Respuestas ejemplo – Capitulo 3

3.23

$$X(z) = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

$$+1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \cdots$$

$$x(n) = \delta(n) + \frac{1}{n!}$$

3.24

a)

$$X(z) = \frac{1}{1 + 1.5z^{-1} - 0.5z^{-2}}$$

$$= \frac{0.136}{1 - 0.28z^{-1}} + \frac{0.864}{1 + 1.78z^{-1}}$$

$$por\ lo\ tanto, x(n) = [0.136(0.28)^n + 0.864(-1.78)^n]u(n)$$

b)

$$X(z) = \frac{1}{1 - 0.5z^{-1} + 0.6z^{-2}}$$

$$= \frac{1 - 0.2.5z^{-1}}{1 - 0.5z^{-1} + 0.6z^{-2}} + 0.3412\,\frac{0.7326z^{-1}}{1 - 0.5z^{-1} + 0.6z^{-2}}$$

$$por\ lo\ tanto, x(n) = (0.7746)^n[cos1.24n + 0.3412sin1.24n]u(n)$$

$$Revisión\ parcial: x(0) = 1, x(1) = 0.5016, x(2) = -0.3476, x(\infty) = 0.$$

$$Para\ la\ ecuación\ de\ diferencia, x(n) - 0.5x(n-1) + 0.6x(n-2) = \delta(n)\ obtenemos:$$

$$x(0) = 1, x(1) = 0.5, x(2) = -0.35, x(\infty) = 0$$

3.25

a)

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

$$= \frac{2}{1 - z^{-1}} - \frac{1}{1 - 0.5z^{-2}}$$

$$para\ |z| < 0.5, x(n) = [(0.5)^n - 2]u(-n-1)$$

$$para\ |z| > 1, x(n) = [2 - (0.5)^n]u(n)$$

$$para\ 0.5 < |z| < 1, x(n) = -(0.5)^n u(n) - 2u(-n-1)$$

b)

$$X(z) = \frac{1}{(1 - 0.5z^{-1})^2}$$

$$= \left[\frac{0.5z^{-1}}{(1 - 0.5z^{-1})^2}\right] 2z$$

$$para\ |z| > 0.5, x(n) = 2(n+1)(0.5)^{n+1}u(n+1)$$

$$= (n+1)\ (0.5)^n u(n)$$

$$para\ |z| < 0.5, x(n) = -2(n+1)(0.5)^{n+1}u(-n-2)$$

$$= -(n+1)(0.5)^n u(-n-1)$$

3.26

$$X(z) = \frac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}}$$

$$= \frac{-\frac{3}{8}}{1 - \frac{1}{3}z^{-1}} + \frac{\frac{27}{8}}{1 - 3z^{-1}}$$

$$ROC: \frac{1}{3} < |z| < 3, x(n) = \frac{3}{8}\left(\frac{1}{3}\right)^n u(n) - \frac{27}{8}\ 3^n u(-n-1)$$

3.34

$$H(z) = \sum_{n=-1}^{-\infty} 3^n\ z^{-n} + \sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n\ z^{-n}$$

$$= \frac{-1}{1-3z^{-1}} + \frac{1}{1-\frac{2}{5}z^{-1}} \text{ }, ROC\text{: } \frac{2}{5} < |z| < 3$$

$$X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{-\frac{13}{5}z^{-1}}{(1-z^{-1})(1-3z^{-1})(1-\frac{2}{5}z^{-1})} \text{ }, ROC\text{: } 1 < |z| < 2$$

$$= \frac{\frac{13}{6}}{1-z^{-1}} - \frac{\frac{3}{2}}{1-3z^{-1}} - \frac{\frac{2}{3}}{1-\frac{2}{5}z^{-1}}$$

$$Por\ lo\ tanto$$

$$y(n) = \frac{3}{2}\ 3^n\ u(-n-1) + \left[\frac{13}{6} - \frac{2}{3}\left(\frac{2}{5}\right)^n\right]u(n)$$

3.35

a)

$$h(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$H(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$x(n) = \left(\frac{1}{2}\right)^n cos\frac{\pi n}{3}u(n)$$

$$X(z) = \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{1-\frac{1}{4}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}$$

$$= \frac{\frac{1}{7}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{6}{7}\left(1-\frac{1}{4}z^{-1}\right)}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} + \frac{3\sqrt{3}}{7}\frac{\frac{\sqrt{3}}{4}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$Por\ lo\ tanto$$

$$y(n) = \left[\frac{1}{7}\left(\frac{1}{3}\right)^n + \frac{6}{7}\left(\frac{1}{2}\right)^n cos\frac{\pi n}{3} + \frac{3\sqrt{3}}{7}\left(\frac{1}{2}\right)^n sin\frac{\pi n}{3}\right]u(n)$$

b)

