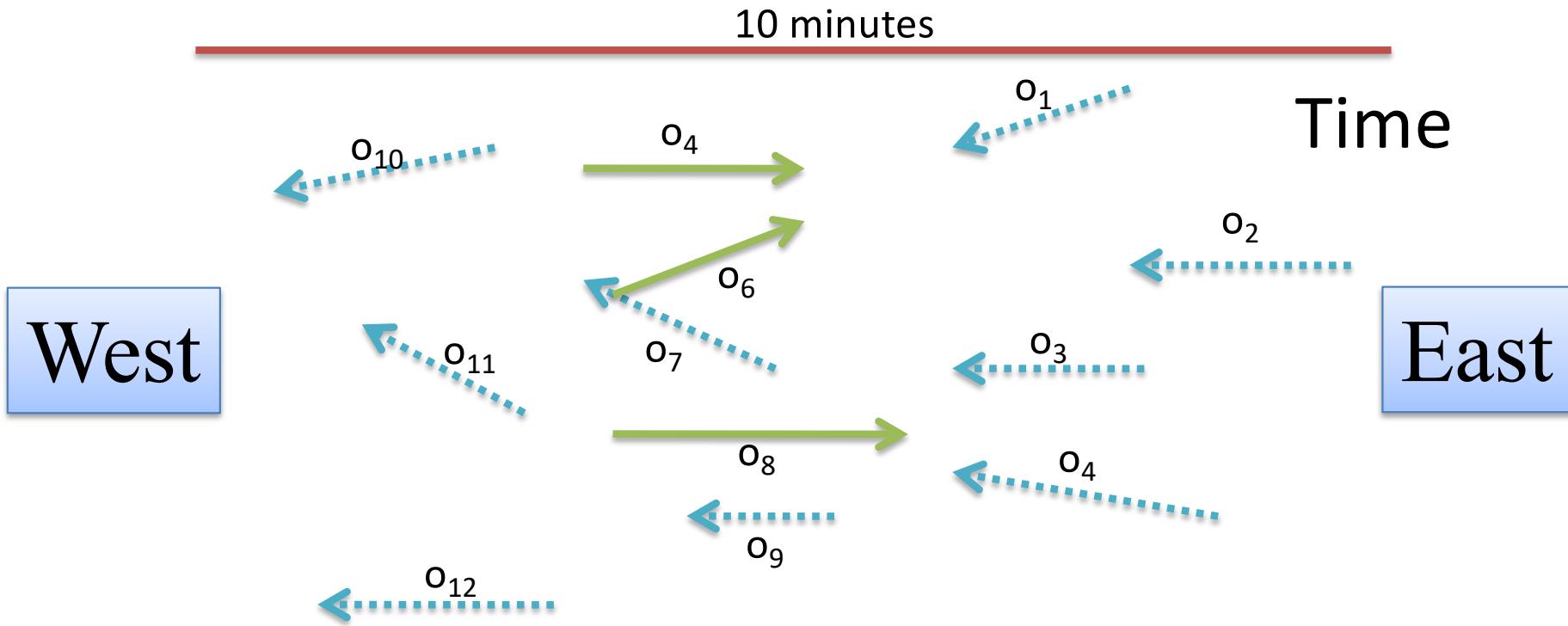
# Gradual Rules

---

- Gradual rules highlight complex order correlations of the form *“The more/less X, then the more/less Y”*
  - The nearer to city center, the higher the rent
- Mining gradual patterns plays a crucial role in many applications (numerical data): biological databases, survey databases, data streams...



# Issues and Motivations



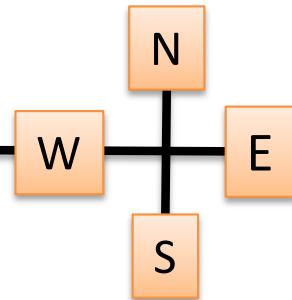
The more time passes  
the greater the number of objects  
that move from east to west

Combination of gradual rule  
and spatio-temporal pattern  
*“gradual-spatio-temporal”* pattern

# Rule Definition

---

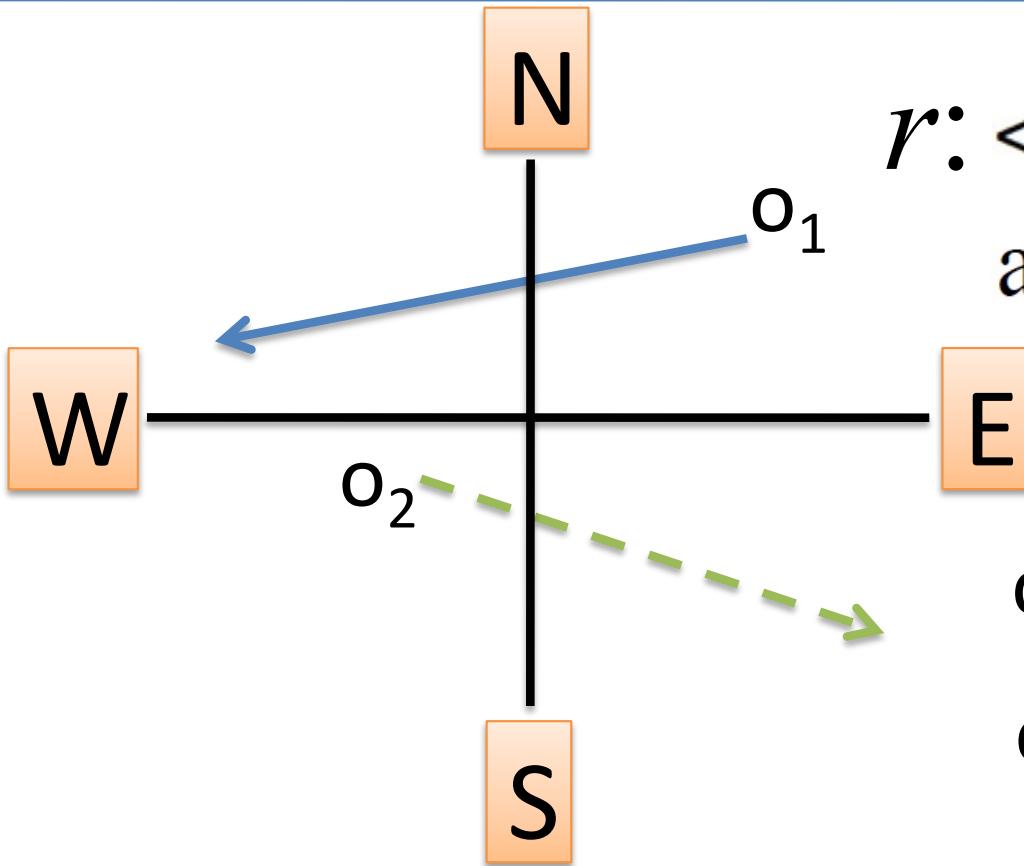
- Set of timestamps:  $T = \{t_1, t_2, \dots, t_n\}$
- Set of objects:  $O = \{o_1, o_2, \dots, o_z\}$
- A variation:  $* \in \{+, -\}$
- Set of directions:  $\Delta = \{\Delta_{EW}, \Delta_{WE}, \Delta_{NS}, \Delta_{SN}\}$

$$< T^+, O^* \rightarrow \Delta_q >$$


“the more time passes  
the greater the number of objects  
that move from east to west”

$$< T^+, O^+ \rightarrow \Delta_{EW} >$$

# Positive Moving Objects



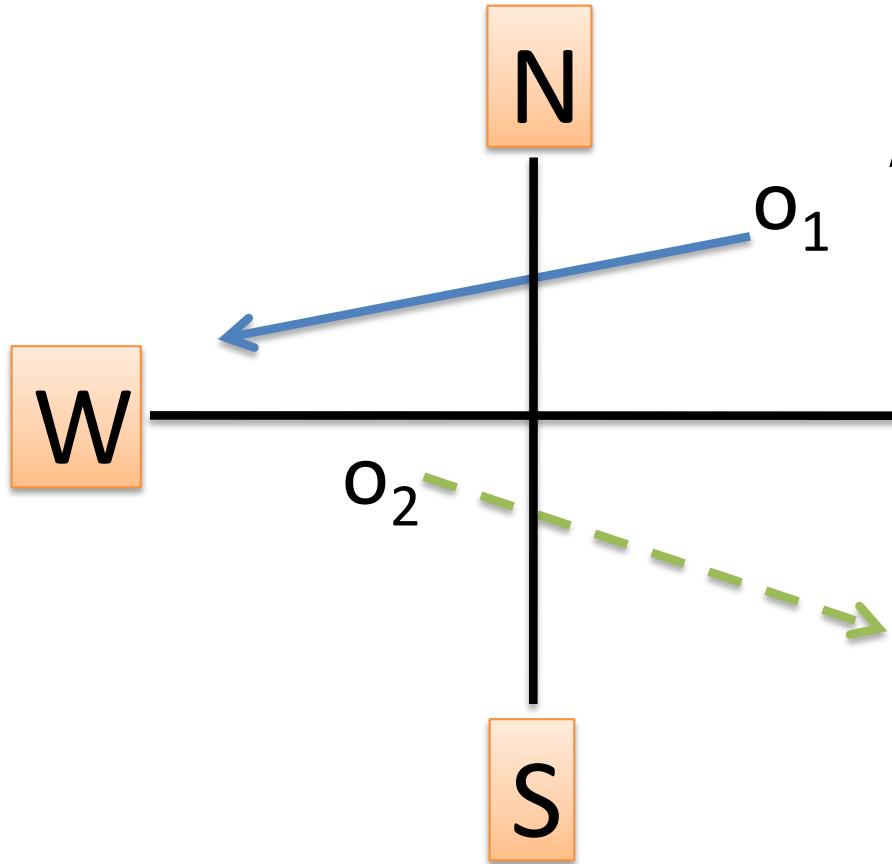
$r: < T^+, O^+ \rightarrow \Delta_{EW} >$

a time segment  $[t_i, t_{i+1}]$

$o_1: p_{moving,r}[t_i, t_{i+1}]$

$o_2: n_{moving,r}[t_i, t_{i+1}]$

# Support of a Rule on a Timestamp



$r: < T^+, O^+ \rightarrow \Delta_{EW} >$

a time segment  $[t_i, t_{i+1}]$

$O_1: p_{moving,r[t_i, t_{i+1}]}$

$O_2: n_{moving,r[t_i, t_{i+1}]}$

$$\sigma(r)_{[t_i, t_{i+1}]} = \frac{p_{moving,r[t_i, t_{i+1}]}}{p_{moving,r[t_i, t_{i+1}]} + n_{moving,r[t_i, t_{i+1}]}}$$

# Supporting Time Segment

---

- Supporting time segment, non supporting time segment

$$r: \langle T^+, O^* \rightarrow \Delta_q \rangle$$

$[t_i, t_{i+1}]$  is a *supporting time segment*

$$\boxed{\text{if } O^* = O^+ \text{ then } \begin{cases} p_{moving,r[t_i,t_{i+1}]} \geq p_{moving,r[t_{i-1},t_i]} & : \text{condition(1)} \\ \sigma(r)_{[t_i,t_{i+1}]} \geq \sigma(r)_{[t_{i-1},t_i]} & : \text{condition(2)} \\ \sigma(r)_{[t_i,t_{i+1}]} \geq \sigma_0 & : \text{condition(3)} \end{cases}}$$

$$\boxed{\text{if } O^* = O^- \text{ then } \begin{cases} p_{moving,r[t_i,t_{i+1}]} \leq p_{moving,r[t_{i-1},t_i]} & : \text{condition (1)} \\ \sigma(r)_{[t_i,t_{i+1}]} \leq \sigma(r)_{[t_{i-1},t_i]} & : \text{condition(2)} \\ \sigma(r)_{[t_i,t_{i+1}]} \geq \sigma_0 & : \text{condition(3)} \end{cases}}$$



# Supporting Time Pattern

---

$$p_s = (t_i, t_{i+k}), |p_s| = k \ (k \geq 1)$$

$p_s$  is a *k-supporting time pattern* if and only if

$\forall j \ (0 \leq j < k), [t_{i+j}, t_{i+j+1}]$  is a *supporting time segment*.

$p_{ns} = (t_i, t_{i+k})$  is a *k-non-supporting time pattern* if and only if

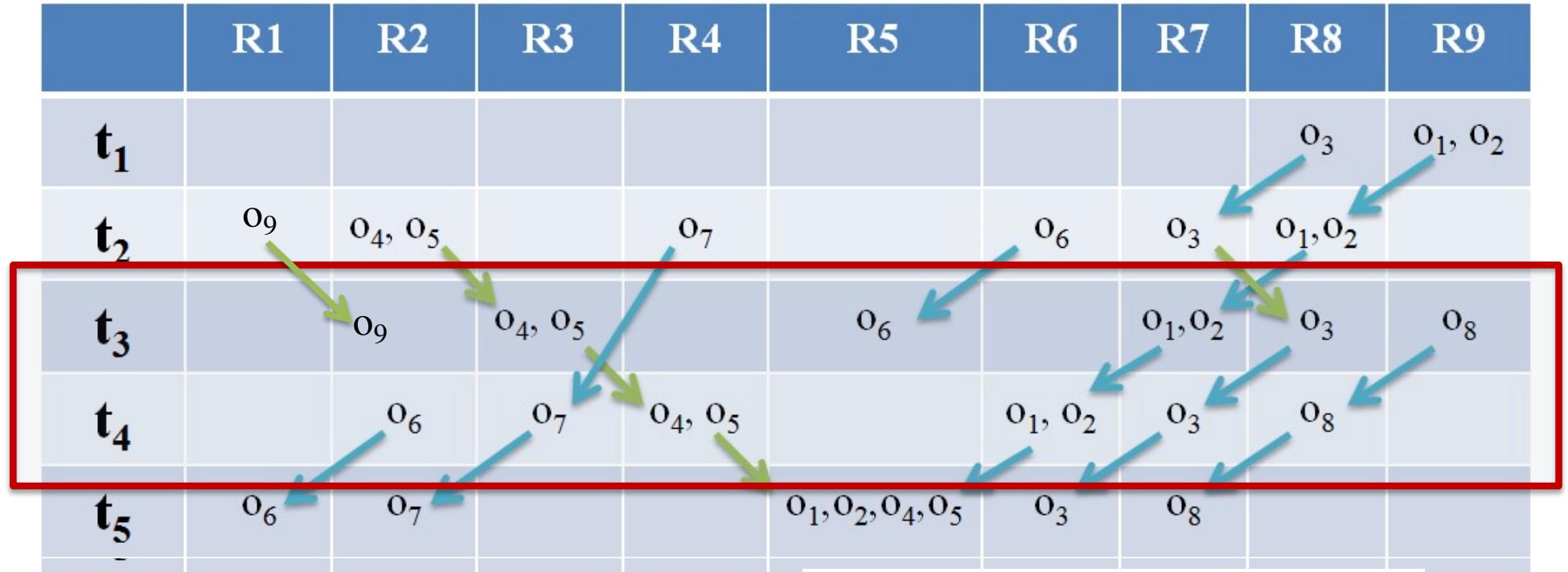
$\forall j \ (0 \leq j < k), [t_{i+j}, t_{i+j+1}]$  is a *non-supporting time segment*.

$p_{nt} = (t_i, t_{i+k})$  is a *k-neutral time pattern* if and only if

$\forall j \ (0 \leq j < k), [t_{i+j}, t_{i+j+1}]$  is a *neutral time segment*.



# A Running Example



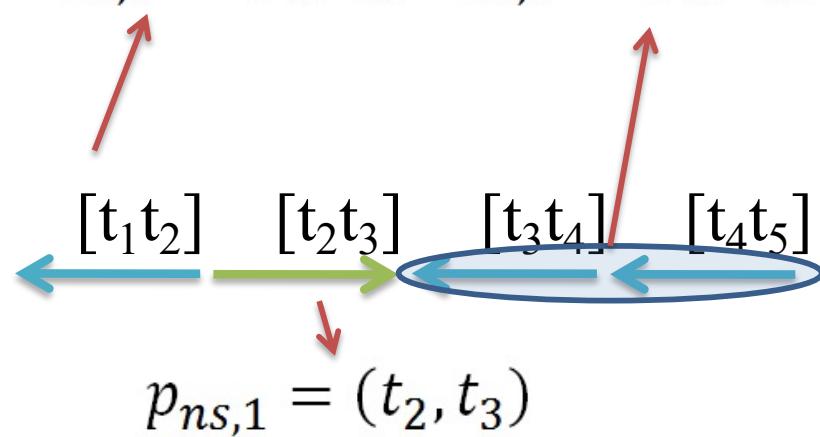
$\langle T^+, O^+ \rightarrow \Delta_{EW} \rangle$

$$p_{s,1} = (t_1, t_2) \quad p_{s,2} = (t_3, t_5)$$

$$p_{moving,r_{[t_3,t_4]}} = 5 \text{ (i.e. } o_7, o_1, o_2, o_3, o_8\text{)}$$

$$n_{moving,r_{[t_3,t_4]}} = 2 \text{ (i.e. } o_4, o_5\text{)}$$

$$\sigma(r)_{[t_3,t_4]} = 5/7 > \sigma_0 = 0.3$$



# Support and Confidence of Rules

---

a set of timestamps  $T = \{t_1, t_2, \dots, t_n\}$

$r: < T^+, O^* \rightarrow \Delta_q >$

- *We have already see the support of a Rule on one Timestamp:*

$$\sigma(r)_{[t_i, t_{i+1}]} = \frac{p_{moving, r_{[t_i, t_{i+1}]}}}{p_{moving, r_{[t_i, t_{i+1}]}} + n_{moving, r_{[t_i, t_{i+1}]}}}$$



# Support and Confidence of Rules

---

a set of timestamps  $T = \{t_1, t_2, \dots, t_n\}$

$r: < T^+, O^* \rightarrow \Delta_q >$

- $p_s$  : is  $k$ -supporting time pattern
- $p_{ns}$  : is  $k$ -non supporting time pattern
- $p_{nt}$  : is  $k$ -neutral time pattern

$$\sigma(r)_T = \sum_{i=1}^n |p_{s,i}|$$
$$c(r)_T = \frac{\sum_{i=1}^n |p_{s,i}|}{\sum_{i=1}^n |p_{s,i}| + \sum_{j=1}^m |p_{ns,j}| + \sum_{k=1}^l |p_{nt,k}|}$$



# A Running Example

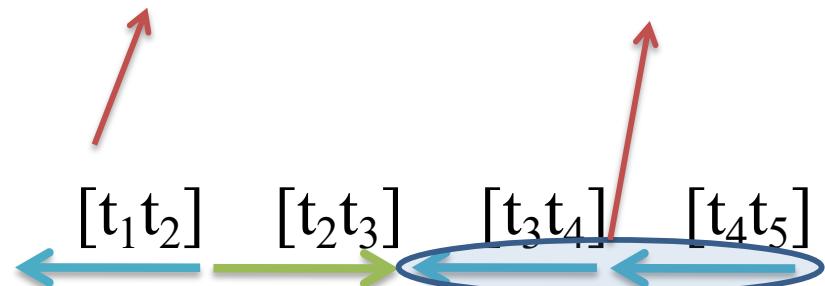
	R1	R2	R3	R4	R5	R6	R7	R8	R9
$t_1$								$o_3$	$o_1, o_2$
$t_2$		$o_4, o_5$		$o_7$		$o_6$	$o_3$	$o_1, o_2$	
$t_3$			$o_4, o_5$				$o_3$	$o_3$	$o_8$
$t_4$		$o_6$	$o_7$	$o_4, o_5$		$o_1, o_2$	$o_3$	$o_8$	
$t_5$	$o_6$	$o_7$			$o_1, o_2, o_4, o_5$	$o_3$	$o_8$		

$\langle T^+, O^+ \rightarrow \Delta_{EW} \rangle$

$$\sigma(r)_T = \sum_{i=1}^2 |p_{s,i}| = 3$$

$$c(r)_T = \frac{\sum_{i=1}^2 |p_{s,i}|}{\sum_{i=1}^2 |p_{s,i}| + \sum_{j=1}^1 |p_{ns,j}|} = \frac{3}{4} = 0.75$$

$$p_{s,1} = (t_1, t_2) \quad p_{s,2} = (t_3, t_5)$$



$$p_{ns,1} = (t_2, t_3)$$

# The Algorithm

Compute support value of each direction at each timestamp.

If the support is larger than predefined threshold then it's considered as a rule candidate.

Compute the supporting time patterns, non-supporting time patterns, neutral time patterns.

Compute support and confidence value for each rule.

**Algorithm GSTD:** Gradual-Spatio-Temporal rule Discovering  
**Input:** a set of objects  $O$ , a set of timestamps  $T = \{t_1, t_2, \dots, t_n\}$ , a set of directions  $\Delta = \{\Delta_{EW}, \Delta_{WE}, \Delta_{NS}, \Delta_{SN}\}$ , a minimum support threshold  $\sigma_0$ , a minimum moving objects threshold  $\theta$ .

