

# AC/DC Optimal Power Flow and Power flow Equations and their Derivatives in Complex Matrix Notation using FUBM for MATPOWER

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# 1 Useful Equations and Derivatives

## 1.1 Optimisation Variables

$$X = [V_a \quad V_m \quad P_g \quad Q_g \quad B_{eq} \quad \theta_{sh} \quad m_a \quad G_{sw}]^\top \quad (1)$$

## 1.2 Voltage Equations

$$V = V_m e^{jV_a} \quad (2)$$

### 1.2.1 Voltage First Derivatives

$$\frac{\partial V}{\partial V_a} = j [V_m e^{jV_a}] = j[V] \quad (3)$$

$$\frac{\partial V}{\partial V_m} = [1e^{jV_a}] \quad (4)$$

### 1.2.2 Voltage Second Derivatives

$$\frac{\partial^2 V}{\partial^2 V_a} = -[V_m e^{jV_a}] = -[V] \quad (5)$$

$$\frac{\partial^2 V}{\partial V_m \partial V_a} = [je^{jV_a}] \quad (6)$$

$$\frac{\partial^2 V}{\partial V_a \partial V_m} = \frac{\partial^2 V}{\partial V_m \partial V_a}^\top \quad (7)$$

$$\frac{\partial V^2}{\partial^2 V_m} = [0] \quad (8)$$

## 1.3 Admittance Equations

$$\begin{bmatrix} i_f \\ i_t \end{bmatrix} = \begin{bmatrix} Y_{ff} & Y_{ft} \\ Y_{tf} & Y_{tt} \end{bmatrix} \begin{bmatrix} v_f \\ v_t \end{bmatrix} = [Y_{br}] \begin{bmatrix} v_f \\ v_t \end{bmatrix} \quad (9)$$

$$Y_{br} = \begin{bmatrix} G_{sw} + (y_s + j\frac{b_c}{2} + jB_{eq})\frac{1}{m_a'^2} & \frac{-y_s}{m_a' e^{-j\theta_{sh}}} \\ \frac{-y_s}{m_a' e^{j\theta_{sh}}} & y_s + j\frac{b_c}{2} \end{bmatrix} \quad (10)$$

$$Y_f = [Y_{ff}]C_f + [Y_{ft}]C_t \quad (11)$$

$$Y_t = [Y_{tf}]C_f + [Y_{tt}]C_t \quad (12)$$

$$Y_{bus} = C_f^\top Y_f + C_t^\top Y_t + [Y_{sh}] \quad (13)$$

### 1.3.1 Admittance First Derivatives

$$\frac{\partial Y_{bus}}{\partial B_{eq}} = C_f^\top \frac{\partial Y_f}{\partial B_{eq}} + C_t^\top \frac{\partial Y_t}{\partial B_{eq}} \quad (14)$$

$$\frac{\partial Y_{bus}}{\partial \theta_{sh}} = C_f^\top \frac{\partial Y_f}{\partial \theta_{sh}} + C_t^\top \frac{\partial Y_t}{\partial \theta_{sh}} \quad (15)$$

$$\frac{\partial Y_{bus}}{\partial m_a} = C_f^\top \frac{\partial Y_f}{\partial m_a} + C_t^\top \frac{\partial Y_t}{\partial m_a} \quad (16)$$

$$\frac{\partial Y_{bus}}{\partial G_{sw}} = C_f^\top \frac{\partial Y_f}{\partial G_{sw}} + C_t^\top \frac{\partial Y_t}{\partial G_{sw}} \quad (17)$$

$$\frac{\partial Y_f}{\partial B_{eq}} = \left[ \frac{\partial Y_{ff}}{\partial B_{eq}} \right] C_f + \left[ \frac{\partial Y_{ft}}{\partial B_{eq}} \right] C_t \quad (18)$$

$$\frac{\partial Y_f}{\partial \theta_{sh}} = \left[ \frac{\partial Y_{ff}}{\partial \theta_{sh}} \right] C_f + \left[ \frac{\partial Y_{ft}}{\partial \theta_{sh}} \right] C_t \quad (19)$$

$$\frac{\partial Y_f}{\partial m_a} = \left[ \frac{\partial Y_{ff}}{\partial m_a} \right] C_f + \left[ \frac{\partial Y_{ft}}{\partial m_a} \right] C_t \quad (20)$$

$$\frac{\partial Y_f}{\partial G_{sw}} = \left[ \frac{\partial Y_{ff}}{\partial G_{sw}} \right] C_f + \left[ \frac{\partial Y_{ft}}{\partial G_{sw}} \right] C_t \quad (21)$$

$$\frac{\partial Y_t}{\partial B_{eq}} = \left[ \frac{\partial Y_{tf}}{\partial B_{eq}} \right] C_f + \left[ \frac{\partial Y_{tt}}{\partial B_{eq}} \right] C_t \quad (22)$$

$$\frac{\partial Y_t}{\partial \theta_{sh}} = \left[ \frac{\partial Y_{tf}}{\partial \theta_{sh}} \right] C_f + \left[ \frac{\partial Y_{tt}}{\partial \theta_{sh}} \right] C_t \quad (23)$$

$$\frac{\partial Y_t}{\partial m_a} = \left[ \frac{\partial Y_{tf}}{\partial m_a} \right] C_f + \left[ \frac{\partial Y_{tt}}{\partial m_a} \right] C_t \quad (24)$$

$$\frac{\partial Y_t}{\partial G_{sw}} = \left[ \frac{\partial Y_{tf}}{\partial G_{sw}} \right] C_f + \left[ \frac{\partial Y_{tt}}{\partial G_{sw}} \right] C_t \quad (25)$$

$$\frac{\partial Y_{ff}}{\partial B_{eq}} = \frac{j}{(k_2 m_a)^2} \quad (26)$$

$$\frac{\partial Y_{ft}}{\partial B_{eq}} = 0 \quad (27)$$

$$\frac{\partial Y_{tf}}{\partial B_{eq}} = 0 \quad (28)$$

$$\frac{\partial Y_{tt}}{\partial B_{eq}} = 0 \quad (29)$$

$$\frac{\partial Y_{ff}}{\partial \theta_{sh}} = 0 \quad (30)$$

$$\frac{\partial Y_{ft}}{\partial \theta_{sh}} = \frac{-j y_s}{k_2 m_a e^{-j\theta_{sh}}} \quad (31)$$

$$\frac{\partial Y_{tf}}{\partial \theta_{sh}} = \frac{j y_s}{k_2 m_a e^{j\theta_{sh}}} \quad (32)$$

$$\frac{\partial Y_{tt}}{\partial \theta_{sh}} = 0 \quad (33)$$

$$\frac{\partial Y_{ff}}{\partial m_a} = \frac{-2(y_s + j \frac{b_c}{2} + j B_{eq})}{k_2^2 m_a^3} \quad (34)$$

$$\frac{\partial Y_{ft}}{\partial m_a} = \frac{y_s}{k_2 m_a^2 e^{-j\theta_{sh}}} \quad (35)$$

$$\frac{\partial Y_{tf}}{\partial m_a} = \frac{y_s}{k_2 m_a^2 e^{j\theta_{sh}}} \quad (36)$$

$$\frac{\partial Y_{tt}}{\partial m_a} = 0 \quad (37)$$

$$\frac{\partial Y_{ff}}{\partial G_{sw}} = 1 \quad (38)$$

$$\frac{\partial Y_{ft}}{\partial G_{sw}} = 0 \quad (39)$$

$$\frac{\partial Y_{tf}}{\partial G_{sw}} = 0 \quad (40)$$

$$\frac{\partial Y_{tt}}{\partial G_{sw}} = 0 \quad (41)$$

### 1.3.2 Admittance Second Derivatives

$$\frac{\partial^2 Y_{bus}}{\partial B_{eq} \partial \theta_{sh}} = C_f^\top \frac{\partial^2 Y_f}{\partial B_{eq} \partial \theta_{sh}} + C_t^\top \frac{\partial^2 Y_t}{\partial B_{eq} \partial \theta_{sh}} \quad (42)$$

$$\frac{\partial^2 Y_{bus}}{\partial B_{eq} \partial m_a} = C_f^\top \frac{\partial^2 Y_f}{\partial B_{eq} \partial m_a} + C_t^\top \frac{\partial^2 Y_t}{\partial B_{eq} \partial m_a} \quad (43)$$

$$\frac{\partial^2 Y_{bus}}{\partial^2 B_{eq}} = C_f^\top \frac{\partial^2 Y_f}{\partial^2 B_{eq}} + C_t^\top \frac{\partial^2 Y_t}{\partial^2 B_{eq}} \quad (44)$$

