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Blind Frequency and Symbol Rate Estimation for MSK Signal under Low Signal-to-Noise Ratio ⋆

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Abstract

We investigate a novel blind estimator combination of carrier frequency and symbol rate of an complex MSK modulation under low signal-to-noise ratio (SNR). The carrier frequency is estimated by using Modified Rife frequency estimator at first and the referenced signal is constructed consequently. Then, the received signal is transformed to baseband by means of correlation with the referenced signal. The symbol rate is estimated by wavelet transform of the baseband signal at a suitable scale. Simulation results show that the proposed method is more accurate than the existing estimators when SNR is low.

Keywords: Frequency Estimation; MSK; Symbol Rate Estimation; Haar Wavelet

1 Introduction

Minimum-shift keying (MSK) signal is a widely used modulation format in communication and radar applications since efficient symbol-by-symbol coherent detectors can be devised for it [1, 2]. It is a classical project to estimate parameters of the received signals under low signal-to-noise ratio (SNR) in non-cooperative aspects ,such as cognitive radio or Electronic Intelligence (Elint) [1,3–7].

Extensive literature focuses on the frequency, timing, phase or symbol rate estimations of MSK signals. In [8, 9], a symbol timing and frequency offset estimation scheme and in [4] an algorithm for timing and carrier phase recovery are proposed respectively by applying maximum-likelihood methods, which are both hang-up free and well-suited for burst-mode transmissions because of its feed forward structure. In [10], the received signal is transformed to baseband by a delay-multiplied auto-frequency-mixed signal and the symbol rate estimation is estimated by using Haar wavelet transform. However, the error of the symbol rate estimation is relatively larger when the SNR is less than 6dB and the performance of carrier frequency estimator is dramatically becoming worse when the SNR is below -2dB. In [11], a blind frequency estimator based on cyclic spectrum is proposed. As it is reported that when SNR is greater than -7dB, the Normalized root

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mean square error (RMSE) of the carrier frequency and symbol rate estimators are approximated to 0 respectively. But the computation of the cyclic spectrum is relatively complex.

In this paper, we propose a novel estimator combination of symbol rate and frequency estimation scheme for MSK signals which is efficient when the SNR is below -9dB. Firstly, the carrier frequency is estimated by Modified Rife method. And then the received signal is transformed to baseband by multiplying with the referenced signal and the symbol rate is estimated by using wavelet transform of the based signal by a suitable scale. The selection rule for the wavelet scale is also discussed in the paper.

The paper is organized as follows. The signal model is presented in Section 2. In Section 3 and Section 4 the frequency and symbol rate estimators are proposed respectively. Simulation results for AWGN channels are illustrated in Section 5. Finally some conclusions are drawn in Section 6.

2 Signal Model

A complex MSK signal (over a finite observation interval) is given by [10]

$$s(t) = Aexp\{j2\pi[f_c + \frac{f_b}{4}a(t)]t + \phi(t)\}, \quad 0 \leq t \leq T \tag{1}$$

where

$$a(t) = \sum_n a_n g(t - nT_c) \tag{2}$$

is the information symbol series and

$$\phi(t) = \sum_n \phi_n g(t - nT_c) \tag{3}$$

is the initial phase series corresponding to each symbol information and ϕ_n is the initial phase of the nth symbol. In (1), A is the amplitude, f_c is carrier frequency, f_b is the symbol rate and T is the signal interval. In (2)-(3), a_n are independent data symbols taking on the values ± 1 with equal probability, T_c is the symbol period, and $g(t)$ is the rectangular pulse.

The observed signals with added noise, is given by

$$r(t) = s(t) + w(t) \tag{4}$$

where $w(t)$ is a complex-valued zero-mean white Gaussian noise process whose real and imaginary parts are independent with the variance σ^2.

3 Frequency Estimation

As discussed in [10], there is no line spectrum in spectrum of the MSK signal [1,2], but the square of the MSK signal will result in line spectrum. The square of the MSK signals is given by

$$s^2(t) = A^2 exp\{j4\pi[f_c + \frac{a(t)}{4T_b}]t + 2\phi_0\} \tag{5}$$

where $T_b = \dfrac{1}{f_b}$ and ϕ_0 is an equivalent phase. From the equation (5), we can see that the square of MSK signal is a FSK signal with the following two carrier frequencies

$$f_1 = 2f_c + \frac{1}{2T_b}, \quad f_2 = 2f_c - \frac{1}{2T_b} \tag{6}$$

The frequency estimation method develops through the following steps:

Step1 Apply square calculation to the observed signal, and then compute DFT to the transformed signal.

Step2 Find the positions corresponding to two local maximum magnitudes in spectrum of squared MSK signal.

Step3 Set the position of the local maximum as the center respectively, and then reserve 10 lines located near the center.

