PGM Assignment -1

Sol a Given, random variables $X_2, X_3 \ldots, X_n$ and $Y_2, Y_3 \ldots, Y_m$

$P(X_2, X_3 \ldots, X_n, Y_2, Y_3, \ldots, Y_m)$

$2^{m+n} - 1$

Sol b

$P(X_2, X_3 \ldots, X_n, Y_2 Y_m) = 3^{m+n} - 1$

Sol c

$P(X_2, X_3 \ldots X_n, Y_2, Y_3 \ldots Y_n)$

$= (2 \times 3 \ldots n \cdot 2 \cdot 3 \ldots n) - 1$

Sol d

$P(Y_2, Y_3 \ldots, Y_m | X_2, X_3 \ldots, X_n)$

$\Rightarrow (2^m - 1) * (2^n)$

Sol e $P(Y_2, Y_3 \ldots X_m | X_2, X_3 \ldots X_n)$

if X_i & Y_i are three possible

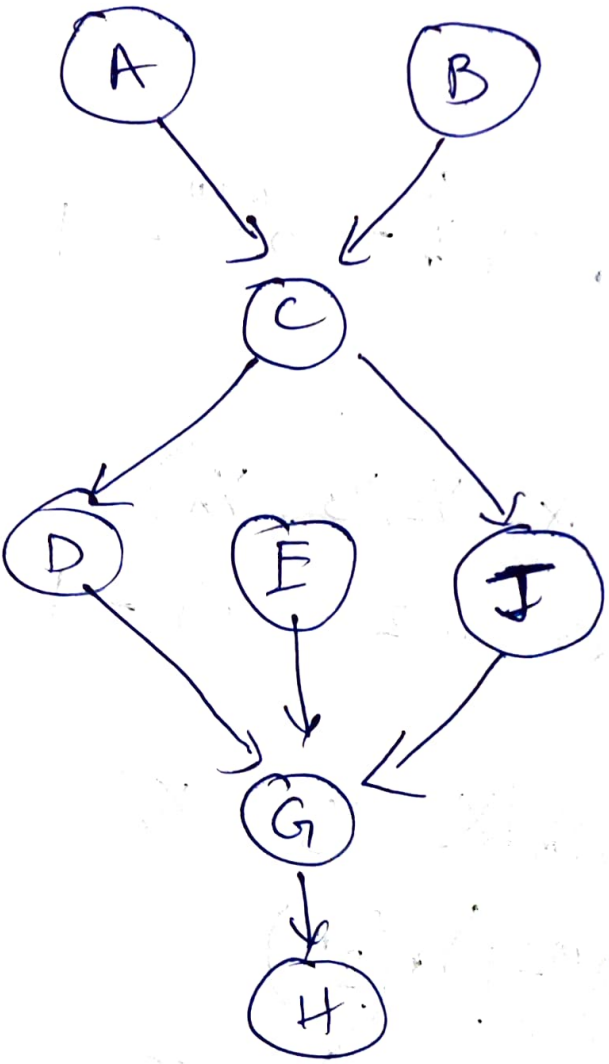
$\Rightarrow (3^m - 1) * (3^n)$

Sol: ~~$P(X_2, X_3, X_4 \ldots X_n)$~~

$$P(Y_2, Y_3 \ldots Y_m \mid X_2, X_3 \ldots X_n)$$

$$\Rightarrow \left((2 \cdot 3 \cdot 4 \ldots m) - 1\right) * (2 \cdot 3 \cdot 4 \ldots n)$$

Sol:



a) $P(A, B, C, D, E, J, G, H)$

$= P(A)\, P(B)\, P(C \mid A, B)\, P(D \mid C)\, P(J \mid C)$
$P(E)\, P(G \mid D, E, J)\, P(H \mid G)$.

Sol 2b So P(A) has no parent and it can take n possible values so it is n-1 parameters