$$\frac{\partial^2 Y_{bus}}{\partial B_{eq} \partial m_a} = C_f^\top \frac{\partial^2 Y_f}{\partial B_{eq} \partial G_{sw}} + C_t^\top \frac{\partial^2 Y_t}{\partial B_{eq} \partial G_{sw}} \quad (45)$$

$$\frac{\partial^2 Y_f}{\partial B_{eq} \partial \theta_{sh}} = \left[ \frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial \theta_{sh}} \right] C_f + \left[ \frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial \theta_{sh}} \right] C_t \quad (46)$$

$$\frac{\partial^2 Y_f}{\partial B_{eq} \partial m_a} = \left[ \frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial m_a} \right] C_f + \left[ \frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial m_a} \right] C_t \quad (47)$$

$$\frac{\partial^2 Y_f}{\partial^2 B_{eq}} = \left[ \frac{\partial^2 Y_{ff}}{\partial^2 B_{eq}} \right] C_f + \left[ \frac{\partial^2 Y_{ft}}{\partial^2 B_{eq}} \right] C_t \quad (48)$$

$$\frac{\partial^2 Y_f}{\partial B_{eq} \partial G_{sw}} = \left[ \frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial G_{sw}} \right] C_f + \left[ \frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial G_{sw}} \right] C_t \quad (49)$$

$$\frac{\partial^2 Y_t}{\partial B_{eq} \partial \theta_{sh}} = \left[ \frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial \theta_{sh}} \right] C_f + \left[ \frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial \theta_{sh}} \right] C_t \quad (50)$$

$$\frac{\partial^2 Y_t}{\partial B_{eq} \partial m_a} = \left[ \frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial m_a} \right] C_f + \left[ \frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial m_a} \right] C_t \quad (51)$$

$$\frac{\partial^2 Y_t}{\partial^2 B_{eq}} = \left[ \frac{\partial^2 Y_{tf}}{\partial^2 B_{eq}} \right] C_f + \left[ \frac{\partial^2 Y_{tt}}{\partial^2 B_{eq}} \right] C_t \quad (52)$$

$$\frac{\partial^2 Y_t}{\partial B_{eq} \partial G_{sw}} = \left[ \frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial G_{sw}} \right] C_f + \left[ \frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial G_{sw}} \right] C_t \quad (53)$$

### 1.3.3 2nd Derivatives of $Y_{ff}$

$$\frac{\partial^2 Y_{ff}}{\partial^2 B_{eq}} = 0 \quad (54)$$

$$\frac{\partial^2 Y_{ff}}{\partial^2 \theta_{sh}} = 0 \quad (55)$$

$$\frac{\partial^2 Y_{ff}}{\partial^2 m_a} = \frac{6(y_s + j\frac{b_c}{2} + jB_{eq})}{k_2^2 m_a^4} \quad (56)$$

$$\frac{\partial^2 Y_{ff}}{\partial^2 G_{sw}} = 0 \quad (57)$$

$$\frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial \theta_{sh}} = \frac{\partial^2 Y_{ff}}{\partial \theta_{sh} \partial B_{eq}} = 0 \quad (58)$$

$$\frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial m_a} = \frac{\partial^2 Y_{ff}}{\partial m_a \partial B_{eq}} = \frac{-2j}{k_2^2 m_a^3} \quad (59)$$

$$\frac{\partial^2 Y_{ff}}{\partial B_{eq} \partial G_{sw}} = \frac{\partial^2 Y_{ff}}{\partial G_{sw} \partial B_{eq}} = 0 \quad (60)$$

$$\frac{\partial^2 Y_{ff}}{\partial \theta_{sh} \partial m_a} = \frac{\partial^2 Y_{ff}}{\partial m_a \partial \theta_{sh}} = 0 \quad (61)$$

$$\frac{\partial^2 Y_{ff}}{\partial \theta_{sh} \partial G_{sw}} = \frac{\partial^2 Y_{ff}}{\partial G_{sw} \partial \theta_{sh}} = 0 \quad (62)$$

$$\frac{\partial^2 Y_{ff}}{\partial m_a \partial G_{sw}} = \frac{\partial^2 Y_{ff}}{\partial G_{sw} \partial m_a} = 0 \quad (63)$$

### 1.3.4 2nd Derivatives of $Y_{ft}$

$$\frac{\partial^2 Y_{ft}}{\partial^2 B_{eq}} = 0 \quad (64)$$

$$\frac{\partial^2 Y_{ft}}{\partial^2 \theta_{sh}} = \frac{y_s}{k_2 m_a e^{-j\theta_{sh}}} \quad (65)$$

$$\frac{\partial^2 Y_{ft}}{\partial^2 m_a} = \frac{-2y_s}{k_2 m_a^3 e^{-j\theta_{sh}}} \quad (66)$$

$$\frac{\partial^2 Y_{ft}}{\partial^2 G_{sw}} = 0 \quad (67)$$

$$\frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial \theta_{sh}} = \frac{\partial^2 Y_{ft}}{\partial \theta_{sh} \partial B_{eq}} = 0 \quad (68)$$

$$\frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial m_a} = \frac{\partial^2 Y_{ft}}{\partial m_a \partial B_{eq}} = 0 \quad (69)$$

$$\frac{\partial^2 Y_{ft}}{\partial B_{eq} \partial G_{sw}} = \frac{\partial^2 Y_{ft}}{\partial G_{sw} \partial B_{eq}} = 0 \quad (70)$$

$$\frac{\partial^2 Y_{ft}}{\partial \theta_{sh} \partial m_a} = \frac{\partial^2 Y_{ft}}{\partial m_a \partial \theta_{sh}} = \frac{jy_s}{k_2 m_a^2 e^{-j\theta_{sh}}} \quad (71)$$

$$\frac{\partial^2 Y_{ft}}{\partial \theta_{sh} \partial G_{sw}} = \frac{\partial^2 Y_{ft}}{\partial G_{sw} \partial \theta_{sh}} = 0 \quad (72)$$

$$\frac{\partial^2 Y_{ft}}{\partial m_a \partial G_{sw}} = \frac{\partial^2 Y_{ft}}{\partial G_{sw} \partial m_a} = 0 \quad (73)$$

### 1.3.5 2nd Derivatives of $Y_{tf}$

$$\frac{\partial^2 Y_{tf}}{\partial^2 B_{eq}} = 0 \quad (74)$$

$$\frac{\partial^2 Y_{tf}}{\partial^2 \theta_{sh}} = \frac{y_s}{k_2 m_a e^{j\theta_{sh}}} \quad (75)$$

$$\frac{\partial^2 Y_{tf}}{\partial^2 m_a} = \frac{-2y_s}{k_2 m_a^3 e^{j\theta_{sh}}} \quad (76)$$

$$\frac{\partial^2 Y_{tf}}{\partial^2 G_{sw}} = 0 \quad (77)$$

$$\frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial \theta_{sh}} = \frac{\partial^2 Y_{tf}}{\partial \theta_{sh} \partial B_{eq}} = 0 \quad (78)$$

$$\frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial m_a} = \frac{\partial^2 Y_{tf}}{\partial m_a \partial B_{eq}} = 0 \quad (79)$$

$$\frac{\partial^2 Y_{tf}}{\partial B_{eq} \partial G_{sw}} = \frac{\partial^2 Y_{tf}}{\partial G_{sw} \partial B_{eq}} = 0 \quad (80)$$

$$\frac{\partial^2 Y_{tf}}{\partial \theta_{sh} \partial m_a} = \frac{\partial^2 Y_{tf}}{\partial m_a \partial \theta_{sh}} = \frac{-j y_s}{k_2 m_a^2 e^{j\theta_{sh}}} \quad (81)$$

$$\frac{\partial^2 Y_{tf}}{\partial \theta_{sh} \partial G_{sw}} = \frac{\partial^2 Y_{tf}}{\partial G_{sw} \partial \theta_{sh}} = 0 \quad (82)$$

$$\frac{\partial^2 Y_{tf}}{\partial m_a \partial G_{sw}} = \frac{\partial^2 Y_{tf}}{\partial G_{sw} \partial m_a} = 0 \quad (83)$$