**Output:** a set of rules  $r$ .

```
candidateSet = Ø;  
for each timestamp  $t_i \in T$   
    for each direction  $\Psi \in \Delta$   
         $r < T^+, O^+ \rightarrow \Psi >. p_{[t_i, t_{i+1}]} \leftarrow \text{ComputePositiveMovingObject}(O);$   
         $r < T^+, O^+ \rightarrow \Psi >. n_{[t_i, t_{i+1}]} \leftarrow \text{ComputeNegativeMovingObject}(O);$   
         $r < T^+, O^+ \rightarrow \Psi >. \sigma_{[t_i, t_{i+1}]} \leftarrow \frac{p}{p+n};$   
        if  $r \notin \text{candidateSet}$  then  
            if  $r < T^+, O^+ \rightarrow \Psi >. \sigma_{[t_i, t_{i+1}]} \geq \sigma_0$  then  
                 $r < T^+, O^+ \rightarrow \Psi >. t_{st} \leftarrow t_i; // t_{st} is the starting point$   
                candidateSet  $\leftarrow$  candidateSet  $\cup r < T^+, O^+ \rightarrow \Psi >;$   
            end if  
        end if  
        for each rule  $r \in \text{candidateSet} // r < T^+, O^+ \rightarrow \Psi >$   
            if  $p_{[t_i, t_{i+1}]} \geq p_{[t_{i-1}, t_i]}$  and  $\sigma(r)_{[t_i, t_{i+1}]} \geq \sigma(r)_{[t_{i-1}, t_i]}$   
                and  $\sigma(r)_{[t_i, t_{i+1}]} \geq \sigma_0$  and  $(p_{[t_i, t_{i+1}]} + n_{[t_i, t_{i+1}]}) \geq \theta$  then  
                     $r.sp \leftarrow r.sp + 1; // r.sp is the supporting time$   
                end if  
        end for  
    end for  
end for  
for each rule  $r \in \text{candidateSet} // r < T^+, O^+ \rightarrow \Psi >$   
     $\sigma(r)_T \leftarrow r.sp;$   
     $c(r)_T \leftarrow \frac{r.sp}{|T|};$   
    return  $r;$   
end for
```



# Experimental Results

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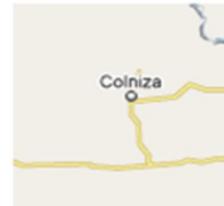
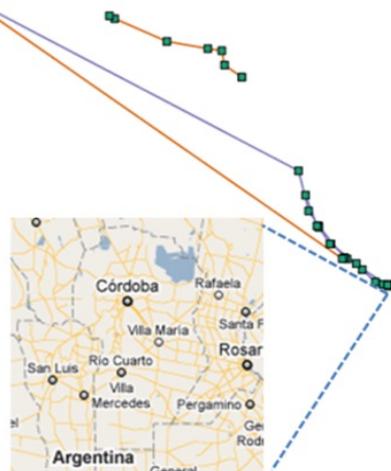
- Swainsoni dataset (43 objects evolving over time and 4225 different timestamps). July 1995 to June 1998.

SUPPORT AND CONFIDENCE FOR EACH RULE.

Rule	$\sigma(r)_T$	$c(r)_T$
$< T^+, O^+ \rightarrow \Delta_{NS} >$	2653	0.6416
$< T^+, O^+ \rightarrow \Delta_{SN} >$	1944	0.4617
$< T^+, O^+ \rightarrow \Delta_{WE} >$	2725	0.6472
$< T^+, O^+ \rightarrow \Delta_{EW} >$	2060	0.4903



# Gradual-Spatio-Temporal Rules vs Convoy, Swarm

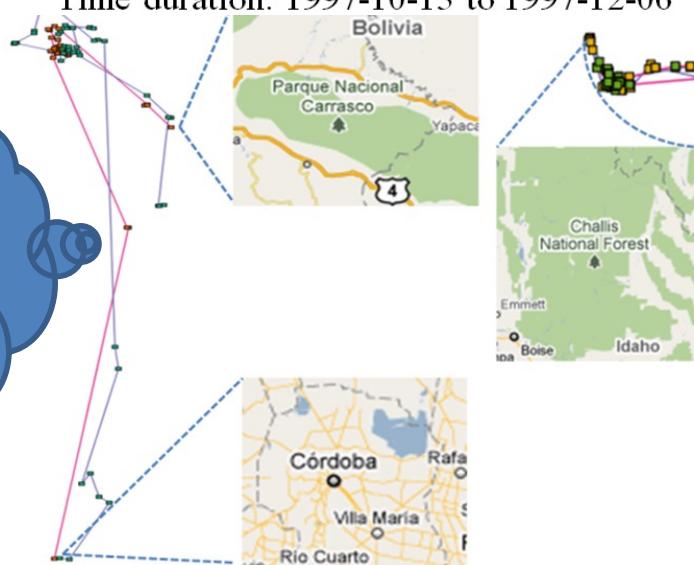


independently

(c)-Longest pattern of  $r < T^+, O^+ \rightarrow \Delta_{WE} >$   
Time duration: 1997-10-13 to 1997-12-06

(d)-Longest pattern of  $r < T^+, O^+ \rightarrow \Delta_{EW} >$   
Time duration: 1997-03-10

together  
exactly



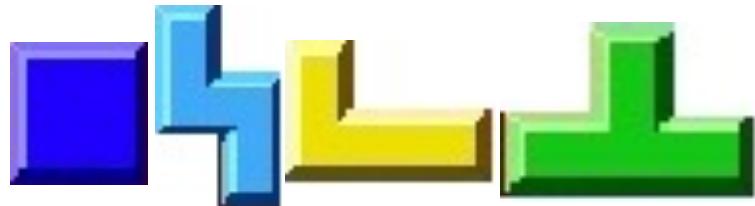
(e)-Convoy  
Time duration: 1996-11-17 to 1997-02-28

Separate some  
time – rejoin  
other time



(f)-Closed swarm  
Time duration: 1996-08-29 to 1997-06-17

# The Most Informative Patterns



Patterns



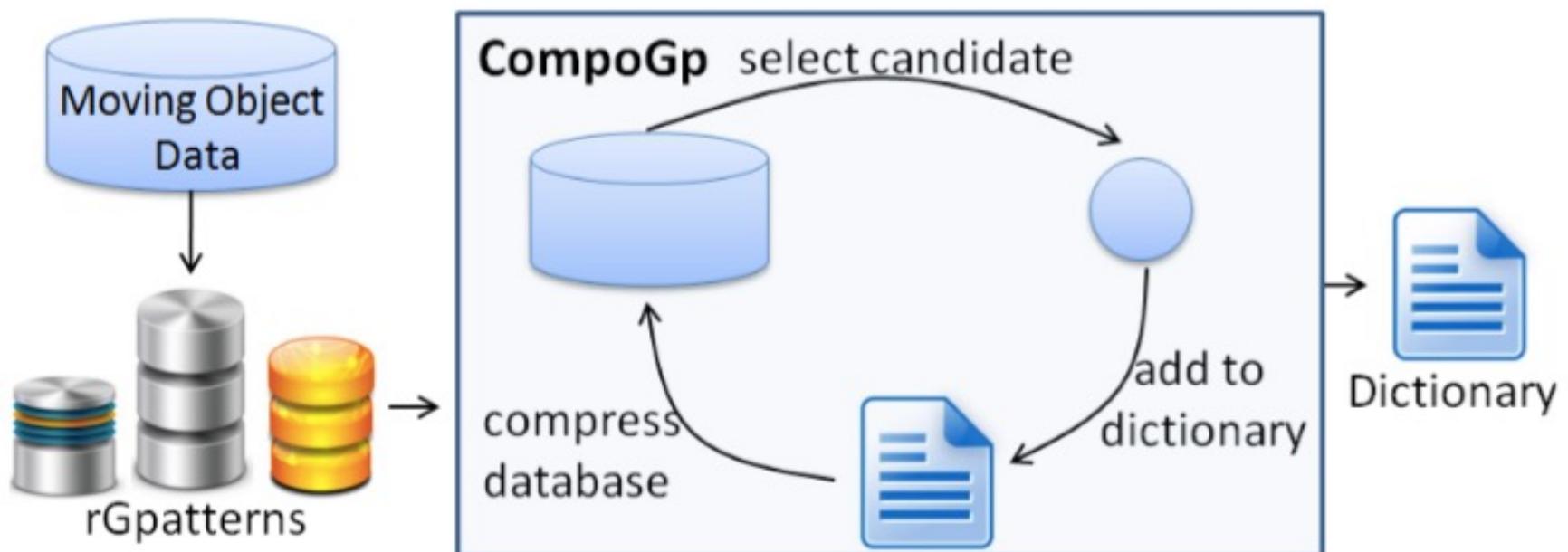
data

Group  
Moving Periodic  
**Pattern** Cluster  
T-Pattern Convoy Evolving  
k-Star Tralus Closed Swarm  
Flock

$T_{DB}$		$t_1$		$t_2$		$t_3$	$t_4$	$t_5$			
Clusters	$C_{DB}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$O_{DB}$	$o_1$	1			1		1	1	1		
	$o_2$			1	1		1	1			1
	$o_3$						1				
	$o_4$		1			1		1	1		
	$o_5$		1			1		1	1		

# The Main Idea

---



# Problem Statement

- Given a spatio-temporal DB  $O_{db}$  and a set of patterns  $F$  (extracted from  $O_{db}$ )
- Discover the **optimal dictionary  $P$**  (subset of  $F$ )
  - compresses the data best w.r.t. the given encoding schema

$$\text{MDL approach: } L_P(O_{db}) = L(P) + L(O_{db}|P)$$

$$\arg \min_{P \subseteq F} \left( \sum_{p \in P} L(p) + L(O_{DB}|P) \right)$$

- $L(p)$ : number of bits to encode the pattern  $p$  + extra bit to encode the type of pattern
- $L(O_{db}|P)$ : number of bits to encode the dataset  $O_{db}$  given  $P$

# Encoding Example

$T_{DB}$		$t_1$		$t_2$		$t_3$	$t_4$	$t_5$			
Clusters	$C_{DB}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$O_{DB}$	$o_1$	1			1	1	1	1			
	$o_2$			1	1	1	1	1			1
	$o_3$						1				
	$o_4$		1	1		1	1	1	1		
	$o_5$		1	1		1	1	1	1		

$O_{DB}$	Patterns $\mathcal{F}$
$o_1 = c_1 c_4 c_6 c_7 c_8$	
$o_2 = c_3 c_4 c_6 c_7 c_{10}$	$p_1 = c_1 c_4 c_6, \bar{1}$
$o_3 = c_6$	$p_2 = c_2 c_5 c_7 c_9, \bar{0}$
$o_4 = c_2 c_5 c_7 c_9$	$p_3 = c_7 c_8, \bar{2}$
$o_5 = c_2 c_5 c_7 c_9$	$P_4 = c_4 c_6 c_7 \bar{0}$



# Encoding Example

$T_{DB}$		$t_1$		$t_2$		$t_3$	$t_4$	$t_5$		
Clusters $C_{DB}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$c_{10}$
$O_{DB}$	$o_1$	1			1	1	1	1		
	$o_2$			1	1	1	1			1
	$o_3$						1			
	$o_4$		1		1	1	1	1		
	$o_5$		1		1	1	1	1		

Encoded $O_{DB}$	Dictionary $\mathcal{P}$
$o_1 = [p_1, 0][p_3, 0]$	
$o_2 = c_3[p_1, 1][p_3, 0]c_{10}$	$p_1 = c_1c_4c_6, \bar{1}$
$o_3 = [p_1, 2]$	$p_2 = c_2c_5c_7c_9, \bar{0}$
$o_4 = p_2$	$p_3 = c_7c_8, \bar{2}$
$o_5 = p_2$	$p_4 = c_4c_6c_7, \bar{0}$

$$L(O_{DB}/P) = 4 + 6 + 2 + 1 + 1 = 14$$

$$L(P) = 4 + 5 + 3 + 4 = 16$$

$$L_P(O_{DB}) = 30$$

Encoded $O_{DB}$	Dictionary $\mathcal{P}$
$o_1 = [p_1, 0]c_7c_8$	
$o_2 = c_3[p_1, 1]c_7c_{10}$	$p_1 = c_1c_4c_6, \bar{1}$
$o_3 = [p_1, 2]$	$p_2 = c_2c_5c_7c_9, \bar{0}$
$o_4 = p_2$	
$o_5 = p_2$	

$$L(O_{DB}/P) = 4 + 5 + 2 + 1 + 1 = 13$$

$$L(P) = 4 + 5 = 9$$

$$L_P(O_{DB}) = 22$$



# The Algorithm

Compute for each pattern its compression size and select the most efficient

Compute the compression size for each object of the DB. This can be done efficiently by using pointers rather than true value

---

## Algorithm 9: NaiveCompo

---

**Input** : Database  $O_{db}$ , set of patterns  $F$ , int  $K$   
**Output**: Compressing patterns  $\mathcal{P}$

1 **begin**  
2      $\mathcal{P} \leftarrow \emptyset;$   
3     **while**  $|\mathcal{P}| < K$  **do**  
4         **foreach**  $p \in F$  **do**  
5              $O_{db}^d \leftarrow O_{db};$   
6              $L^*(O_{db}^d | p) \leftarrow \text{CompressionSize}(O_{db}^d, p);$   
7              $p^* \leftarrow \arg\min_p L^*(O_{db}^d | p);$   
8              $\mathcal{P} \leftarrow p^*; F \leftarrow F \setminus \{p^*\};$   
9             Replace all instances of  $p^*$  in  $O_{db}$  by its pointers;  
10             Replace all instances of  $p^*$  in  $F$  by its pointers;  
11         **output**  $\mathcal{P};$   
12     **CompressionSize**( $O_{db}^d, p)$   
13     **begin**  
14         size  $\leftarrow 0;$   
15         **foreach**  $o \in O_{db}$  **do**  
16             **if**  $p.\text{involved}(o) = \text{true}$  **then**  
17                 Replace instance of  $p$  in  $o$  by its pointers;  
18         **foreach**  $o \in O_{db}$  **do**  
19             size  $\leftarrow size + |o|;$   
20         size  $\leftarrow size + |p| + 1;$   
21         **output** size;

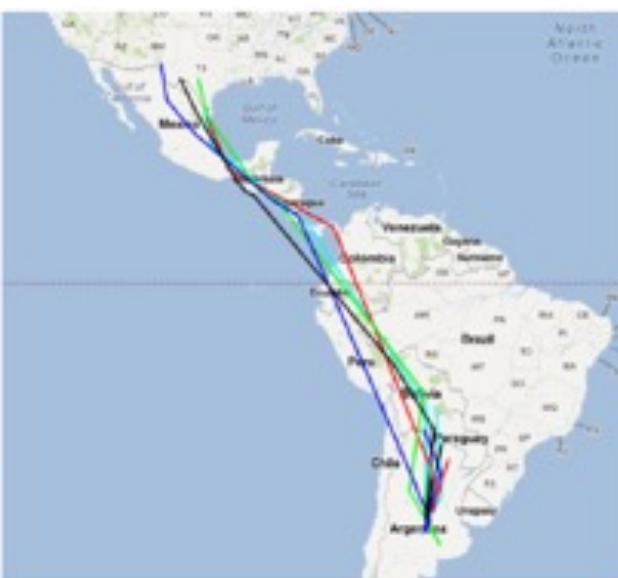


# The 3-patterns

---



(a)  $rG\text{pattern} \geq$



(b) Closed swarm



(c)  $rG\text{pattern} \leq$

From the running Swanson example

# Outline

---

- A quick reminder on pattern discovery
- The different kinds of patterns from trajectory
- Towards a unified approach for extracting trajectories
- **Deal with and without a spatial component?**
- A concrete illustration
- Conclusion



# Spatial or not spatial?

---

- Trajectories have been defined for dealing with spatio-temporal data
- What are exactly trajectories?



# Web Usage Mining

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- Web Usage Mining is the application of data mining techniques to discover interesting usage patterns from Web data in order to understand and better serve the needs of Web-based applications. (wikipedia)
- Use log entries

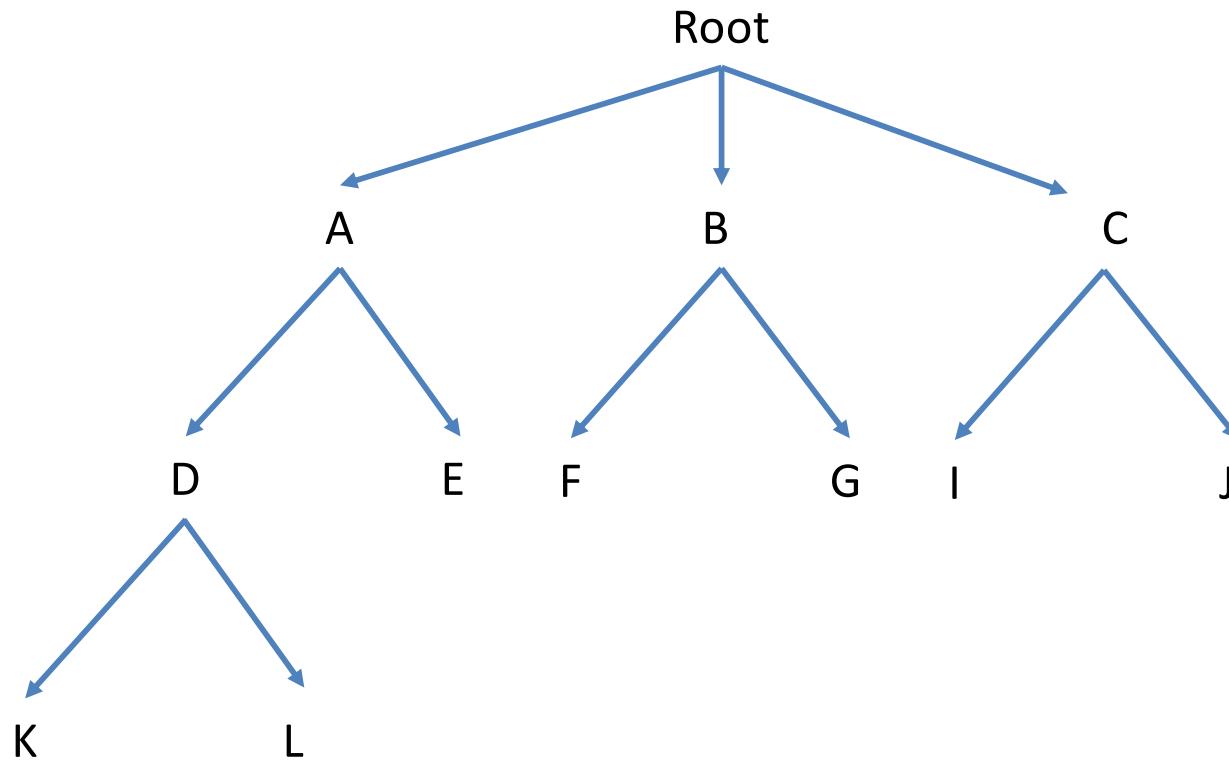
123.456.78.9 -- [24/Apr/2018:19:13:44 -0400] "GET /Images/tagline.gif HTTP/1.0"

200 1449 http://www.teced.com/ "Mozilla/4.51 [en] (Win98;I)"



# An Illustrative Example

---



# The Cluster Matrix

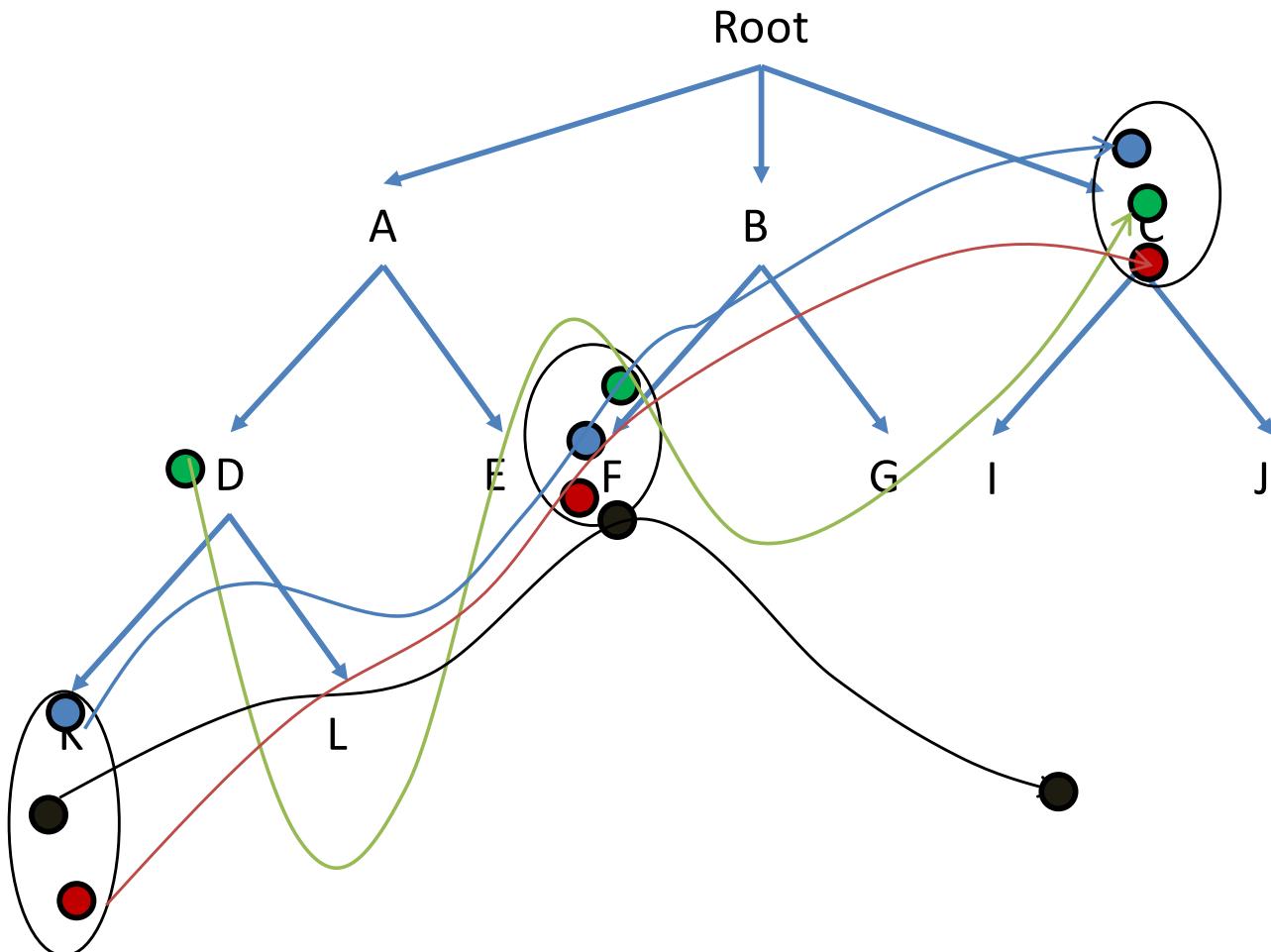
---

	T1												T2												T3																				
	A	B	C	D	E	F	G	H	I	J	K	L	A	B	C	D	E	F	G	H	I	J	K	L	A	B	C	D	E	F	G	H	I	J	K	L	A	..							
C1	1																																												
C2	1																																												
C3		1																																											
C4		1																																											
C5			1																																										
C6			1																																										
C7			1																																										
C8				1																																									
...																																													

