

2021
Gour Mahavidyalaya

MATHEMATICS (General)

Paper Code: MTMGII-DC-2/GE-2

[CBCS]

Full Marks : 32

Time : Two hours

The figures in the margin indicate full marks.

Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any four questions.

1×4=4

(a) Find the order and degree of the differential equation $\left[1 + \frac{d^2y}{dx^2}\right]^{3/2} = a \frac{d^2y}{dx^2}$

(b) State the Archimedean property.

(c) The sequence $\left\{\frac{n^2+1}{n}\right\}$ is convergent. Find its limit?

(d) Define Gamma (Γ) function.

(e) If $\int_0^1 \frac{x^{p-1}}{1+x} dx$ is convergent, then find the value of p.

(f) Verify the Rolle's theorem for $x^2 - 5x + 10$ on $[2,3]$

(h) Find differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constant

Group - B

(10 Marks)

Answer any two questions

5×2=10

2. Prove that the sequence $\{x_n\}$ defined by

5

$$x_1 = \sqrt{2}, x_{n+1} = \sqrt{2x_n}, \forall n \geq 1$$

converges to 2.

3.(a) Show between any two real roots of the equation $e^x \cos x + 1 = 0$ there is atleast one real root of the equation $e^x \sin x + 1 = 0$.

3

(b) What is the geometrical interpretation of mean value theorem?

2

- 4.(a) Prove that $\int \tan^n x . dx = \frac{1}{n-1} . \tan^{n-1} x - \int \tan^{n-2} x . dx$ 3
- (b) State the fundamental theorem of calculus. Let $F(x) = \int_{-5}^x (t^2 + \sin t) dt$ then value of $F'(x)$. 2
5. Solve $(D^2 - 3D + 4) y = 16xe^{3x}$,where $D = \frac{d}{dx}$ 5

Group - C

(18 Marks)

Answer any two questions

9×2=18

- 6.(a) Show that,

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \cdots + \frac{1}{\sqrt{n^2 + n}} \right) = 1$$
 5

- (b) Let $c \in R$ and a real function f be such that f is continuous on some neighbourhood of c . Prove that

$$\lim_{h \rightarrow 0} \frac{f(c+h) - 2f(c) + 2f(c-h)}{h^2} = f''(c)$$
 4

- 7.(a) Show that $\int_0^{\frac{\pi}{2}} \frac{x^m}{\sin^n x} dx$ is convergent if $n < 1+m$ 4

- (b) Evaluate

$$\lim_{h \rightarrow 0} \frac{e^x - e^{-x} - 2 \log(1)}{x \sin x}$$
 3

- (c) State De Morgan's law for sets. 2

- 8.(a) Solve the Bernoulli's equation

$$6 \frac{dy}{dx} - \frac{2y}{x} = -x^2 y^2$$
 5

- (b) Find a general solution of the differential equation 4

$$\frac{d^2 y}{dx^2} - (x+1) \frac{dy}{dx} + y = x^2$$

$$\text{given } y_1(x) = e^x, y_2(x) = x+1$$

-the end-