### 1.3.6 2nd Derivatives of $Y_{tt}$

$$\frac{\partial^2 Y_{tt}}{\partial^2 B_{eq}} = 0 \quad (84)$$

$$\frac{\partial^2 Y_{tt}}{\partial^2 \theta_{sh}} = 0 \quad (85)$$

$$\frac{\partial^2 Y_{tt}}{\partial^2 m_a} = 0 \quad (86)$$

$$\frac{\partial^2 Y_{tt}}{\partial^2 G_{sw}} = 0 \quad (87)$$

$$\frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial \theta_{sh}} = \frac{\partial^2 Y_{tt}}{\partial \theta_{sh} \partial B_{eq}} = 0 \quad (88)$$

$$\frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial m_a} = \frac{\partial^2 Y_{tt}}{\partial m_a \partial B_{eq}} = 0 \quad (89)$$

$$\frac{\partial^2 Y_{tt}}{\partial B_{eq} \partial G_{sw}} = \frac{\partial^2 Y_{tt}}{\partial G_{sw} \partial B_{eq}} = 0 \quad (90)$$

$$\frac{\partial^2 Y_{tt}}{\partial \theta_{sh} \partial m_a} = \frac{\partial^2 Y_{tt}}{\partial m_a \partial \theta_{sh}} = 0 \quad (91)$$

$$\frac{\partial^2 Y_{tt}}{\partial \theta_{sh} \partial G_{sw}} = \frac{\partial^2 Y_{tt}}{\partial G_{sw} \partial \theta_{sh}} = 0 \quad (92)$$

$$\frac{\partial^2 Y_{tt}}{\partial m_a \partial G_{sw}} = \frac{\partial^2 Y_{tt}}{\partial G_{sw} \partial m_a} = 0 \quad (93)$$

## 2 Bus Injections

Consider the power balance equation  $G^s(X) = 0$ , where:

$$G^s(X) = S^{bus} + S_d - C_g S_g \quad (94)$$

and

$$S^{bus} = [V] I^{bus} = [V] (Y_{bus} V)^* \quad (95)$$

### 2.1 First Derivatives

$$\begin{aligned} \tilde{G}_X^s &= \frac{\partial G^s}{\partial X} \\ &= \begin{bmatrix} G_{V_a}^s & G_{V_m}^s & G_{P_g}^s & G_{Q_g}^s & G_{B_{eq}}^s & G_{\theta_{sh}}^s & G_{m_a}^s & G_{G_{sw}}^s \end{bmatrix} \end{aligned} \quad (96)$$

where:

$$\begin{aligned} \tilde{G}_{V_a}^s &= \frac{\partial S^{bus}}{\partial V_a} \\ &= \frac{\partial V}{\partial V_a} (Y_{bus} V)^* + [V] \left( Y_{bus} \frac{\partial V}{\partial V_a} \right)^* \end{aligned} \quad (97)$$

$$\begin{aligned} \tilde{G}_{V_m}^s &= \frac{\partial S^{bus}}{\partial V_m} \\ &= \frac{\partial V}{\partial V_m} (Y_{bus} V)^* + [V] \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* \end{aligned} \quad (98)$$

$$G_{P_g}^s = \frac{\partial S^{bus}}{\partial P_g} = -C_g \quad (99)$$

$$G_{Q_g}^s = \frac{\partial S^{bus}}{\partial Q_g} = -jC_g \quad (100)$$

$$G_{B_{eq}}^s = \frac{\partial S^{bus}}{\partial B_{eq}} = [V] \left( \frac{\partial Y_{bus}}{\partial B_{eq}} V \right)^* \quad (101)$$

$$G_{\theta_{sh}}^s = \frac{\partial S^{bus}}{\partial \theta_{sh}} = [V] \left( \frac{\partial Y_{bus}}{\partial B_{eq} \theta_{sh}} V \right)^* \quad (102)$$



$$G_{m_a}^s = \frac{\partial S^{bus}}{\partial m_a} = [V] \left( \frac{\partial Y_{bus}}{\partial m_a} V \right)^* \quad (103)$$

$$G_{G_{sw}}^s = \frac{\partial S^{bus}}{\partial G_{sw}} = [V] \left( \frac{\partial Y_{bus}}{\partial G_{sw}} V \right)^* \quad (104)$$

## 2.2 Second Derivatives

$$G_{XX}^s(\lambda) = \frac{\partial}{\partial X} (G_X^s{}^\top \lambda) = \quad (105)$$

$G^s_V V_a$	$G^s_V V_m$	$G^s_V P_g$	$G^s_V Q_g$	$G^s_V B_{eqz}$	$G^s_V B_{eqv}$	$G^s_V \theta_{sh}$	$G^s_V qtm_a$	$G^s_V vtm_a$	$G^s_V G_{sw}$
$G^s_{V_m} V_a$	$G^s_{V_m} V_m$	$G^s_{V_m} P_g$	$G^s_{V_m} Q_g$	$G^s_{V_m} B_{eqz}$	$G^s_{V_m} B_{eqv}$	$G^s_{V_m} \theta_{sh}$	$G^s_{V_m} qtm_a$	$G^s_{V_m} vtm_a$	$G^s_{V_m} G_{sw}$
$G^s_{P_g} V_a$	$G^s_{P_g} V_m$	$G^s_{P_g} P_g$	$G^s_{P_g} Q_g$	$G^s_{P_g} B_{eqz}$	$G^s_{P_g} B_{eqv}$	$G^s_{P_g} \theta_{sh}$	$G^s_{P_g} qtm_a$	$G^s_{P_g} vtm_a$	$G^s_{P_g} G_{sw}$
$G^s_{Q_g} V_a$	$G^s_{Q_g} V_m$	$G^s_{Q_g} P_g$	$G^s_{Q_g} Q_g$	$G^s_{Q_g} B_{eqz}$	$G^s_{Q_g} B_{eqv}$	$G^s_{Q_g} \theta_{sh}$	$G^s_{Q_g} qtm_a$	$G^s_{Q_g} vtm_a$	$G^s_{Q_g} G_{sw}$
$G^s_{B_{eqz}} V_a$	$G^s_{B_{eqz}} V_m$	$G^s_{B_{eqz}} P_g$	$G^s_{B_{eqz}} Q_g$	$G^s_{B_{eqz}} B_{eqz}$	$G^s_{B_{eqz}} B_{eqv}$	$G^s_{B_{eqz}} \theta_{sh}$	$G^s_{B_{eqz}} qtm_a$	$G^s_{B_{eqz}} vtm_a$	$G^s_{B_{eqz}} G_{sw}$
$G^s_{B_{eqv}} V_a$	$G^s_{B_{eqv}} V_m$	$G^s_{B_{eqv}} P_g$	$G^s_{B_{eqv}} Q_g$	$G^s_{B_{eqv}} B_{eqz}$	$G^s_{B_{eqv}} B_{eqv}$	$G^s_{B_{eqv}} \theta_{sh}$	$G^s_{B_{eqv}} qtm_a$	$G^s_{B_{eqv}} vtm_a$	$G^s_{B_{eqv}} G_{sw}$
$G^s_{\theta_{sh}} V_a$	$G^s_{\theta_{sh}} V_m$	$G^s_{\theta_{sh}} P_g$	$G^s_{\theta_{sh}} Q_g$	$G^s_{\theta_{sh}} B_{eqz}$	$G^s_{\theta_{sh}} B_{eqv}$	$G^s_{\theta_{sh}} \theta_{sh}$	$G^s_{\theta_{sh}} qtm_a$	$G^s_{\theta_{sh}} vtm_a$	$G^s_{\theta_{sh}} G_{sw}$
$G^s_{qtm_a} V_a$	$G^s_{qtm_a} V_m$	$G^s_{qtm_a} P_g$	$G^s_{qtm_a} Q_g$	$G^s_{qtm_a} B_{eqz}$	$G^s_{qtm_a} B_{eqv}$	$G^s_{qtm_a} \theta_{sh}$	$G^s_{qtm_a} qtm_a$	$G^s_{qtm_a} vtm_a$	$G^s_{qtm_a} G_{sw}$
$G^s_{vtm_a} V_a$	$G^s_{vtm_a} V_m$	$G^s_{vtm_a} P_g$	$G^s_{vtm_a} Q_g$	$G^s_{vtm_a} B_{eqz}$	$G^s_{vtm_a} B_{eqv}$	$G^s_{vtm_a} \theta_{sh}$	$G^s_{vtm_a} qtm_a$	$G^s_{vtm_a} vtm_a$	$G^s_{vtm_a} G_{sw}$
$G^s_{G_{sw}} V_a$	$G^s_{G_{sw}} V_m$	$G^s_{G_{sw}} P_g$	$G^s_{G_{sw}} Q_g$	$G^s_{G_{sw}} B_{eqz}$	$G^s_{G_{sw}} B_{eqv}$	$G^s_{G_{sw}} \theta_{sh}$	$G^s_{G_{sw}} qtm_a$	$G^s_{G_{sw}} vtm_a$	$G^s_{G_{sw}} G_{sw}$