P(B) → n-1 parameters; ~~P(C|A,B) so n^2 = 1 parameter~~

~~P(D|C) has~~ P(C|A,B) ⇒ $(n-1)(n^2)$

P(D|C) ⇒ $(n-1)(n)$ | P(E) ⇒ n-1

P(J|C) ⇒ $(n-1)(n)$ | P(H|G) ⇒ $(n-1)n$

P(G|D,E,J) ⇒ $(n-1)n^3$

So adding all these we get,

$(n-1)(n^2) + 3(n-1)(n) + n - 1 + (n-1)n^3$

$n^3 - n^2 + 3n^2 - 3n + n - 1 + n^4 - n^3$

$\boxed{n^4 + 2n^2 - 2n - 1}$

Sol 2c

(i) A ⊥ B ⇒ Yes

(ii) A ⊥ B | C ⇒ No

(iii) A ⊥ B | J ⇒ No

(iv) A ⊥ B | G ⇒ No

(v) A ⊥ B | E ⇒ No

(vi) A ⊥ B | H ⇒ No

(vii) A ⊥ H ⇒ No

(viii) $A \perp H \mid J \Rightarrow$ ~~No~~ Yes

(ix) $A \perp H \mid D, J \Rightarrow$ No

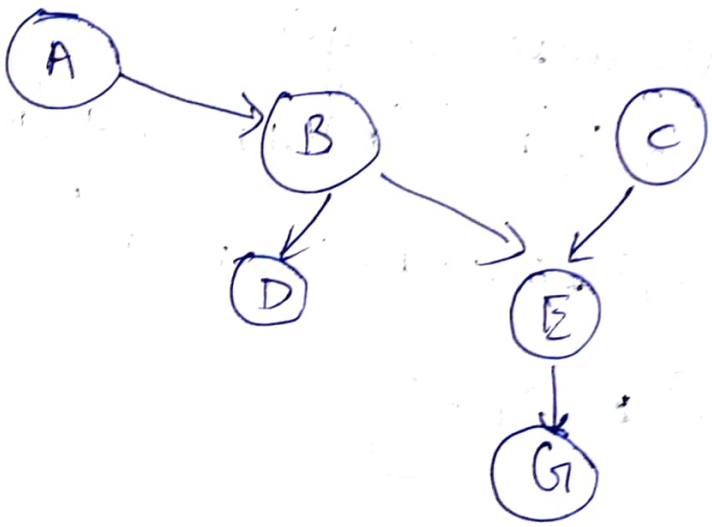
(x) $D \perp J \Rightarrow$ Yes

(xi) $B \perp E \Rightarrow$ ~~No~~ Yes

(xii) $B \perp E \mid J \Rightarrow$ No

(xiii) $B \perp E \mid J, H \Rightarrow$ No

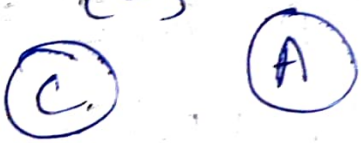
Sol:



(a) C, A, B, E, D, G.

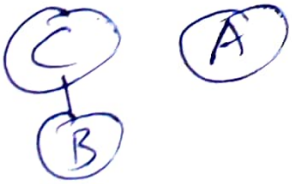
(1) C $G_1 = \{\phi\}$ $U = \{\phi\}$

(2) C, A $U = \{A\}$ $G_1 = \{C\}$



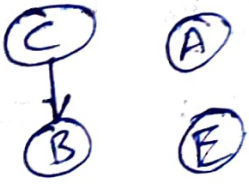
(3) C, A, B

$G_1 = \{C, A\}$ $U = \{C\}$



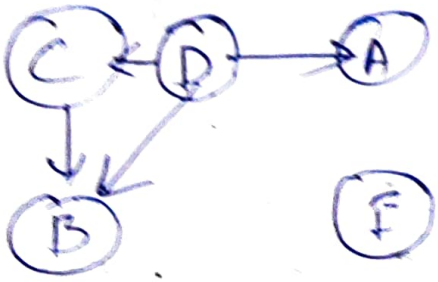
(4) C, A, B, E

$G_1 = \{C, A, B, E\}$



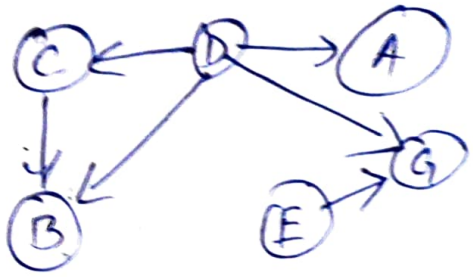
5) ~~D~~ ~~B~~ C, A, B, ~~E~~, D

$G = \{C, A, B, E\}$



6) C, A, B, E, D, G

$G = \{C, A, B, E, D, G\}$



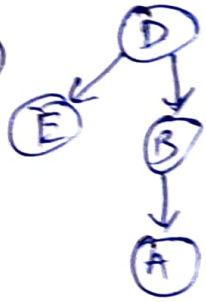
b) D, B, A, E, C, G.

1)

2)

3)

4)



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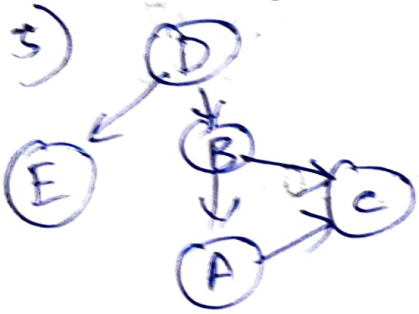
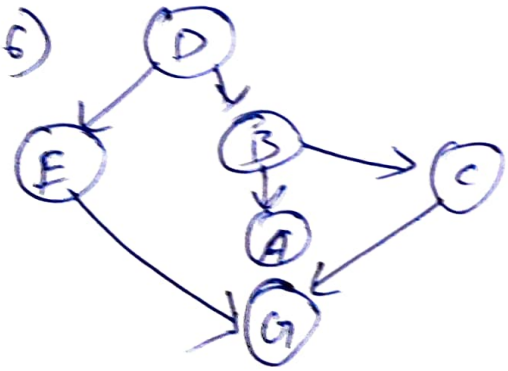


6)



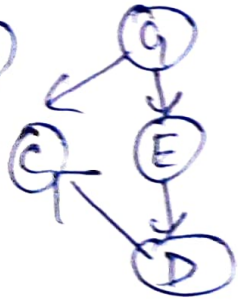
C) G, E, D, C, B, A.

1)

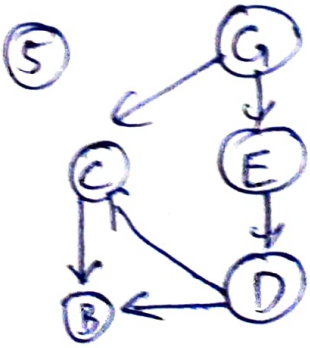
2)

3)

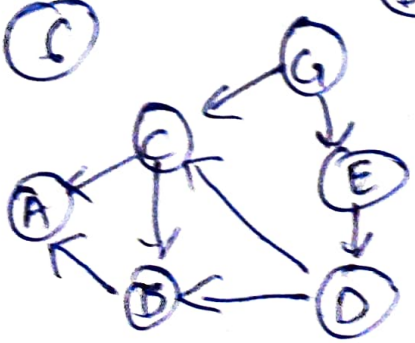
4)



5)



C)



d) G, A, C, E, D, B