18.335 Problem Set 2

Due Friday March 1, 2019, at 3pm via Stellar.

Problem 1: Stability

(a) Trefethen, exercise 15.1. [In parts (e) and (f), assume that $\frac{1}{k!}$ can be computed to $O(\epsilon_{\text{machine}})$ and concentrate on the accumulation of errors in the summations.]

(b) Trefethen, exercise 16.1.

Problem 2: Norms

(a) Derive Trefethen eq. (3.10) (for which Trefethen only writes "by much the same argument"). Find the code that computes the induced $\|A\|_\infty$ norm in Julia, the `opnorm(A, Inf)` function, on `github.com/JuliaLang/julia` in stdlib/LinearAlgebra/src/generic.jl and satisfy yourself that it is equivalent to (3.10).

(b) Trefethen, problem 3.4. Check your result for a random 10×7 matrix A in Julia, constructed by `A=randn(10,7)` with the induced $p = 2$ norm as computed by `opnorm(A)` in Julia.

Problem 3: Conditioning

In class, we found the condition number of $f(x) = Ax$, in the L_2 norm, to be $\|A\|_2\|x\|_2/\|Ax\|_2$ for some $A \in \mathbb{C}^{m \times n}$ and $x \in \mathbb{C}^n$. Instead, compute the condition number of $g(A) = Ax$, using any convenient choice of norm (e.g. the Frobenius norm for the input matrices A).

Problem 4: SVD and low-rank approximations

(a) Trefethen, probem 4.5.

(b) Trefethen, problem 5.2.

(c) Trefethen, problem 5.4.