$\boxed{1}$

$f(x) = 11x - 5$, $g(x) = 2x^2 + 7$

a) $f(7) = 11 \times 7 - 5 = \boxed{72}$ (Ans)

b) $g(-2) = 2(-2)^2 + 7 = \boxed{15}$ (Ans)

c) $f(x) + g(x) = 11x - 5 + 2x^2 + 7$

$= \boxed{2x^2 + 11x + 2}$ (Ans)

d) $gof(x) = g[f(x)] = 2 \times (11x - 5)^2 + 7$

$= 2[(11x)^2 - 2 \times 11 \times 5x + (5)^2] + 7$

$= 242x^2 - 220x + 50 + 7$

$= \boxed{242x^2 - 220x + 57}$ (Ans

e) Let, $f(x) = y$ $\therefore$ $x = f^{-1}(y)$ ...(i)

$\Rightarrow y = 11x - 5$

$\Rightarrow x = \frac{y+5}{11}$

$\Rightarrow f^{-1}(y) = \frac{y+5}{11}$ [from eq^n (i)]

$\Rightarrow \boxed{f^{-1}(x) = \frac{x+5}{11}}$ (Ans)

$\boxed{2}$

a) $5x + 21 = 11$

$\Rightarrow 5x = -10 \qquad \Rightarrow \boxed{x = -2}$

b) $3x^3 - 6x - 20 = 4$

$\Rightarrow 3x^3 - 6x - 24 = 0$

$\Rightarrow x^3 - 2x = 8 \; \cancel{=0} \qquad \Leftrightarrow \quad x^3 + px = q$

$\downarrow$

$\underbrace{(a-b)^3}_{x} + \underbrace{3ab}_{p}\underbrace{(a-b)}_{x} = \underbrace{a^3 - b^3}_{q}$

$\Rightarrow (a-b)^3 + p(a-b) = q$ where $x = a - b \; \ldots$ (ii)

$p = 3ab$

$q = a^3 - b^3$

here, $p = -2$ and $q = 8$

$= 3ab \qquad\qquad = a^3 - b^3$

$\Rightarrow \boxed{\frac{-2}{3b} = a} \; \ldots$ (i) $\Rightarrow \left(\frac{-2}{3b}\right)^3 - b^3 = 8$

$\Rightarrow \frac{-8}{27b^3} - b^3 = 8$

$\Rightarrow -8 - 27b^6 = 216b^3$

$\Rightarrow -27b^6 - 216b^3 - 8 = 0$

$\Rightarrow 27b^6 + 216b^3 + 8 = 0$

let $u = b^3$

$\Rightarrow 27b^6 + 216b^3 + 8 = 0$

$\Rightarrow 27u^2 + 216u + 8 = 0$

$$u = \frac{-216 \pm \sqrt{(216)^2 - 4\times 27\times 8}}{2\times 27}$$

$$\Rightarrow b^3 = \frac{-216 \pm \sqrt{45792}}{\cancel{2\times 27}\; 54}$$

$$b = \sqrt[3]{\frac{-216 \pm \sqrt{45792}}{54}}$$

from eq^n (i), $a = \dfrac{-2}{3b}$

$$\therefore \quad a = \frac{-2}{3} \times \sqrt[(-3)]{\frac{-216 \pm \sqrt{45792}}{54}}$$

from eq^n (ii), $x = a - b$

$$\Rightarrow \boxed{x = \frac{-2}{3}\left(\sqrt[3]{\frac{-216 \pm \sqrt{45792}}{54}}\right)^{-1} - \sqrt[3]{\frac{-216 \pm \sqrt{45792}}{54}}}$$

(Ans)

$\boxed{3}$

$$
\begin{aligned}
x + 2y + 3z &= 2 \\
2x + 2y + 0z &= 7 \\
x - 4y - 3z &= 3
\end{aligned}
$$

a) Gaussian Elimination:

Rewriting the system in a conjugated matrix:

$$
\left[\begin{array}{ccc|c}
1 & 2 & 3 & 2 \\
2 & 2 & 0 & 7 \\
1 & -4 & -3 & 3
\end{array}\right]
$$

$$
= \left[\begin{array}{ccc|c}
1 & 2 & 3 & 2 \\
0 & -2 & -6 & 3 \\
0 & -6 & -6 & 1
\end{array}\right]
\left(\begin{array}{l}
R_2 - 2R_1 \rightarrow R_2 \\
R_3 - R_1 \rightarrow R_3
\end{array}\right)
$$

$$
= \left[\begin{array}{ccc|c}
1 & 2 & 3 & 2 \\
0 & 1 & 3 & -1.5 \\
0 & -6 & -6 & 1
\end{array}\right]
\left(\frac{R_2}{-2} \rightarrow R_2\right)
$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 3 & -1.5 \\ 0 & 0 & 12 & -8 \end{array}\right] \begin{pmatrix} R_1 - 2R_2 \to R_1 \\ R_3 + 6R_2 \to R_3 \end{pmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 3 & -1.5 \\ 0 & 0 & 1 & -2/3 \end{array}\right] \left(\frac{R_3}{12} \to R_3\right)$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & -2/3 \end{array}\right] \begin{pmatrix} R_1 + 3R_3 \to R_1 \\ R_2 - 3R_3 \to R_2 \end{pmatrix}$$

$$\therefore \boxed{x = 3, \quad y = \frac{1}{2}, \quad z = \frac{-2}{3}}$$

(Ans)

