A Another Rock-Paper-Scissors Problem

Sonny uses a very peculiar pattern when it comes to playing rock-paper-scissors. He likes to vary his moves so that his opponent can't beat him with his own strategy.

Sonny will play rock (R) on his first game, followed by paper (P) and scissors (S) for his second and third games, respectively. But what if someone else is using the same strategy? To thwart those opponents, he'll then play paper to beat rock, scissors to beat paper, and rock to beat scissors, in that order, for his 4th through 6th games. After that, he'll play scissors, rock, and paper for games 7–9 to beat anyone copying his last set of moves. Now we're back to the original order—rock, paper, scissors—but instead of being predictable and using the same moves, do you know what would be better? You guessed it! Sonny then plays the sequence of moves that would beat anyone trying to copy his whole strategy from his first move, and on it goes...

To recap, in symbolic form, Sonny's rock-paper-scissors moves look like this:

```
R P S PSR SRP PSRSRPRPS SRPRPSPSR PSRSRPRPSSRPRPSPSRRPSPSRSRP ...
```

The spaces are present only to help show Sonny's playing pattern and do not alter what move he'll play on a certain game.

Naturally, your job is to beat Sonny at his own game! If you know the number of the game that you'll be playing against Sonny, can you figure out what move you would need to play in order to beat him?

A.1 Input

Each line of the input contains a single integer N, $1 \leq N \leq 10^{12}$, the number of the game you'll be playing against Sonny. An integer $N = 1$ indicates it would be Sonny's first game, $N = 7$ indicates it would be the 7th game, and so forth. The input terminates with a line with $N = 0$. For example:

```
1
7
33
0
```

Warning: N may be large enough to overflow a 32-bit integer, so be sure to use a larger data type (i.e. `long` in Java or `long long` in C/C++) in your program.

A.2 Output

For each test case, output a single line which contains the letter corresponding to the move you would need to play to beat Sonny on that game. For example, the correct output for the sample input above would be:

```
P
R
S
```

B Ball Painting

There are $2N$ white balls on a table in two rows, making a nice 2-by-N rectangle. Jon has a big paint bucket full of black paint. (Don't ask why.) He wants to paint all the balls black, but he would like to have some math fun while doing it. (Again, don't ask why.) First, he computed the number of different ways to paint all the balls black. In no time, he figured out that the answer is $(2N)!$ and thought it was too easy. So, he introduced some rules to make the problem more interesting.

- The first ball that Jon paints can be any one of the $2N$ balls.
- After that, each subsequent ball he paints must be adjacent to some black ball (that was already painted). Two balls are assumed to be adjacent if they are next to each other horizontally, vertically, or diagonally.

Jon was quite satisfied with the rules, so he started counting the number of ways to paint all the balls according to them. Can you write a program to find the answer faster than Jon?

B.1 Input

The input consists of multiple test cases. Each test case consists of a single line containing an integer N, where $1 \leq N \leq 1{,}000$. The input terminates with a line with $N = 0$. For example:

```
1
2
3
0
```

B.2 Output

For each test case, print out a single line that contains the number of possible ways that Jon can paint all the $2N$ balls according to his rules. The number can become very big, so print out the number modulo 1,000,000,007. For example, the correct output for the sample input above would be:

```
2
24
480
```

C City Driving

You recently started frequenting San Francisco in your free time and realized that driving in the city is a huge pain. There are only N locations in the city that interest you, though, and you have decided to try to improve your driving experience. Since you lack a GPS and cannot remember too many different routes, you wrote down the directions and how long it takes to get between N different pairs of locations (the same in both directions), ensuring that using only these paths you can get from any location to any other one.

Now you are planning your trip for the weekend and you need to figure out the fastest way to get between Q pairs of locations in the city using only the routes you have written down.

C.1 Input

The input consists of multiple test cases. The first line contains a single integer N, $3 \leq N \leq 100{,}000$, the number of locations of interest and the number of routes you wrote down. The next N lines each contain three integers u, v, and w ($1 \leq w \leq 1{,}000$), indicating that you have directions from location u to location v and vice-versa (0-indexed) which take w time. The following line contains a single integer Q, $1 \leq Q \leq 10{,}000$, the number of pairs of locations you need to compute the travel time for. The next Q lines each contain two integers u and v, indicating that you should find the minimum time to get from location u to location v. The input terminates with a line with $N = 0$. For example:

```
7
0 1 2
0 2 3
1 3 2
2 3 8
2 4 3
3 5 1
1 6 7
3
4 5
0 6
1 2
0
```

C.2 Output

For each test case, print out Q lines, one for each pair of locations u and v you are finding the fastest routes for. Each line should simply contain the minimum time it takes to travel from location u to location v. For example, the correct output for the sample input above would be:

```
11
9
5
```

D Drunken Walk

After having a bit too much to drink in the evening, you find yourself going on a long walk on a directed graph—but not too long, as there are no cycles. You start at vertex 0, and whenever you are at a vertex, you will randomly leave the vertex along one of its outgoing edges with probability proportional to the weight of the edge.