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$x(n) = \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{2}\right)^{-n} u(-n-1)$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{-\frac{5}{3}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)(1-2z^{-1})}$$

$$= \frac{\frac{10}{3}}{1-\frac{1}{2}z^{-1}} + \frac{-2}{1-\frac{1}{3}z^{-1}} + \frac{-\frac{4}{3}}{1-2z^{-1}}$$

$$Por\ lo\ tanto$$

$$y(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right]u(n) + \frac{4}{3}2^n u(-n-1)$$

c)

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + x(n) + x(n-1)$$

$$H(z) = \frac{2+z^{-1}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$x(n) = \left(\frac{1}{3}\right)^n u(n)$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{1+z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)(1+0.1z^{-1}-0.2z^{-2})}$$

$$= \frac{-8}{1-\frac{1}{3}z^{-1}} + \frac{\frac{28}{3}}{1-0.4z^{-1}} + \frac{\frac{-1}{3}}{1+0.5z^{-1}}$$

$$Por\ lo\ tanto$$

$$y(n) = \left[-8\left(\frac{1}{3}\right)^n + \frac{28}{3}\left(\frac{2}{5}\right)^n - \frac{1}{3}\left(\frac{1}{2}\right)^n\right]u(n)$$

d)

$$y(n) = \frac{1}{2}x(n) - \frac{1}{2}x(n-1)$$

$$Y(z) = \frac{1}{2}(1-z^{-1})X(z)$$

$$X(z) = \frac{10}{1+z^{-2}}$$

$$Entonces, Y\ (z) = \frac{10(1-z^{-1})/2}{1+z^{-2}}$$

$$y(n) = 5cos\frac{\pi n}{2}u(n) - 5cos\frac{\pi(n-1)}{2}u(n-1)$$

$$= \left[5cos\frac{\pi n}{2} - 5sin\frac{\pi n}{2}\right]u(n-1) + 5\delta(n)$$

$$= 5\delta(n) + \frac{10}{\sqrt{2}}sin\left(\frac{\pi n}{2}+\frac{\pi}{4}\right)u(n-1)$$

$$= \frac{10}{\sqrt{2}}sin\left(\frac{\pi n}{2}+\frac{\pi}{4}\right)u(n)$$

e)

$$y(n) = -y(n-2) + 10x(n)$$

$$Y(z) = \frac{10}{1+z^{-2}}\ X(z)$$

$$X(z) = \frac{10}{1+z^{-2}}$$

$$Y(z) = \frac{100}{(1+z^{-2})^2}$$

$$= \frac{50}{1+jz^{-1}} + \frac{50}{1-jz^{-1}} + \frac{-25jz^{-1}}{(1+jz^{-1})^2} + \frac{25jz^{-1}}{(1-jz^{-1})^2}$$

$$Entonces,$$

$$y(n) = \{50[j^n + (-j)^n] - 25n\,[j^n + (-j)^n]\}\,u\,(n)$$

$$= (50 - 25n)(j^n + (-j)^n u(n)$$

$$= (50 - 25n)2cos\frac{\pi n}{2}u(n)$$

f)

$$h(n) = \left(\frac{2}{5}\right)^n u(n)$$

$$H(z) = \frac{1}{1 - \frac{2}{5}z^{-1}}$$

$$x(n) = u(n) - u(n-7)$$

$$X(z) = \frac{1 - z^{-n}}{1 - z^{-1}}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{1 - z^{-n}}{\left(1 - \frac{2}{5}z^{-1}\right)(1 - z^{-1})}$$

$$= \frac{\frac{5}{3}}{1 - z^{-1}} + \frac{\frac{-2}{3}}{1 - \frac{2}{5}z^{-1}} - \left[\frac{\frac{5}{3}}{1 - z^{-1}} + \frac{\frac{-2}{3}}{1 - \frac{2}{5}z^{-1}}\right]$$

$$por\ lo\ tanto$$

$$y(n) = \frac{1}{3}\left[5 - 2\left(\frac{2}{5}\right)^n\right]u(n) - \frac{1}{3}\left[5 - 2\left(\frac{2}{5}\right)^{n-7}\right]u(n-7)$$

g)

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$x(n) = (-1)^n, -\infty < n < \infty$$

$$= \cos\pi n, -\infty < n < \infty$$

$$x(n) es\ una\ secuencia\ periódica\ y\ su\ transformada\ z\ no\ existe$$

$$y(n) = |H(\omega_0)|cos[\pi n + \Theta(\omega_0)], \omega_0 = \pi$$

$$H(z) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

$$H(\pi) = \frac{1}{1 + \frac{1}{2}}$$

$$= \frac{2}{3}, \quad \Theta = 0$$

$$entonces, \ \ y(n) = \frac{2}{3} cos\pi n, \quad -\infty < n < \infty$$

h)