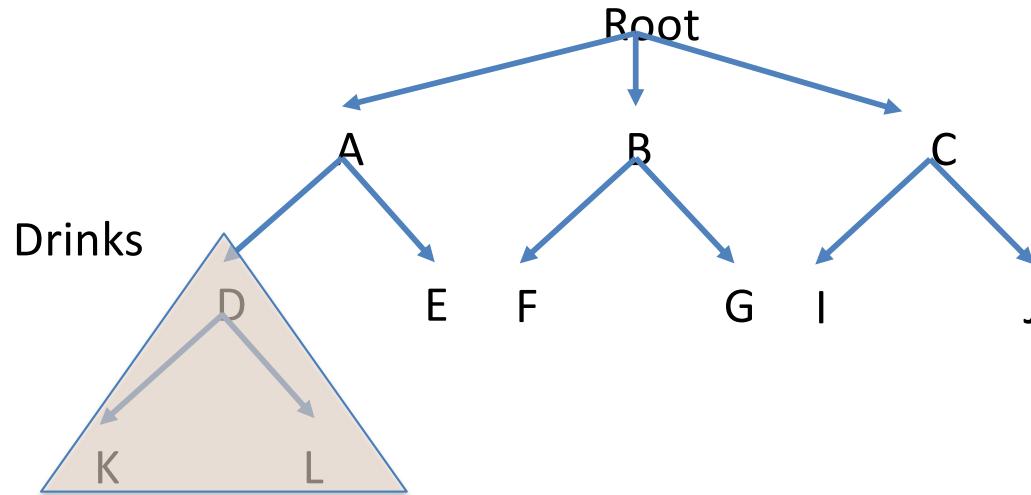


# An Illustrative Example

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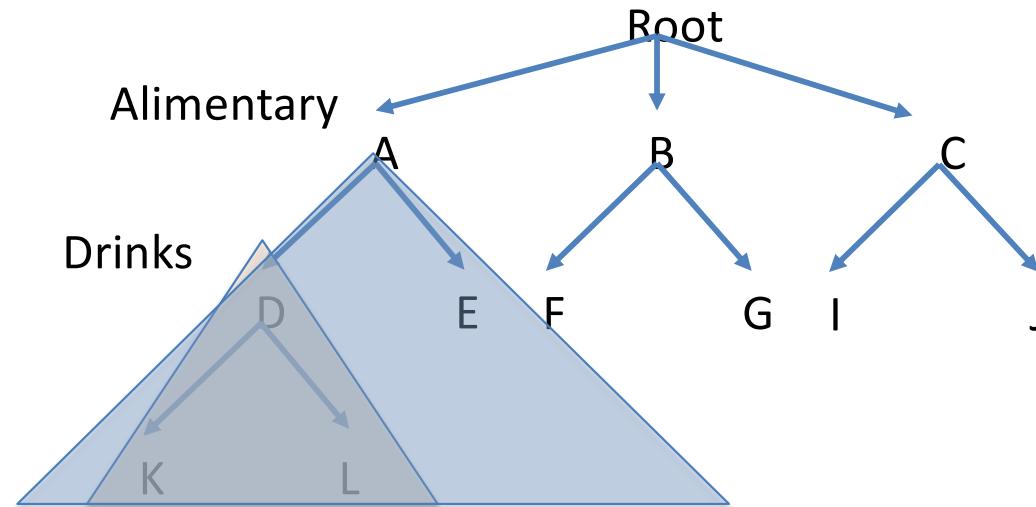


# At Different Levels of Granularities



	T1										T2					T3					A ..
	A	B	C	D	E	F	G	H	I	J	A	B	C	D	E	F	G	H	I	J	
C1	1																			1	
C2	1											1								1	
C3		1										1								1	
C4		1											1							1	
C5			1										1							1	
C6			1											1						1	
C7			1											1						1	
C8					1										1					1	
...																				130	

# At Different Levels of Granularities



Group at the upper level

	T1			T2			T3			..	
	A	B	C	A	B	C	A	B	C		
C1	1					1			1		
C2	1				1				1		
C3		1			1			1			
C4		1				1			1		
C5	1					1		1			
C6	1				1				1		
C7	1			1				1			
C8	1			1					1		
...											

# Usefulness

---

- Personalization: Probabilistic Latent Semantic Analysis
- Efficiency: preload pages, site organization
- E-commerce: Ad placements



# PMSI Data

---

- Using data from the French hospital database (PMSI) to estimate **coronary thrombosis**
- PMSI is a coding system mandatory for hospital public service and private care facilities

## Patient general status

D 001	A S A 1	001 BE
D 002	A S A 2	012 BE
D 003	A S A 3	014 BE
D 004	A S A 4	020 BE

## Anaesthesia type and anaesthesia recovery

### 1) General anaesthesia

No addition between all these items

D 010	General anaesthesia	000 BE
D 011	General anaesthesia with intubation	030 BE
D 018	General anaesthesia with blood transfusion	075 BE
D 012	General anaesthesia in Trendelenbourg position	030 BE

D 040	30 min	025 BE	D 049	5 h	250 BE
D 041	1 h	050 BE	D 050	5 h 30 min	275 BE
D 042	1 h 30 min	075 BE	D 051	6 h	300 BE
D 043	2 h	100 BE	D 052	6 h 30 min	325 BE
D 044	2 h 30 min	125 BE	D 053	7 h	350 BE



# Clusters

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- Clusters are the different types of cares (main diagnosis)

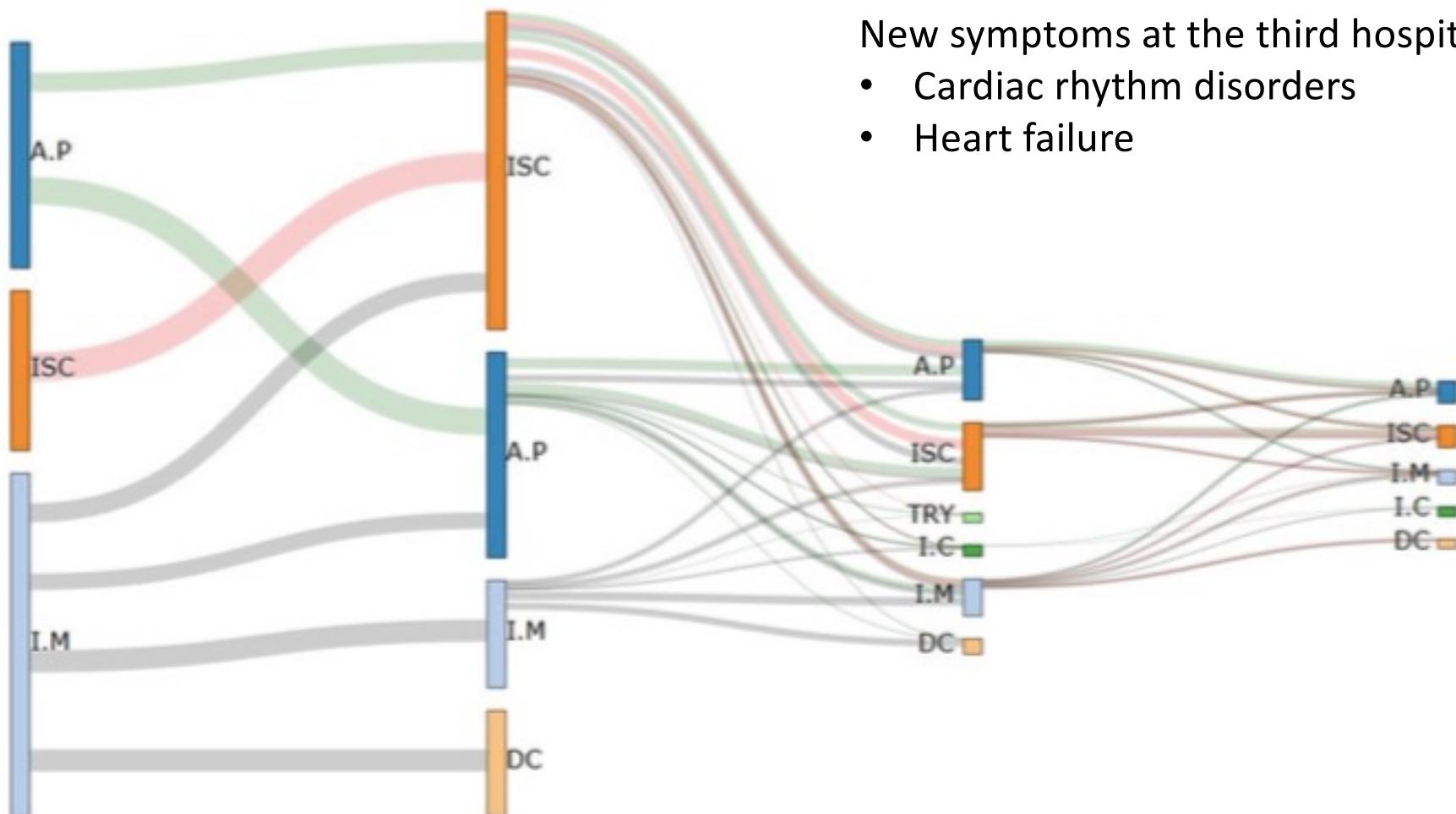
I21: coronary thrombosis  
I20: Angina pectoris  
R07: Chest pain  
E14: Diabetes  
I70: Atherosclerosis

<b>Patient</b>	$t_0$	$t_1$	$t_2$	$t_3$
$P_1$	I21	E14	I20	I70
$P_2$	I21	R07	I20	
$P_3$	I21	E14	I20	I70
$P_4$	I21	R07	I20	

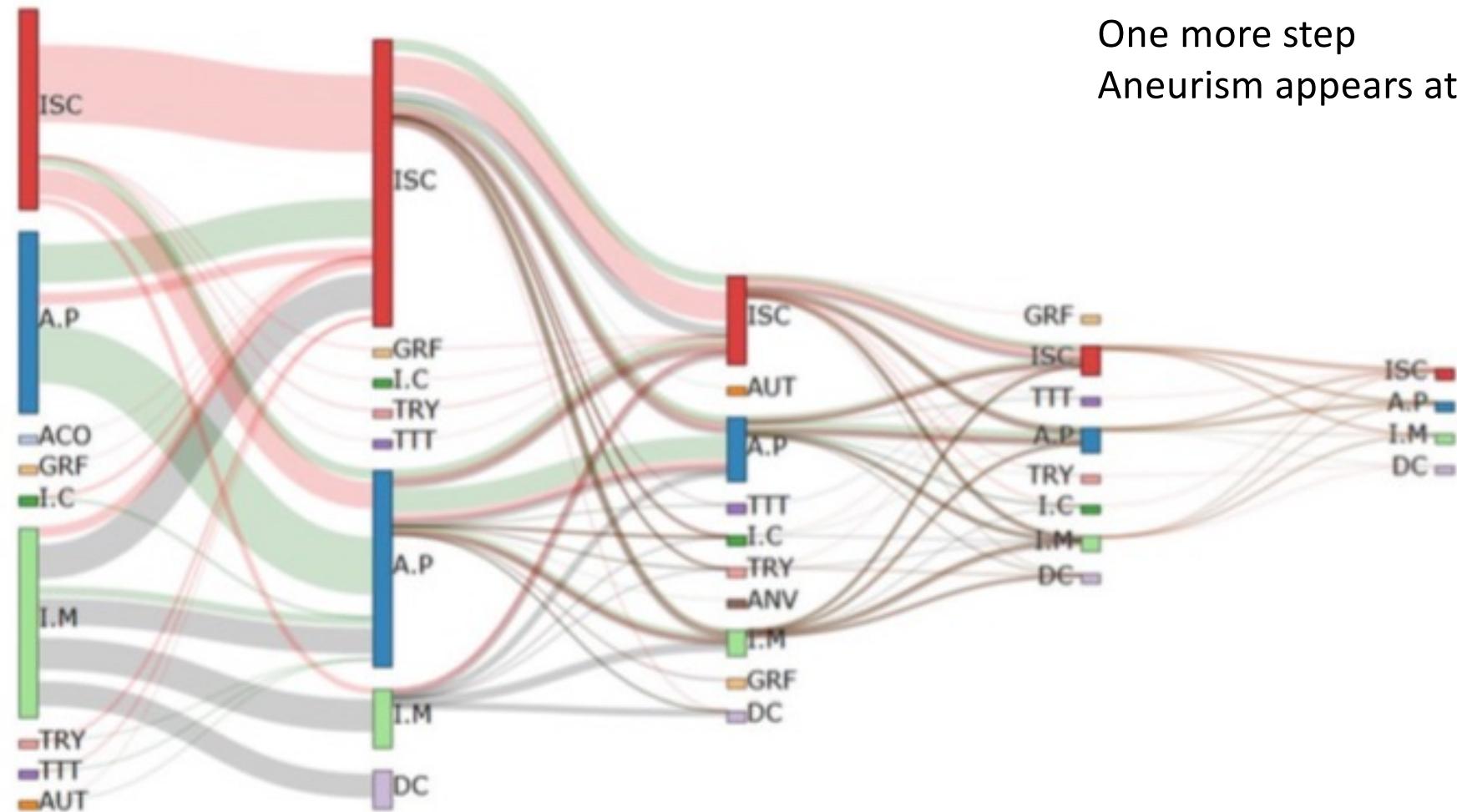
- From 2009 to 2014, all patients having coronary thrombosis as main diagnosis



# Women > +65

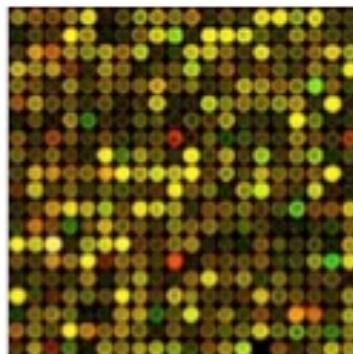


# Men >+65

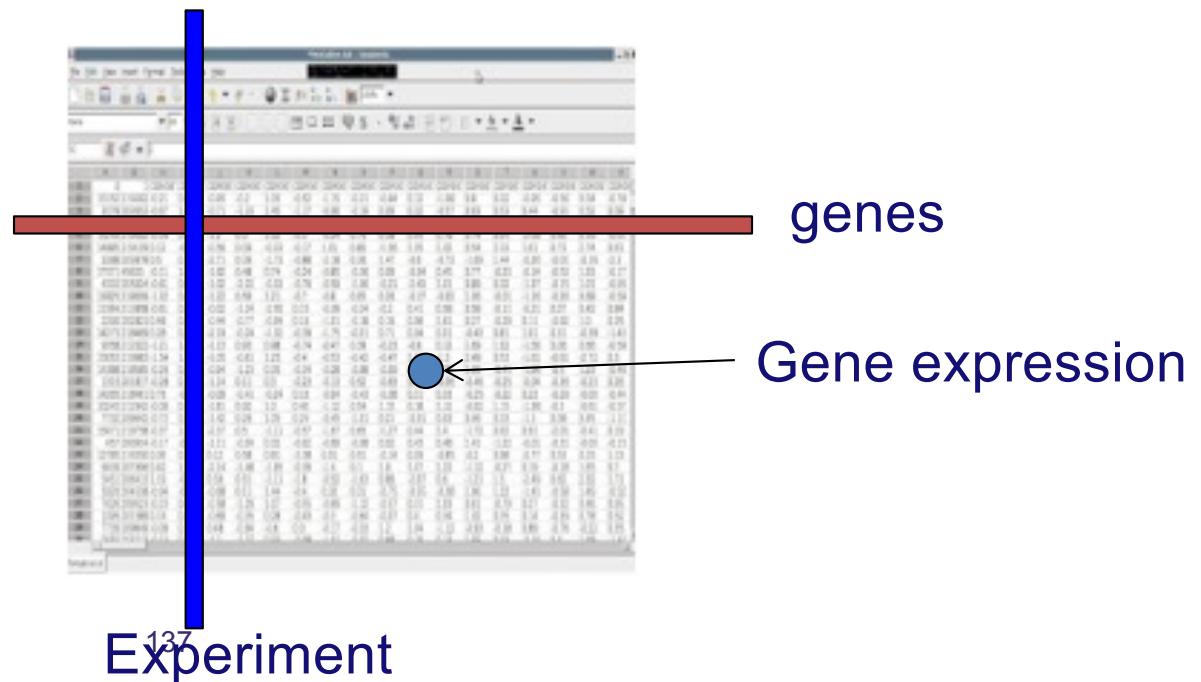


# DNA Arrays Analysis

- DNA Arrays analysis: understanding the reaction of genes on different conditions
- Experiments conducted on Alzheimer disease, Cancer and HIV



Affymetrix U-133 plus 2.0 Array  
54,675 probesets



# Dealing with HIV Data

---

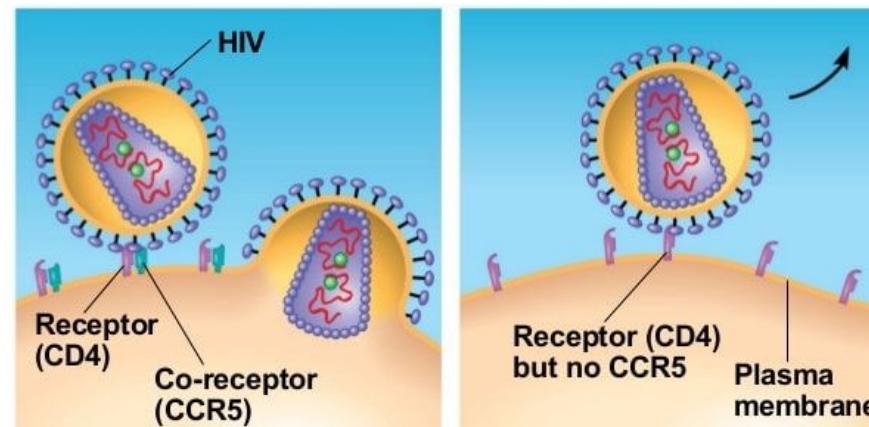
- Two major types of HIV viruses exist: type-1 (HIV-1) and type-2 (HIV-2)
- HIV-2 is a close relative of the prototype AIDS virus HIV-1
- HIV-2 is biologically similar to HIV-1, but precision concerning clinical outcomes of HIV-2 infected individuals are lacking



# Co-receptor CXCR4 – CCR5

- HIV-1 cells invasion is enabled by the binding of envelope glycoproteins to the receptor CD4 and a co-receptor, principally CXCR4 or CCR5, according to the viral strain (X4 or R5, respectively).