(b) Solving with inverse matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 0 \\ 1 & -4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$$

$$\underset{A}{\downarrow} \qquad \underset{X}{\downarrow} = \underset{B}{\downarrow}$$

$$\Rightarrow \quad AX = B$$

$$\Rightarrow \quad A^{-1}AX = A^{-1}B$$

$$\Rightarrow \quad X = A^{-1}B$$

We'll use linear row reduction to find A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 1 & -4 & -3 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{5}{12} & -\frac{1}{4} & \frac{1}{12} \end{array}\right]$$

(with calculator)

$$\Rightarrow \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{5}{12} & -\frac{1}{4} & \frac{1}{12} \end{bmatrix} \rightarrow A^{-1}$$

now, $X = A^{-1}B$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{5}{12} & -\frac{1}{4} & \frac{1}{12} \end{bmatrix} \times \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{4} + \frac{3}{4} + \frac{7}{4} \\ \frac{-3}{4} + \frac{-2}{4} + \frac{7}{4} \\ \frac{-7}{4} + \frac{3}{12} + \frac{2\cdot 5}{12} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ \frac{1}{2} \\ -\frac{2}{3} \end{bmatrix}$$

$$\boxed{\therefore (x, y, z) = \left(3, \frac{1}{2}, -\frac{2}{3}\right)}$$

(Ans)

(c) With Cramers Rule:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 0 \\ 1 & -4 & -3 \end{bmatrix}$$

$\det(A) = -24$ (with calculator)

$$A_x = \begin{bmatrix} 2 & 2 & 3 \\ 7 & 2 & 0 \\ 3 & -4 & -3 \end{bmatrix}, \quad \det(A_x) = -72$$

$$A_y = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 0 \\ 1 & 3 & -3 \end{bmatrix}, \quad \det(A_y) = -12$$

$$A_z = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 7 \\ 1 & -4 & 3 \end{bmatrix}, \quad \det(A_z) = 16$$

now, $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-72}{-12} = 3$

$y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-12}{-24} = \dfrac{1}{2}$

$z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{16}{-24} = -\dfrac{2}{3}$

$$\therefore \boxed{\begin{aligned} x &= 3 \\ y &= \tfrac{1}{2} \\ z &= \tfrac{-2}{3} \end{aligned}}$$

(Ans

The 3 method each has their characteristics and for diffirent linear systems, we can apply various methods flexibly and finally Solve the problem.

However, I prefer Gaussian Elimination over the other two methods. From a computational standpoint, it offers the best algorithmic complexity.

Cramer's rule roughly takes $n!(n-1)$ multiplications to evaluate each $(n+1)$ dets.

So, complexity is $O((n+1)!(n-1))$. Which is worse than $O(n^3)$ offered by other two methods.

Between Gaussian Elimination and inverse method, GE has more optimized ~~calculations~~ methods. (like Chinese fang cheng reduction, etc).

So, from a computational point of view, I prefer Gaussian Eleminatism over the other two methods.

$\boxed{4}$

$$A = \begin{bmatrix} 5 & 3 & -1 \\ 1 & 0 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & -1 & 2 \\ 4 & -3 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

a) $A + B = \begin{bmatrix} 4 & 2 & 1 \\ 5 & -3 & -5 \end{bmatrix}$ (Ans)

b) AC = not possible, dimention ~~dism~~ mismatch

it's impossible to multiply a

2×3 mat with a 2×2 one. (Ans)

c) $$CB = \begin{bmatrix} 1\times(-1)+(-1)\cdot 4 & 1\times(-1)+(-1)(-3) & (1)(2)+(-1)0 \\ (-3)(-1)+2\times 4 & (-3)(-1)+2(-3) & (-3)(2)+(2)\times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 2 & 2 \\ 11 & -3 & 6 \end{bmatrix}$$ (Ans)

d) $|C| = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = 1\times 2 - (-1)(-3)$

$= 2 - 3$

$= -1$

$\therefore |C| = -1$ (Ans)

e) $C^{-1} = \frac{1}{|C|} \cdot (C^{adj})^{t}$

$= \frac{1}{-1}\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & -1 \\ -3 & -1 \end{bmatrix}$ (Ans

f) $CC^{-1} = I$ (by the identity rule)

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (Ans