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}}$$

$$x(n) = (n+1) \left(\frac{1}{4}\right)^n u(n)$$

$$X(z) = \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{\frac{1}{4} z^{-1}}{\left(1 - \frac{1}{4} z^{-1}\right)^2}$$

$$Y(z) = H(z)X(z)$$

$$= \frac{1}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)^2}$$

$$= \frac{4}{1 - \frac{1}{2} z^{-1}} + \frac{\frac{-1}{4} z^{-1}}{\left(1 - \frac{1}{4} z^{-1}\right)^2} + \frac{-3}{1 - \frac{1}{4} z^{-1}}$$

$$por \ lo \ tanto$$

$$y(n) = \left[4\left(\frac{1}{2}\right)^n - n\left(\frac{1}{4}\right)^n - 3\left(\frac{1}{4}\right)^n\right] u(n)$$

3.36

$$H(z) = \frac{1 - 2z^{-1} + 2z^{-2} - z^{-3}}{(1 - z^{-1})\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{5} z^{-1}\right)}, \quad \frac{1}{2} < |z| < 1$$

$$= \frac{1 - z^{-1} + z^{-2}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{5} z^{-1}\right)}, \qquad \frac{1}{2} < |z| < 1$$

a)

$Z_{1,2} = \frac{1 \pm j\sqrt{3}}{2}$, $P_1 = \frac{1}{2}$, $P_2 = \frac{1}{5}$

b) $H(z) = 1 + \left[\frac{\frac{5}{2}}{1 - \frac{1}{2} z^{-1}} + \frac{-2.8}{1 - \frac{1}{5} z^{-1}}\right] z^{-1}$

$$h(n) = \delta(n) + \left[5 \left(\frac{1}{2}\right)^n - 14 \left(\frac{1}{5}\right)^n\right] u(n)$$

3.37

$$y(n) = 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$$

$$Y(z) = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}} X(z)$$

$$x(n) = nu(n)$$

$$X(z) = \frac{z^{-1}}{(1 - z^{-1})^2}$$

$$Y(z) = \frac{z^{-2} + z^{-3}}{(1 - z^{-1})^2 \left(1 - \frac{3}{10} z^{-1}\right)\left(1 - \frac{2}{50} z^{-2}\right)}$$

$\rightarrow El\ sistema\ es\ estable$

$$Y(z) = \frac{4.76\, z^{-1}}{(1 - z^{-1})^2} + \frac{-12.36}{(1 - z^{-1})} + \frac{-26.5}{\left(1 - \frac{3}{10} z^{-1}\right)} + \frac{38.9}{\left(1 - \frac{2}{5} z^{-1}\right)}$$

$$y(n) = \left[4.76n - 12.36 - 26.5 \left(\frac{3}{10}\right)^n + 38.9 \left(\frac{2}{5}\right)^n\right] u(n)$$

3.38

a)

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n)$$

$$Y(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} X(z)$$

Respuesta al impulso: $X(z) = 1$

$$Y(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$\rightarrow h(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$$

como el polo de $H(z)$ *se encuentra adentro del circulo unitario, el sistema es estable*

$\left(polos\ a\ z = \frac{1}{2}, \frac{1}{4} \right)$

respuesta al escalón: $X(z) = \frac{1}{1 - z^{-1}}$

$$Y(z) = \frac{\frac{8}{3}}{1 - z^{-1}} - \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1 - \frac{1}{4}z^{-1}}$$

$$y(n) = \left[\frac{8}{3} - 2\left(\frac{1}{2}\right)^n + \frac{1}{3}\left(\frac{1}{4}\right)^n\right] u(n)$$

b)

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) + x(n-1)$$

$$Y(z) = \frac{1 + z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}} X(z)$$

$H(z)$ *tiene ceros para* $z = 0, 1$ *y polos para* $z = \frac{1 \pm j}{2}$. *Por lo tanto, el sistema es estable*

Respuesta al impulso: $X(z) = 1$

$$Y(z) = \frac{1 - \left(\sqrt{2}\right)^{-1} \cos\frac{\pi}{4}\, z^{-1}}{1 - 2\left(\sqrt{2}\right)^{-1} \cos\frac{\pi}{4} z^{-1} + \left(\frac{1}{\sqrt{2}}\right)^2 z^{-2}} + \frac{\frac{3}{2}\, z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

$$\rightarrow y(n) = h(n) = \left(\frac{1}{\sqrt{2}}\right)^n \left[\cos\frac{\pi}{4}n + \sin\frac{\pi}{4}n + \right] u(n)$$

