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Problem 1.

1. M_1 - expand x by a factor of 1.5

$\rightarrow A' = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.5\cdot 0 + 0\cdot 0 \\ 0\cdot 0 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$\rightarrow B' = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5\cdot 0 + 0\cdot 1 \\ 0\cdot 0 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ✓

$\rightarrow C' = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5\cdot 1 + 0\cdot 1 \\ 0\cdot 1 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$ ✓

$\rightarrow D' = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.5\cdot 1 + 0\cdot 0 \\ 0\cdot 1 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$ ✓

2. M_2 - contract y by a factor of 0.75

$\rightarrow A' = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 0\cdot 0 \\ 0\cdot 0 + 0\cdot 0.75 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$\rightarrow B' = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 0\cdot 1 \\ 0\cdot 0 + 0.75\cdot 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}$ ✓

$\rightarrow C' = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot 1 \\ 0\cdot 1 + 0.75\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.75 \end{bmatrix}$ ✓

$\rightarrow D' = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot 0 \\ 0\cdot 1 + 0.75\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

3. M_3 - shear along x by a factor of 2

$\rightarrow A' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 1\cdot 0 \\ 0\cdot 0 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$\rightarrow B' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 1\cdot 1 \\ 0\cdot 0 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓

$\rightarrow C' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 1\cdot 1 \\ 0\cdot 1 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ✓

$\rightarrow D' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 1\cdot 0 \\ 0\cdot 1 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

3. M_3 - shear along x by a factor of 2

$\rightarrow A' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 2\cdot 0 \\ 0\cdot 0 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$\rightarrow B' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 2\cdot 1 \\ 0\cdot 0 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ✓

$\rightarrow C' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 1 \\ 0\cdot 1 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ ✓

$\rightarrow D' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 0 \\ 0\cdot 1 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓

4. M_4 - shear along y by a factor of 1.25

$\rightarrow A' = \begin{bmatrix} 1 & 0 \\ 1.25 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 0\cdot 0 \\ 1.25\cdot 0 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ✓

$\rightarrow B' = \begin{bmatrix} 1 & 0 \\ 1.25 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 0 + 0\cdot 1 \\ 1.25\cdot 0 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ✓

$\rightarrow C' = \begin{bmatrix} 1 & 0 \\ 1.25 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot 1 \\ 1.25\cdot 1 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2.25 \end{bmatrix}$ ✓

$\rightarrow D' = \begin{bmatrix} 1 & 0 \\ 1.25 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 0\cdot 0 \\ 1.25\cdot 1 + 1\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25 \end{bmatrix}$ ✓

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5. M_5 - counterclockwise rotation by $\pi/4$

$$\rightarrow A' = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4}\cdot 0 & -\sin\frac{\pi}{4}\cdot 0 \\ \sin\frac{\pi}{4}\cdot 0 + \cos\frac{1}{4}\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightarrow B' = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4}\cdot 0 & -\sin\frac{1}{4}\cdot 1 \\ \sin\frac{\pi}{4}\cdot 0 + \cos\frac{\pi}{4}\cdot 1 \end{bmatrix} = \begin{bmatrix} -\sin\frac{\pi}{4} \\ \cos\frac{\pi}{4} \end{bmatrix}$$

$$\rightarrow C' = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} - \sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} + \cos\frac{\pi}{4} \end{bmatrix}$$

$$\rightarrow D' = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4}\cdot 1 - \sin\frac{\pi}{4}\cdot 0 \\ \sin\frac{\pi}{4}\cdot 1 + \cos\frac{\pi}{4}\cdot 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{bmatrix}$$

