Assignment

$\int \sqrt{\tan x}\, dx$

Solve: Let,

$$\sqrt{\tan x} = t$$

$$\Rightarrow \tan x = t^2$$

Differentiat both sides,

$$\sec^2 x\, dx = 2t\, dt$$

but. $\sec^2 x = 1 + \tan^2 x = 1 + t^4$,

so,

$$dx = \frac{2t\, dt}{1+t^4}$$

$$\therefore \int \sqrt{\tan x}\, dx = \int t \cdot dx = \int t \cdot \frac{2t\, dt}{1+t^4} = \int \frac{2t^2}{1+t^4}\, dt$$

Now,

$$\frac{2t^2}{1+t^4} = \frac{t^2+1}{1+t^4} + \frac{t^2-1}{1+t^4}$$

$$\therefore \int \frac{t^2+1}{1+t^4}\,dt = \frac{1}{2}\ln\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right|$$

$$\int \frac{t^2-1}{1+t^4}\,dt = \tan^{-1}\cdot\left(\frac{\sqrt{2}t}{1-t^2}\right)$$

$$\therefore \int \sqrt{\tan x}\,dx = \frac{1}{2}\ln\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right| + \tan^{-1}\left(\frac{\sqrt{2}t}{1-t^2}\right) + c$$

Substituting, $t = \sqrt{\tan x}$.

$$\int \sqrt{\tan x}\,dx = \frac{1}{2}\ln\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right| + \tan^{-1}\left(\frac{\sqrt{2\tan x}}{1-\tan x}\right) + c$$

(Ans.)