[illegible]

$$= \begin{bmatrix} G_{V_a V_a}^s(\lambda) & G_{V_a V_m}^s(\lambda) & 0 & 0 & G_{V_a B_{eq}}^s(\lambda) & G_{V_a \theta_{sh}}^s(\lambda) & G_{V_a m_a}^s(\lambda) & G_{V_a G_{sw}}^s(\lambda) \\ G_{V_m V_a}^s(\lambda) & G_{V_m V_m}^s(\lambda) & 0 & 0 & G_{V_m B_{eq}}^s(\lambda) & G_{V_m \theta_{sh}}^s(\lambda) & G_{V_m m_a}^s(\lambda) & G_{V_m G_{sw}}^s(\lambda) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ G_{B_{eq} V_a}^s(\lambda) & G_{B_{eq} V_m}^s(\lambda) & 0 & 0 & G_{B_{eq} B_{eq}}^s(\lambda) & G_{B_{eq} \theta_{sh}}^s(\lambda) & G_{B_{eq} m_a}^s(\lambda) & G_{B_{eq} G_{sw}}^s(\lambda) \\ G_{\theta_{sh} V_a}^s(\lambda) & G_{\theta_{sh} V_m}^s(\lambda) & 0 & 0 & G_{\theta_{sh} B_{eq}}^s(\lambda) & G_{\theta_{sh} \theta_{sh}}^s(\lambda) & G_{\theta_{sh} m_a}^s(\lambda) & G_{\theta_{sh} G_{sw}}^s(\lambda) \\ G_{m_a V_a}^s(\lambda) & G_{m_a V_m}^s(\lambda) & 0 & 0 & G_{m_a B_{eq}}^s(\lambda) & G_{m_a \theta_{sh}}^s(\lambda) & G_{m_a m_a}^s(\lambda) & G_{m_a G_{sw}}^s(\lambda) \\ G_{G_{sw} V_a}^s(\lambda) & G_{G_{sw} V_m}^s(\lambda) & 0 & 0 & G_{G_{sw} B_{eq}}^s(\lambda) & G_{G_{sw} \theta_{sh}}^s(\lambda) & G_{G_{sw} m_a}^s(\lambda) & G_{G_{sw} G_{sw}}^s(\lambda) \end{bmatrix} \quad (108)$$

### 2.3 Bus Injection Detailed Hessian

$$G_{V_a V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{V_a}^s{}^\top \lambda) \quad (109)$$

$$= \frac{\partial}{\partial V_a} \left( \left( \frac{\partial V}{\partial V_a} (Y_{bus} V)^* + [V] \left( Y_{bus} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \right) \quad (110)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( Y_{bus} \frac{\partial V}{\partial V_a} \right)^* + \frac{\partial^2 V}{\partial^2 V_a} (Y_{bus} V)^* + \left[ \frac{\partial V}{\partial V_a} \right] \left( Y_{bus} \frac{\partial V}{\partial V_a} \right)^* + [V] \left( Y_{bus} \frac{\partial^2 V}{\partial^2 V_a} \right)^* \right)^\top \lambda \quad (111)$$

$$G_{V_m V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{V_m}^s{}^\top \lambda) \quad (112)$$

$$= \frac{\partial V}{\partial V_a} \left( \left( \frac{\partial V}{\partial V_m} (Y_{bus} V)^* + [V] \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \right) \quad (113)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( Y_{bus} \frac{\partial V}{\partial V_a} \right)^* + \frac{\partial^2 V}{\partial V_m \partial V_a} (Y_{bus} V)^* + \left[ \frac{\partial V}{\partial V_a} \right] \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* + [V] \left( Y_{bus} \frac{\partial^2 V}{\partial V_m \partial V_a} \right)^* \right)^\top \lambda \quad (114)$$

$$G_{P_g V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{P_g}^s{}^\top \lambda) = 0 \quad (115)$$

$$G_{Q_g V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (116)$$

$$G_{B_{eqz} V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{B_{eqz}}^s{}^\top \lambda) \quad (117)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial B_{eqz}} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial B_{eqz}} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (118)$$

$$G_{B_{eqv} V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{B_{eqv}}^s{}^\top \lambda) \quad (119)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial B_{eqv}} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial B_{eqv}} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (120)$$

$$G_{\theta_{sh} V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{\theta_{sh}}^s{}^\top \lambda) \quad (121)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial \theta_{sh}} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial \theta_{sh}} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (122)$$

$$G_{qtm_a V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{qtm_a}^s{}^\top \lambda) \quad (123)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial qtm_a} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial qtm_a} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (124)$$

$$G_{vtm_a V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{vtm_a}^s{}^\top \lambda) \quad (125)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial vtm_a} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial vtm_a} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (126)$$

$$G_{G_{sw} V_a}^s(\lambda) = \frac{\partial}{\partial V_a} (G_{G_{sw}}^s{}^\top \lambda) \quad (127)$$

$$= \left( \frac{\partial V}{\partial V_a} \left( \frac{\partial Y_{bus}}{\partial G_{sw}} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial G_{sw}} \frac{\partial V}{\partial V_a} \right)^* \right)^\top \lambda \quad (128)$$

$$G_{V_a V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{V_a}^s{}^\top \lambda) \quad (129)$$

$$= G_{V_m V_a}^s{}^\top(\lambda) \quad (130)$$

$$G_{V_m V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{V_m}^s{}^\top \lambda) \quad (131)$$

$$= \frac{\partial}{\partial V_m} \left( \left( \frac{\partial V}{\partial V_m} (Y_{bus} V)^* + [V] \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \right) \quad (132)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* + \frac{\partial^2 V}{\partial^2 V_m} (Y_{bus} V)^* + \left[ \frac{\partial V}{\partial V_m} \right] \left( Y_{bus} \frac{\partial V}{\partial V_m} \right)^* + [V] \left( Y_{bus} \frac{\partial^2 V}{\partial^2 V_m} \right)^* \right)^\top \lambda \quad (133)$$

$$G_{P_g V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{P_g}^s{}^\top \lambda) = 0 \quad (134)$$

$$G_{Q_g V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (135)$$

$$G_{B_{eqz} V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{B_{eqz}}^s{}^\top \lambda) \quad (136)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\delta Y_{bus}}{\delta B_{eqz}} V \right)^* + [V] \left( \frac{\delta Y_{bus}}{\delta B_{eqz}} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (137)$$

$$G_{B_{eqv} V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{B_{eqv}}^s{}^\top \lambda) \quad (138)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\delta Y_{bus}}{\delta B_{eqv}} V \right)^* + [V] \left( \frac{\delta Y_{bus}}{\delta B_{eqv}} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (139)$$

$$G_{\theta_{sh} V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{\theta_{sh}}^s{}^\top \lambda) \quad (140)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\delta Y_{bus}}{\delta \theta_{sh}} V \right)^* + [V] \left( \frac{\delta Y_{bus}}{\delta \theta_{sh}} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (141)$$

$$G_{qtm_a V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{qtm_a}^s{}^\top \lambda) \quad (142)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\delta Y_{bus}}{\delta qtm_a} V \right)^* + [V] \left( \frac{\delta Y_{bus}}{\delta qtm_a} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (143)$$

$$G_{vtm_a V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{vtm_a}^s{}^\top \lambda) \quad (144)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\delta Y_{bus}}{\delta vtm_a} V \right)^* + [V] \left( \frac{\delta Y_{bus}}{\delta vtm_a} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (145)$$

$$G_{G_{sw} V_m}^s(\lambda) = \frac{\partial}{\partial V_m} (G_{G_{sw}}^s{}^\top \lambda) \quad (146)$$

$$= \left( \frac{\partial V}{\partial V_m} \left( \frac{\partial Y_{bus}}{\partial G_{sw}} V \right)^* + [V] \left( \frac{\partial Y_{bus}}{\partial G_{sw}} \frac{\partial V}{\partial V_m} \right)^* \right)^\top \lambda \quad (147)$$

$$G_{V_a P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{V_a}^s{}^\top \lambda) = 0 \quad (148)$$

$$G_{V_m P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{P_g}^s{}^\top \lambda) = 0 \quad (149)$$