Respuesta al esalón: $X(z) = \frac{1}{1 - z^{-1}}$

$$Y(z) = \frac{1+z^{-1}}{(1-z^{-1})\left(1-z^{-1}+\frac{1}{2}z^{-2}\right)}$$

$$= \frac{-(1-\frac{1}{2}z^{-1})}{1-z^{-1}+\frac{1}{2}z^{-2}} + \frac{\frac{1}{2}z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}} + \frac{2}{1-z^{-1}}$$

$$y(n) = \left(\frac{1}{\sqrt{2}}\right)^n \left[\sin\frac{\pi}{4}n - \cos\frac{\pi}{4}n\right] u(n) + 2u(n)$$

c)

$$H(z) = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

$$\rightarrow h(n) = n^2 u(n)$$

$Polo\ triple\ en\ el\ circulo\ unitario\ \rightarrow el\ sistema\ es\ inestable$

$Respuesta\ al\ escalón{:}\ X(z) = \dfrac{1}{1-z^{-1}}$

$$Y(z) = \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^4}$$

$$= \frac{1}{3}\frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} + \frac{1}{2}\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} + \frac{1}{6}\frac{z^{-1}}{(1-z^{-1})^2}$$

$$y(n) = \left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right) u(n)$$

$$= \frac{1}{6}n(n+1)(2n+1)u(n)$$

d)

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$Y(z) = \frac{1}{1-0.6z^{-1}+0.08z^{-2}}\ X(z)$$

$Respuesta\ al\ impulso{:}\ X(z) = 1$

$$H(z) = \frac{1}{\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{2}{5}z^{-1}\right)}$$

$\rightarrow ceros\ en\ z = 0, polos\ en\ p_1 = \dfrac{1}{2}, p_2 = \dfrac{2}{5}\ el\ sistema\ es\ estable$

$$H(z) = \frac{-1}{1-\frac{1}{5}z^{-1}} + \frac{2}{1-\frac{2}{5}z^{-1}}$$

$$\rightarrow h(n) = \left[2\left(\frac{2}{5}\right)^n - \left(\frac{1}{5}\right)^n\right]u(n)$$

$$Respuesta\ al\ escalón:\ X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{1}{\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{2}{5}z^{-1}\right)(1-z^{-1})}$$

$$Y(z) = \frac{\frac{25}{12}}{1-z^{-1}} + \frac{\frac{1}{4}}{1-\frac{1}{5}z^{-1}} + \frac{-\frac{4}{3}}{1-\frac{2}{5}z^{-1}}$$

$$y(n) = \left[\frac{25}{12} + \frac{1}{4}\left(\frac{1}{5}\right)^n - \frac{4}{3}\left(\frac{2}{5}\right)^n\right]u(n)$$

e)

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$$

$$Y(z) = \frac{2-z^{-2}}{1-0.7z^{-1}+0.1\,z^{-2}}\ X(z)$$

$$= \frac{2-z^{-2}}{\left(1-\frac{1}{5}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}\ X(z)$$

$$con\ ceros\ en\ z = 0{,}2\ \ y\ polos\ en\ z = \frac{1}{2}, \frac{1}{5}.\ Por\ lo\ tanto\ el\ sistema\ es\ estable$$

$$Respuesta\ al\ impulso: X(z) = 1$$

$$H(z) = 2 + \left(\frac{\frac{-5}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{46}{15}}{1-\frac{1}{5}z^{-1}}\right)z^{-1}$$

$$\rightarrow h(n) = 2\delta(n) - \frac{5}{3}\left(\frac{1}{2}\right)^{n-1} u(n-1) + \frac{46}{15}\left(\frac{1}{5}\right)^{n-1} u(n-1)$$

$$Respuesta\ al\ escalón:\ X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{2 - z^{-2}}{(1 - z^{-1})\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{5}z^{-1}\right)}$$

$$= \frac{\frac{5}{2}}{1 - z^{-1}} + \frac{\frac{10}{3}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{-23}{6}}{1 - \frac{1}{5}z^{-1}}$$

$$y(n) = \left[\frac{5}{2} + \frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{23}{6}\left(\frac{1}{5}\right)^n\right]u(n)$$

3.39

$$X(z) = \frac{(1 + z^{-1})}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - pz^{-1})(1 - p * z^{-1})}, \quad p = -\frac{1}{2} + \frac{j}{2}$$

a)

$$x_1(n) = x(-n + 2)$$

$$X_1(z) = z^{-2}X(z^{-1})$$

$$= \frac{z^{-2}(1 + z)}{\left(1 - \frac{1}{2}z\right)(1 - pz)(1 - p * z)}, \qquad ROC: |z| < 2$$

$$Cero\ en\ \ z = -1, polos\ en\ z = 2, \frac{1}{p}\ , \frac{1}{p *}\ y\ en\ z = 0$$

b)