Step4 Calculate IDFT of the reserved spectrum, and then get two sinusoid signals in time domain.

Step5 Estimate the frequency of sinusoid signal using modified Rife frequency estimator [12].

So if we estimate f_1 and f_2 at first respectively, the carrier frequency can be calculated by

$$\hat{f_c} = \frac{1}{4}(\hat{f_1} + \hat{f_2}) \tag{7}$$

As shown in equation (7), the accuracy of $\hat{f_c}$ is depend on the estimation performances of $\hat{f_1}$ and $\hat{f_2}$. However, the nonlinear transformation for the MSK signal will lead to the declining of the SNR. Next we will assess the loss of SNR due to the square operation. As for MSK signal, the square of it is given by equation (5). If the noise part is considered,

$$r^2(t) = s^2(t) + 2s(t)w(t) + w^2(t) = s_1(t) + w_1(t) \tag{8}$$

The noise part of $r^2(t)$ is

$$w_1(t) = 2s(t)w(t) + w^2(t) \tag{9}$$

Its mean and variance can be given by

$$E[w_1(t)] = 0$$

and

$$Var[w_1(t)] = 4A^2\sigma^2 + \sigma^4$$

respectively.

Hence we get the output SNR as

$$SNR_0 = \frac{A^4}{4A^2\sigma^2 + \sigma^4} = \frac{SNR}{4SNR + 1} SNR = l_1 SNR \tag{10}$$

As we see, the output SNR of the square of the MSK signal is the l_1 times of the original SNR and $l_1 < 1$. For example, if the input SNR is 0dB, the output SNR is about declined to -6.99dB. Clearly, the output SNR will decrease dramatically with the declining of the input SNR.

In order to improve the processing SNR, preprocessing method called short time filtering is presented in this paper. The filtering procedures are given by:

Step1 Dividing the original signal in discrete form into several segments, then it can be written as

$$r_i(n) \approx Aexp[j(2\pi f_i n\Delta t + \theta_i)] + w_i(n), i(N_0 - 1) \leq n \leq (i+1)(N_0 - 1)$$

where $r_i(n)$ is the ith segment of the signals in discrete and A, f_i, θ_i represent envelope function, carrier frequency and initial phase respectively; $w_i(n)$ is the complex additive white Gaussian noise with zero mean and variance σ^2; N_0 is the length of signal segment, Δt is the sampling interval, T_0 is pulse period.

Step2 Computing N_0-point DFT of $r_i(n)$, we can arrive at $R_i(k) = \mathrm{DFT}[r_i(n)]$.

Step3 Design a band-pass filter with the transmission function as follow

$$H(k) = \begin{cases} 1, & k_0 - \delta \leq k \leq k_0 + \delta \\ 0, & \text{others} \quad k \end{cases} \tag{11}$$

where k_0 is the position of the maximum magnitude of $R(k)$, δ is the filtering point.

Step4 Considering $R_i^{,}(k)$=$H(k)R_i(k)$, then compute N_0-point IDFT of $R_i^{,}(k)$, we can get $r_i^{,}(n)$= IDFT($R_i^{,}(k)$).

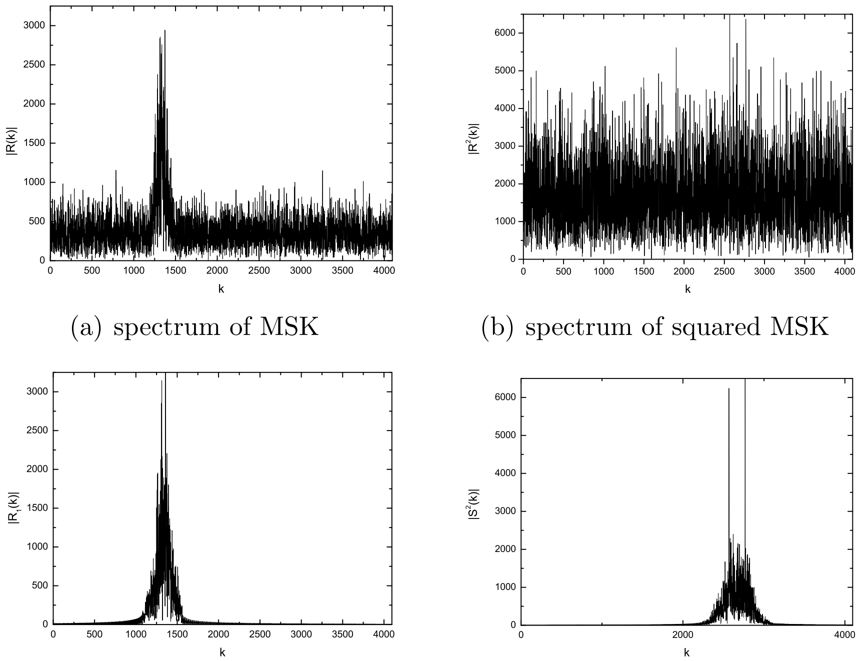
Step5 Finally, combine each segment $r_i^{,}(n)$ into $\hat{r}(n)$.

