Approximate Solution of Kuramoto-Sivashinsky Equation using Reduced Differential Transform Method

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Keywords

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Approximate Solution of Kuramoto–Sivashinsky Equation using Reduced Differential Transform Method

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Abstract. In this study, approximate solution of Kuramoto–Sivashinsky Equation, by the reduced differential transform method, are presented.We apply this method to an example.Thus, we have obtained numerical solution Kuramoto–Sivashinsky equation. Comparisons are made between the exact solution and the reduced differential transform method. The results show that this method is very effective and simple.

Keywords: Reduced differential transform method (RDTM), Kuramoto–Sivashinsky(KS) Equations,Approximate solution.
PACS: 02.60.Cb.

INTRODUCTION

Partial differential equations (PDEs) have numerous essential applications in various fields of science and engineering such as fluid mechanic, thermodynamic, heat transfer, physics [10].

Most new nonlinear PDEs do not have a precise analytic solution. So, numerical methods have largely been used to handle these equations. It is difficult to handle nonlinear part of these equations. Although most of scientists applied numerical methods to find the solution of these equations, solving such equations analytically is of fundamental importance since the existent numerical methods which approximate the solution of PDEs don't result in such an exact and analytical solution which is obtained by analytical methods.

Many researchers have paid attention by studying to the solutions of nonlinear PDEs by various methods[11-19]. RDTM [1-4], devised by YıldırayKeskin in 2009, is a numerical method to obtain approximate solutions of various types of nonlinear partial differential equations. It has received much attentionsince it has applied to solve a wide variety of problems by many authors [20-24].

In this study, RDTM is used to obtain approximate solution of KS equation. The Generalized Kuramoto-Sivashinsky equation is given [5,6] by

$$u_t + \alpha u^{\beta} u_x + \gamma u^{\tau} u_{xx} + \lambda u_{xxxx} = 0, \tag{1}$$

Where $\alpha, \beta, \gamma, \lambda, \tau \neq 0$ and $\alpha, \beta, \gamma, \lambda, \tau \in R$.

When $\alpha = \beta = 1$ and $\tau = 0$, eq. (1) reduces to originalKS equationof the form[6-8]

$$u_t + uu_x + \gamma u_{xx} + \lambda u_{xxxx} = 0. \tag{2}$$

In the past several decades, many researchers have used various methods to solve KS equation.We apply the RDTM to solve KS equation of the form (2).

ANALYSIS OF THE RDTM

The basic definitions of RDTM werealready given in [1].

For the purpose of illustration of the methodology to the proposed method, we write the KS equation in the standard operator form

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$$L_t\left(u(x,t)\right)+L_x\left(u(x,t)\right)+N\left(u(x,t)\right)=0 \tag{3}$$

with initial condition

$$u(x,0)=f(x) \tag{4}$$

where $L_t\left(u(x,t)\right)=u_t$ and $L_x\left(u(x,t)\right)=\gamma u_{xx}+\lambda u_{xxxx}$ arelinear operators which have partial derivatives, $N\left(u(x,t)\right)=uu_x$ is a nonlinear term.

According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x)=-\gamma\frac{\partial^2}{\partial x^2}U_k(x)-\lambda\frac{\partial^4}{\partial x^4}U_k(x)-\sum_{r=0}^{k}U_{k-r}(x)\left(\frac{\partial^2}{\partial x^2}U_r(x)\right). \tag{5}$$

From initial condition (4), we write

$$U_0(x)=f(x) \tag{6}$$

substituting (6) into (5) and by a straight forward iterative calculations, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ gives approximation solution as,

$$\tilde{u}_n(x,t)=\sum_{k=0}^{n}U_k(x)t^k \tag{7}$$

where$n$ is order of approximation solution.

