UoPeople MATH 1302 U5

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Question 1: Validity Check Using Logical Laws

Given argument: $A \to B, A \lor C, C \to \neg B, D \to B, \neg C \land D \Rightarrow B \land \neg C$

Step-by-Step Validity Check

Step 1: From premise $\neg C \land D$, we can derive:

$$\neg C \quad \text{(Conjunction Elimination)} \tag{1}$$

$$D \quad \text{(Conjunction Elimination)} \tag{2}$$

Step 2: From D and premise $D \to B$:

$$B \quad \text{(Modus Ponens on } D \to B \text{ and } D\text{)} \tag{3}$$

Step 3: We now have $\neg C$ from Step 1 and B from Step 2.

$$B \land \neg C \quad \text{(Conjunction Introduction)} \tag{4}$$

Verification of consistency: Let's verify that our derived conclusion is consistent with all premises:

- $A \to B$: We have B is true, so this is satisfied regardless of A's truth value
- $A \lor C$: Since $\neg C$ is true (i.e., C is false), A must be true for this disjunction to hold
- $C \to \neg B$: Since C is false, this conditional is vacuously true
- $D \to B$: We have both D and B true, so this is satisfied
- $\neg C \land D$: Both conjuncts are true as established

Conclusion: The argument is **VALID**. We successfully derived $B \land \neg C$ from the given premises using valid logical inference rules.

Question 2: Quantifiers and Their Types

Definition and Types of Quantifiers

Quantifiers are logical operators that specify the quantity of specimens in a domain for which a predicate is true. The two main types are:

1. **Universal Quantifier ($\forall$):** "For all" or "for every"
2. **Existential Quantifier ($\exists$):** "There exists" or "for some"

Original Examples with Symbolic Representation

Example 1 - Universal Quantifier:

- **Natural Language:** "All students in the mathematics department own a calculator."
- **Symbolic:** $\forall x(S(x) \rightarrow C(x))$
- Where: $S(x) =$ "x is a student in the mathematics department", $C(x) =$ "x owns a calculator"
- **Truth Analysis:** This predicate is true if every individual who is a mathematics student also owns a calculator. It's false if we can find even one mathematics student who doesn't own a calculator.

Example 2 - Existential Quantifier:

- **Natural Language:** "There exists a planet in our solar system that has liquid water on its surface."
- **Symbolic:** $\exists x(P(x) \wedge W(x))$
- Where: $P(x) =$ "x is a planet in our solar system", $W(x) =$ "x has liquid water on its surface"
- **Truth Analysis:** This predicate is true because Earth satisfies both conditions (it's a planet in our solar system and has liquid water). The statement only requires at least one such planet to exist.

Example 3 - Mixed Quantifiers:

- **Natural Language:** "For every programming language, there exists a compiler that can process it."
- **Symbolic:** $\forall x(L(x) \rightarrow \exists y(C(y) \wedge P(y, x)))$
- Where: $L(x) =$ "x is a programming language", $C(y) =$ "y is a compiler", $P(y, x) =$ "y can process x"
- **Truth Analysis:** This predicate is true if for each programming language we can identify, there's at least one compiler capable of processing it. The statement would be false if we found a programming language with no available compiler.

Example 4 - Negated Quantifier:

- **Natural Language:** "Not all birds can fly."
- **Symbolic:** $\neg\forall x(B(x) \rightarrow F(x))$ or equivalently $\exists x(B(x) \wedge \neg F(x))$
- Where: $B(x) =$ "x is a bird", $F(x) =$ "x can fly"
- **Truth Analysis:** This predicate is true because there exist birds (like penguins, ostriches) that cannot fly. The equivalent existential form explicitly states that there exists at least one bird that cannot fly.

Truth Value Discussion

The truth values of these predicates depend on the domain of discourse and the actual state of the world:

- **Example 1** is likely false in reality, as not all mathematics students may own calculators
- **Example 2** is true (Earth has liquid water)
- **Example 3** is generally true for mainstream programming languages
- **Example 4** is true (flightless birds exist)