PGM – Assignment (4)

Sol:

(a)

X	Counts	P(X)
T	60	60/150 = 2/5
F	90	90/150 = 3/5
	150	

Y | X=T	Counts	P(Y | X=T)
R	30	30/60 = 1/2
G	0	0
B	30	30/60 = 1/2

Y | X=F	Counts	P(Y | X=F)
R	40	40/90 = 4/9
G	0	0
B	50	50/90 = 5/9

Z | Y=R	Counts	P(Z | Y=R)
T	10	10/70 = 1/7
F	60	60/70 = 6/7

Z | Y=G	Counts	P(Z | Y=G)
T	0	0
F	0	0

Z | Y=B	Counts	P(Z | Y=B)
T	80	80/80 = 1
F	0	0

$P(X) = \langle 2/5, 3/5 \rangle$

$P(Y|X=T) = \langle 1/2, 0, 1/2 \rangle$

$P(Y|X=F) = \langle 4/9, 0, 5/9 \rangle$

$P(Z|Y=R) = \langle 1/7, 6/7 \rangle$

$P(Z|Y=G) = \langle 0, 0 \rangle$

$P(Z|Y=B) = \langle 1, 0 \rangle$

b)

X	Counts	P(X)	P(X_{nextID})
T	60	60/150	61/152
F	90/150	90/150	91/152

Y	X=F	Counts	P(Y|X=F)	P(Y_{nextID})
R	40	40/90	41/93	
G	0	0	1/93	
B	50	50/90	51/93	

X	X=T	Counts	P(Y|X=T)	P(X_{nextID})
R	30	30/60	31/63	
G	0	0	1/63	
B	30	30/60	31/63	

Z	Y=R	Counts	P(Z|Y=R)	P(Z_{nextID})
T	10	10/70	11/72	
F	60	60/70	61/72	

Z	Y=G	Counts	P(Z|Y=G)	P(Z_{nextID})
T	0	0	1/2	
F	0	0	1/2	

Z	Y=B	Counts	P(Z|Y=B)	P(Z next
T	80	80/80	81/82	
F	0	0/80	1/82	

$$P(X) = \langle 61/152 , 91/152 \rangle$$

$$P(Y|X=T) = \langle 31/63 , 1/63 , 31/63 \rangle$$

$$P(Y|X=F) = \langle 41/93 , 1/93 , 51/93 \rangle$$

$$P(Z|Y=R) = \langle 11/72 , 61/72 \rangle$$

$$P(Z|Y=G) = \langle 1/2 , 1/2 \rangle$$

$$P(Z|Y=B) = \langle 81/82 , 1/82 \rangle$$

c) $|D'|$ = probabilities 12 & P' is uniform

X	Counts	P(X)	P(X next ID)
T	60	$60/150 = 2/5$	$66/162 = 33/81$
F	90 / 150	$90/150 = 3/5$	$96/162 = 48/81$

Y | X=T	Counts	P(Y | X=T)	P(X next ID)
R	30	$30/60 = 1/2$	$32/66 = 16/33$
G	0	$0/60 = 0$	$2/66 = 1/33$
B	30	$30/60 = 1/2$	$32/66 = 16/33$

Y | X=F	Counts	P(Y | X=F)	P(Y next ID
R	40	$40/90 = 4/9$	$48/96 = 21/48$
G	0	0	$2/96 = 1/48$
B	50	$50/90 = 5/9$	$52/96 = 26/48$

Z | Y=R	Counts	P(Z | Y=R)	P(Z next ID)
T	10	$10/70 = 1/7$	$12/74 = 6/37$
F	60	$60/70 = 6/7$	$62/74 = 31/37$

Z | Y=G	Counts	P(Z | Y=G)	P(Z next ID)
T	0	0	$2/4 = 1/2$
F	0	0	$2/4 = 1/2$

Z | Y=B	Counts	P(Z | Y=B)	P(Z next ID)
T	80	$80/80 = 1$	$80/84 = 41/42$
F	0	0	$2/84 = 1/42$

$$P(X) = \langle 33/81, 48/81 \rangle$$

$$P(Y \mid X = T) = \langle 16/33, 1/33, 16/33 \rangle$$

$$P(Y \mid X = F) = \langle 21/48, 1/48, 26/48 \rangle$$

$$P(Z \mid Y = R) = \langle 6/37, 31/37 \rangle$$

$$P(Z \mid Y = G) = \langle 1/2, 1/2 \rangle$$

$$P(Z \mid Y = B) = \langle 41/42, 1/42 \rangle$$