6. M_6 - clockwise rotation by $\pi/6$

$$\rightarrow A' = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\pi/6\cdot 0 + \sin\pi/6\cdot 0 \\ -\sin\pi/6\cdot 0 + \cos\pi/6\cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\rightarrow B' = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\pi/6\cdot 0 + \sin\pi/6\cdot 1 \\ -\sin\pi/6\cdot 0 + \cos\pi/6\cdot 1 \end{bmatrix} = \begin{bmatrix} \sin\pi/6 \\ \cos\pi/6 \end{bmatrix}$$

$$\rightarrow C' = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\pi/6\cdot 1 + \sin\pi/6\cdot 1 \\ -\sin\pi/6\cdot 1 + \cos\pi/6\cdot 1 \end{bmatrix} = \begin{bmatrix} \cos\pi/6 + \sin\pi/6 \\ -\sin\pi/6 + \cos\pi/6 \end{bmatrix}$$

$$\rightarrow D' = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\pi/6\cdot 1 + \sin\pi/6\cdot 0 \\ -\sin\pi/6\cdot 1 + \cos\pi/6\cdot 0 \end{bmatrix} = \begin{bmatrix} \cos\pi/6 \\ -\sin\pi/6 \end{bmatrix}$$

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Problem 2. Inverse for each matrix $m^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

1. $M_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow M_1^{-1} = \frac{1}{1.5-0}\begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix} = \begin{bmatrix} 2/3 & 0 \\ 0 & 1 \end{bmatrix}$ ⇒A

2. $M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.75 \end{bmatrix} \rightarrow M_2^{-1} = \frac{1}{0.75-0}\begin{bmatrix} 0.75 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4/3 \end{bmatrix}$ ⇒A

3. $M_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \rightarrow M_3^{-1} = \frac{1}{1-0}\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ ⇒A

4. $M_4 = \begin{bmatrix} 1 & 0 \\ 1.25 & 1 \end{bmatrix} \rightarrow M_4^{-1} = \frac{1}{1-0}\begin{bmatrix} 1 & 0 \\ -1.25 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1.25 & 1 \end{bmatrix}$ ⇒A

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Continue Problem 2

5. $M_5 = \begin{bmatrix} \cos\pi/4 & -\sin\pi/4 \\ \sin\pi/4 & \cos\pi/4 \end{bmatrix} \rightarrow M_5^{-1} = \frac{1}{\cos\pi/4 \cdot \cos\pi/4 - (-\sin\pi/4 \cdot \sin\pi/4)} \begin{bmatrix} \cos\pi/4 & \sin\pi/4 \\ -\sin\pi/4 & \cos\pi/4 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} \cos\pi/4 & \sin\pi/4 \\ -\sin\pi/4 & \cos\pi/4 \end{bmatrix}$

$\rightarrow M_5^{-1} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} \end{bmatrix}$

6. $M_6 = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix} \rightarrow M_6^{-1} = \frac{1}{(\cos\pi/6 \cdot \cos\pi/6) - (\sin\pi/6 \cdot -\sin\pi/6)} \begin{bmatrix} \cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6 \end{bmatrix} = M_6^{-1} = \frac{1}{\frac{3}{4} - (-\frac{1}{4})} \begin{bmatrix} \cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6 \end{bmatrix}$

$\rightarrow M_6^{-1} = \begin{bmatrix} \cos\pi/6 & -\sin\pi/6 \\ \sin\pi/6 & \cos\pi/6 \end{bmatrix} \rightarrow \begin{bmatrix} \sqrt{3}/2 & -0.5 \\ 0.5 & \sqrt{3}/2 \end{bmatrix}$

Problem 3 Apply transformations

1 $(M_1^2)(M_3^3)M_5$

$\rightarrow M_1^2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.25 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow M_3^3 = M_3^2 M \rightarrow M_3^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$\rightarrow M_3^3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \rightarrow (M_1^2)(M_3^3) = \begin{bmatrix} 2.25 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2.25 & 13.5 \\ 0 & 1 \end{bmatrix}$

$\rightarrow (M_1^2)(M_3^3)M_5 = \begin{bmatrix} 2.25 & 13.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\pi/4 & -\sin\pi/4 \\ \sin\pi/4 & \cos\pi/4 \end{bmatrix} = \begin{bmatrix} 2.25\cos\pi/4 + 13.5\sin\pi/4 & -2.25\sin\pi/4 + 13.5\cos\pi/4 \\ 0 + \sin\pi/4 & 0 \times \cos\pi/4 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 11.13569318 & 7.95495129 \\ 0.70710678 & 0.70710678 \end{bmatrix} \approx \begin{bmatrix} \frac{63\sqrt{2}}{8} & \frac{45\sqrt{2}}{8} \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