$$G_{P_g P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{P_g}^s{}^\top \lambda) = 0 \quad (150)$$

$$G_{Q_g P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (151)$$

$$G_{B_{eqz} P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{B_{eqz}}^s{}^\top \lambda) = 0 \quad (152)$$

$$G_{B_{eqv} P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{B_{eqv}}^s{}^\top \lambda) = 0 \quad (153)$$

$$G_{\theta_{sh} P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{\theta_{sh}}^s{}^\top \lambda) = 0 \quad (154)$$

$$G_{qtm_a P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{qtm_a}^s{}^\top \lambda) = 0 \quad (155)$$

$$G_{vtm_a P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{vtm_a}^s{}^\top \lambda) = 0 \quad (156)$$

$$G_{G_{sw} P_g}^s(\lambda) = \frac{\partial}{\partial P_g} (G_{G_{sw}}^s{}^\top \lambda) = 0 \quad (157)$$

$$G_{V_a Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{V_a}^s{}^\top \lambda) = 0 \quad (158)$$

$$G_{V_m Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (159)$$

$$G_{P_g Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{P_g}^s{}^\top \lambda) = 0 \quad (160)$$

$$G_{Q_g Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (161)$$

$$G_{B_{eqz} Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{B_{eqz}}^s{}^\top \lambda) = 0 \quad (162)$$

$$G_{B_{eqv}Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{B_{eqv}}^s{}^\top \lambda) = 0 \quad (163)$$

$$G_{\theta_{sh}Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{\theta_{sh}}^s{}^\top \lambda) = 0 \quad (164)$$

$$G_{qtm_aQ_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{qtm_a}^s{}^\top \lambda) = 0 \quad (165)$$

$$G_{vtm_aQ_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{vtm_a}^s{}^\top \lambda) = 0 \quad (166)$$

$$G_{G_{sw}Q_g}^s(\lambda) = \frac{\partial}{\partial Q_g} (G_{G_{sw}}^s{}^\top \lambda) = 0 \quad (167)$$

$$G_{V_aB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{V_a}^s{}^\top \lambda) \quad (168)$$

$$= G_{B_{eqz}V_a}^s{}^\top(\lambda) \quad (169)$$

$$G_{V_mB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{V_m}^s{}^\top \lambda) \quad (170)$$

$$G_{V_mB_{eqz}}^s(\lambda) = G_{B_{eqz}V_m}^s{}^\top(\lambda) \quad (171)$$

$$G_{P_gB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{P_g}^s{}^\top \lambda) = 0 \quad (172)$$

$$G_{Q_gB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (173)$$

$$G_{B_{eqz}B_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{B_{eqz}}^s{}^\top \lambda) \quad (174)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial^2 B_{eqz}} V \right)^* \right)^\top \lambda \quad (175)$$

$$G_{B_{eqv}B_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{B_{eqv}}^s{}^\top \lambda) \quad (176)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial B_{eqv}B_{eqz}} V \right)^* \right)^\top \lambda \quad (177)$$

$$G_{\theta_{sh}B_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{\theta_{sh}}^s{}^\top \lambda) \quad (178)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial \theta_{sh}B_{eqz}} V \right)^* \right)^\top \lambda \quad (179)$$

$$G_{qtm_aB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{qtm_a}^s{}^\top \lambda) \quad (180)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial qtm_aB_{eqz}} V \right)^* \right)^\top \lambda \quad (181)$$

$$G_{vtm_aB_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{vtm_a}^s{}^\top \lambda) \quad (182)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial v t m_a B_{eqz}} V \right)^* \right)^\top \lambda \quad (183)$$

$$G_{G_{sw} B_{eqz}}^s(\lambda) = \frac{\partial}{\partial B_{eqz}} (G_{G_{sw}}^s{}^\top \lambda) \quad (184)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial G_{sw} B_{eqz}} V \right)^* \right)^\top \lambda \quad (185)$$

$$G_{V_a B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{V_a}^s{}^\top \lambda) \quad (186)$$

$$= G_{B_{eqv} V_a}^s{}^\top(\lambda) \quad (187)$$

$$G_{V_m B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{P_g}^s{}^\top \lambda) \quad (188)$$

$$= G_{B_{eqv} V_m}^s{}^\top(\lambda) \quad (189)$$

$$G_{P_g B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{P_g}^s{}^\top \lambda) = 0 \quad (190)$$

$$G_{Q_g B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (191)$$

$$G_{B_{eqz} B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{B_{eqz}}^s{}^\top \lambda) \quad (192)$$

$$= G_{B_{eqv} B_{eqz}}^s{}^\top(\lambda) \quad (193)$$

$$G_{B_{eqv} B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{B_{eqv}}^s{}^\top \lambda) \quad (194)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial^2 B_{eqv}} V \right)^* \right)^\top \lambda \quad (195)$$

$$G_{\theta_{sh} B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{\theta_{sh}}^s{}^\top \lambda) \quad (196)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial \theta_{sh} B_{eqv}} V \right)^* \right)^\top \lambda \quad (197)$$

$$G_{q t m_a B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{q t m_a}^s{}^\top \lambda) \quad (198)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial q t m_a B_{eqz}} V \right)^* \right)^\top \lambda \quad (199)$$

$$G_{v t m_a B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{v t m_a}^s{}^\top \lambda) \quad (200)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial v t m_a B_{eqz}} V \right)^* \right)^\top \lambda \quad (201)$$

$$G_{G_{sw} B_{eqv}}^s(\lambda) = \frac{\partial}{\partial B_{eqv}} (G_{G_{sw}}^s{}^\top \lambda) \quad (202)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial G_{sw} B_{eqz}} V \right)^* \right)^\top \lambda \quad (203)$$

$$G_{V_a \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{V_a}^s{}^\top \lambda) \quad (204)$$

$$= G_{\theta_{sh} V_a}^s{}^\top(\lambda) \quad (205)$$

$$G_{V_m \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{P_g}^s{}^\top \lambda) \quad (206)$$

$$= G_{\theta_{sh} V_m}^s(\lambda)^\top(\lambda) \quad (207)$$

$$G_{P_g \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{P_g}^s{}^\top \lambda) = 0 \quad (208)$$

$$G_{Q_g \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{Q_g}^s{}^\top \lambda) = 0 \quad (209)$$

$$G_{B_{eqz} \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{B_{eqz}}^s{}^\top \lambda) \quad (210)$$

$$= G_{\theta_{sh} B_{eqz}}^s{}^\top(\lambda) \quad (211)$$

$$G_{B_{eqv} \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{B_{eqv}}^s{}^\top \lambda) \quad (212)$$

$$= G_{\theta_{sh} B_{eqv}}^s{}^\top(\lambda) \quad (213)$$

$$G_{\theta_{sh} \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{\theta_{sh}}^s{}^\top \lambda) \quad (214)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial^2 \theta_{sh}} V \right)^* \right)^\top \lambda \quad (215)$$

$$G_{qtm_a \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{qtm_a}^s{}^\top \lambda) \quad (216)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial qtm_a \theta_{sh}} V \right)^* \right)^\top \lambda \quad (217)$$

$$G_{vtm_a \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{vtm_a}^s{}^\top \lambda) \quad (218)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial vtm_a \theta_{sh}} V \right)^* \right)^\top \lambda \quad (219)$$

$$G_{G_{sw} \theta_{sh}}^s(\lambda) = \frac{\partial}{\partial \theta_{sh}} (G_{G_{sw}}^s{}^\top \lambda) \quad (220)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial G_{sw} \theta_{sh}} V \right)^* \right)^\top \lambda \quad (221)$$

$$G_{V_a qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} (G_{V_a}^s{}^\top \lambda) \quad (222)$$

$$= G_{qtm_a V_a}^s{}^\top(\lambda) \quad (223)$$

$$G_{V_m qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} (G_{P_g}^s{}^\top \lambda) \quad (224)$$



$$= G_{qtm_a V_m}^s{}^\top(\lambda) \quad (225)$$

$$G_{P_g qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{P_g}^s{}^\top \lambda \right) = 0 \quad (226)$$

$$G_{Q_g qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{Q_g}^s{}^\top \lambda \right) \quad (227)$$

$$G_{B_{eqz} qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{B_{eqz}}^s{}^\top \lambda \right) \quad (228)$$

$$= G_{qtm_a B_{eqz}}^s{}^\top(\lambda) \quad (229)$$

$$G_{B_{eqv} qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{B_{eqv}}^s{}^\top \lambda \right) \quad (230)$$