Figure 7.11



# Experiments

---

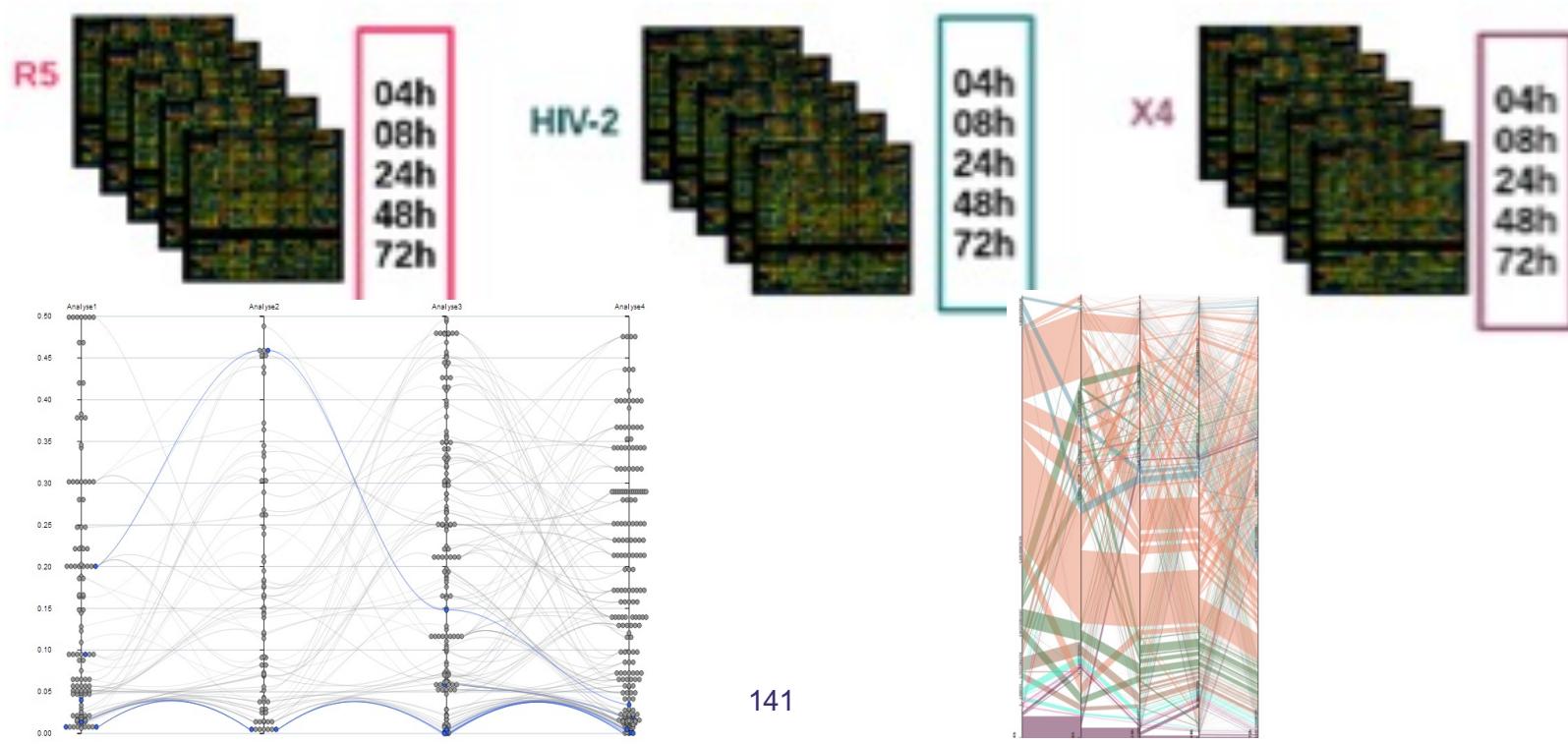
- Expression profile of about 19.000 human genes at 5 points timestamps after infection by one of 3 strains of HIV (HIV-2, X4, R5)

$T_{db}$	04h	08h	24h	48h	72h	
Clusters $C_{db}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	
Genes	gene <sub>1</sub>	1			1	1
	gene <sub>2</sub>	1		1	1	1
	gene <sub>3</sub>	1	1	1	1	1
	gene <sub>4</sub>	1	1	1	1	



# Extracting patterns

- « Real unknown and interesting phenomena. Correlations between biological functions » dixit Biologist. Under investigation



# Trajectories for Tweets

---

- A tweet:
  - 140 characters
  - Tags (e.g. #saopaulo18, @user), RT (retweet)
  - Metatags (location, date, user-id, ...)
- Cluster of users
- Cluster of words



# Using Tweets

---

- Identification of patterns of retweets and to understand how information spreads over time in Twitter
- Tweetprofile a research platform for extracting, storing and analyzing the Portuguese Twittosphere for research and journalistic purposes

**RetweetPatterns: detection of spatio-temporal patterns of retweets**

Tomy Rodrigues<sup>1</sup>, Tiago Cunha<sup>1</sup>, Dino Ienco<sup>2</sup>, Pascal Poncelet<sup>3</sup>, and Carlos Soares<sup>1</sup>



# Experiments

---

- A set of retweets extracted at the time of the protests in Brazil
- From June 2013 and July 2013 - 17083 tweets extracted from Twitter during a protests period in Brazil
- After a preprocessing : 260 retweets



# Results

- 18 closed swarms (84 retweets), 5 convoys, 5 moving clusters
- the majority of paths found are located in Rio de Janeiro and São Paulo

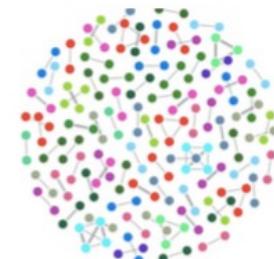


Closed Swarm patterns found

bomba brasil curitiba dilma gente manifestai  
ogiganteacordou orgulho  
pessoas pra protesto protestorj  
**protestos**  
protestosp rj rua ruas  
**vemprarua**



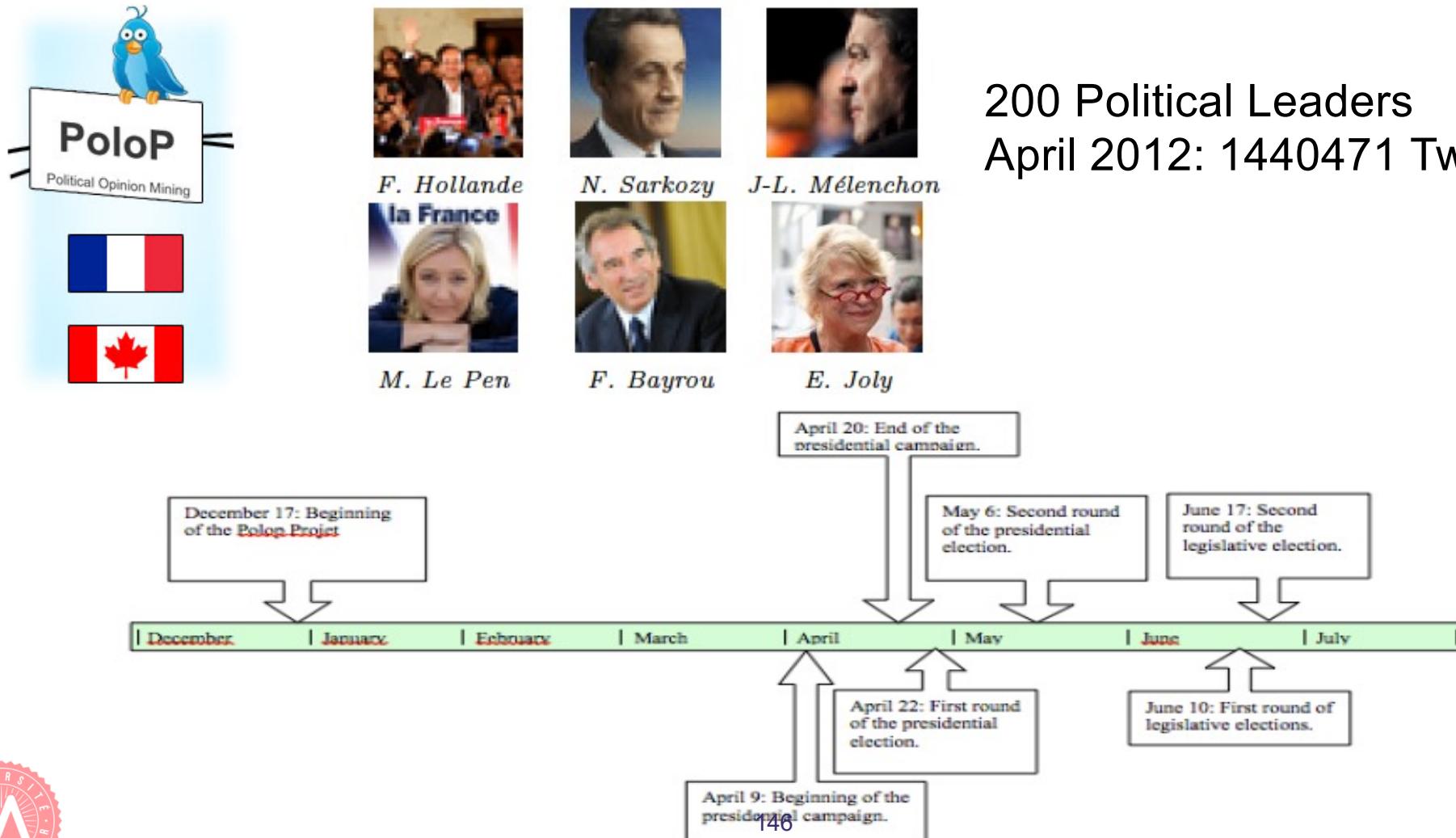
8:23 PM  
June 20, 2013  
**@DionisioJuliana**  
RT @papelop: Av. Paulista agora... #vemprarua  
#popovaacordou @ Avenida Paulista  
<http://t.co/HvoFymXj99>



Timeline and wordcloud for the Closed Swarm patterns found

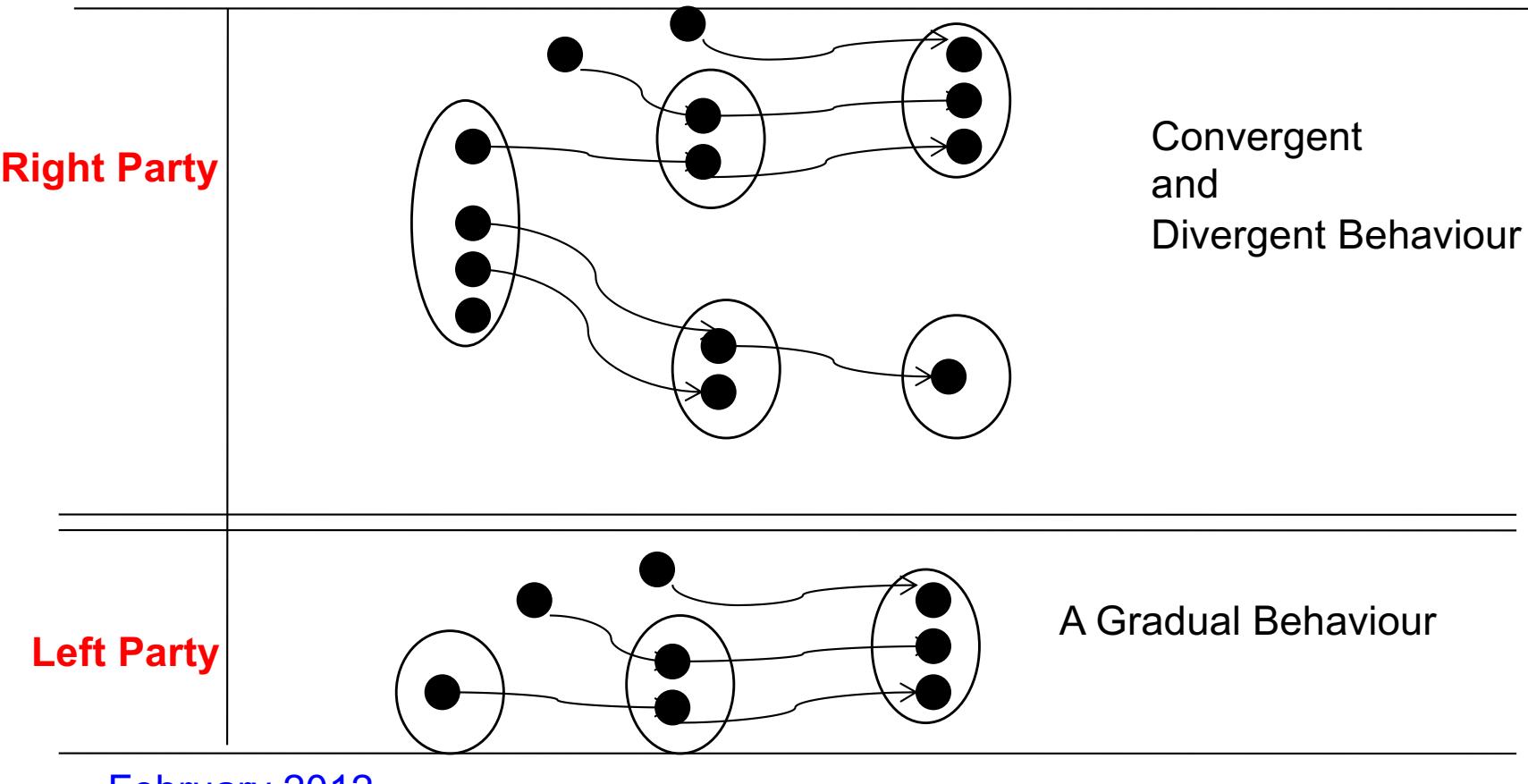
# Tweets for Communities

- A set of users sharing the same interest



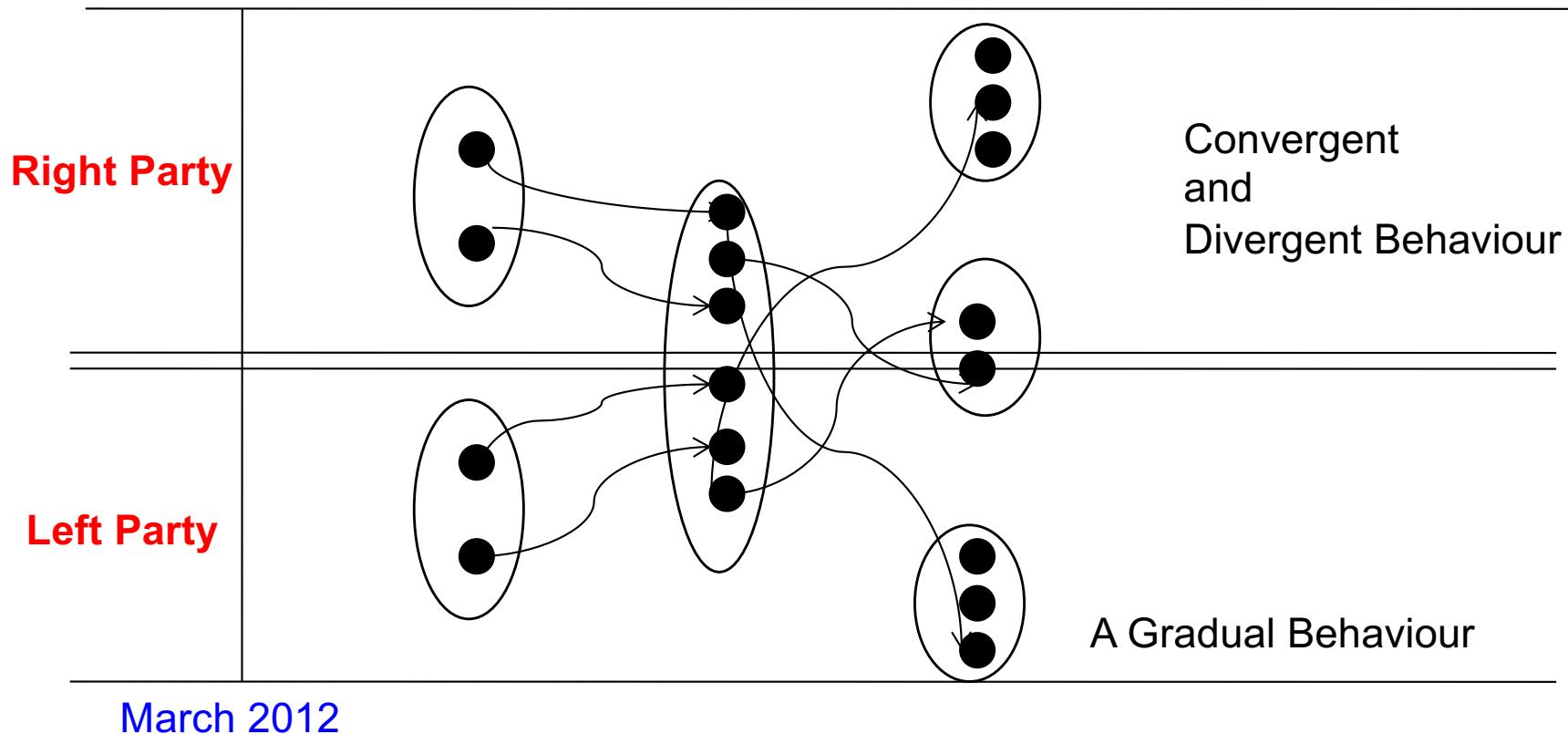
# One trajectory

- “*The Debate - The focus of the French election campaign suddenly shifts from the economy to racism and national identity*” (07/02/2012)



# One trajectory

- "Shootings in Toulouse and Montauban: The victims - Seven people have been killed and two wounded in serial gun attacks in south-western France" (3/2012)



# A typical French trajectory



President  
May 2012 – May 2017



1980 - 2007

**Ségolène Royal**



2005 - 2014

**Valerie Trierweiler**

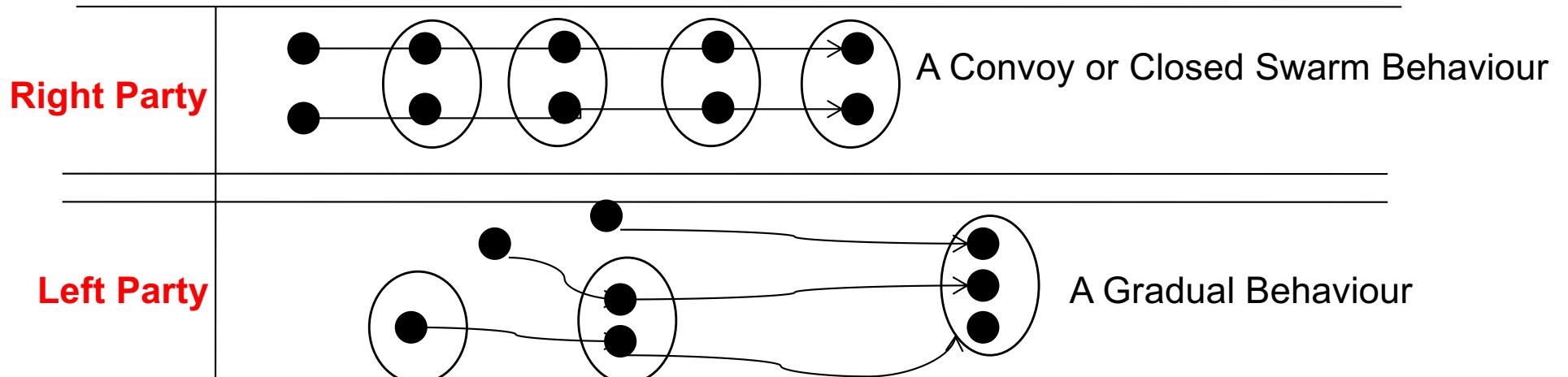


2013 -

**Julie Gayet**

# One trajectory

- “Courage to Olivier Falorni who has not been unworthy, who has battled alongside La Rochelle residents for so many years with unselfish commitment” Valerie Trierweiler, First Lady/Girlfriend of the President François Hollande



June 2012

Left Party



Left Party

Valerie Trierweiler

Ségolène Royal



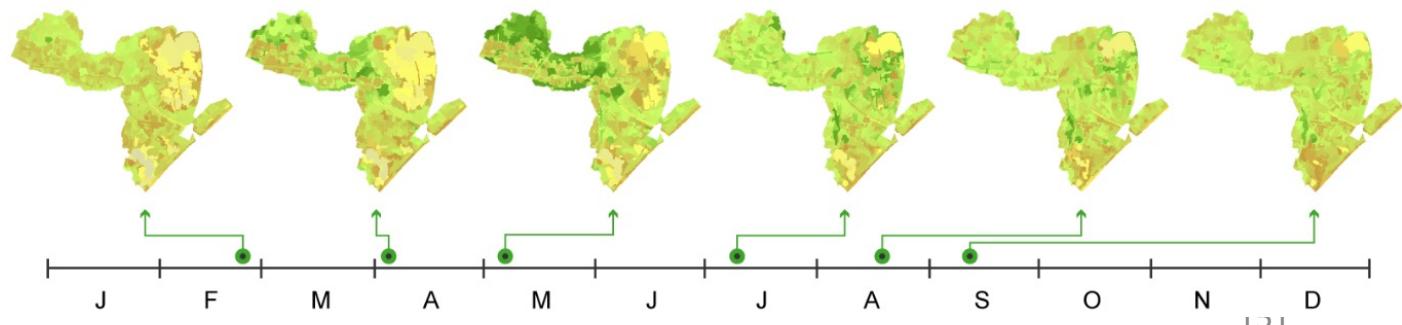
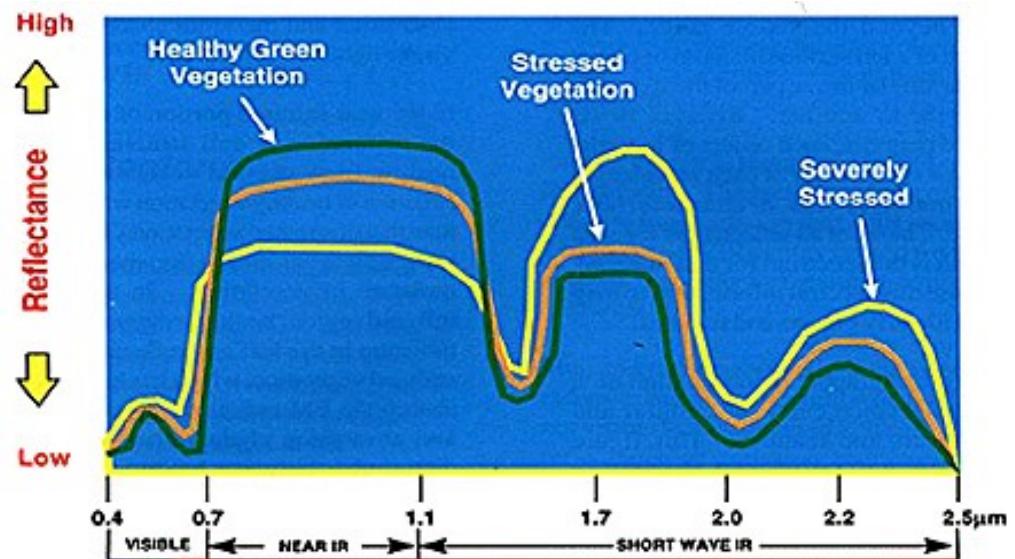
Julie Gayet