Commonly, the output signal to noise ratio of the filter is given by

$$SNR^{,} = \frac{A^2}{2\sigma_f^2} = \frac{A^2 N_0}{2\sigma^2(2\delta+1)} = SNR\frac{N_0}{2\delta+1} \tag{12}$$

Equation (12) indicates that processing SNR increases $\frac{N_0}{2\delta}$ times in comparison with the original one. Generally we choose the parameter δ=6∼10 according to our experiences.

Fig. 1 shows the spectrum comparisons of MSK signal when SNR is -10dB. From the figure, we can see that there are no distinct spectrum line in the spectrum of the squared MSK signal when SNR is -10dB and after filtering the two line of the spectrum is more clearly than that of without filtering. Hence, the above mentioned short time filtering is effective and simple.



(a) spectrum of MSK

(b) spectrum of squared MSK

(c) spectrum of MSK after short time filtering

(d) spectrum of squared MSK after short time filtering

Fig. 1: Spectrum of MSK signal when SNR is -10dB

4 Symbol Rate Estimation

The symbol rate estimation method proposed in [10] was described as follows:

Step1 Delay-multiplied auxiliary signal is generated to obtain the sequence composed of baseband information.

Step2 Get the baseband sequence by low-pass filtering.

Step3 Apply wavelet transform to the baseband sequence.

Step4 Calculate the coefficient modules of wavelet transform, and then detect the symbol rate from the line in the spectrum of the coefficient modules.

However, the drawbacks of the above mentioned estimation method can be concluded as follows:

(1) The multiply operation of the observed signal will increase the noise power, and it will lead to the decrease of SNR. In [10], the symbol rate estimation is based on the auto-delay correlation as follows

$$r(t) = s(t)s(t+\tau) + w(t)s(t+\tau) + w(t)w(t+\tau) + s(t)w(t+\tau) \tag{13}$$

the noise part of $r(t)$ is expressed by

$$w_2(t) = w(t)s(t+\tau) + w(t)w(t+\tau) + s(t)w(t+\tau) \tag{14}$$

Its mean and variance, respectively, are given as

$$E[w_2(t)] = 0$$

and

$$Var[|w_2(t)|^2] = 2A^2\sigma^2 + \sigma^4$$

Hence, the output SNR can be expressed as

$$SNR_0 = \frac{A^4}{2A^2\sigma^2 + \sigma^4} = \frac{SNR}{2SNR + 1} SNR = l_2 SNR \tag{15}$$

As we see from the above expression the output SNR of the auto-correlation of the MSK signal is the l_2 fold of the original SNR and l_2<1. For example, if the input SNR is 0dB, the output SNR is about declined to -4.77dB.

(2) Auto-correlation of the signal will reduce the sample size, so the delay-time can not be too large, otherwise the signal energy will be reduced.

(3) The choice of parameters of low pass filter needs *a priori* knowledge of the observed signal such as carrier frequency, which is unknown for blind processing.

In order to improve the estimating performance, we proposed a new scheme as follows:

Step1 Constructing the referenced signals by using the estimated carrier frequency, we can get

$$y(t) = exp[-j(2\pi \hat{f}_c t)], \ \ 0 < t < T \tag{16}$$

Step2 Multiplying the received signal $r(t)$ with $y(t)$, the baseband signal can be expressed as

$$z(t) = r(t)y(t) = Aexp\{j2\pi\Delta f_c t + \frac{f_b}{4} a(t)t + \phi(t)\} + w_3(t), \ \ 0 \le t \le T \tag{17}$$

where Δf_c=f_c−$\hat{f}_c$ is the carrier estimating error and the equivalent noise part $w_3(t)$=$exp[-j(2\pi \hat{f}_c t)]w(t)$ whose mean and variance, respectively, are given as

