

## 18.335 Problem Set 5

Due Monday, 15 November 2010.

### Problem 1:

- (a) Trefethen, 36.3. Plot the error in this eigenvalue as a function of how many  $Ax$  matrix-vector multiplies you perform (use a semilog or log-log scale as appropriate). (The files `lanczos.m` and `A363.m` posted on the web page are helpful.)
- (b) Same problem, but use restarted Lanczos: after every 10 iterations of Lanczos, restart with the best Ritz vector from those 10 iterations. Again, plot the error vs. matrix-vector multiply count.
- (c) The above questions asked for the minimum- $\lambda$  eigenvalue (which may be negative). Plot what happens if, instead, you try to get the minimum- $|\lambda|$  eigenvalue by these techniques. (Aside: a better way is to use Lanczos on  $A^{-1}$ , but that requires a fast way to solve  $Ax = b$  in order to multiply by  $A^{-1}$ .)

### Problem 2:

Trefethen, problem 38.6. (The files `SD.m` and `A386.m` on the web page are helpful.)

### Problem 3:

In problem 3 of the Fall 2008 midterm for 18.335, it was claimed that you could use the conjugate-gradient algorithm for a Hermitian positive semidefinite matrix  $A$ , with a random starting guess, to find a vector in the null space (see the midterm solutions). Demonstrate this by means of an example, in Matlab, and plot the norm of the residual vs. iteration. (You can construct a random positive-semidefinite matrix  $A$  via, for example, `B=rand(198,200); A = B' * B`).