150

# Satellite Remote Sensing Time Series

## Remote sensing possibilities

- Spectral response of natural vegetation
  - Vegetation types
- Textural response
  - Vegetation structure (mapping physiognomic classes, e.g. shrubland and grassland)
- Temporal response
  - Vegetation structure dynamics



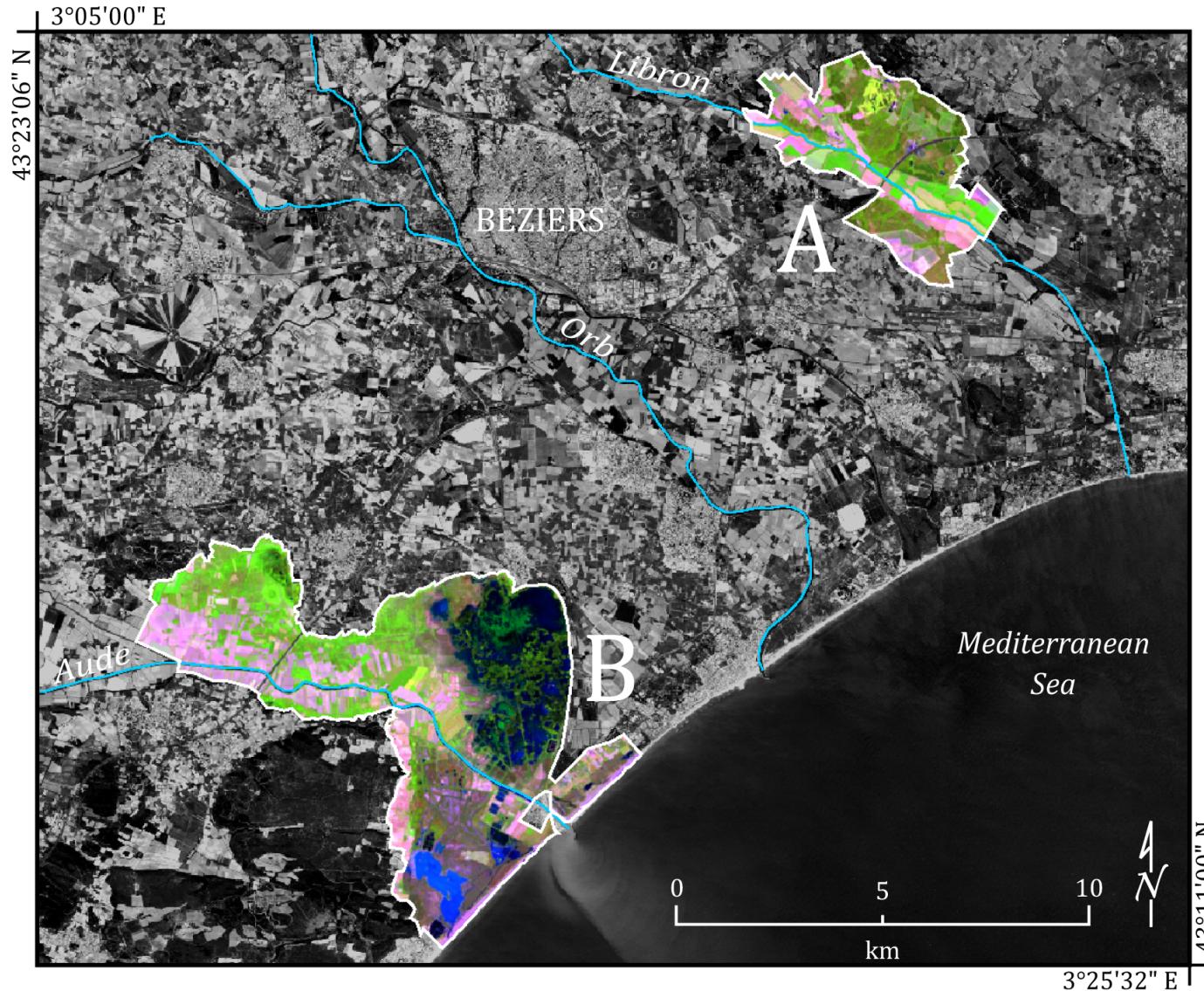
# Motivations

---

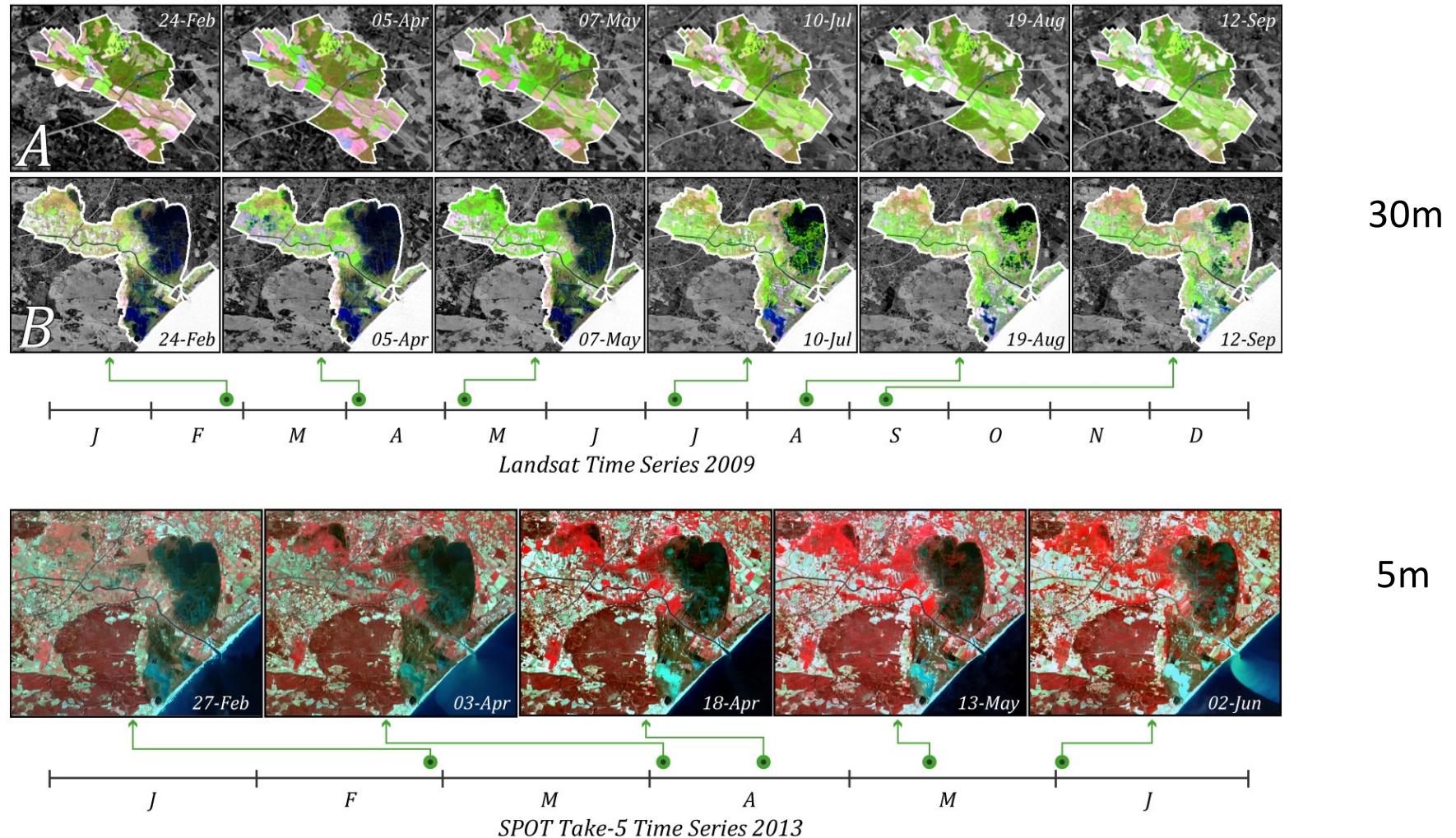
- Automatically detect spatiotemporal evolutions (and their related dynamics)
  - how an entity (i.e. a lake, a saltmarsh area, a crop field) evolves along the time
- Provided useful information for natural habitats monitoring and mapping



# A – Libron Valley | B – Low Aude Plane



# A – Libron Valley | B – Low Aude Plane



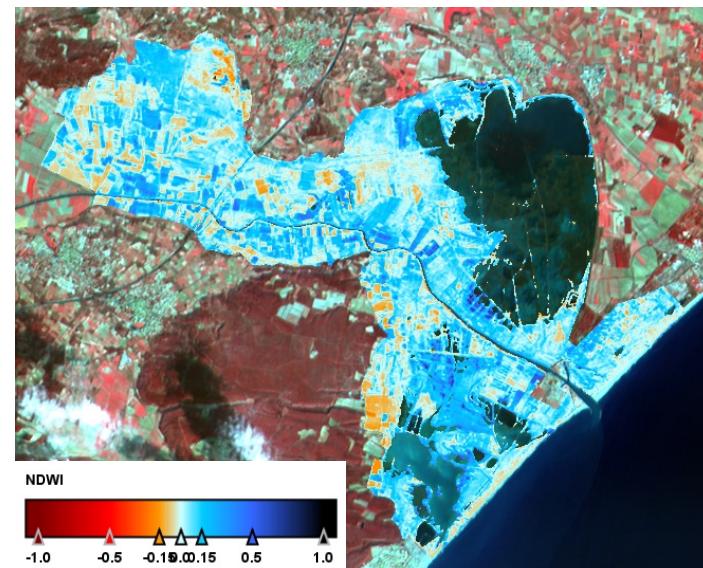
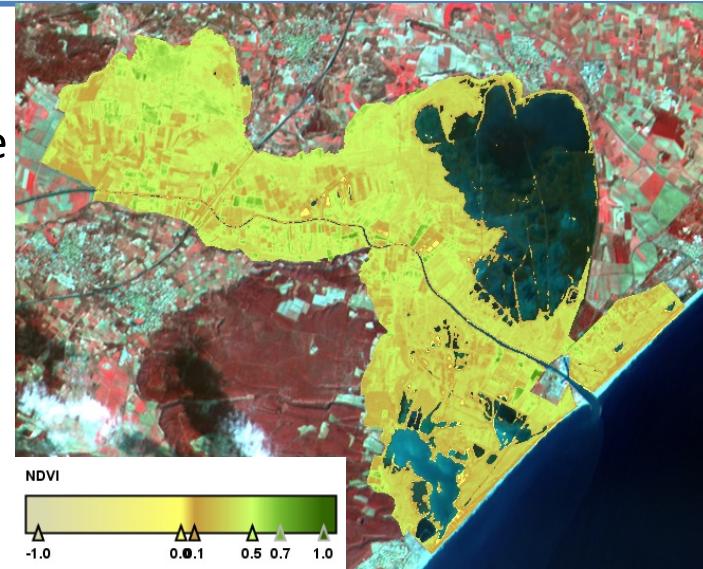
# General Evolution of the Vegetation

## NDVI: Normalized Difference Vegetation Index

Sensitive to the amount of photosynthetically active vegetation present in the plant canopy

## NDWI: Normalized Difference Water Index

sensitive to changes in liquid water content of vegetation canopies

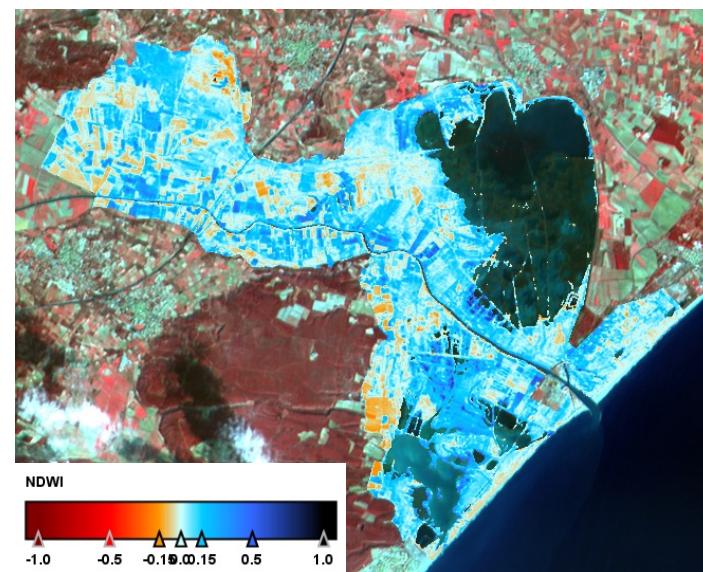
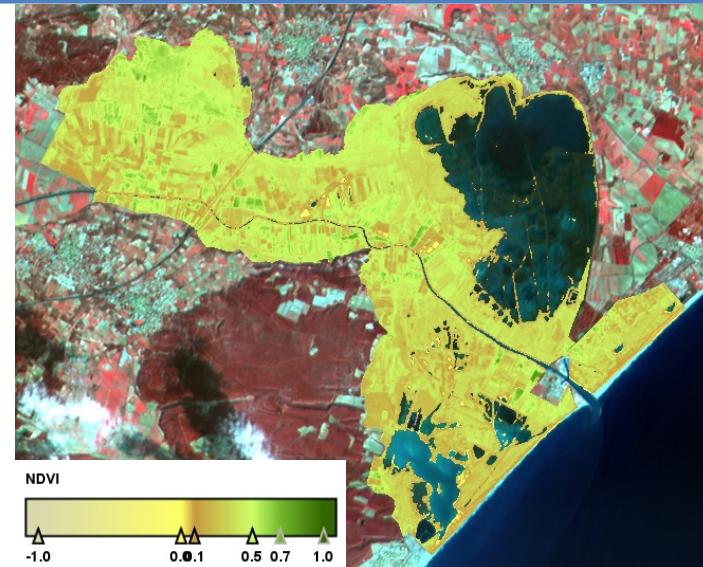


# General Evolution of the Vegetation

NDVI and NDWI Evolutions



Feb Mar Apr May Jun

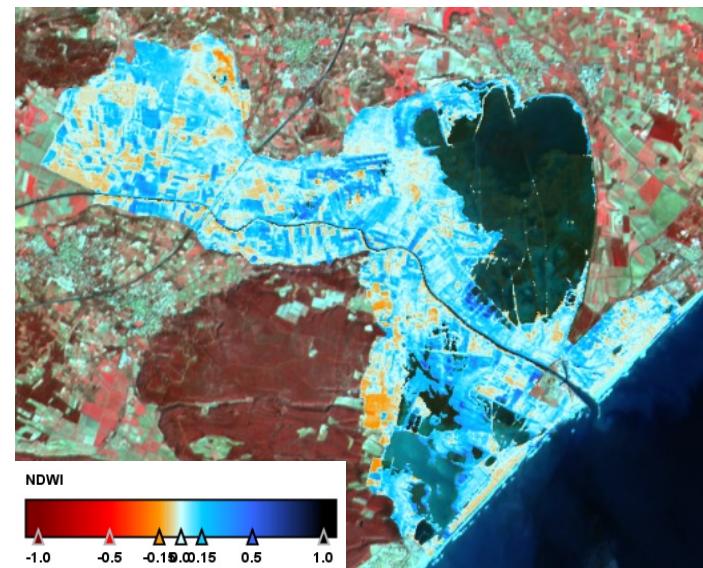
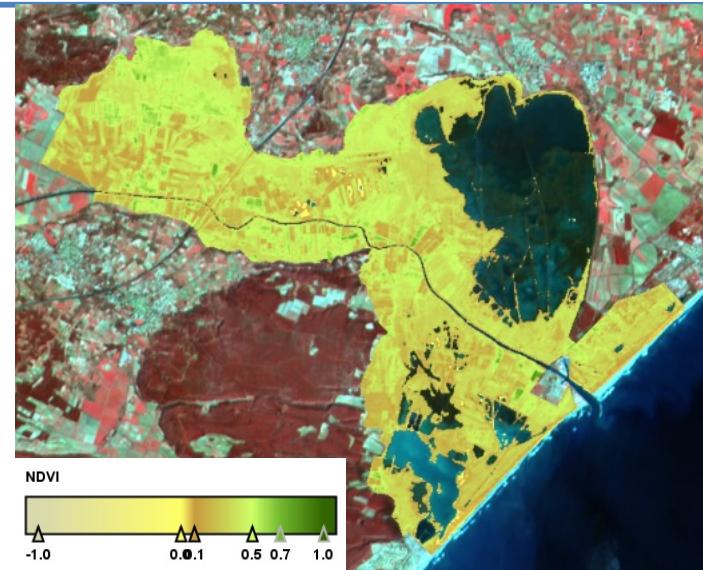


# General Evolution of the Vegetation

NDVI and NDWI Evolutions



27-Feb

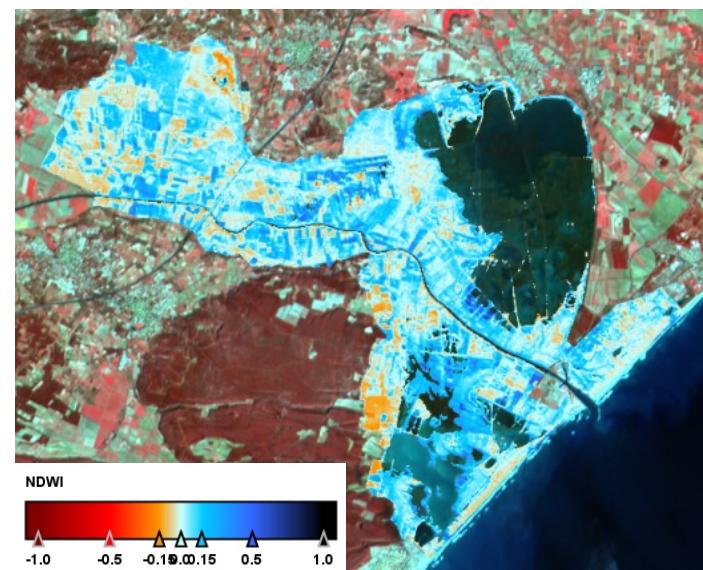
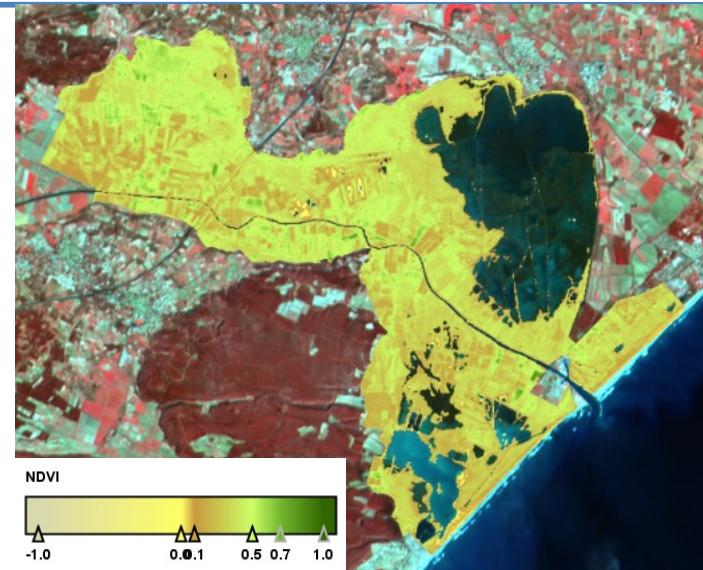


# General Evolution of the Vegetation

NDVI and NDWI Evolutions



27-Feb



# General Evolution of the Vegetation

NDVI and NDWI Evolutions



Université  
Montpellier 1

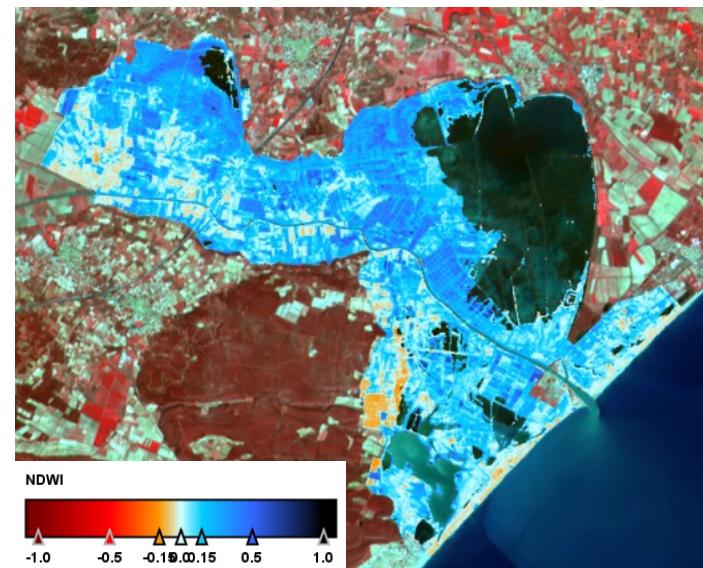
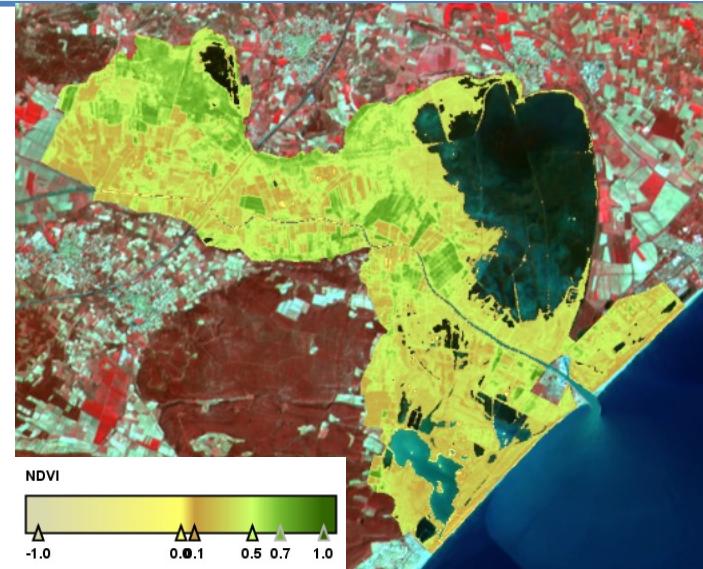
Feb