$$x_2(n) = e^{\frac{j\pi n}{3}}x(n)$$

$$X_2(z) = X\left(e^{\frac{-j\pi}{3}}z\right)$$

$$= \frac{1 + e^{\frac{j\pi}{3}}z^{-1}}{\left(1 - \frac{1}{2}e^{\frac{j\pi}{3}}z^{-1}\right)\left(1 - pe^{\frac{j\pi}{3}}z^{-1}\right)\left(1 - p * e^{\frac{j\pi}{3}}z^{-1}\right)},$$

$$Todos\ los\ polos\ y\ ceros\ se\ encuentran\ rotados\ por\ \frac{\pi}{3}\ en\ una\ dirección\ sinistrorsum\ (hacia\ la\ izquierda)$$

$$El\ ROC\ para\ \ X_2(z)\ es\ el\ mismo\ ROC\ que\ para\ X(z).$$

3.40

$$x(n) = \left(\frac{1}{2}\right)^n u(n) - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u(n - 1)$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{4}\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{1-\frac{1}{4}z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$y(n) = \left(\frac{1}{3}\right)^{n} u(n)$$

$$Y(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

a)

$$H(z) = Y(z)X(z)$$

$$= \frac{1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$= \frac{3}{1-\frac{1}{4}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

$$h(n) = \left[3\left(\frac{1}{4}\right)^{n} - 2\left(\frac{1}{3}\right)^{n}\right]u(n)$$

b)

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{7}{12}z^{-1}+\frac{1}{12}z^{-2}}$$

$$y(n) = \frac{7}{12}y(n-1) - \frac{1}{12}y(n-2) + x(n) - \frac{1}{2}x(n-1)$$

c) $Referirse\ a\ la\ figura\ 3.40 - 1$
d) $Los\ polos\ en\ el\ sistema\ se\ encuentran\ dentro\ del\ circulo\ unitario, por\ lo\ tanto\ el\ sistema\ es\ estable$

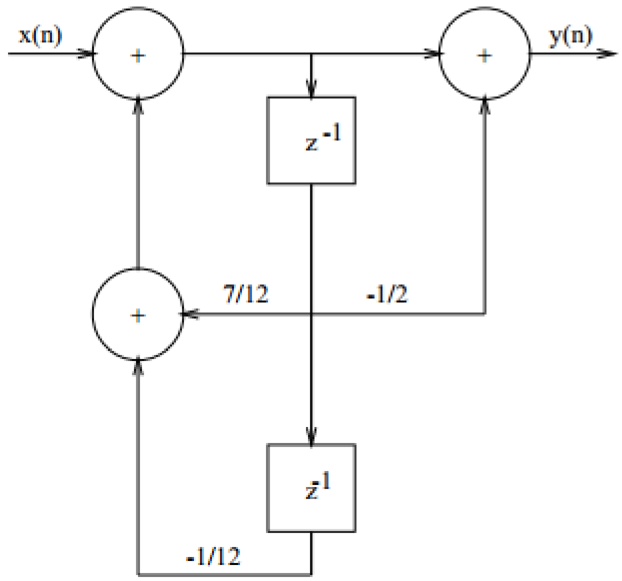

Figure 3.40-1:

3.41

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$si\ a_1{}^2 - 4a_2 < 0, existen\ dos\ polos\ complejos$

$$p_{1,2} = \frac{-a_1 \pm j\sqrt{4a_2 - a_1{}^2}}{2}$$

$$|p_{1,2}|^2 = \left(\frac{a_1}{2}\right)^2 + \left(\frac{\sqrt{4a_2 - a_1{}^2}}{2}\right)^2 < 1$$

