Homework 9

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- Problem 1. $Q(x,y,z) = 2x^2+5y^2+11z^2+20xy-4xz+16yz$

a) matrix representation A $A = \begin{bmatrix} 2 & 10 & -2 \\ 10 & 5 & 8 \\ -2 & 8 & 11 \end{bmatrix}$

b) $eig(A) = [-9, 9, 18]$

c) $Q(x,y,z)$ is not positive definite because not all eigenvalues are more than 0.

- Problem 2. $f(x,y) = x^2+2y^2$, $g(x,y) = x^2-2y^2$

b) $f' = 2x+4y$, $H_f = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

c) H_f pos def? $H_f - \lambda = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 \\ 0 & 4-\lambda \end{bmatrix} \rightarrow \lambda^2-6\lambda+8 = (\lambda-4)(\lambda-2)$, $\lambda=4$, $\lambda=2$

H_f is positive definite

e) $g' = 2x-4y$, $H_g = \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix}$

f) H_g pos def? $H_g - \lambda = \begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 \\ 0 & -4-\lambda \end{bmatrix} \rightarrow \lambda^2+2\lambda-8 = (\lambda+4)(\lambda-2)$, $\lambda=-4$, $\lambda=2$

H_g is not positive definite

- Problem 3. Get min value

a) $f(x) = e^{-(x+1)^2} \rightarrow f'(x) = -2xe^{-(x+1)^2} - 2e^{-(x+1)^2} \rightarrow (-2x-2)e^{-x^2-2x-1} = 0$

$\rightarrow e^{-x^2-2x+1} = 0$ (no solution) $\rightarrow -2x-2=0 \rightarrow -2x = 2$ $x = -1$

b) $g(x) = \log(-(x+1)^2) \rightarrow g'(x) = \frac{1}{-2x-x^2-1} \frac{d}{dx}(-2x-2x) \rightarrow \frac{1}{-x^2-2x-1}(-2x-2) = \frac{2}{x+1}$

$\frac{2}{x+1} = 0 \rightarrow$ no solution

c) $h(x) = (x-2)^4+(x-1)^2$ $h'(x) = 4(x-2)^3+2(x-1) \rightarrow 4x^3-24x^2+50x-34=0$

$x = 1.41$

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Problem 6. $f(x,y) = x^2+2y^2$ constraint to $g(x,y) = x-2y+1=0$

a) write Lagrangian L

$\rightarrow L(x,y,\lambda) = f(x,y) + \lambda g(x,y) = x^2+2y^2+\lambda(x-2y+1)$

c) equation for $\nabla L = 0$

$\rightarrow \frac{\partial L}{\partial x} = 2x+\lambda$ $\rightarrow \frac{\partial L}{\partial y} = 4y-2\lambda$ $\rightarrow \frac{\partial L}{\partial \lambda} = x-2y+1$

$$\rightarrow \begin{cases} 2x+\lambda=0 \\ 4y-2\lambda=0 \\ x-2y+1=0 \end{cases}$$

d) Solve equations

$\rightarrow 2x+\lambda=0 \rightarrow 2x=-\lambda \rightarrow x=-\lambda/2$

$\rightarrow 4y-2\lambda=0 \rightarrow 4y=2\lambda \rightarrow y=\lambda/2$

$\rightarrow x-2y+1=0 \rightarrow x=2y-1$

$\rightarrow 2x+\lambda=0 \rightarrow \lambda=-2x$

$\rightarrow 4y-2\lambda=0 \rightarrow \lambda=2y$