Therefore, the exact solution of problem is given by

$$u(x,t)=\lim_{n\to\infty}\tilde{u}_n(x,t)\,. \tag{8}$$

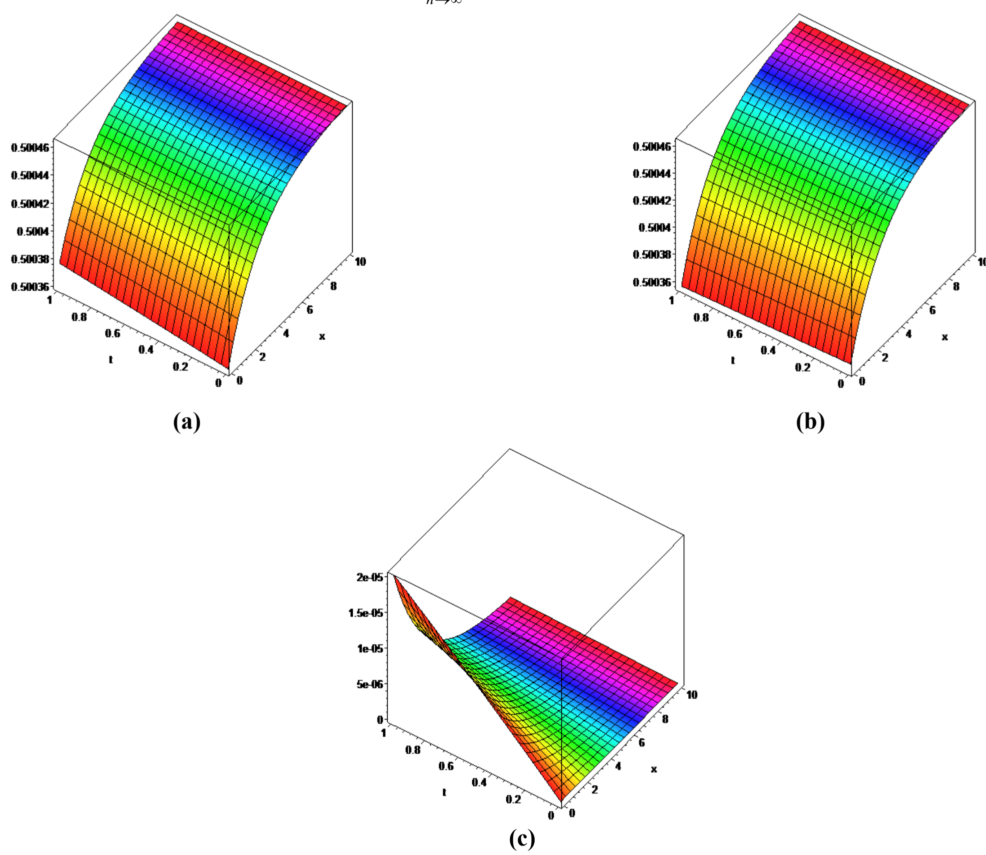

FIGURE 1. The surface shows the solution $u(x,t)$ for eq.(9) when $c=0.1$, $k=\left(\sqrt{11/19}\right)/4$, $x_0=-30$ (a) exact solution (11), (b) 2th order of approximate solution for eq.(9) and (c) the absolute error between exact and numerical solution.

NUMERICAL APPLICATIONS

In this section, we test the RDTM for KS equation. We consider the following KS equation [6,9]:

$$u_t + uu_x + u_{xx} + u_{xxxx} = 0 \tag{9}$$

subjectto initial condition

$$u(x,0) = c + \frac{5}{19}\sqrt{\frac{11}{19}}\left(11\tanh^3\left(k\left(x-x_0\right)\right) - 9\tanh\left(k\left(x-x_0\right)\right)\right). \tag{10}$$

In [6,9], the exact solution of (9) is given as

$$u(x,t) = c + \frac{5}{19}\sqrt{\frac{11}{19}}\left(11\tanh^3\left(k\left(x-ct-x_0\right)\right) - 9\tanh\left(k\left(x-ct-x_0\right)\right)\right). \tag{11}$$

RESULTS AND DISCUSSION

When analysed the solution of the above an exampleby RDTM, the following results are obtained:

TABLE 1. The exact and 2th order of approximate solution for(9) and absolute error between exact and numerical solutions

x	t	RDTM Solution	Exact Solution	Abs-Error
0.0	0.0	0.5003600908	0.500360093	$0.5420022964\times10^{-9}$
0.0	0.5	0.5003685212	0.500358052	$0.1046850860\times10^{-4}$
0.0	1.0	0.5003762240	0.500355975	$0.2024957166\times10^{-4}$
0.5	0.0	0.5003784827	0.500378483	$0.6041276398\times10^{-9}$
0.5	0.5	0.5003854531	0.500376798	$0.8655395022\times10^{-5}$
0.5	1.0	0.5003918215	0.500375080	$0.1674210244\times10^{-4}$
1.0	0.0	0.5003936888	0.500393690	$0.5954475010\times10^{-9}$
1.0	0.5	0.5003799452	0.500392296	$0.7156111486\times10^{-5}$
1.0	1.0	0.5004047174	0.500390875	$0.1384174699\times10^{-4}$

In this study, reduced differential transform method has been successfully applied to KSequation. This example shows that the results of the presented method are in excellent agreement with those of absolute errors and the obtained solutions shown in table.

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