You continue walking until you reach a vertex that has no edges leaving it, after which you fall asleep. The length of your walk is the number of edges you traverse. Moreover, before leaving vertex 0, you may choose one edge from anywhere in the graph that you do not like and ignore it during your walk (or you may choose to not ignore any of them). Determine the longest possible expected length of your walk.

D.1 Input

The input consists of multiple test cases. Each test case begins with a line containing two integers N, $2 \leq N \leq 10{,}000$, and M, $1 \leq M \leq 100{,}000$, the number of vertices and edges in the graph, respectively. The next M lines each contain three integers u, v, and w ($1 \leq w \leq 1{,}000$), indicating that there is a directed edge from vertex u to vertex v (numbered from 0 to $N-1$) with weight w. The input terminates with a line with $N = M = 0$. For example:

```
4 5
0 1 2
0 2 1
0 3 3
1 3 1
2 3 4
9 8
0 1 1
1 2 1
2 3 1
0 4 2
4 5 10
4 6 1
6 7 1
7 8 1
0 0
```

D.2 Output

For each test case, print out a single line that contains the longest possible expected length of your walk. Your answer will be considered correct if it is within 10^{-6} absolute or relative precision. In the first sample case, ignoring the edge from vertex 0 to vertex 3 gives the maximum possible expected length of 2. (Without ignoring it, the expected length is 1.5.) The correct output for the sample input above would be:

```
2.00000000
3.66666667
```

E Empty Triangles

Do you know how easy it is to make a very simple problem into a brutally hard one? Here is an example. How many triangles can you make with N straight lines in the plane? As long as they have different slopes and no three of them meet at a single point, there will be $\binom{N}{3}$ triangles, which is the maximum possible you can get.

Okay, that wasn't too bad. But let's see what happens if we only count triangles that are empty (that is, none of the lines pass through the interior of the triangle). Then, the number of triangles suddenly becomes very small. For example, with 4 straight lines, we can only make 2 empty triangles, whereas the total number of triangles can be as big as 4. Refer to the diagram.

In fact, a general formula for the maximum number of empty triangles that can be drawn with N lines is not known. The hard part, however, is to find the right configuration of the lines. Your job is much easier; given N straight lines on the plane, count the number of empty triangles.

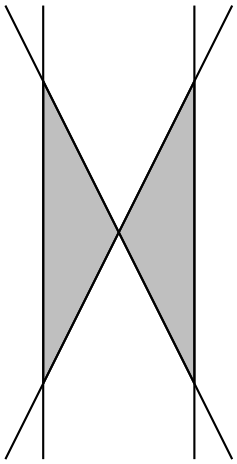


Figure 1: Four lines making two empty triangles (shaded).

E.1 Input

The input consists of multiple test cases. Each test case begins with a line containing an integer N, $1 \leq N \leq 500$, which indicates the number of lines on the plane. The next N lines each contain four integers x_1, y_1, x_2, and y_2 (between $-1{,}000$ and $1{,}000$), representing a straight line that goes through (x_1, y_1) and (x_2, y_2). It is guaranteed that no three lines meet at the same point, and all the lines are distinct. The input terminates with a line with $N = 0$. For example:

```
4
0 0 1 2
0 0 -1 2
1 0 1 1
-1 0 -1 -1
5
0 0 1 0
0 1 1 1
0 2 1 2
0 3 1 3
0 4 1 4
0
```

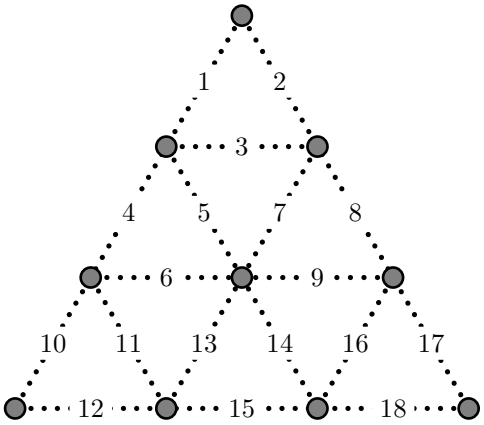
E.2 Output

For each test case, print out a single line that contains the number of empty triangles formed by the given lines. For example, the correct output for the sample input above would be:

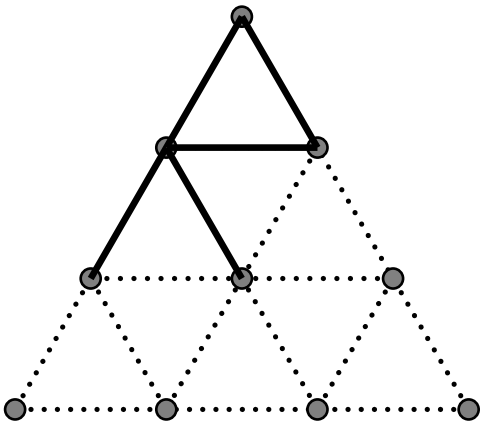
```
2
0
```

F Fighting for Triangles

Andy and Ralph are playing a two-player game on a triangular board that looks like the following:



At each turn, a player must choose two adjacent vertices and draw a line segment that connects them. If the newly drawn edge results in a triangle on the board (only the smallest ones count), then the player claims the triangle and draws another edge. Otherwise, the turn ends and the other player plays. The objective of the game is to claim as many triangles as possible. For example, assume that it is Andy's turn, where the board has fives edges as shown in the picture below. If Andy draws edge 6, then he will claim the triangle formed by edge 4, 5, and 6, and continue playing.



Given a board that already has some edges drawn on it, decide the winner of the game assuming that both Andy and Ralph play optimally. Andy always goes first. Note that if a triangle exists on the board before the first move, neither player claims it.

F.1 Input

The input consists of multiple test cases. Each test case begins with a line containing an integer N, $5 \leq N \leq 10$, which indicates the number of edges that are already present on the board before the game begins. The next line contains N integers, indicating the indices of these edges. The input terminates with a line with $N = 0$. For example:

```
6
1 2 3 4 5 6
5
4 5 6 7 8
0
```

F.2 Output

For each test case, print out a single line that contains the result of the game. If Andy wins, then print out "`Andy wins`". If Ralph wins, then print out "`Ralph wins`". If both players get the same number of triangles, then print out "`Draw`". Quotation marks are used for clarity and should not be printed. For example, the correct output for the sample input above would be:

```
Andy wins
Ralph wins
```

G Guessing Game

Jaehyun has two lists of integers, namely $a_1, \ldots, a_N$ and $b_1, \ldots, b_M$. Jeffrey wants to know what these numbers are, but Jaehyun won't tell him the numbers directly. So, Jeffrey asks Jaehyun a series of questions of the form "How big is $a_i + b_j$?" Jaehyun won't even tell him that, though; instead, he answers either "It's at least c," or "It's at most c." (Right, Jaehyun simply doesn't want to give his numbers for whatever reason.) After getting Jaehyun's responses, Jeffrey tries to guess the numbers, but he cannot figure them out no matter how hard he tries. He starts to wonder if Jaehyun has lied while answering some of the questions. Write a program to help Jeffrey.

G.1 Input

The input consists of multiple test cases. Each test case begins with a line containing three positive integers N, M, and Q, which denote the lengths of the Jaehyun's lists and the number of questions that Jeffrey asked. These numbers satisfy $2 \leq N + M \leq 1{,}000$ and $1 \leq Q \leq 10{,}000$. Each of the next Q lines is of the form `i j <= c` or `i j >= c`. The former represents $a_i + b_j \leq c$, and the latter represents $a_i + b_j \geq c$. It is guaranteed that $-1{,}000 \leq c \leq 1{,}000$. The input terminates with a line with $N = M = Q = 0$. For example:

```
2 1 3
1 1 <= 3
2 1 <= 5
1 1 >= 4
2 2 4
1 1 <= 3
2 1 <= 4
1 2 >= 5
2 2 >= 7
0 0 0
```

G.2 Output

For each test case, print a single line that contains "`Possible`" if there exist integers $a_1, \ldots, a_N$ and $b_1, \ldots, b_M$ that are consistent with Jaehyun's answers, or "`Impossible`" if it can be proven that Jaehyun has definitely lied (quotes added for clarity). The correct output for the sample input above would be:

```
Impossible
Possible
```

H Hidden Code

It's time to put your hacking skills to the test! You've been called upon to help crack enemy codes in the current war on... something or another. Anyway, the point is that you have discovered the encryption technique used by the enemy; it is quite simple, and proceeds as follows. Note that all strings contain only uppercase letters of the alphabet.