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# General Evolution of the Vegetation

NDVI and NDWI Evolutions

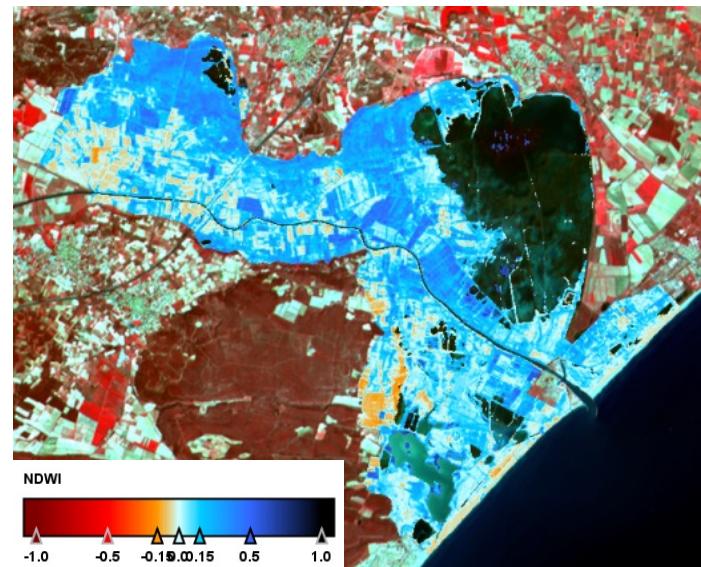
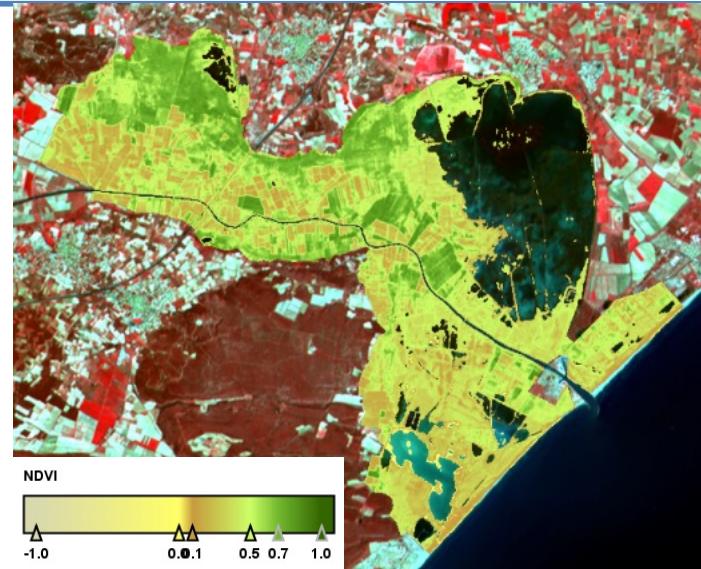


Feb Mar Apr May Jun

Apr

May

Jun



# General Evolution of the Vegetation

NDVI and NDWI Evolutions

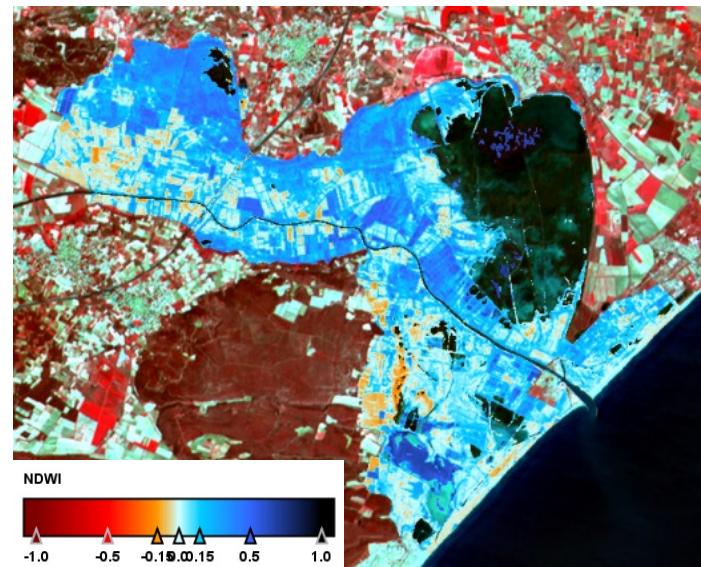
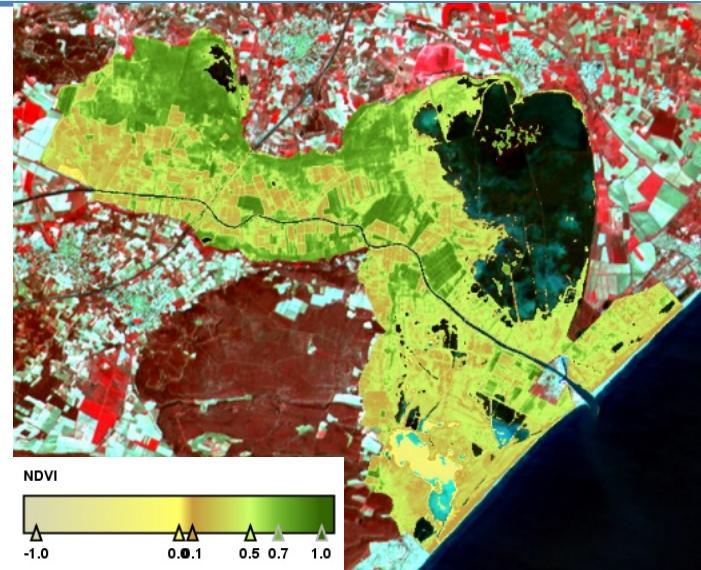


Feb Mar Apr May Jun

Apr

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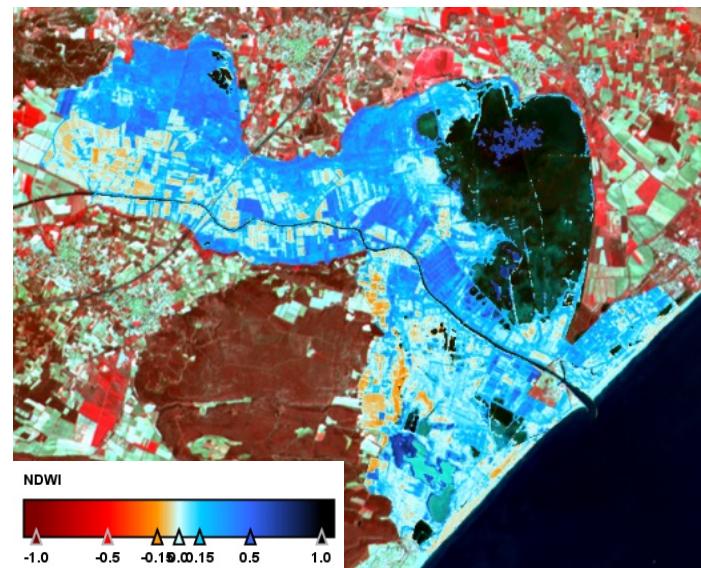
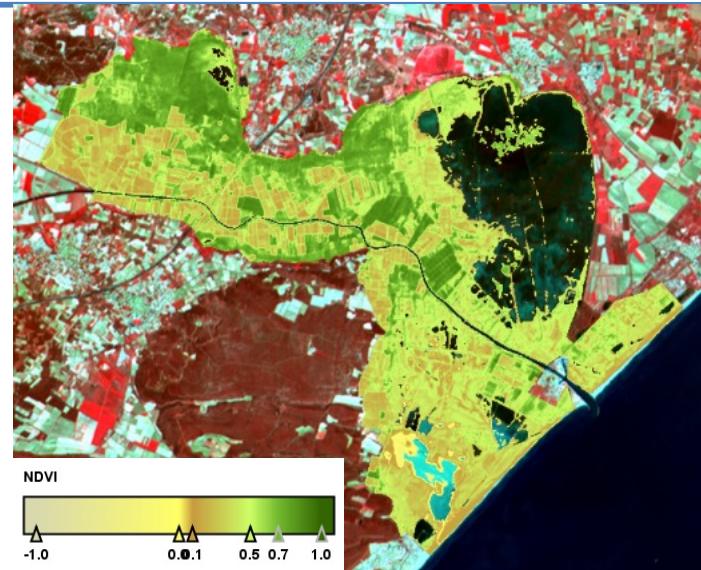


# General Evolution of the Vegetation

NDVI and NDWI Evolutions



Feb Mar Apr May Jun

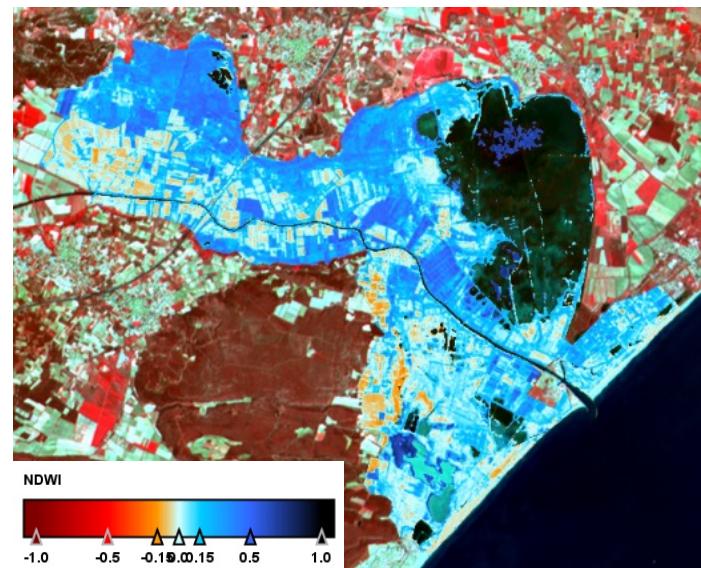
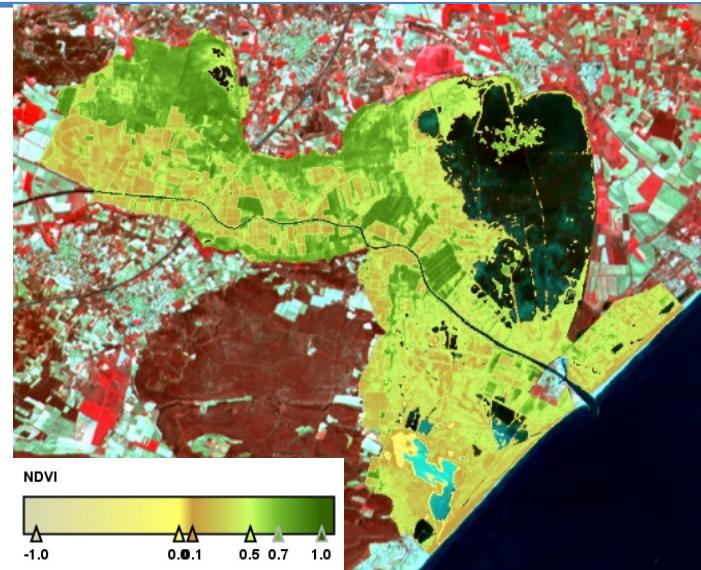


# General Evolution of the Vegetation

NDVI and NDWI Evolutions

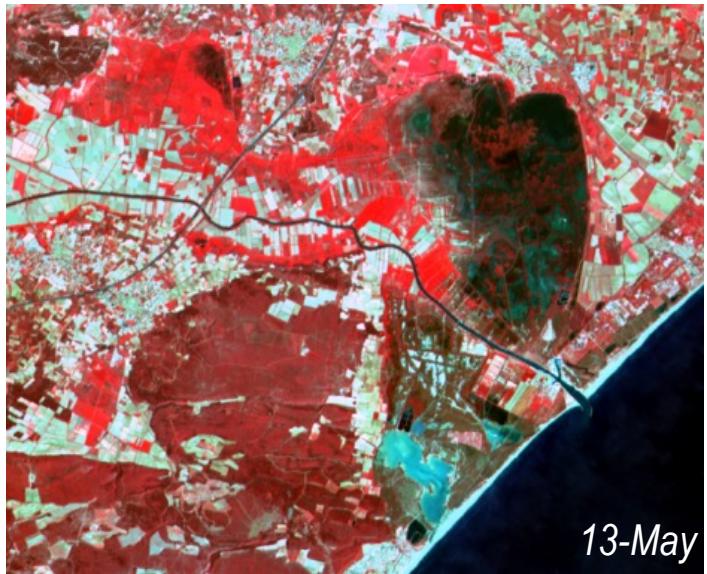


Feb Mar Apr May Jun



# General Evolution of the Vegetation

NDVI and NDWI Evolutions



Université  
Montpellier 1

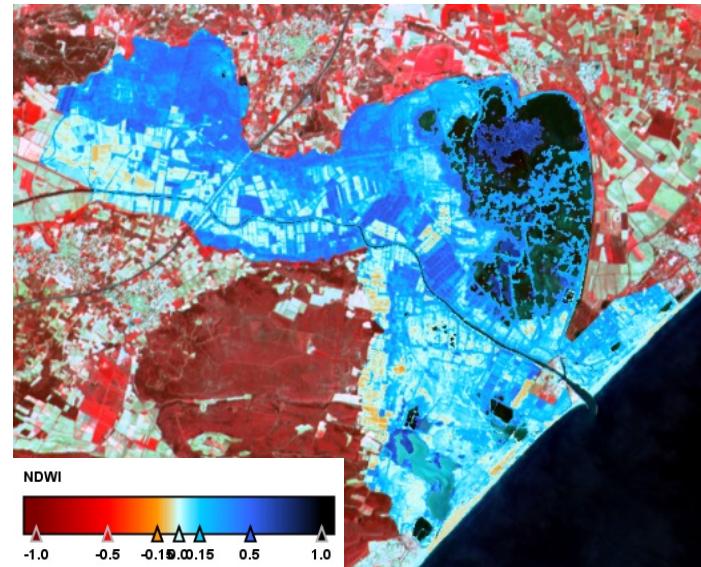
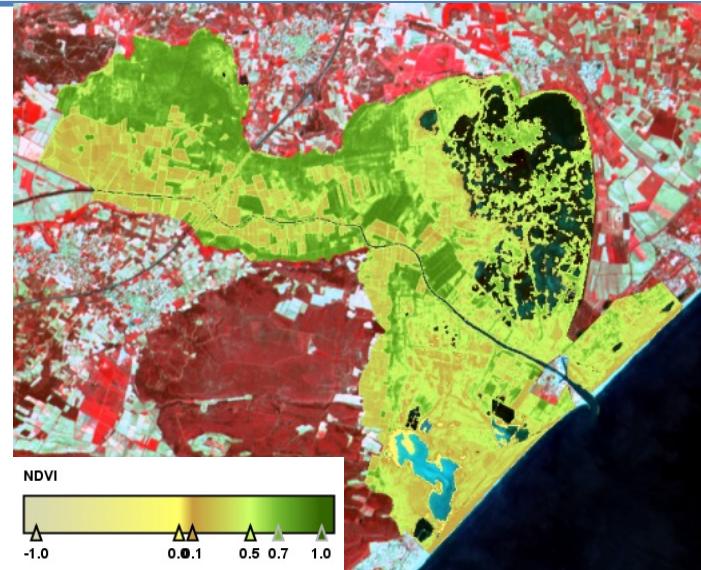
Feb

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May

Jun

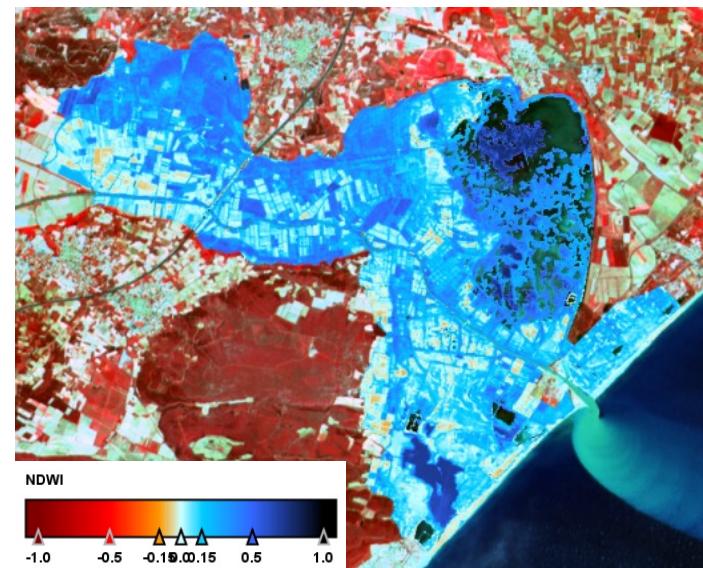
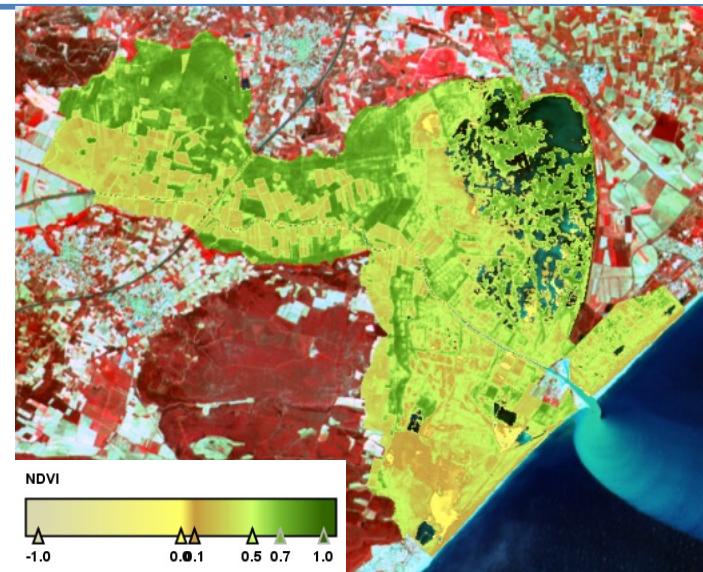