$\rightarrow A^1 = \begin{bmatrix} 63\sqrt{2}/8 & 45\sqrt{2}/8 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\rightarrow B^1 = \begin{bmatrix} 63\sqrt{2}/8 & 45\sqrt{2}/8 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 45\sqrt{2}/8 \\ \sqrt{2}/2 \end{bmatrix}$

$\rightarrow C^1 = \begin{bmatrix} 63\sqrt{2}/8 & 45\sqrt{2}/8 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 63\sqrt{2}/8 + 45\sqrt{2}/8 \\ \sqrt{2}/2 + \sqrt{2}/2 \end{bmatrix} = \begin{bmatrix} 108\sqrt{2}/8 \\ \sqrt{2} \end{bmatrix} = \begin{bmatrix} 27/\sqrt{2} \\ \sqrt{2} \end{bmatrix}$

$\rightarrow D^1 = \begin{bmatrix} 63\sqrt{2}/8 & 45\sqrt{2}/8 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 63\sqrt{2}/8 \\ \sqrt{2}/2 \end{bmatrix}$

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Continue Problem 3

2. $(M_2^{-1})(M_4^{-2})M_6$

→ $M_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 4/3 \end{bmatrix}$ → $M_4^{-2} = M_4^{-1} M_4^{-1} = \begin{bmatrix} 1 & 0 \\ -1.25 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1.25 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2.5 & 1 \end{bmatrix}$

→ $(M_2^{-1})(M_4^{-2}) = \begin{bmatrix} 1 & 0 \\ 0 & 4/3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2.5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3\frac{1}{3} & 4/3 \end{bmatrix}$

→ $(M_2^{-1})(M_4^{-2})(M_6) = \begin{bmatrix} 1 & 0 \\ -3\frac{1}{3} & \frac{4}{3} \end{bmatrix}\begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -\sin\pi/6 & \cos\pi/6 \end{bmatrix} = \begin{bmatrix} \cos\pi/6 & \sin\pi/6 \\ -3\frac{1}{3}\cos\pi/6 + \frac{4}{3}(-\sin\pi/6) & -3\frac{1}{3}\sin\pi/6 + \frac{4}{3}\cos\pi/6 \end{bmatrix}$

→ $\begin{bmatrix} \cos\pi/6 & 0.5 \\ -3.553431801 & -0.51196613 \end{bmatrix}$

→ $A' = \begin{bmatrix} \cos\pi/6 & 0.5 \\ -3.553431801 & -0.51196613 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ → $B' = \begin{bmatrix} \cos\pi/6 & 0.5 \\ -3.553431801 & -0.51196613 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.51196613 \end{bmatrix}$

→ $C' = \begin{bmatrix} \cos\pi/6 & 0.5 \\ -3.553431801 & -0.51196613 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\pi/6 + 0.5 \\ -3.553431801 - 0.51196613 \end{bmatrix} = \begin{bmatrix} 1.3660254 \\ -4.065538414 \end{bmatrix}$

→ $D' = \begin{bmatrix} \cos\pi/6 & 0.5 \\ -3.553431801 & -0.51196613 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\pi/6 \\ -3.553431801 \end{bmatrix}$

Problem 4. Invert matrices with Gauss-Jordan method

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ → $[A\ I] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{bmatrix}$ → Row[2] = Row[2] − 3Row[1]

→ Row[2] = $\begin{bmatrix} 3-3 & 4-6 & 0-3 & 1-0 \end{bmatrix}$ → $[A\ I] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 0 \end{bmatrix}$ → Row[2] = Row[2] / −2

→ Row[2] = $\begin{bmatrix} 0/-2 & -2/-2 & -3/-2 & 0/-2 \end{bmatrix}$ → $[A\ I] = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -0.5 \end{bmatrix}$

→ Row[1] = Row[1] − 2 Row[2] → Row[1] = $\begin{bmatrix} 1-0 & 2-2 & 1-3 & 0+1 \end{bmatrix}$ → Row[1] = $\begin{bmatrix} 1 & 0 & -2 & 1 \end{bmatrix}$

→ $[A\ I] = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1.5 & -0.5 \end{bmatrix}$ → [redacted]

→ $A' = \begin{bmatrix} -2 & 1 \\ 1.5 & 0.5 \end{bmatrix}$

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Continue Problem 4.