$$= G_{qtm_a B_{eqv}}^s{}^\top(\lambda) \quad (231)$$

$$G_{\theta_{sh} qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{\theta_{sh}}^s{}^\top \lambda \right) \quad (232)$$

$$= G_{qtm_a \theta_{sh}}^s{}^\top(\lambda) \quad (233)$$

$$G_{qtm_a qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{qtm_a}^s{}^\top \lambda \right) \quad (234)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial^2 qtm_a} V \right)^* \right)^\top \lambda \quad (235)$$

$$G_{vtm_a qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{vtm_a}^s{}^\top \lambda \right) \quad (236)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial vtm_a qtm_a} V \right)^* \right)^\top \lambda \quad (237)$$

$$G_{G_{sw} qtm_a}^s(\lambda) = \frac{\partial}{\partial qtm_a} \left( G_{G_{sw}}^s{}^\top \lambda \right) \quad (238)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial G_{sw} qtm_a} V \right)^* \right)^\top \lambda \quad (239)$$

$$G_{V_a vtm_a}^s(\lambda) = \frac{\partial}{\partial vtm_a} \left( G_{V_a}^s{}^\top \lambda \right) \quad (240)$$

$$= G_{vtm_a V_a}^s{}^\top(\lambda) \quad (241)$$

$$G_{V_m vtm_a}^s(\lambda) = \frac{\partial}{\partial vtm_a} \left( G_{P_g}^s{}^\top \lambda \right) \quad (242)$$

$$= G_{vtm_a V_m}^s{}^\top(\lambda) \quad (243)$$

$$G_{P_g vtm_a}^s(\lambda) = \frac{\partial}{\partial vtm_a} \left( G_{P_g}^s{}^\top \lambda \right) = 0 \quad (244)$$

$$G_{Q_g vtm_a}^s(\lambda) = \frac{\partial}{\partial vtm_a} \left( G_{Q_g}^s{}^\top \lambda \right) \quad (245)$$

$$G_{B_{eqz}v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{B_{eqz}}^s{}^\top \lambda \right) \quad (246)$$

$$= G_{v t m_a B_{eqz}}^s{}^\top(\lambda) \quad (247)$$

$$G_{B_{eqv}v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{B_{eqv}}^s{}^\top \lambda \right) \quad (248)$$

$$= G_{v t m_a B_{eqv}}^s{}^\top(\lambda) \quad (249)$$

$$G_{\theta_{sh}v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{\theta_{sh}}^s{}^\top \lambda \right) \quad (250)$$

$$= G_{v t m_a \theta_{sh}}^s{}^\top(\lambda) \quad (251)$$

$$G_{q t m_a v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{q t m_a}^s{}^\top \lambda \right) \quad (252)$$

$$= G_{v t m_a q t m_a}^s{}^\top(\lambda) \quad (253)$$

$$G_{v t m_a v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{v t m_a}^s{}^\top \lambda \right) \quad (254)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial^2 v t m_a} V \right)^* \right)^\top \lambda \quad (255)$$

$$G_{G_{sw}v t m_a}^s(\lambda) = \frac{\partial}{\partial v t m_a} \left( G_{G_{sw}}^s{}^\top \lambda \right) \quad (256)$$

$$= \left( [V] \left( \frac{\partial^2 Y_{bus}}{\partial G_{sw} \partial v t m_a} V \right)^* \right)^\top \lambda \quad (257)$$

### 3 Branch Flows

Consider the branch flow constraints for Loadability, Zero Constraint, Active power control and Reactive power Control of equations Equations (258) to (262).

*Branch Loadability*

$$\begin{aligned} |F_f(X)| - F_{max} &\leq 0 \\ |F_t(X)| - F_{max} &\leq 0 \end{aligned} \quad (258)$$

where:

$$F_f(X) = \begin{cases} S_f & \text{apparent power} \\ P_f & \text{real power} \\ I_f & \text{current} \end{cases} \quad (259)$$

*Zero Constraint*

$$\Im(S_f) = 0 \quad (260)$$

*Active Power Control*

$$\Re(S_f) - P_{set} = 0 \quad (261)$$

$$\Im(S_f) - Q_{set} = 0 \quad (262)$$

Which are in function of  $S_f$  and  $S_t$ . Where:

$$\begin{aligned} S_f &= C_f[V][Y_f V]^* \\ S_t &= C_t[V][Y_t V]^* \end{aligned} \quad (263)$$

### 3.1 First Derivatives

$$\begin{aligned} S_X^f &= \frac{\partial S^f}{\partial X} = \\ & \begin{bmatrix} S_{V_a}^f & S_{V_m}^f & S_{P_g}^f & S_{Q_g}^f & S_{B_{eq}}^f & S_{\theta_{sh}}^f & S_{m_a}^f & S_{G_{sw}}^f \end{bmatrix} \end{aligned} \quad (264)$$

$$\begin{aligned} S_X^t &= \frac{\partial S^t}{\partial X} = \\ & \begin{bmatrix} S_{V_a}^t & S_{V_m}^t & S_{P_g}^t & S_{Q_g}^t & S_{B_{eq}}^t & S_{\theta_{sh}}^t & S_{m_a}^t & S_{G_{sw}}^t \end{bmatrix} \end{aligned} \quad (265)$$

$$\begin{aligned} S_{V_a}^f &= \frac{\partial S_f}{\partial V_a} = \\ & [C_f V] \left( Y_f \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] (Y_f V)^* \end{aligned} \quad (266)$$

$$\begin{aligned} S_{V_a}^t &= \frac{\partial S_t}{\partial V_a} = \\ & [C_t V] \left( Y_t \frac{\partial V}{\partial V_a} \right)^* + [C_t] \left[ C_t \frac{\partial V}{\partial V_a} \right] (Y_t V)^* \end{aligned} \quad (267)$$

$$\begin{aligned} S_{V_m}^f &= \frac{\partial S_f}{\partial V_m} = \\ & [C_f V] \left( Y_f \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] (Y_f V)^* \end{aligned} \quad (268)$$

$$\begin{aligned} S_{V_m}^t &= \frac{\partial S_t}{\partial V_m} = \\ & [C_t V] \left( Y_t \frac{\partial V}{\partial V_m} \right)^* + [C_t] \left[ C_t \frac{\partial V}{\partial V_m} \right] (Y_t V)^* \end{aligned} \quad (269)$$

$$S_{P_g}^f = \frac{\partial S_f}{\partial P_g} = 0 \quad (270)$$

$$S_{P_g}^t = \frac{\partial S_t}{\partial P_g} = 0 \quad (271)$$

$$S_{Q_g}^f = \frac{\partial S_f}{\partial Q_g} = 0 \quad (272)$$

$$S_{Q_g}^t = \frac{\partial S_t}{\partial Q_g} = 0 \quad (273)$$

$$S_{B_{eq}}^f = \frac{\partial S_f}{\partial B_{eq}} = [C_f V] \left( \frac{\partial Y_f}{\partial B_{eq}} V \right)^* \quad (274)$$

$$S_{B_{eq}}^t = \frac{\partial S_t}{\partial B_{eq}} = [C_t V] \left( \frac{\partial Y_t}{\partial B_{eq}} V \right)^* \quad (275)$$

$$S_{\theta_{sh}}^f = \frac{\partial S_f}{\partial \theta_{sh}} = [C_f V] \left( \frac{\partial Y_f}{\partial \theta_{sh}} V \right)^* \quad (276)$$

$$S_{\theta_{sh}}^t = \frac{\partial S_t}{\partial \theta_{sh}} = [C_t V] \left( \frac{\partial Y_t}{\partial \theta_{sh}} V \right)^* \quad (277)$$

$$S_{m_a}^f = \frac{\partial S_f}{\partial m_a} = [C_f V] \left( \frac{\partial Y_f}{\partial m_a} V \right)^* \quad (278)$$

$$S_{m_a}^t = \frac{\partial S_t}{\partial m_a} = [C_t V] \left( \frac{\partial Y_t}{\partial m_a} V \right)^* \quad (279)$$

