

## 18.335 Problem Set 2

Due Fri. Feb 28, 2020, at 3pm via Stellar.

### Problem 1: Stability

- (a) Trefethen, exercise 15.1. You will need to review the definition of “stability,” of which backwards stability is a (common) special case, in that chapter. [In parts (e) and (f), assume that  $\frac{1}{k!}$  can be computed to  $O(\epsilon_{\text{machine}})$  and concentrate on the accumulation of errors in the summations.]
- (b) Trefethen, exercise 16.1.
- (c) Consider the function  $f(x, y) = x^*y = \sum_{i=1}^n \overline{x_i}y_i$  for  $x, y \in \mathbb{C}^n$ , computed by naive (left-to-right) summation of the floating-point products  $\overline{x_i} \otimes y_i$ . Show that this algorithm is backwards stable (very similar to the proof of backwards stability of summation from class). Here, the inputs  $(x, y)$  live in the *Cartesian product* space  $\mathbb{C}^n \times \mathbb{C}^n$ , which is a vector space using the obvious operations  $(x, y) + (z, w) = (x + z, y + w)$  and  $\alpha(x, y) = (\alpha x, \alpha y)$ . You will need to choose a convenient norm of  $\mathbb{C}^n \times \mathbb{C}^n$  to apply the backwards-stability definition.

(Food for thought: Your solution could easily be adapted to establish backwards stability of matrix products  $f(A, B) = AB$  computed by the familiar “row · column” algorithm. Not required here, though.)

### Problem 2: Norms

- (a) Derive Trefethen eq. (3.10) (for which Trefethen only writes “by much the same argument”). Find the code that computes the induced  $\|A\|_\infty$  norm in Julia, the `opnorm(A, Inf)` function, on [github.com/JuliaLang/julia](https://github.com/JuliaLang/julia) in `stdlib/LinearAlgebra/src/generic.jl` and satisfy yourself that it is equivalent to (3.10).
- (b) Trefethen, problem 3.4. Check your result for a random  $10 \times 7$  matrix  $A$  in Julia, constructed by `A=randn(10,7)` with the induced  $p = 2$  norm as computed by `opnorm(A)` in Julia.

### Problem 3: SVD and low-rank approximations

- (a) Trefethen, problem 4.5.
- (b) Trefethen, problem 5.2.
- (c) Trefethen, problem 5.4.

### Problem 4: Least squares

Trefethen, problem 11.2. Note that the  $\Gamma(x)$  function is provided as `gamma(x)` by the `SpecialFunctions` package in Julia (execute `] add SpecialFunctions` to install this package).