2. $B=\begin{bmatrix}5 & 9\\ 2 & 5\end{bmatrix}$ → $[BI]=\begin{bmatrix}5 & 9 & 1 & 0\\ 2 & 5 & 0 & 1\end{bmatrix}$ → Row[1]= Row[1]/5

$\frac{2}{5}\div\frac{7}{5}\quad\frac{10}{35}$ 2/7

→ Row[1]=$\begin{bmatrix}5/5 & 9/5 & 1/5 & 0/5\end{bmatrix}$ → $[BI]=\begin{bmatrix}1 & 9/5 & 1/5 & 0\\ 2 & 5 & 0 & 1\end{bmatrix}$ → Row[2]= Row[2] - 2 Row[1]

$\frac{5}{5}\div\frac{7}{5}=\frac{25}{35}$ 5/7

→ Row[2]=$\begin{bmatrix}2-2 & 5-18/5 & 0-2/5 & 1-0\end{bmatrix}$ → $[BI]=\begin{bmatrix}1 & 9/5 & 1/5 & 0\\ 0 & +1.4 & -2/5 & 1\end{bmatrix}$ → Row[2]= Row[2]/+1.4

$\frac{9}{5}\cdot\frac{7}{7}=\frac{18}{35}$

→ Row[2]=$\begin{bmatrix}0/1.4 & +1.4/+1.4 & -2/5/+1.4 & 1/1.4\end{bmatrix}$ → $[BI]=\begin{bmatrix}1 & 9/5 & 1/5 & 0\\ 0 & 1 & -2/7 & +5/7\end{bmatrix}$

$\frac{9}{5}\quad\frac{5}{7}\quad\frac{45}{35}\quad\frac{9}{7}$

→ Row[1] = Row[1] - 9/5 Row[2] → Row[1]=$\begin{bmatrix}1-0 & \frac{9}{5}-\frac{9}{5} & \frac{1}{5}+\frac{18}{35} & 0-\frac{9}{7}\end{bmatrix}$

$\frac{7}{35}+\frac{18}{35}=\frac{25}{35}$ 5/7

→ $[BI]=\begin{bmatrix}1 & 0 & 5/7 & -9/7\\ 0 & 1 & -2/7 & 5/7\end{bmatrix}$ $B'=\begin{bmatrix}5/7 & -9/7\\ -2/7 & 5/7\end{bmatrix}$ ✓

3. $C=\begin{bmatrix}2 & 1 & 0\\ 3 & 6 & 1\\ 5 & 7 & 1\end{bmatrix}$ → $[CI]=\begin{bmatrix}2 & 1 & 0 & 1 & 0 & 0\\ 3 & 6 & 1 & 0 & 1 & 0\\ 5 & 7 & 1 & 0 & 0 & 1\end{bmatrix}$ → Row[1] = Row[1]/2

$\frac{2}{2}\cdot\frac{9}{2}\quad\frac{4}{19}=\frac{2}{9}$

→ Row[1]=$\begin{bmatrix}2/2 & 1/2 & 0/2 & 1/2 & 0/2 & 0/2\end{bmatrix}$ → $[CI]=\begin{bmatrix}1 & 0.5 & 0 & 0.5 & 0 & 0\\ 3 & 6 & 1 & 0 & 1 & 0\\ 5 & 7 & 1 & 0 & 0 & 1\end{bmatrix}$

$\frac{9}{2}\cdot\frac{2}{9}=\frac{18}{18}$

→ Row[2] = Row[2] - 3 Row[1] → Row[2]=$\begin{bmatrix}3-3 & 6-1.5 & 1-0 & 0-0.5 & 1-0 & 0-0\end{bmatrix}=\begin{bmatrix}0 & 4.5 & 1 & -1.5 & 1 & 0\end{bmatrix}$