# General Evolution of the Vegetation

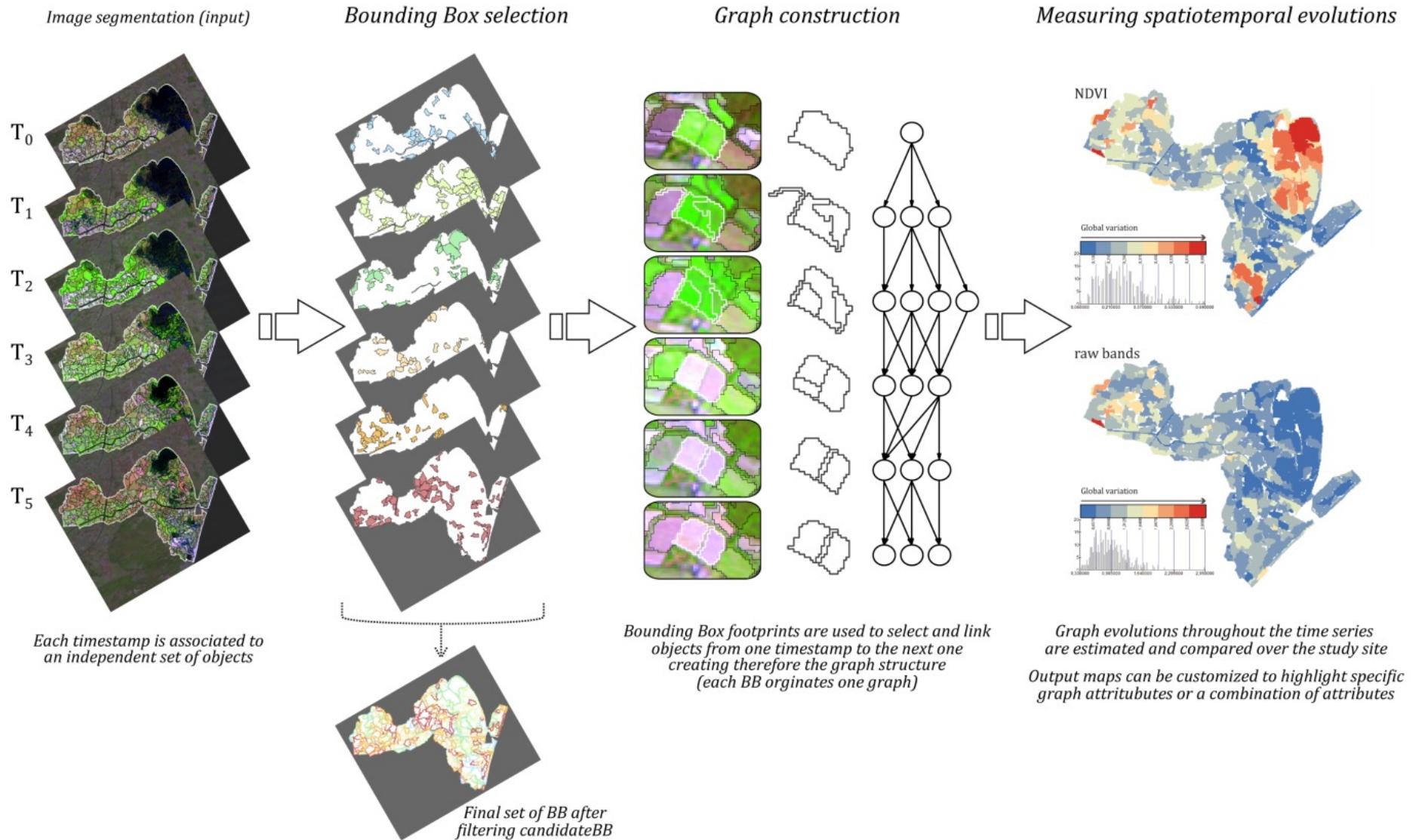
NDVI and NDWI Evolutions



Feb Mar Apr May Jun



# The Main Approach

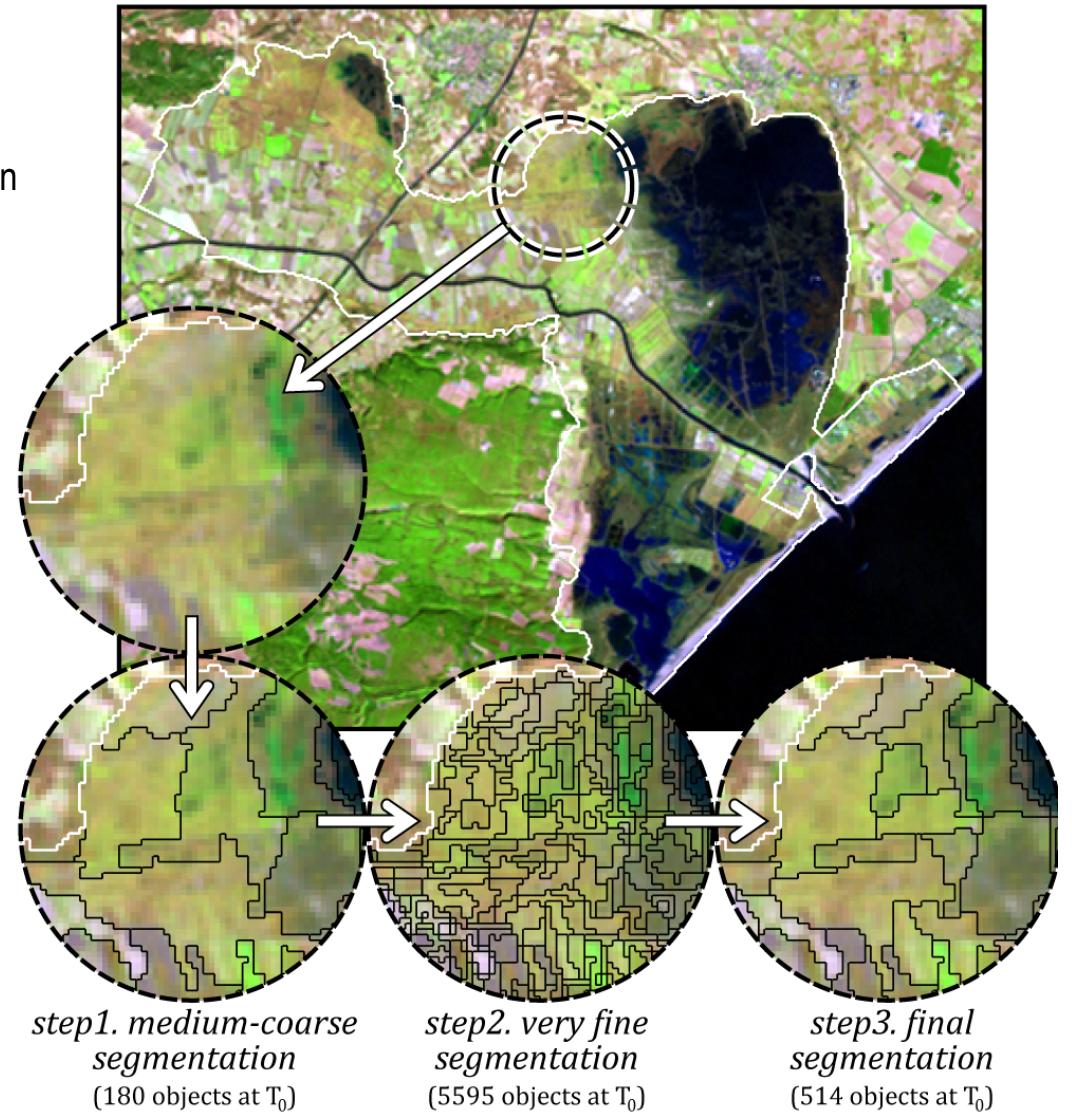


# The Main Steps

## Step 1

- *Preprocessing & Segmentation*

- data standardization
- fine geometrical registration
- spatial subset
- spectral indices
- ...



# The Main Steps

---



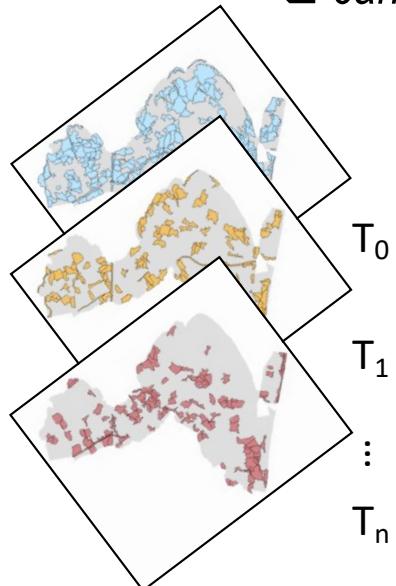
- *Preprocessing & Segmentation*

- ***BB selection***

- Retrieval of the Maximal Spatial Extents

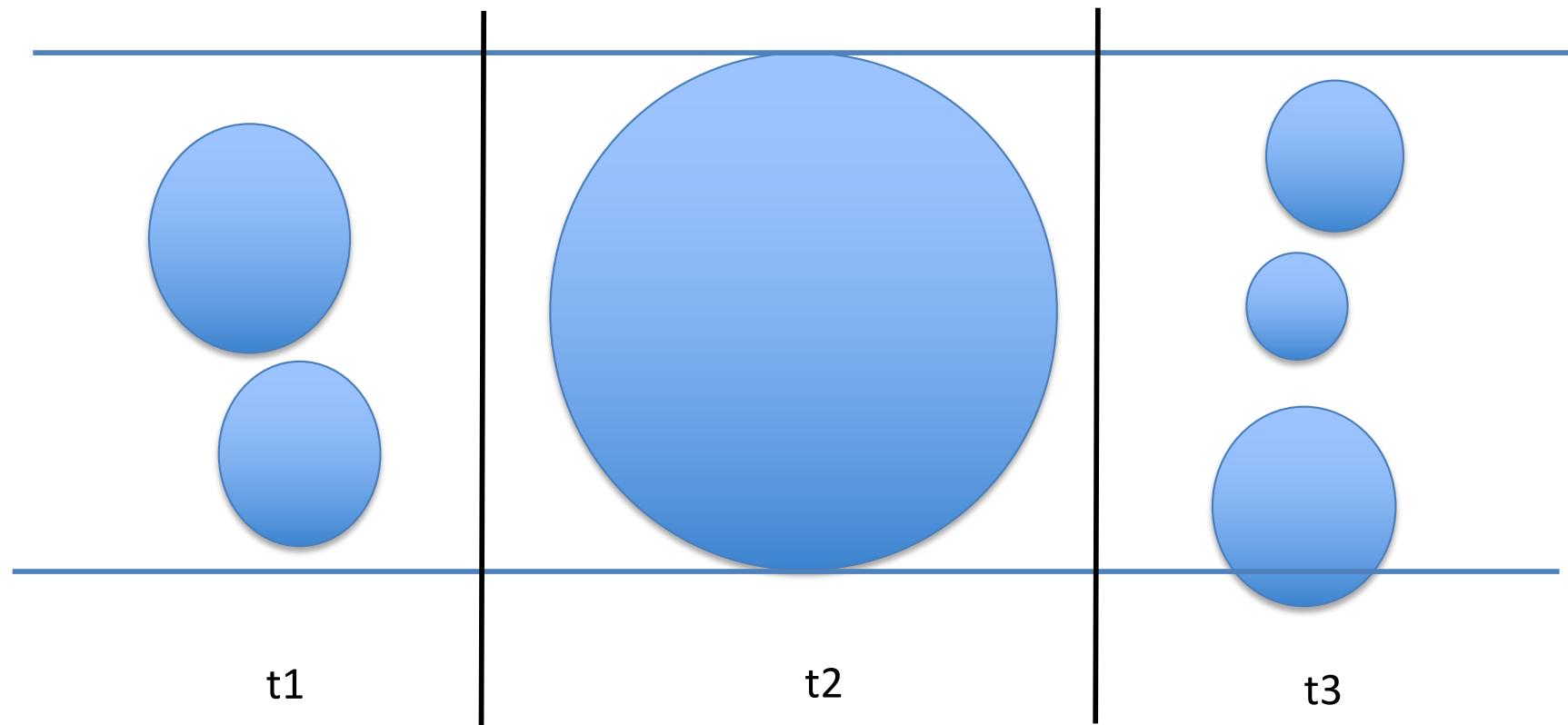
- On the whole study area
- Along the 6 timestamps
- The result is a list of objects

➔ *candidateBB*



# Selection of a Bounding Box

---



A temporary lake: At t2 it reaches its maximal spatial extent

# The Main Steps



- *Preprocessing & Segmentation*

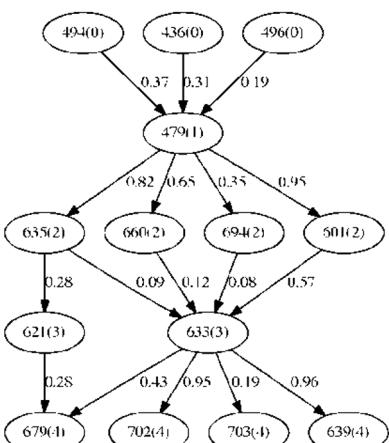
- *BB selection*

- *Graph construction*

- Each *BB* is used to build an evolution graph

- The objects of each graph are chosen following 2 conditions:

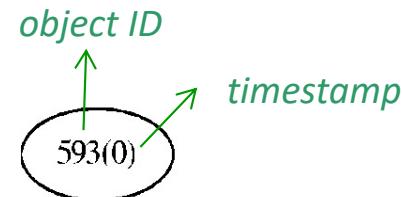
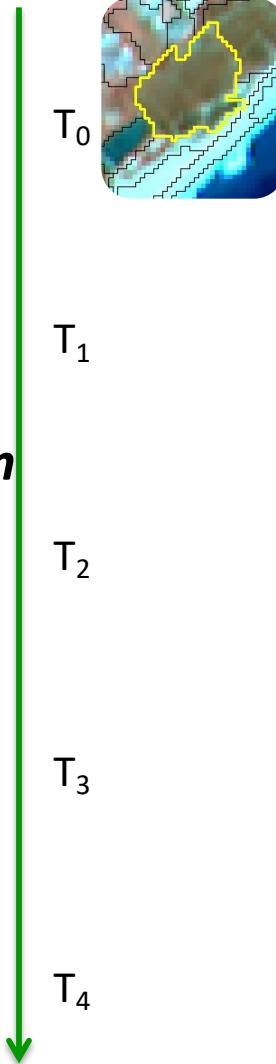
- $\text{objet} \cap \text{BB} \geq 25\%$  of the object surface
- $\text{objet} \cap \text{BB} \geq 20\%$  of the BB surface



# The Main Steps



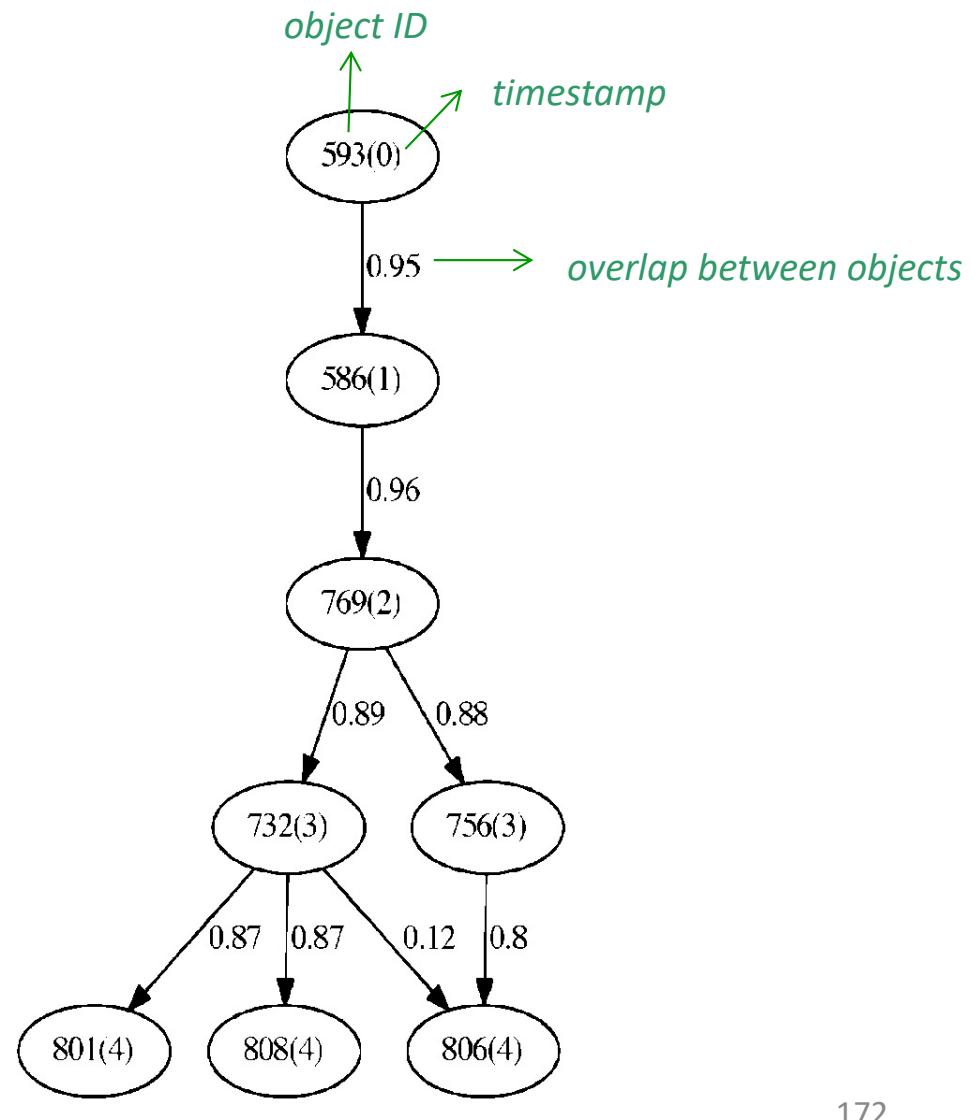
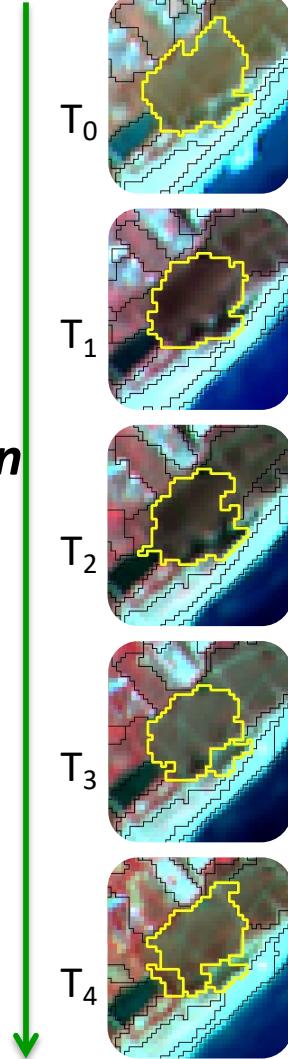
- *Preprocessing & Segmentation*
- *BB selection*
- *Graph construction*



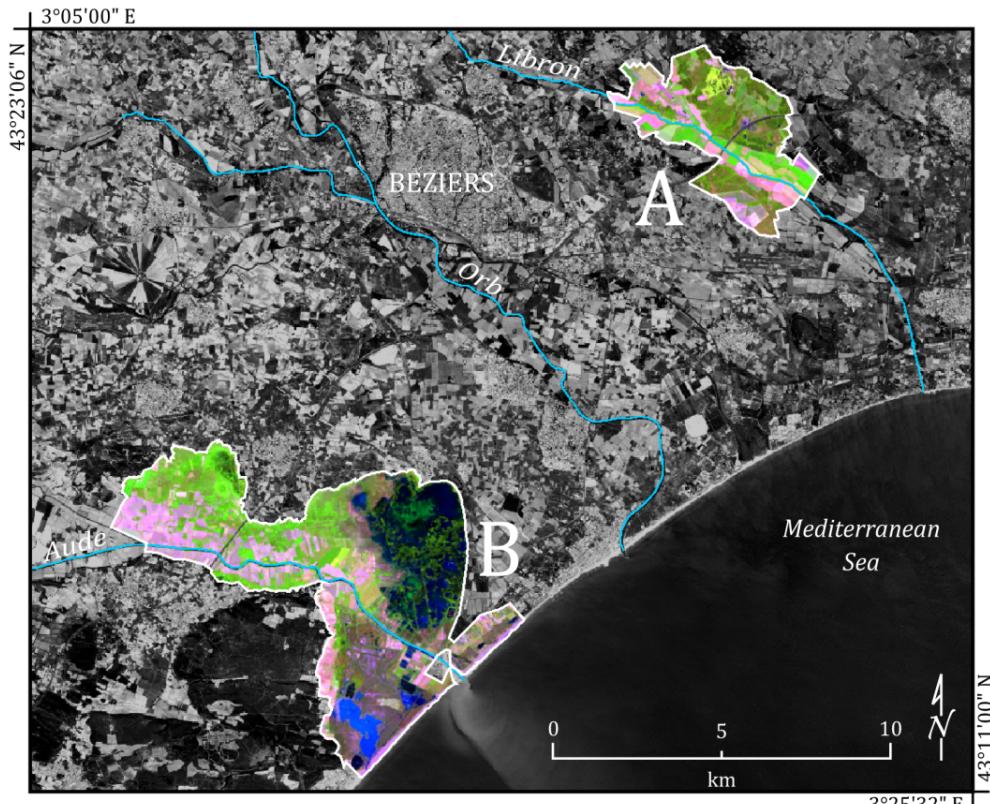
# The Main Steps



- Preprocessing & Segmentation
- BB selection
- Graph construction



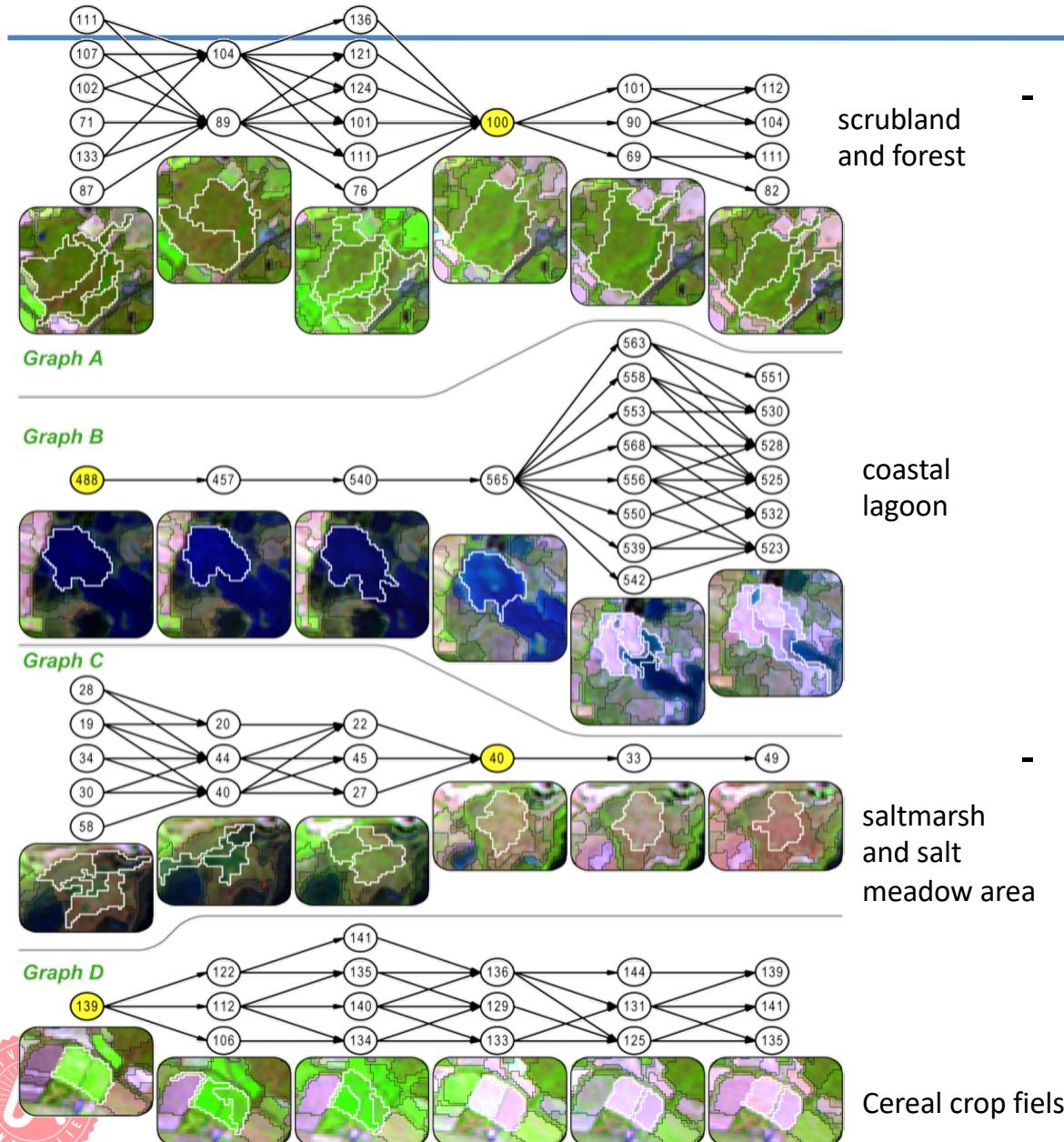
# Results



Landsat 2009

- Number of objects
  - Libron = 1218 (~ 203 per timestamps)
  - BPA = 3 373 (~ 562 per timestamps)
- Number of extracted graphs (evolutions)
  - Libron = 142
  - BPA = 340

