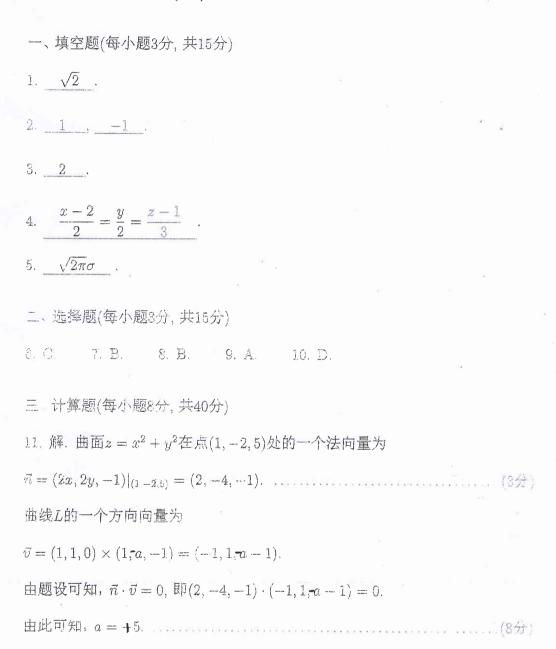
安徽大学2017-2018学年第二学期 《高等数学A(二)》期中考试参考答案与评分标准



15. 解. 由积分区域的对称性和被积函数的奇偶性,

四、应用题(每小题10分,共20分)

$$\begin{cases} L_x = 2x - 4\lambda = 0, \\ L_y = \frac{\sqrt{3}}{2}y - 3\lambda, \\ L_\lambda = -(4x + 3y - 2) = 0. \end{cases}$$

解得
$$x_0 = \frac{2}{4+3\sqrt{3}}, y_0 = \frac{2\sqrt{3}}{4+3\sqrt{3}}. f(x_0, y_0) = \frac{1}{4+3\sqrt{3}}.$$
 (**9**分)
当 $x = 0, y = \frac{2}{3}$ 时, $f(0, \frac{2}{3}) = \frac{\sqrt{3}}{9} > f(x_0, y_0);$
当 $x = \frac{1}{2}, y = 0$ 时, $f(\frac{1}{2}, 0) = \frac{1}{4} > f(x_0, y_0).$
故两个图形的面积之和存在最小值,且最小值为 $\frac{1}{4+3\sqrt{3}}$. (10分)

17. 解: 区域
$$D$$
的面积为 $S = \iint dxdy$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -\frac{v}{u^2} & \frac{1}{u} \\ -\frac{v^2}{u^2} & \frac{2v}{u} \end{vmatrix} = -\frac{v^2}{u^3}.$$
 $(\rat{7}\%)$

于是
$$S = \iint_{D_1} \frac{v^2}{u^3} du dv = \int_1^2 v^2 dv \int_1^2 \frac{1}{u^3} du = \frac{7}{3} \times \frac{3}{8} = \frac{7}{8}.$$
 (10分)

五、证明题(每小题5分,共10分)

18. 证明:
$$\frac{\partial z}{\partial x} = f'(\sqrt{x^2 + y^2}) \frac{x}{\sqrt{x^2 + y^2}}$$
. (2分)

$$\frac{\partial^2 z}{\partial x^2} = f''(\sqrt{x^2 + y^2}) \frac{x^2}{x^2 + y^2} + f'(\sqrt{x^2 + y^2}) (\frac{1}{\sqrt{x^2 + y^2}} - \frac{x^2}{(x^2 + y^2)\sqrt{x^2 + y^2}}),$$

同理可得

$$\frac{\partial^2 z}{\partial y^2} = f''(\sqrt{x^2 + y^2}) \frac{y^2}{x^2 + y^2} + f'(\sqrt{x^2 + y^2}) (\frac{1}{\sqrt{x^2 + y^2}} - \frac{y^2}{(x^2 + y^2)\sqrt{x^2 + y^2}}).$$

由题设可知,

$$f''(\sqrt{x^2 + y^2}) + f'(\sqrt{x^2 + y^2}) \frac{1}{\sqrt{x^2 + y^2}} = 0.$$

$$\Leftrightarrow u = \sqrt{x^2 + y^2}, \ \text{Dif} uf''(u) + f'(u) = 0. \tag{5}$$

19. 证明:设
$$I = \iint_{D} \frac{1}{100 + \cos^{2} x + \cos^{2} y} dxdy$$
. 对任意 $(x, y) \in D$, 显然有

$$\frac{1}{102} \le \frac{1}{100 + \cos^2 x + \cos^2 y} \le \frac{1}{100}.$$
 (25)

由此可知,
$$\frac{206}{102} \le I \le 2.$$
 (5分)