$\frac{9}{2}\cdot\frac{9}{6}=\frac{3}{2}$

→ Row[3] = Row[3] - 5 Row[1] → Row[3]=$\begin{bmatrix}5-5 & 7-2.5 & 1-0 & 0-2.5 & 0-0 & 1-0\end{bmatrix}=\begin{bmatrix}0 & 4.5 & 1 & -2.5 & 0 & 1\end{bmatrix}$

→ $[CI]=\begin{bmatrix}1 & 0.5 & 0 & 0.5 & 0 & 0\\ 0 & 4.5 & 1 & -1.5 & 1 & 0\\ 0 & 4.5 & 1 & -2.5 & 0 & 1\end{bmatrix}$ → Row[2]/4.5 =$\begin{bmatrix}0 & 4.5/4.5 & 1/4.5 & -1.5/4.5 & 1/4.5 & 0\end{bmatrix}$

→ $[CI]=\begin{bmatrix}1 & 0.5 & 0 & 0.5 & 0 & 0\\ 0 & 1 & 2/9 & -1/3 & 2/9 & 0\\ 0 & 4.5 & 1 & -2.5 & 0 & 1\end{bmatrix}$ → [illegible]

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Continue Problem 4

3. $[C1] = \begin{bmatrix} 1 & 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & 2/9 & -1/3 & 2/9 & 0 \\ 0 & 4.5 & 1 & -2.5 & 0 & 1 \end{bmatrix}$ → Row[3] = Row[3] − 0.5 Row[2]

→ Row[3] = [1 0.5−0.5 0−1/9 0.5+1/6 0−1/9 0] = [1 0 −1/9 2/3 −1/9 0]

$[C1] = \begin{bmatrix} 1 & 0 & -1/9 & 2/3 & -1/9 & 0 \\ 0 & 1 & 2/9 & -1/3 & 2/9 & 0 \\ 0 & 4.5 & 1 & -2.5 & 0 & 1 \end{bmatrix}$ → Row[3] = Row[3] − 4.5 Row[2]

→ Row[3] = [0 4.5−4.5 1−1 −2.5+1.5 0−1 1]

→ Row[3] = [0 0 0 −1 −1 1]

→ $[C1] = \begin{bmatrix} 1 & 0 & -1/9 & 2/3 & -1/9 & 0 \\ 0 & 1 & 2/9 & -1/3 & 2/9 & 0 \\ 0 & 0 & 0 & -1.0 & -1 & 1 \end{bmatrix}$ zero row (matrix is not invertible)

Problem 5. Logarithm function (3 terms)

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 9 \\ 2 & 6 \end{bmatrix}$

1. Log(A) → $(A-I) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix}$ → $(A-I)^2 = \begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 9 & 15 \end{bmatrix}$

→ $\frac{(A-I)^2}{2} = \begin{bmatrix} 6/2 & 6/2 \\ 9/2 & 15/2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 4.5 & 7.5 \end{bmatrix}$ → $(A-I)^3 = \begin{bmatrix} 6 & 6 \\ 9 & 15 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 30 \\ 45 & 63 \end{bmatrix}$

→ $\frac{(A-I)^3}{3} = \begin{bmatrix} 18/3 & 30/3 \\ 45/3 & 63/3 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 15 & 21 \end{bmatrix}$ → $(A-I) - \frac{(A-I)^2}{2} = \begin{bmatrix} 0 & 2 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 4.5 & 7.5 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -1.5 & -4.5 \end{bmatrix}$

→ $(A-I) - \frac{(A-I)^2}{2} + \frac{(A-I)^3}{3} = \begin{bmatrix} -3 & -1 \\ -1.5 & -4.5 \end{bmatrix} + \begin{bmatrix} 6 & 10 \\ 15 & 21 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 13.5 & 16.5 \end{bmatrix}$

2. Log(A') = $A' = \frac{1}{4-6}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$ → $(A'-I) = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1.5 & -1.5 \end{bmatrix}$