$\to a_2 < 1$

$si\ a_1{}^2 - 4a_2 \geq 0, entonces\ hay\ dos\ polos\ reales$

$$p_{1,2} = \frac{-a_1 \pm j\sqrt{a_1{}^2 - 4a_2}}{2}$$

$$\frac{-a_1 \pm \sqrt{a_1{}^2 - 4a_2}}{2} < 1\ y$$

$$\frac{-a_1 - \sqrt{a_1{}^2 - 4a_2}}{2} > -1$$

$\rightarrow\ a_1 - a_2\ < 1\ y$

$a_1 + a_2 > 1$

$Referirse\ a\ la\ figura\ 3.41 - 1$

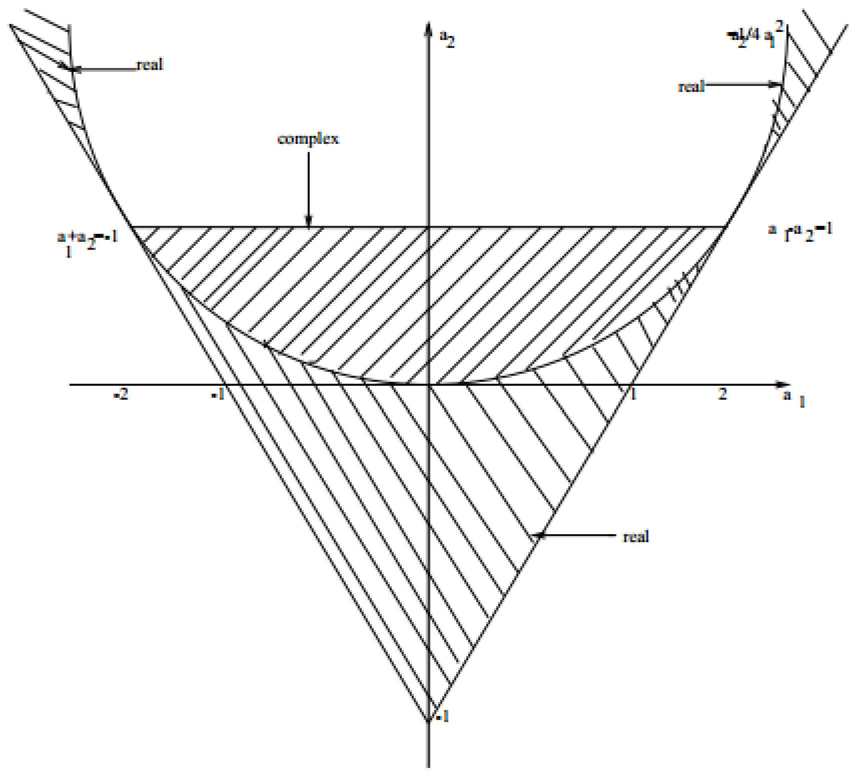

Figure 3.41-1:

3.42

$$H(z) = \frac{z^{-1} + \frac{1}{2}z^{-2}}{1 - \frac{3}{5}z^{-1} + \frac{2}{25}z^{-2}}$$

$$H(z) = \ z^{-1}\left[\frac{-\frac{7}{2}}{1 - \frac{1}{5}z^{-1}} + \frac{\frac{9}{2}}{1 - \frac{2}{5}z^{-1}}\right]$$

$$h(n) = \left[-\frac{7}{2}\left(\frac{1}{5}\right)^{n-1} + \frac{9}{2}\left(\frac{2}{5}\right)^{n-1}\right]u(n-1)$$

$$Y(z) = H(z)X(z)$$

$$X(z) = \frac{1}{1 - z^{-1}}$$

$$Y(z) = \frac{\frac{25}{8}}{1 - z^{-1}} + \frac{\frac{7}{8}}{1 - \frac{1}{5}z^{-1}} + \frac{-3}{1 - \frac{2}{5}z^{-1}}$$

$$y(n) = \left[\frac{25}{8} + \frac{7}{8}\left(\frac{1}{5}\right)^n - 3\left(\frac{2}{5}\right)^n\right]u(n)$$

c) Determine la respuesta causada por las condiciones iniciales y agrégala en la respuesta en b

$$y(n) - \frac{3}{5}y(n-1) + \frac{2}{25}y(n-2) = 0$$

$$Y^+(z) - \frac{3}{5}[Y^+(z)z^{-1} + 1] + \frac{2}{25}[Y^+(z)z^{-2} + z^{-1} + 2] = 0$$

$$Y^+(z) = \frac{\frac{2}{25}z^{-1} - \frac{11}{25}}{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 - \frac{2}{5}z^{-1}\right)}$$

$$= \frac{\frac{1}{25}}{1 - \frac{1}{5}z^{-1}} + \frac{\frac{-12}{25}}{1 - \frac{2}{5}z^{-1}}$$

$$y^+(n) = \left[\frac{1}{25}\left(\frac{1}{5}\right)^n - \frac{12}{25}\left(\frac{2}{5}\right)^n\right]u(n)$$

$Por\ lo\ tanto, la\ respuesta\ total\ al\ escalon\ es$

$$y(n) = \left[\frac{25}{8} + \frac{33}{200}\left(\frac{1}{5}\right)^n - \frac{87}{25}\left(\frac{2}{5}\right)^n\right]u(n)$$

3.43

$$[aY(z) + X(z)]z^{-2} = Y(z)$$

$$Y(z) = \frac{z^{-2}}{1 - az^{-2}}X(z)$$

$Asumiendo\ que\ a > 0.\ Entonces$

$$H(z) = -\frac{1}{a} + \frac{\frac{1}{a}}{\left(1 - \sqrt{a}z^{-1}\right)\left(1 + \sqrt{a}z^{-1}\right)}$$

$$= -\frac{1}{a} + \frac{1}{2a}\frac{1}{1-\sqrt{a}z^{-1}} + \frac{1}{2a}\frac{1}{1+\sqrt{a}z^{-1}}$$