1. We are given a key K and a plaintext P which is encrypted character-by-character to produce a ciphertext C of the same length.
2. If $|K|$ is the length of the key K, then the first $|K|$ characters of C are obtained by adding the first $|K|$ characters of P to the characters of K, where adding two letters means interpreting them as numbers (`A` $= 0$, `B` $= 1$, and so on) and taking the sum modulo 26. That is, $C_i = (P_i + K_i) \bmod 26$ for $i = 1, \ldots, |K|$. If $|K| > |P|$, then the extra characters in K are ignored.
3. The remaining characters of P, i.e. P_i for $i > |K|$, are encrypted using the previous ciphertext characters by $C_i = (P_i + C_{i-|K|}) \bmod 26$ for $i = |K| + 1, \ldots, |P|$.

As an example, consider the encryption of the string "`STANFORD`" using the key "`ACM`":

```
  STA  NFORD
+ ACM  SVMFA
  ----------
  SVM  FAAWD
```

Knowing this, you are well on your way to being able to read the enemy's communications. Luckily, you also have several pairs of plaintexts and ciphertexts which your team recovered, all of which are known to be encrypted with the same key. Help find the key that the enemy is using.

H.1 Input

The input consists of multiple test cases. Each test case begins with a line containing a single integer N, $1 \leq N \leq 100$, the number of plaintext and ciphertext pairs you will receive. The next N lines each contain two strings P and C, the plaintext and ciphertext, respectively. P and C will contain only uppercase letters (`A-Z`) and have the same length (at most 100 characters). The input terminates with a line with $N = 0$. For example:

```
1
A B
3
STANFORD SVMFAAWD
AVOWIENR AXAWFEJW
VAMRI VCYMK
3
ABCDEFGHIJKLMNOPQRSTUVWXYZ AAAAAAAAAAAAAAAAAAAAAAAAAA
Y Y
Z Z
2
A B
B A
0
```

H.2 Output

For each test case, print a single line that contains the shortest possible key or "`Impossible`" (quotes added for clarity) if no possible key could have produced all of the encryptions. For example, the correct output for the sample input above would be:

```
B
ACM
AZYXWVUTSRQPONMLKJIHGFEDCB
Impossible
```

I Identity Checker

You likely have seen that $x(\sin^2 x + \cos^2 x) - x = 0$, and you may have seen that $\sin(2x) - 2\sin x \cos x = 0$. But did you know that $\tan(2x)(x - x\tan^2 x) - 2x\tan x = 0$? Would you believe that $\sin(2x) - 2\cos x = 0$? That last one is false, but don't just take our word for it; you should write a program that determines whether an algebraic expression simplifies to zero (whenever it is defined).

I.1 Input

The input consists of multiple test cases, each on one line. Each test case starts with an integer N, the number of tokens that describes a formula. The next N tokens describe a formula in reverse polish notation. The notation works as follows. There is a stack that begins empty, and the following commands manipulate the contents of the stack:

- "`x`" pushes the variable x to the stack.
- "`sin`", "`cos`", and "`tan`" replace the top element of the stack with its sin, cos, and tan, respectively.
- "`+`", "`-`", and "`*`" replace the top two elements of the stack (a on top, followed by b) with their sum $(b + a)$, difference $(b - a)$, and product $(b * a)$, respectively.

You may assume that the input is valid, and results in a single item on the stack, which is the desired expression. The length of a line will be at most 300 characters. Function arguments can contain functions, so `x sin sin` is valid, but the recursion will not go any deeper than this. The input terminates with a line with $N = 0$. For example:

```
15 x sin x sin * x cos x cos * + x * x -
16 x sin x cos * x sin x cos * + x x + sin -
24 x x + tan x x tan x tan * x * - * x tan x * - x tan x * -
10 x x + sin x cos - x cos -
0
```

I.2 Output

For each test case, print out a single line that contains "`Identity`" if the expression is always zero, and "`Not an identity`" otherwise (quotes added for clarity). For example, the correct output for the sample input above would be:

```
Identity
Identity
Identity
Not an identity
```