→ $(A'-I)^2 = \begin{bmatrix} -3 & 1 \\ 1.5 & -1.5 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 1.5 & -1.5 \end{bmatrix} = \begin{bmatrix} 10.5 & -4.5 \\ -6.75 & 3.25 \end{bmatrix}$ → $\frac{(A'-I)^2}{2} = \begin{bmatrix} 10.5/2 & -4.5/2 \\ -6.75/2 & 3.25/2 \end{bmatrix} = \begin{bmatrix} 5.25 & -2.25 \\ -3.375 & 1.875 \end{bmatrix}$

→ $(A'-I)^3 = \begin{bmatrix} 10.5 & -4.5 \\ -6.75 & 3.25 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 1.5 & -1.5 \end{bmatrix} = \begin{bmatrix} -38.25 & 17.25 \\ 25.875 & -12.375 \end{bmatrix}$ → $\frac{(A'-I)^3}{3} = \begin{bmatrix} -38.25/3 & 17.25/3 \\ 25.875/3 & -12.375/3 \end{bmatrix} = \begin{bmatrix} -12.75 & 5.75 \\ 8.625 & -4.125 \end{bmatrix}$

→ $(A'-I) - \frac{(A'-I)^2}{2} = \begin{bmatrix} -3 & 1 \\ 1.5 & -1.5 \end{bmatrix} - \begin{bmatrix} 5.25 & -2.25 \\ -3.375 & 1.875 \end{bmatrix} = \begin{bmatrix} -8.25 & 3.25 \\ 4.875 & -3.375 \end{bmatrix}$

→ $(A'-I) - \frac{(A'-I)^2}{2} + \frac{(A'-I)^3}{3} = \begin{bmatrix} -8.25 & 3.25 \\ 4.875 & -3.375 \end{bmatrix} + \begin{bmatrix} -12.75 & 5.75 \\ 8.625 & -4.125 \end{bmatrix} = \begin{bmatrix} -21 & 9 \\ 13.5 & -7.5 \end{bmatrix}$

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Continue Problem 5

3. Log (B) $\rightarrow (B-I)=\begin{bmatrix}5&9\\2&5\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}4&9\\2&4\end{bmatrix} \rightarrow (B-I)^2=\begin{bmatrix}4&9\\2&4\end{bmatrix}\begin{bmatrix}4&9\\2&4\end{bmatrix}=\begin{bmatrix}34&72\\16&34\end{bmatrix}$

$\rightarrow \frac{(B-I)^2}{2}=\begin{bmatrix}34/2&72/2\\16/2&34/2\end{bmatrix}=\begin{bmatrix}17&36\\8&17\end{bmatrix} \rightarrow (B-I)^3=\begin{bmatrix}34&72\\16&34\end{bmatrix}\begin{bmatrix}4&9\\2&4\end{bmatrix}=\begin{bmatrix}280&594\\132&280\end{bmatrix}$

$\rightarrow \frac{(B-I)^3}{3}=\begin{bmatrix}280/3&594/3\\132/3&280/3\end{bmatrix}=\begin{bmatrix}93\frac{1}{3}&198\\44&93\frac{1}{3}\end{bmatrix} \rightarrow (B-I)-\frac{(B-I)^2}{2}=\begin{bmatrix}4&9\\2&4\end{bmatrix}-\begin{bmatrix}17&36\\8&17\end{bmatrix}=\begin{bmatrix}-13&-27\\-6&-13\end{bmatrix}$

$\rightarrow (B-I)-\frac{(B-I)^2}{2}+\frac{(B-I)^3}{3}=\begin{bmatrix}-13&-27\\-6&-13\end{bmatrix}+\begin{bmatrix}93\frac{1}{3}&198\\44&93\frac{1}{3}\end{bmatrix}-\begin{bmatrix}80\frac{1}{3}&171\\38&80\frac{1}{3}\end{bmatrix}$

4. Log (B') $\rightarrow B'=\frac{1}{25-18}\begin{bmatrix}5&-9\\2&5\end{bmatrix}=\begin{bmatrix}5/7&-9/7\\2/7&5/7\end{bmatrix} \rightarrow (B'-I)=\begin{bmatrix}5/7&-9/7\\2/7&5/7\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}-2/7&-9/7\\2/7&-2/7\end{bmatrix}$