$$h(n) = -\frac{1}{a}\delta(n) + \frac{1}{2a}\left[\left(\sqrt{a}^{n}\right) + \left(\sqrt{a}^{n}\right)\right]u(n)$$

$$Respuesta\ al\ escalón: X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{z^{-2}}{(1-z^{-1})\left(1-\sqrt{a}z^{-1}\right)\left(1+\sqrt{a}z^{-1}\right)}$$

$$= \frac{\frac{1}{(a-1)}}{1-z^{-1}} + \frac{\frac{1}{2\left(a-\sqrt{a}\right)}}{1-\sqrt{a}z^{-1}} + \frac{\frac{1}{2\left(a+\sqrt{a}\right)}}{1+\sqrt{a}z^{-1}}$$

$$y(n) = \left[\frac{1}{a-1} + \frac{1}{2(a-\sqrt{a})}\left(\sqrt{a}\right)^{n} + \frac{1}{2(a+\sqrt{a})}\left(-\sqrt{a}\right)^{n}\right]u(n)$$

3.44

$$y(n) = -a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$

$$Y(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}\ X(z)$$

a)

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}\ \rightarrow h(n) = b_0(-a_1)^n u(n) + b_1(-a_1)^{n-1}u(n-1)$$

$$= b_0 + \frac{(b_1 - b_0 a_1)z^{-1}}{1 + a_1 z^{-1}}\ \rightarrow h(n) = b_0\delta(n) + (b_1 - b_0 a_1)(-a_1)^{n-1}u(n-1)$$

b)

$$Respuesta\ al\ escalón:\ X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{b_0 + b_1 z^{-1}}{(1-z^{-1})(1 + a_1 z^{-1})}$$

$$= \frac{b_0 + b_1}{1 + a_1}\frac{1}{1-z^{-1}} + \frac{a_1 b_0 - b_1}{1 + a_1}\frac{1}{1 + a_1 z^{-1}}$$

$$y(n) = \left[\frac{b_0 + b_1}{1 + a_1} + \frac{a_1 b_0 - b_1}{1 + a_1}(-a_1)^n\right]u(n)$$

c) $Calculando\ la\ respuesta\ al\ inpulso\ cero\ y\ agregandolo\ a\ la\ respuesta\ en\ b, tenemos.$

$$Y^{+}(z) + a_1[z^{-1}Y^{+}(z) + A] = 0$$

$$Y^{+}(z) = \frac{-a_1 A}{1 + a_1 z^{-1}}$$

$$\rightarrow y_{zi}(n) = -a_1 A(-a_1)^n u(n)$$

$La\ respuesta\ total\ a\ un\ escalón\ unitario\ es$

$$y(n) = \left[\frac{b_0 + b_1}{1 + a_1} + \frac{a_1 b_0 - b_1 - a_1 A(1 + a_1)}{1 + a_1}(-a_1)^n\right] u(n)$$

d)

$$x(n) = \cos \omega_0 n u(n)$$

$$X(z) = \frac{1 - z^{-1} cos\omega_0}{1 - 2z^{-1} cos\omega_0 + z^{-2}}$$

$$Y(z) = \frac{(b_0 + b_1 z^{-1})(1 - z^{-1} cos\omega_0)}{(1 + a_1 z^{-1})(1 - 2z^{-1} cos\omega_0 + z^{-2})}$$

$$= \frac{A}{1 + a_1 z^{-1}} + \frac{B(1 - z^{-1} cos\omega_0)}{1 - 2z^{-1} cos\omega_0 + z^{-2}} + \frac{C(z^{-1} cos\omega_0)}{1 - 2z^{-1} cos\omega_0 + z^{-2}}$$

$$cuando, y(n) = [A(-a_1)^n + Bcos\omega_0 + Csin\omega_0] u(n)$$

$Donde\ A, B\ y\ C\ son\ determinados\ por\ las\ ecuaciones$

$$A + B = b_0$$

$$(2cos\omega_0)A + (a_1 - cos\omega_0)B + (sin\omega_0)C = b_1 - b_0 cos\omega_0$$

$$A - (a_1 - cos\omega_0)B + (sin\omega_0)C = -b_1 cos\omega_0$$

3.45

$$y(n) = \frac{1}{2} y(n-1) + 4x(n) + 3x(n-1)$$

$$Y(z) = \frac{4 + 3z^{-1}}{1 - \frac{1}{2} z^{-1}} X(z)$$

$x(n) = e^{j\omega_0 n}u(n)$

$$X(z) = \frac{1}{1 - e^{j\omega_0}z^{-1}}$$

$$Y(z) = \frac{4 + 3z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - e^{j\omega_0}z^{-1})}$$

$$Y(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - e^{j\omega_0}z^{-1}}$$