### 3.2 Second Derivatives

$$S_{XX}^f(\mu) = \frac{\partial}{\partial X} \left( S_X^{f\top} \mu \right) = \quad (280)$$

$$\begin{bmatrix} S_{V_a V_a}^f(\mu) & S_{V_a V_m}^f(\mu) & S_{V_a B_{eqz}}^f(\mu) & S_{V_a B_{eqv}}^f(\mu) & S_{V_a qtm_a}^f(\mu) & S_{V_a vtm_a}^f(\mu) & S_{V_a G_{sw}}^f(\mu) \\ S_{V_m V_a}^f(\mu) & S_{V_m V_m}^f(\mu) & S_{V_m B_{eqz}}^f(\mu) & S_{V_m B_{eqv}}^f(\mu) & S_{V_m qtm_a}^f(\mu) & S_{V_m vtm_a}^f(\mu) & S_{V_m G_{sw}}^f(\mu) \\ S_{B_{eqz} V_a}^f(\mu) & S_{B_{eqz} V_m}^f(\mu) & S_{B_{eqz} B_{eqz}}^f(\mu) & S_{B_{eqz} B_{eqv}}^f(\mu) & S_{B_{eqz} qtm_a}^f(\mu) & S_{B_{eqz} vtm_a}^f(\mu) & S_{B_{eqz} G_{sw}}^f(\mu) \\ S_{B_{eqv} V_a}^f(\mu) & S_{B_{eqv} V_m}^f(\mu) & S_{B_{eqv} B_{eqz}}^f(\mu) & S_{B_{eqv} B_{eqv}}^f(\mu) & S_{B_{eqv} qtm_a}^f(\mu) & S_{B_{eqv} vtm_a}^f(\mu) & S_{B_{eqv} G_{sw}}^f(\mu) \\ S_{\theta_{sh} V_a}^f(\mu) & S_{\theta_{sh} V_m}^f(\mu) & S_{\theta_{sh} B_{eqz}}^f(\mu) & S_{\theta_{sh} B_{eqv}}^f(\mu) & S_{\theta_{sh} qtm_a}^f(\mu) & S_{\theta_{sh} vtm_a}^f(\mu) & S_{\theta_{sh} G_{sw}}^f(\mu) \\ S_{qtm_a V_a}^f(\mu) & S_{qtm_a V_m}^f(\mu) & S_{qtm_a B_{eqz}}^f(\mu) & S_{qtm_a B_{eqv}}^f(\mu) & S_{qtm_a qtm_a}^f(\mu) & S_{qtm_a vtm_a}^f(\mu) & S_{qtm_a G_{sw}}^f(\mu) \\ S_{vtm_a V_a}^f(\mu) & S_{vtm_a V_m}^f(\mu) & S_{vtm_a B_{eqz}}^f(\mu) & S_{vtm_a B_{eqv}}^f(\mu) & S_{vtm_a qtm_a}^f(\mu) & S_{vtm_a vtm_a}^f(\mu) & S_{vtm_a G_{sw}}^f(\mu) \\ S_{G_{sw} V_a}^f(\mu) & S_{G_{sw} V_m}^f(\mu) & S_{G_{sw} B_{eqz}}^f(\mu) & S_{G_{sw} B_{eqv}}^f(\mu) & S_{G_{sw} qtm_a}^f(\mu) & S_{G_{sw} vtm_a}^f(\mu) & S_{G_{sw} G_{sw}}^f(\mu) \end{bmatrix}$$

(281)

$$= \begin{bmatrix} S_{V_a V_a}^f(\mu) & S_{V_a V_m}^f(\mu) & S_{V_a B_{eq}}^f(\mu) & S_{V_a \theta_{sh}}^f(\mu) & S_{V_a G_{sw}}^f(\mu) \\ S_{V_m V_a}^f(\mu) & S_{V_m V_m}^f(\mu) & S_{V_m B_{eq}}^f(\mu) & S_{V_m \theta_{sh}}^f(\mu) & S_{V_m G_{sw}}^f(\mu) \\ S_{B_{eq} V_a}^f(\mu) & S_{B_{eq} V_m}^f(\mu) & S_{B_{eq} B_{eq}}^f(\mu) & S_{B_{eq} \theta_{sh}}^f(\mu) & S_{B_{eq} G_{sw}}^f(\mu) \\ S_{\theta_{sh} V_a}^f(\mu) & S_{\theta_{sh} V_m}^f(\mu) & S_{\theta_{sh} B_{eq}}^f(\mu) & S_{\theta_{sh} \theta_{sh}}^f(\mu) & S_{\theta_{sh} G_{sw}}^f(\mu) \\ S_{m_a V_a}^f(\mu) & S_{m_a V_m}^f(\mu) & S_{m_a B_{eq}}^f(\mu) & S_{m_a \theta_{sh}}^f(\mu) & S_{m_a G_{sw}}^f(\mu) \\ S_{G_{sw} V_a}^f(\mu) & S_{G_{sw} V_m}^f(\mu) & S_{G_{sw} B_{eq}}^f(\mu) & S_{G_{sw} \theta_{sh}}^f(\mu) & S_{G_{sw} G_{sw}}^f(\mu) \end{bmatrix} \quad (282)$$

### 3.3 Branch Flows Detailed Hessian

$$S_{V_a V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{V_a}^{f \top} \mu \right) \quad (283)$$

$$= \left( \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( Y_f \frac{\partial V}{\partial V_a} \right)^* + [C_f V] \left( Y_f \frac{\partial^2 V}{\partial^2 V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( Y_f \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial^2 V}{\partial^2 V_a} \right] (Y_f V)^* \right)^\top \lambda \quad (284)$$

$$S_{V_m V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{V_m}^{f \top} \mu \right) \quad (285)$$

$$= \left( \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( Y_f \frac{\partial V}{\partial V_m} \right)^* + [C_f V] \left( Y_f \frac{\partial^2 V}{\partial V_m \partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( Y_f \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial^2 V}{\partial V_m \partial V_a} \right] (Y_f V)^* \right)^\top \mu \quad (286)$$

$$S_{B_{eqz} V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{B_{eqz}}^{f \top} \mu \right) \quad (287)$$

$$= \frac{\partial}{\partial V_a} \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqz}} V \right)^* \right)^\top \mu \quad (288)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqz}} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (289)$$

$$S_{B_{eqv} V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{B_{eqv}}^{f \top} \mu \right) \quad (290)$$

$$= \frac{\partial}{\partial V_a} \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqv}} V \right)^* \right)^\top \mu \quad (291)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqv}} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (292)$$

$$S_{\theta_{sh} V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{\theta_{sh}}^{f \top} \mu \right) \quad (293)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial \theta_{sh}} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (294)$$

$$S_{qtm_a V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{qtm_a}^{f \top} \mu \right) \quad (295)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial qtm_a} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial qtm_a} V \right)^* \right)^\top (\mu) \quad (296)$$

$$S_{vtm_a V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{vtm_a}^{f \top} \mu \right) \quad (297)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial vtm_a} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial vtm_a} V \right)^* \right)^\top (\mu) \quad (298)$$

$$S_{G_{sw} V_a}^f(\mu) = \frac{\partial}{\partial V_a} \left( S_{G_{sw}}^{f \top} \mu \right) \quad (299)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial G_{sw}} \frac{\partial V}{\partial V_a} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_a} \right] \left( \frac{\partial Y_f}{\partial G_{sw}} V \right)^* \right)^\top (\mu) \quad (300)$$

$$S_{V_a V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{V_a}^{f \top} \mu \right) \quad (301)$$

$$= S_{V_a V_m}^f{}^\top(\mu) \quad (302)$$

$$S_{V_m V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{V_m}^f{}^\top \mu \right) \quad (303)$$

$$= \frac{\partial}{\partial V_m} \left( \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( Y_f \frac{\partial V}{\partial V_m} \right)^* + [C_f V] \left( Y_f \frac{\partial^2 V}{\partial^2 V_m} \right)^* + \right. \\ \left. [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( Y_f \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial^2 V}{\partial^2 V_m} \right] (Y_f V)^* \right)^\top \mu \quad (304)$$

$$S_{B_{eqz} V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (305)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqz}} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( \frac{\partial Y_f}{\partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (306)$$

$$S_{B_{eqv} V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (307)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial B_{eqv}} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( \frac{\partial Y_f}{\partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (308)$$

$$S_{\theta_{sh} V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (309)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial \theta_{sh}} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( \frac{\partial Y_f}{\partial \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (310)$$

$$S_{qtm_a V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (311)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial qtm_a} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \right)^\top (\mu) \left( \frac{\partial Y_f}{\partial qtm_a} V \right)^* \quad (312)$$

$$S_{vtm_a V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (313)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial vtm_a} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( \frac{\partial Y_f}{\partial vtm_a} V \right)^* \right)^\top (\mu) \quad (314)$$

$$S_{G_{sw} V_m}^f(\mu) = \frac{\partial}{\partial V_m} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (315)$$