$\rightarrow (B'-I)^2=\begin{bmatrix}-2/7&-9/7\\2/7&-2/7\end{bmatrix}\begin{bmatrix}-2/7&-9/7\\2/7&-2/7\end{bmatrix}=\begin{bmatrix}22/49&36/49\\8/49&22/49\end{bmatrix} \rightarrow \frac{(B'-I)^2}{2}=\begin{bmatrix}11/49&18/49\\4/49&11/49\end{bmatrix}$

$\rightarrow (B'-I)^3=\begin{bmatrix}22/49&36/49\\8/49&22/49\end{bmatrix}\begin{bmatrix}-2/7&-9/7\\2/7&-2/7\end{bmatrix}=\begin{bmatrix}-116/343&-270/343\\-60/343&-116/343\end{bmatrix}$

$\rightarrow \frac{(B'-I)^3}{3}=\begin{bmatrix}-116/1029&-90/343\\-20/343&-116/1029\end{bmatrix} \rightarrow (B'-I)-\frac{(B'-I)^2}{2}=\begin{bmatrix}-2/7&-9/7\\2/7&-2/7\end{bmatrix}-\begin{bmatrix}11/49&18/49\\4/49&11/49\end{bmatrix}$

$\rightarrow (B'-I)-\frac{(B'-I)^2}{2}=\begin{bmatrix}-25/49&-81/49\\-18/49&-25/49\end{bmatrix}$

$\rightarrow (B'-I)-\frac{(B'-I)^2}{2}+\frac{(B'-I)^3}{3}=\begin{bmatrix}-25/49&-81/49\\-18/49&-25/49\end{bmatrix}+\begin{bmatrix}-116/1029&-90/343\\-20/343&-116/1029\end{bmatrix}$

$\rightarrow \begin{bmatrix}-641/1029&-657/343\\-146/343&-641/1029\end{bmatrix}$

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Problem 6. Systems of equations with Gaussian Elimination

1. $\begin{cases} 2x+3y=8 \\ 2x-y=0 \end{cases}$ $m=\begin{bmatrix} 2 & 3 & 8 \\ 2 & -1 & 0 \end{bmatrix}\begin{matrix}(L_1)\\(L_2)\end{matrix}$ $L_2 \to L_2 - 2L_1$ $L_2=[2-2 \quad -1-6 \mid -16]$

$\to m=\begin{bmatrix} 2 & 3 & 8 \\ 0 & -7 & -16 \end{bmatrix}$ $L_1 \to L_1/2 = L_1=[1 \quad 3/2 \quad 4]$ $m=\begin{bmatrix} 1 & 3/2 & 4 \\ 0 & -7 & -16 \end{bmatrix}$

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Problem 6. Systems of equations with Gaussian Elimination

1. $\begin{cases} 2x+3y=8 \\ 2x-y=0 \end{cases}$ $m=\begin{bmatrix} 2 & 3 & 8 \\ 2 & -1 & 0 \end{bmatrix}\begin{matrix}(L_1)\\(L_2)\end{matrix}$ $L_2 \to L_2/2 = [1 \quad 3/2 \quad 4]$

$m=\begin{bmatrix} 1 & 3/2 & 4 \\ 2 & -1 & 0 \end{bmatrix} \to L_2 \to L_2 - 2L_1 = [0 \quad -4 \quad -8]$ $m=\begin{bmatrix} 1 & 3/2 & 4 \\ 0 & -4 & -8 \end{bmatrix}$

$\to L_2 \to L_2/-4$ $=[0 \quad 1 \quad 2]$ $\to m=\begin{bmatrix} 1 & 3/2 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $\to L_1 \to L_1 - \frac{3}{2}L_2$

$\to L_1=[1 \quad 0 \quad 1]$ $m=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ $x=1, \; y=2$