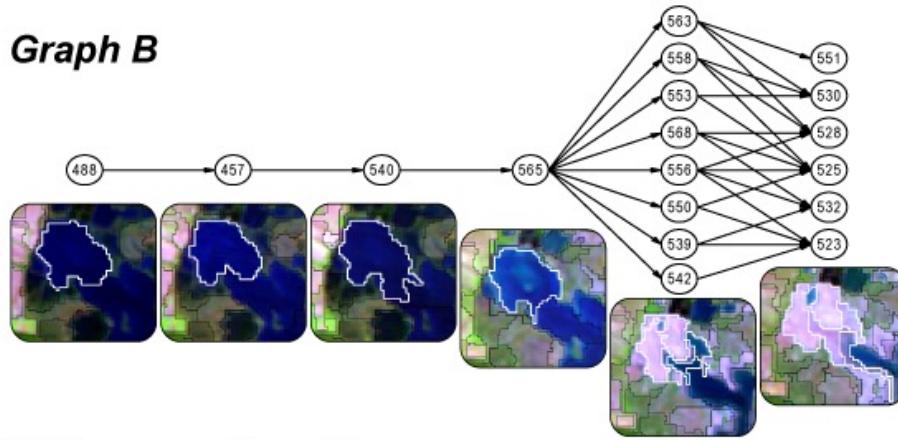
# Evolution Graphs



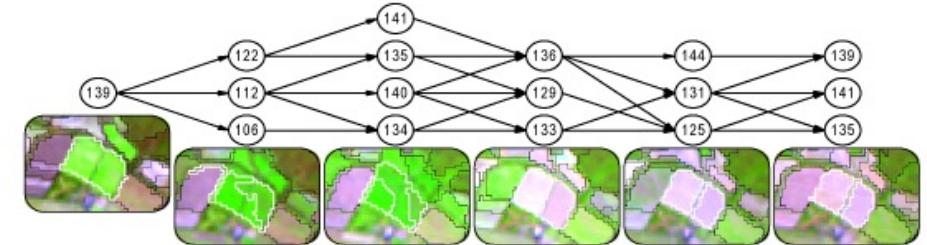
- Each node contains a set of attributes
    - Red
    - Green
    - ...
    - NDVI
    - NDWI
    - VSDI (index conceived to monitor drought)
    - ...
  - The graphs are also described by statistics
    - # of nodes
    - # of edges
    - # of sequences
    - ...

# Evolution Graphs

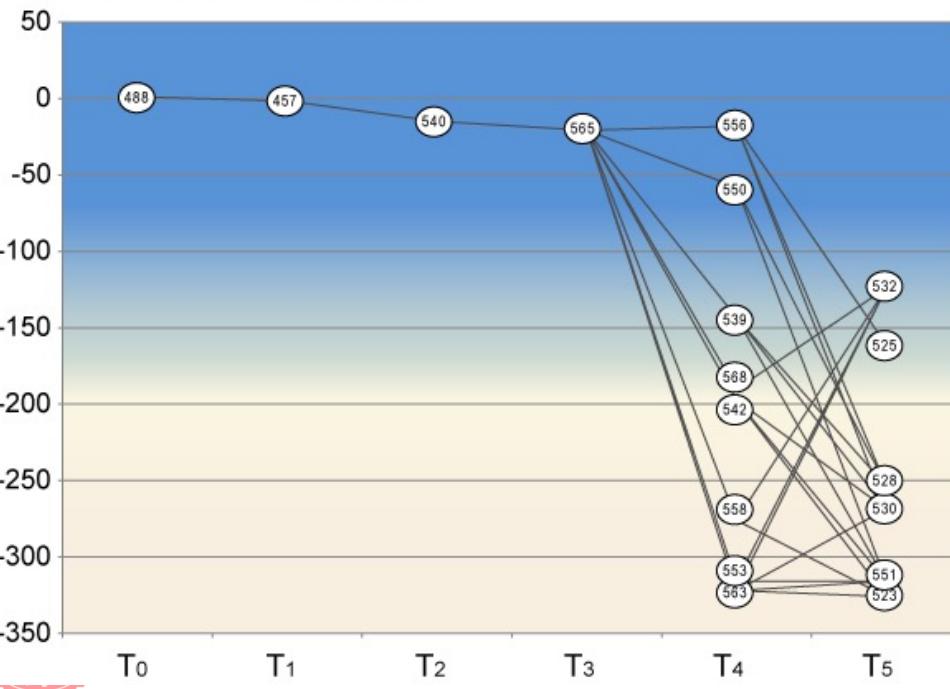
**Graph B**



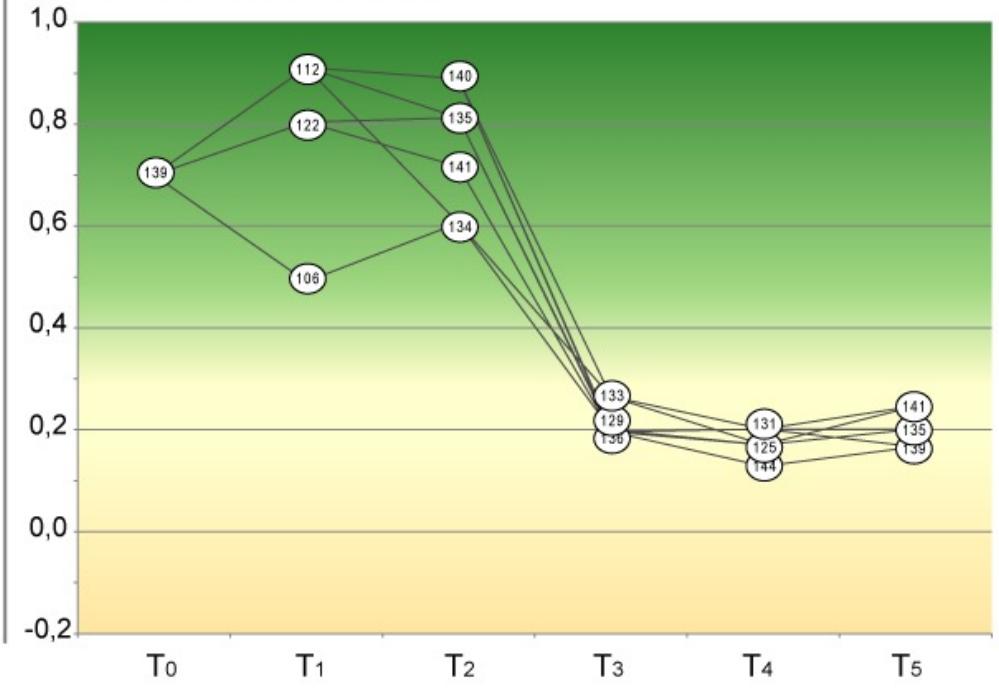
**Graph D**



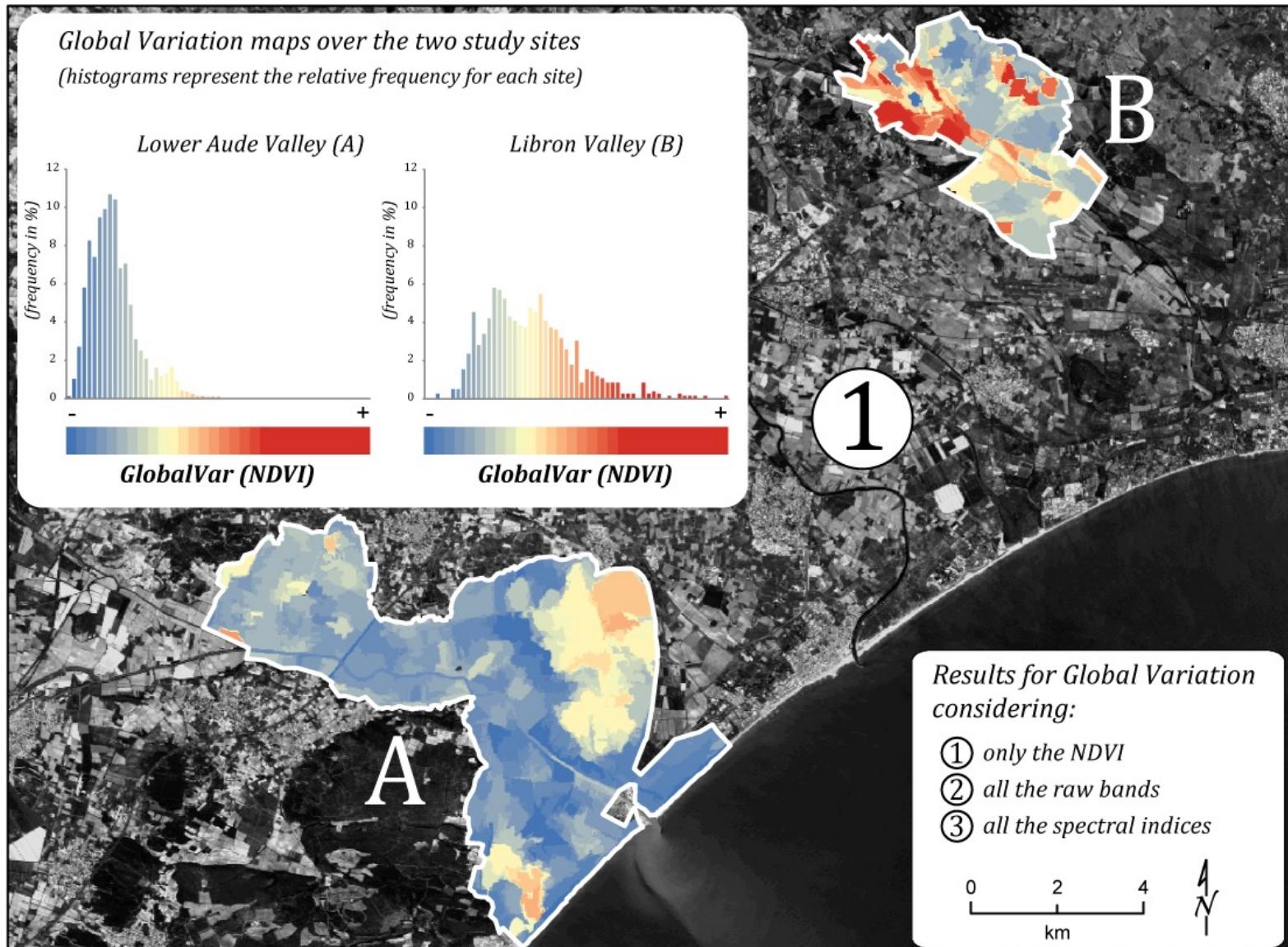
**VSDI temporal profile**



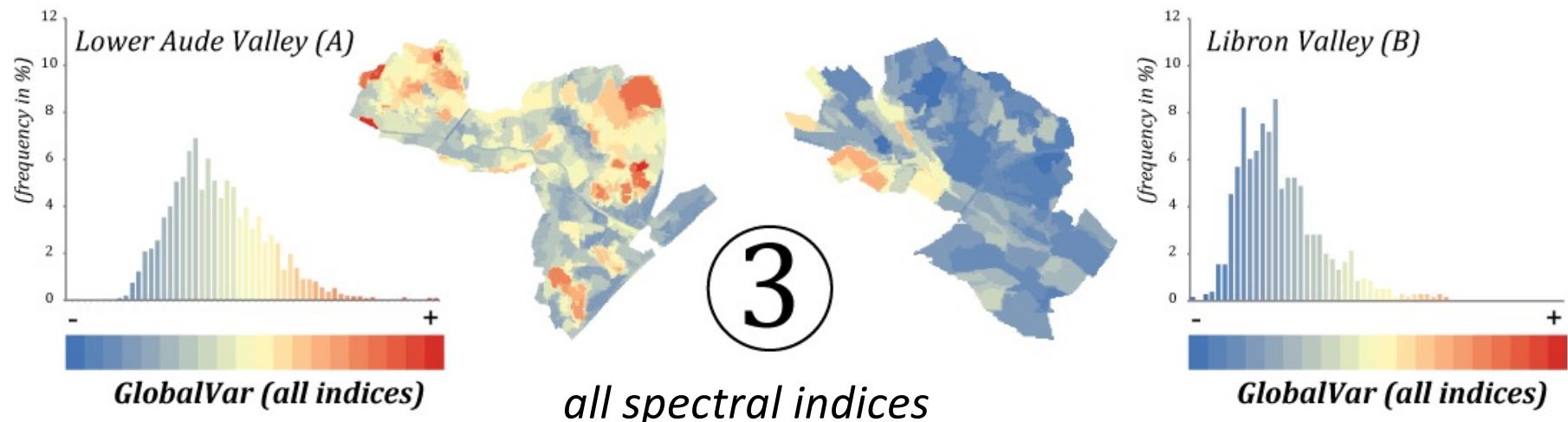
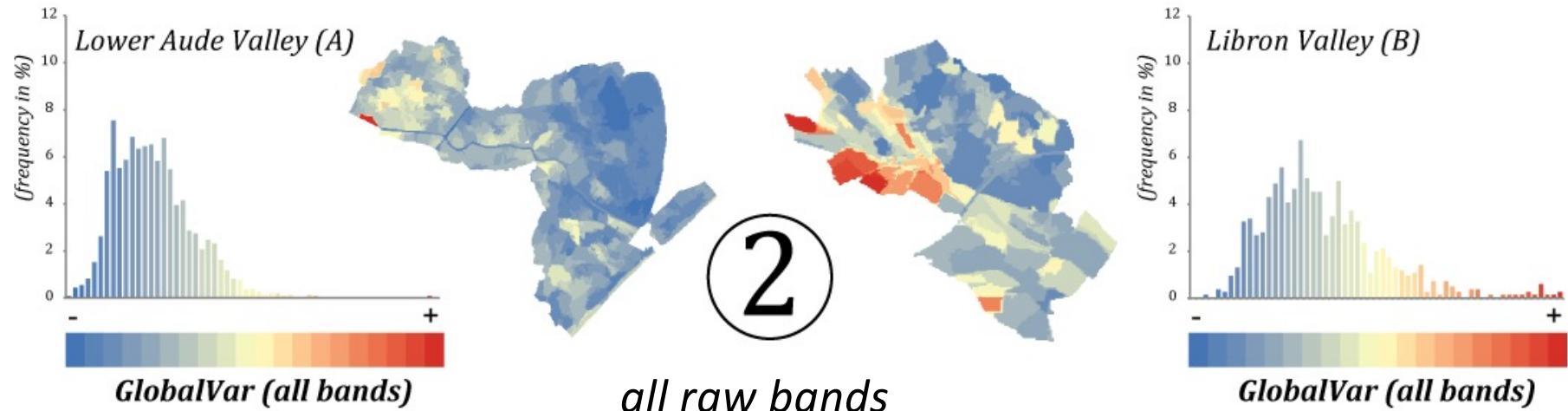
**NDVI temporal profile**



# Evolution Intensity



# Evolution Intensity



# Similar Behaviors

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- Clustering approaches to find the region having the same behaviors
- At every timestamp in the graph, the set of attributes is available
- Considering the number of objects that may appear in a graph



# Outline

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- A quick reminder on pattern discovery
- The different kinds of patterns from trajectory
- Towards a unified approach for extracting trajectories
- Deal with and without a spatial component?
- **A concrete illustration**
- Conclusion



# Now let's have a concrete example

jupyter TrajectoriesNoteBook Last Checkpoint: il y a 4 heures (autosaved)

File Edit View Insert Cell Kernel Widgets Help

Run Markdown

This notebook is an example of applying getmove on real trajectories.

Installation [...]

Running trajectories

```
In [ ]: import matplotlib.pyplot as plt
import matplotlib.cm as cm
from mpl_toolkits.basemap import Basemap
import numpy as np
import datetime
from sklearn.cluster import DBSCAN
import pandas as pd
from matplotlib import interactive
import json
import os
from pprint import pprint
```



# Outline

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- A quick reminder on pattern discovery
- The different kinds of patterns from trajectory
- Towards a unified approach for extracting trajectories
- Deal with and without a spatial component?
- A concrete illustration
- **Conclusion**



# Conclusions

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- Trajectories are useful patterns for ...lots of applications
- They propose new kinds of patterns even when the spatial component is not present
- They have been really well studied and many approaches exist
- There are more and more applications



# Conclusions

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- Considering the Pattern Mining issue:
  - Incremental (Objects)
  - Stream Mining
  - New measures for selecting the most interesting patterns
  - Outlier detection
  - Privacy
  - Classification/Clustering of trajectories
  - Interactive Mining
  - Considering the clustering during the process rather than before
  - ...



# Conclusions

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- Big Data Issues:
  - Volume
  - Variety
  - Velocity
  - Veracity



# Special Thanks ...

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- And many others ....



# Some Related Publications

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- N. Phan, P. Poncelet and M. Teisseire. "*All in One: Mining Multiple Movement Patterns*". International Journal of Information Technology and Decision Making (IJITDM), June 2016, pp. 1-42
- P. N. Hai, Pascal Poncelet, Maguelonne Teisseire. "*An Efficient Spatio-Temporal Mining Approach to Really Know Who Travels with Whom!*". ISI special issue, selected, 2013
- P. N. Hai, Dino Ienco, Pascal Poncelet, Maguelonne Teisseire. "*Mining Representative Movement Patterns through Compression*". PAKDD 2013.
- A.Z.E. Aabidine, A. Sallaberry, S. Bringay, M. Fabregue, C. Lecellier, P. N. Hai, P. Poncelet. "*Co<sup>2</sup>Vis: A Visual Analytics Tool for Mining Co-expressed And Co-regulated Genes Implied in HIV Infections*". IEEE BioVis 2013.
- P. N. Hai, Dino Ienco, Pascal Poncelet, Maguelonne Teisseire. "*Mining Fuzzy Moving Object Clusters*". ADMA 2012.
- P. N. Hai, Dino Ienco, Pascal Poncelet, Maguelonne Teisseire. "*Mining Time Relaxed Gradual Moving Object Clusters*". ACM GIS 2012.
- F. Bouillot, P. N. Hai, N. Béchet, S. Bringay, D. Ienco, S. Matwin, P. Poncelet, M. Roche, and M. Teisseire. "*How to Extract Relevant Knowledge from Tweets?*". ISIP 2012.
- P. N. Hai, Pascal Poncelet, Maguelonne Teisseire. "*GET\_MOVE: An Efficient and Unifying Spatio-Temporal Pattern Mining Algorithm for Moving Objects*". IDA 2012.
- P. N. Hai, Pascal Poncelet, Maguelonne Teisseire. "*An Efficient Spatio-Temporal Mining Approach to Really Know Who Travels with Whom!*". BDA 2012.
- P. N. Hai, Dino Ienco, Pascal Poncelet, Maguelonne Teisseire. "*Extracting Trajectories through an Efficient and Unifying Spatio-Temporal Pattern Mining System*". ECML-PKDD 2012.
- P. N. Hai, Pascal Poncelet, Maguelonne Teisseire. "*MovingObjects: Combining Gradual Rules and Spatio-Temporal Patterns*". IEEE ICSDM 2011.



All the papers are available on my web page

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- Y. Zheng. "Trajectory Data Mining: An Overview". *ACM Transactions on Intelligent Systems and Technology (TIST)*, 6(3), 2015.



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- Questions ?

