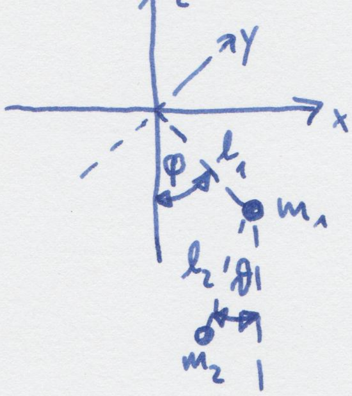

Force: $\vec{F} = g \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$, $g = 9.8 \, m/s^2$

Simplification: $l_1 = l_2 =: l$
$m_1 = m_2 =: m$

Can rescale units of time, length, mass such that $l = 1\,LU$, $m = 1\,MU$, $g = 1 \frac{LU}{(TU)^2}$

Lagrange method:

Kinetic energy: $T = \frac{m_1}{2}\left(l_1^2 \dot{\varphi}^2\right) + \frac{m_2}{2}\left(l_1^2 \dot{\varphi}^2 + l_2^2 \dot{\vartheta}^2 + 2 l_1 l_2 \dot{\varphi}\dot{\vartheta}\cos(\varphi - \vartheta)\right)$

$= \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\left(\dot{\varphi}^2 + \dot{\vartheta}^2 + 2\dot{\varphi}\dot{\vartheta}\cos(\varphi - \vartheta)\right)$

Potential energy: $V = -m_1 g l_1 \cos\varphi - m_2 g\left(l_1 \cos\varphi + l_2 \cos\vartheta\right)$

$= -2\cos\varphi - \cos\vartheta$

Momenta: $p_\varphi = \frac{\partial(T-V)}{\partial \dot{\varphi}} = 2\dot{\varphi} + \dot{\vartheta}\cos(\varphi - \vartheta)$

$p_\vartheta = \frac{\partial(T-V)}{\partial \dot{\vartheta}} = \dot{\vartheta} + \dot{\varphi}\cos(\varphi - \vartheta)$

$\begin{pmatrix} p_\varphi \\ p_\vartheta \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & \cos(\varphi-\vartheta) \\ \cos(\varphi-\vartheta) & 1 \end{pmatrix}}_{=:M} \begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \end{pmatrix}$, The Inverse matrix: $\frac{1}{\det}\begin{pmatrix} 1 & -c \\ -c & 2 \end{pmatrix}$

$\det = 2 - c^2$, $c = \cos(\varphi - \vartheta)$

<u>EOM</u>: $\begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \end{pmatrix} = M^{-1}\begin{pmatrix} p_\varphi \\ p_\vartheta \end{pmatrix}$, $\dot{p}_\varphi, \dot{p}_\vartheta$ from Euler Lagrange eq.:

$\dot{p}_\varphi = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = \frac{\partial L}{\partial \varphi} = \frac{\partial(T-V)}{\partial \varphi} = -\dot{\varphi}\dot{\vartheta}\sin(\varphi - \vartheta) - 2\sin\varphi$

$\dot{p}_\vartheta = \frac{\partial(T-V)}{\partial \vartheta} = \dot{\varphi}\dot{\vartheta}\sin(\varphi - \vartheta) - \sin\vartheta$

$\dot{\varphi}, \dot{\vartheta}$ are the expressions below.

$\dot{\varphi} = \left(2 - \cos^2(\varphi - \vartheta)\right)^{-1} \cdot \left(p_\varphi - \cos(\varphi - \vartheta)\, p_\vartheta\right)$

$\dot{\vartheta} = \left(2 - \cos^2(\varphi - \vartheta)\right)^{-1} \cdot \left(2 p_\vartheta - \cos(\varphi - \vartheta)\, p_\varphi\right)$