$$= \left( [C_f V] \left( \frac{\partial Y_f}{\partial G_{sw}} \frac{\partial V}{\partial V_m} \right)^* + [C_f] \left[ C_f \frac{\partial V}{\partial V_m} \right] \left( \frac{\partial Y_f}{\partial G_{sw}} V \right)^* \right)^\top (\mu) \quad (316)$$

$$S_{V_a B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{V_a}^f{}^\top \mu \right) \quad (317)$$

$$= S_{B_{eqz} V_a}^f{}^\top(\mu) \quad (318)$$

$$S_{V_m B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{P_g}^f{}^\top \mu \right) \quad (319)$$

$$= S_{B_{eqz} V_m}^f{}^\top(\mu) \quad (320)$$

$$S_{B_{eqz} B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (321)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 B_{eqz}} V \right)^* \right)^\top (\mu) \quad (322)$$

$$S_{B_{eqv} B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (323)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial B_{eqv} \partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (324)$$

$$S_{\theta_{sh} B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (325)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial \theta_{sh} \partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (326)$$

$$S_{qtm_a B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (327)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial qtm_a \partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (328)$$

$$S_{vtm_a B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (329)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial vtm_a \partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (330)$$

$$S_{G_{sw} B_{eqz}}^f(\mu) = \frac{\partial}{\partial B_{eqz}} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (331)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial G_{sw} \partial B_{eqz}} V \right)^* \right)^\top (\mu) \quad (332)$$

$$S_{V_a B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{V_a}^f{}^\top \mu \right) \quad (333)$$

$$= S_{B_{eqv} V_a}^f{}^\top (\mu) \quad (334)$$

$$S_{V_m B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{P_g}^f{}^\top \mu \right) \quad (335)$$

$$= S_{B_{eqv} V_m}^f{}^\top (\mu) \quad (336)$$

$$S_{B_{eqz} B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (337)$$

$$= S_{B_{eqv} B_{eqz}}^f{}^\top (\mu) \quad (338)$$

$$S_{B_{eqv} B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (339)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 B_{eqv}} V \right)^* \right)^\top (\mu) \quad (340)$$

$$S_{\theta_{sh} B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (341)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial \theta_{sh} \partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (342)$$



$$S_{qtm_a B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (343)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{qtm_a \partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (344)$$

$$S_{vtm_a B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (345)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{vtm_a \partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (346)$$

$$S_{G_{sw} B_{eqv}}^f(\mu) = \frac{\partial}{\partial B_{eqv}} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (347)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{G_{sw} \partial B_{eqv}} V \right)^* \right)^\top (\mu) \quad (348)$$

$$S_{V_a \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{V_a}^f{}^\top \mu \right) \quad (349)$$

$$= S_{\theta_{sh} V_a}^f{}^\top (\mu) \quad (350)$$

$$S_{V_m \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{P_g}^f{}^\top \mu \right) \quad (351)$$

$$= S_{\theta_{sh} V_m}^f{}^\top (\mu) \quad (352)$$

$$S_{B_{eqz} \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (353)$$

$$= S_{\theta_{sh} B_{eqz}}^f{}^\top (\mu) \quad (354)$$

$$S_{B_{eqv} \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (355)$$

$$= S_{\theta_{sh} B_{eqv}}^f{}^\top (\mu) \quad (356)$$

$$S_{\theta_{sh} \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (357)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (358)$$

$$S_{qtm_a \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (359)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial qtm_a \partial \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (360)$$

$$S_{vtm_a \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (361)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial vtm_a \partial \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (362)$$

$$S_{G_{sw} \theta_{sh}}^f(\mu) = \frac{\partial}{\partial \theta_{sh}} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (363)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial G_{sw} \partial \theta_{sh}} V \right)^* \right)^\top (\mu) \quad (364)$$

$$S_{V_a qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{V_a}^{f \top} \mu \right) \quad (365)$$

$$= S_{qtm_a V_a}^{f \top}(\mu) \quad (366)$$

$$S_{V_m qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{P_g}^{f \top} \mu \right) \quad (367)$$

$$= S_{qtm_a V_m}^{f \top}(\mu) \quad (368)$$

$$S_{B_{eqz} qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{B_{eqz}}^{f \top} \mu \right) \quad (369)$$

$$= S_{qtm_a B_{eqz}}^{f \top}(\mu) \quad (370)$$

$$S_{B_{eqv} qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{B_{eqv}}^{f \top} \mu \right) \quad (371)$$

$$= S_{qtm_a B_{eqv}}^{f \top}(\mu) \quad (372)$$

$$S_{\theta_{sh} qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{\theta_{sh}}^{f \top} \mu \right) \quad (373)$$

$$= S_{qtm_a \theta_{sh}}^{f \top}(\mu) \quad (374)$$

$$S_{qtm_a qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{qtm_a}^{f \top} \mu \right) \quad (375)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 qtm_a} V \right)^* \right)^\top (\mu) \quad (376)$$

$$S_{vtm_a qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{vtm_a}^{f \top} \mu \right) \quad (377)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial vtm_a \partial qtm_a} V \right)^* \right)^\top (\mu) \quad (378)$$

$$S_{G_{sw} qtm_a}^f(\mu) = \frac{\partial}{\partial qtm_a} \left( S_{G_{sw}}^{f \top} \mu \right) \quad (379)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial G_{sw} \partial qtm_a} V \right)^* \right)^\top (\mu) \quad (380)$$

$$S_{V_a vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{V_a}^{f \top} \mu \right) \quad (381)$$

$$= S_{vtm_a V_a}^{f \top}(\mu) \quad (382)$$

$$S_{V_m vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{P_g}^{f \top} \mu \right) \quad (383)$$

$$= S_{vtm_a V_m}^{f \top}(\mu) \quad (384)$$

$$S_{B_{eqz}vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (385)$$

$$= S_{vtm_a B_{eqz}}^f{}^\top(\mu) \quad (386)$$

$$S_{B_{eqv}vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (387)$$

$$= S_{vtm_a B_{eqv}}^f{}^\top(\mu) \quad (388)$$

$$S_{\theta_{sh}vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (389)$$

$$= S_{vtm_a \theta_{sh}}^f{}^\top(\mu) \quad (390)$$

$$S_{qtm_a vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (391)$$

$$= S_{vtm_a qtm_a}^f{}^\top(\mu) \quad (392)$$

$$S_{vtm_a vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (393)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 vtm_a} V \right)^* \right)^\top(\mu) \quad (394)$$

$$S_{G_{sw} vtm_a}^f(\mu) = \frac{\partial}{\partial vtm_a} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (395)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial G_{sw} \partial vtm_a} V \right)^* \right)^\top(\mu) \quad (396)$$

$$S_{V_a G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{V_a}^f{}^\top \mu \right) \quad (397)$$

$$= S_{G_{sw} V_a}^f{}^\top(\mu) \quad (398)$$

$$S_{V_m G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{P_g}^f{}^\top \mu \right) \quad (399)$$

$$= S_{G_{sw} V_m}^f{}^\top(\mu) \quad (400)$$

$$S_{B_{eqz} G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{B_{eqz}}^f{}^\top \mu \right) \quad (401)$$

$$= S_{G_{sw} B_{eqz}}^f{}^\top(\mu) \quad (402)$$

$$S_{B_{eqv} G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{B_{eqv}}^f{}^\top \mu \right) \quad (403)$$

$$= S_{G_{sw} B_{eqv}}^f{}^\top(\mu) \quad (404)$$

$$S_{\theta_{sh} G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{\theta_{sh}}^f{}^\top \mu \right) \quad (405)$$

$$= S_{G_{sw}\theta_{sh}}^f{}^\top(\mu) \quad (406)$$

$$S_{qtm_a G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{qtm_a}^f{}^\top \mu \right) \quad (407)$$

$$= S_{vtm_a G_{sw}}^f{}^\top(\mu) \quad (408)$$

$$S_{vtm_a G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{vtm_a}^f{}^\top \mu \right) \quad (409)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial vtm_a \partial G_{sw}} V \right)^* \right)^\top(\mu) \quad (410)$$

$$S_{G_{sw} G_{sw}}^f(\mu) = \frac{\partial}{\partial G_{sw}} \left( S_{G_{sw}}^f{}^\top \mu \right) \quad (411)$$

$$= \left( [C_f V] \left( \frac{\partial^2 Y_f}{\partial^2 G_{sw}} V \right)^* \right)^\top(\mu) \quad (412)$$