2. $\begin{cases} x+2y=-2 \\ 2x+y=2 \end{cases}$ $m=\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}\begin{matrix}(L_1)\\(L_2)\end{matrix}$ $L_2 \to L_2 - 2L_1 = [0 \quad -3 \quad 6]$

$m=\begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & -6 \end{bmatrix} \to L_2 \to L_2/-3 = [0 \quad 1 \quad -2]$ $m=\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \end{bmatrix}$

$\to L_1 \to L_1 - 2L_2 = [1 \quad 0 \quad 2]$ $m=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \end{bmatrix}$ $x=2, \; y=-2$

3. $\begin{cases} 2x+3y+4z=4 \\ x-y+z=1 \\ 2x-y+z=1 \end{cases}$ $m=\begin{bmatrix} 2 & 3 & 4 & 4 \\ 1 & -1 & 1 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}\begin{matrix}(L_1)\\(L_2)\\(L_3)\end{matrix}$ $\to L_1/2 = [1 \quad 3/2 \quad 2 \quad 2]$ $m=\begin{bmatrix} 1 & 3/2 & 2 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$

$\to L_2 = L_2 - L_1 = [0 \quad -5/2 \quad -1 \quad -1]$ $m=\begin{bmatrix} 1 & 3/2 & 2 & 2 \\ 0 & -5/2 & -1 & -1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$ $\to L_3 = L_3 - 2L_1 = [0 \quad -4 \quad -3 \quad -3]$

$\to m=\begin{bmatrix} 1 & 3/2 & 2 & 2 \\ 0 & -5/2 & -1 & -1 \\ 0 & -4 & -3 & -3 \end{bmatrix} \to L_2/-2.5 = [0 \quad 1 \quad 2/5 \quad 2/5]$ $m=\begin{bmatrix} 1 & 3/2 & 2 & 2 \\ 0 & 1 & 2/5 & 2/5 \\ 0 & -4 & -3 & -3 \end{bmatrix}$ $\to L_1 = L_1 - \frac{3}{2}L_2$

$\to L_1=[1 \quad 0 \quad 7/5 \quad 7/5]$ $m=\begin{bmatrix} 1 & 0 & 7/5 & 7/5 \\ 0 & 1 & 2/5 & 2/5 \\ 0 & -4 & -3 & -3 \end{bmatrix}$ $L_3 = L_3 - 4L_2$ $\to [0 \quad 0 \quad -7/5 \quad -7/5]$ $m=\begin{bmatrix} 1 & 0 & 7/5 & 7/5 \\ 0 & 1 & 2/5 & 2/5 \\ 0 & 0 & -7/5 & -7/5 \end{bmatrix}$

$\to L_3 = L_3/-7/5 = [0 \quad 0 \quad 1 \quad 1]$ $m=\begin{bmatrix} 1 & 0 & 7/5 & 7/5 \\ 0 & 1 & 2/5 & 2/5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $\to L_1 = L_1 - 7/5 L_3 = [1 \quad 0 \quad 0 \quad 0]$

$\to m=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/5 & 2/5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \to L_2 = L_2 - \frac{2}{5}L_3 = [0 \quad 1 \quad 0 \quad 0]$ $m=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $x=0, \; y=0, \; z=1$

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Continue Problem 6.

4. $\begin{cases} x+y+z=1 \\ 2x+y+z=2 \\ x+2y+2z=1 \end{cases}$ $m=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \end{array}\right]$ $\rightarrow L_2 = L_2 - 2L_1 \rightarrow L_2 = [0 \ -1 \ -1 \ 0]$

$\rightarrow$ ~~[0 -1 -1 0]~~ $\rightarrow m=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 1 & 2 & 2 & 1 \end{array}\right]$ $\rightarrow L_3 = L_3 - L_1 = [0 \ 1 \ 1 \ 0]$ $m=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$

$\rightarrow L_2/_{-1} = [0 \ 1 \ 1 \ 0]$ $m=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$ $\rightarrow L_1 = L_1 - L_2 = [1 \ 0 \ 0 \ 1] \rightarrow L_3 = L_3 - L_2 = [0 \ 0 \ 0 \ 0]$

$m=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ zero row (no solution with Gauss method)