$$E[w_3(t)] = 0$$

and

$$Var[|w_3(t)|^2] = \sigma^2$$

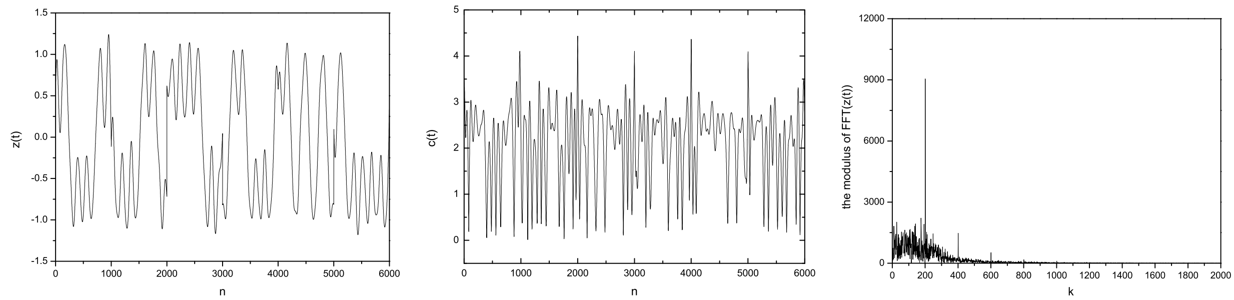
Therefore, the transformation by Eq. (17) does not lead to the loss of the input SNR. Fig. 2(a) shows the wave of $z(t)$ in the time domain when SNR is 5dB.

Step3 Calculating the Haar wavelet transformation of $z(t)$,we can get

$$c(t) = |CWT_d(z(t))| \tag{18}$$

where d is the selected wavelet scale. Fig. 2(b) illustrates the Haar wavelet transformation of $z(t)$ when SNR is 5dB.

Step4 Taking FFT of $c(t)$, and we can find that there is a line spectrum in the frequency domain. Then the location of the maximum of the spectrum indicates to the symbol rate as shown in Fig. 2(c).



(a) The time domain wave of z(t) (b) The wavelet transformation of z(t) by using Haar wavelet (c) The spectrum of the wavelet transformation of z(t)

Fig. 2: Symbol rate estimation by using Haar wavelet transformation (SNR=5dB)

5 Simulation

5.1 Simulation setup

In this section the performance of the proposed recognition method is analyzed using simulated data. The corresponding modulated signals are generated based on the following parameters summarized in Table 1. One thousand of Monte Carlo trials are performed for each SNR condition.

Table 1: The parameters used in simulation

Symbol rate	Sampling frequency	Sample size	Carrier frequency	Haar wavelet scale
600bps	48kHz	16000	4kHz	0.9Tb

5.2 Performance analysis

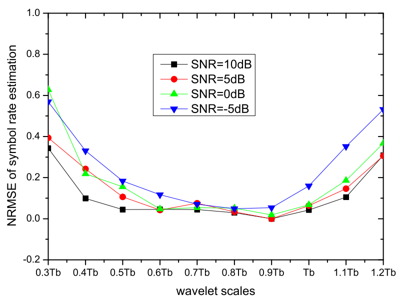


Fig. 3: NRMSE of symbol rate estimation versus wavelet scale

Fig. 3 shows the relationship between the NRMSE of the symbol rate and the selected wavelet scales under different SNRs. From Fig. 3, we find that the NRMSE of symbol rate is relatively small when the wavelet scale varies from $0.6T_b$ to $0.9T_b$. So in practice, we chose the wavelet scale by

$$d = (0.6 \sim 0.9)\,\hat{T}_b \tag{19}$$

Since the true T_b is not be obtained in practice, we estimated it roughly by using equation (6) as

$$\hat{T_b} = (f_1 - f_2)^{-1} \tag{20}$$

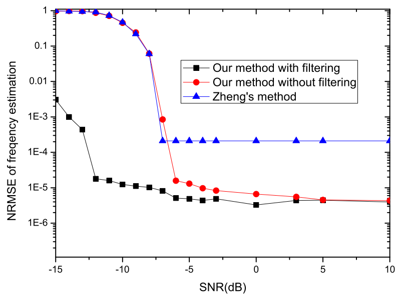


Fig. 4: NRMSE of frequency estimation versus SNR

Fig. 4 shows the NRMSE of carrier frequency estimation versus SNR by using Zheng's Method in [10] and the proposed method in this article. From the figure, we can see that our method is more accurate than Zheng's method in the whole SNR scope especially when the SNR is low.

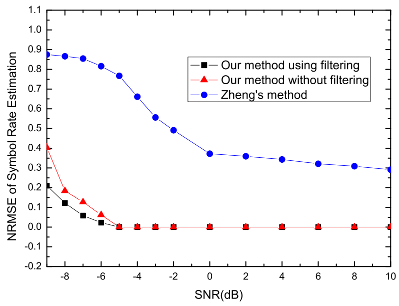


Fig. 5: NRMSE of symbol rate estimation versus SNR

Fig. 5 illustrates the NRMSE of symbol rate estimation versus SNR by using the Zheng's Method and the proposed method in this article. It shows that NRMSE of the symbol rate estimation by using our method is clearly less than that of Zheng's when SNR varies from -9dB to 10dB.

6 Conclusion

A novel estimator combination of carrier frequency and symbol rate estimation for MSK-type signals has been derived in this article. The short time filtering is used as a preprocessing operation to enhance the SNR in processing. Simulations indicate that the novel estimation algorithm provides better NRMSE performance than the method in [10] especially under low SNRs.

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