$$Donde\ A = \frac{5}{\frac{1}{2} - e^{j\omega_0}}$$

$$B = \frac{4e^{j\omega_0} + 3}{e^{j\omega_0} - \frac{1}{2}}$$

$$Entonces\ y(n) = [A\left(\frac{1}{2}\right)^n + Be^{j\omega_0 n}]u(n)$$

La respuesta al estado estacionario es

$$\lim_{n\to\infty} y(n) \equiv y_{ss}(n) = Be^{j\omega_0 n}$$

3.46

a)

$$H(z) = C\frac{(z - re^{j\Theta})(z - re^{-j\Theta})}{z(z + 0.8)}$$

$$= C\frac{1 - 2rcos\Theta z^{-1} + r^2z^{-2}}{(1 + 0.8z^1)}$$

$$H(z)|_{z=1} = 1 \to C = \frac{1.8}{1 - 2rcos\Theta + r^2} = 2.77$$

b) *Los polos se encuentran dentro del circulo unitario, entonces el sistema es estable.*

c) $y(n) = -0.8y(n-1) + C_x(n) - 1.5\sqrt{3}C_x(n-1) + 2.25\ C_x(n-2).$

$Referirse\ a\ la\ fig. 3.46 - 1$

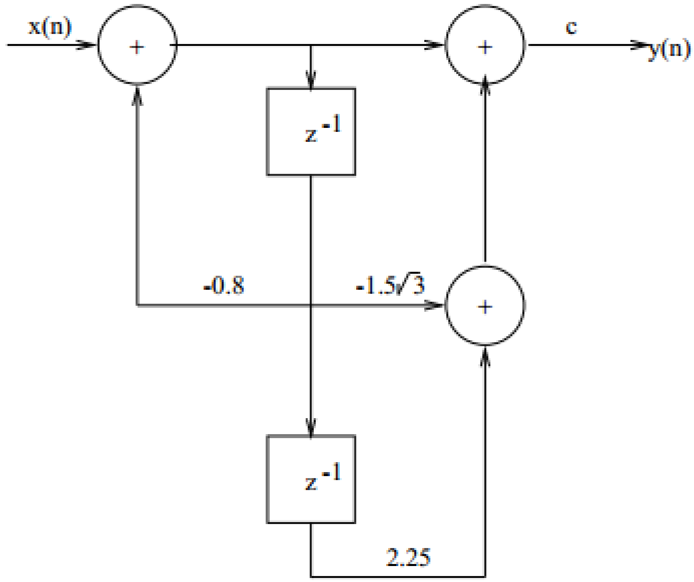

Figure 3.46-1:

3.47

a)

$X_1(z) = z^2 + z + 1 + z^{-1} + z^{-2}$

$X_2(z) = 1 + z^{-1} + z^{-2}$

$Y(z) = X_1(z)X_2(z)$

$= z^2 + 2z + 3 + 3z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

$Por\ lo\ tanto, x_1(z) * x_2(z) = y(n)$

$= \{1,2,3,3,3,2,1\}$

$Por\ transformada\ de\ un\ lado$:

$X_1{}^+(z) = 1 + z^{-1} + z^{-2}$

$X_2{}^+(z) = 1 + z^{-1} + z^{-2}$

$Y^+(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

$Por\ lo\ tanto, y(n) = \{1,2,3,2,1\}$

b) $como\ x_1(n)\ y\ \ x_2(n)\ son\ ambas\ causales, las\ transformadas\ de\ un\ lado\ y\ de\ dos\ lados$

$tienen\ resultados\ de\ campos\ identicos. Entonces$

$Y(z) = X_1(z)\ \ X_2(z)$

$$= \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= \frac{3}{1-\frac{1}{2}z^{-1}} - \frac{2}{1-\frac{1}{3}z^{-1}}$$

$Por\ lo\ tanto, y\ (n) = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n\right]u(n)$

c)

$Por\ convolución$

$y(n) = x_1(n) * \ \ x_2(n)$

$= \{4,11,20,30,20,11,4\}$

$Con\ la\ transformada\ z\ de\ un\ lado,$

$X_1{}^+(z) = 2 + 3z^{-1} + 4z^{-2}$

$X_2{}^+(z) = 2 + z^{-1}$

$Y^+(z) = X_1{}^+(z)X_2{}^+(z)$

$= 4 + 8z^{-1} + 11z^{-2} + 4z^{-3}$

$Por\ lo\ tanto, y(n) = \{4,8,11,4\}$

d)

$como\ x_1(n)\ y\ \ x_2(n)\ son\ causales, por\ lo\ tanto\ ambas\ transformadas\ producen\ el\ mismo\ resultado$

$X_1(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$

$X_2(z) = 1 + z^{-1} + z^{-2}$

$Entonces, Y(z) = X_1(z)X_2(z)$

$= 1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}$

$Por\ lo\ tanto, y(n)\ \ = \{1,2,3,3,3,2,1\}$