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## Reduced Differential Transform Method for (2+1) Dimensional type of the Zakharov-Kuznetsov ZK(n,n) Equations

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### Abstract

In this paper, reduced differential transform method is employed to approximate the solutions of (2+1) dimensional type of the Zakharov-Kuznetsov partial differential equations. We apply this method to two examples. Thus, we have obtained numerical solution Zakharov-Kuznetsov equations. These examples are prepared to show the efficiency and simplicity of this method.

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# Reduced Differential Transform Method for (2+1) Dimensional type of the Zakharov–Kuznetsov ZK(n,n) Equations

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**Abstract.** In this paper, reduced differential transform method is employed to approximate the solutions of (2+1) dimensional type of the Zakharov–Kuznetsov partial differential equations. We apply this method to two examples. Thus, we have obtained numerical solution Zakharov–Kuznetsov equations. These examples are prepared to show the efficiency and simplicity of this method.

**Keywords:** Reduced differential transform method (RDTM), ZK(n,n) Equations.

**PACS:** 02.60.Cb.

## INTRODUCTION

Partial differential equations (PDEs) have numerous essential applications in various fields of science and engineering such as fluid mechanic, thermodynamic, heat transfer, physics [1]. Most new nonlinear PDEs do not have a precise analytic solution. So, numerical methods have largely been used to handle these equations. It is difficult to handle nonlinear part of these equations. Although most of scientists applied numerical methods to find the solution of these equations, solving such equations analytically is of fundamental importance since the existent numerical methods which approximate the solution of PDEs don't result in such an exact and analytical solution which is obtained by analytical methods.

In recent years, many researchers have paid attention to studying the solutions of nonlinear PDEs by various methods[2-13]. The reduced differential transform method(RDTM) was first proposed by Keskin and Oturanc[14-17]. It has received much attentionsince it has applied to solve a wide variety of problems by many authors [24–28].

Zakharov–Kuznetsov (ZK) equation,

$$u_t + auu_x + (\nabla^2 u)_x = 0 \quad (1)$$

where  $\nabla^2 = \partial_x^2 + \partial_y^2$  or  $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$  is the isotropic Laplacian[19-21]. This means that the ZK equationsare given by

$$u_t + auu_x + (u_{xx} + u_{yy})_x = 0, \quad (2)$$

and

$$u_t + auu_x + (u_{xx} + u_{yy} + u_{zz})_x = 0 \quad (3)$$

in (2+1) and (3+1)-dimensional spaces. The ZK equation was first derived for describing weakly nonlinear ion-acoustic wave in a strongly magnetized lossless plasma in two dimensions [19]. A further discussion of the analytical properties of the ZK equation and some constructive results were given.

Recently, in 2005, A.M. Wazwaz [22] studied the type of Zakharov–Kuznetsov equation, that is, the (2+1) dimensional and (3+1) dimensional ZK(n,n) equations of the form

$$u_t + a(u^n)_x + b(u^n)_{xxx} + k(u^n)_{yyx} = 0, \quad b, k > 0 \quad (4)$$

and

$$u_t + a(u^n)_x + b(u^n)_{xxx} + k(u^n)_{yyx} + r(u^n)_{zzx} = 0, \quad b, k, r > 0 \quad (5)$$

and in 2007, C. Lin, X. Zhang, [23] studied the (3+1) dimensional modified ZK equation of the form

$$u_t + au^p u_x + (u_{xx} + u_{yy} + u_{zz})_x = 0. \quad (6)$$

We apply the RDTM to solve types of ZK(n,n) equations of the form (4)

## ANALYSIS OF THE RDTM

The basic definitions of RDTM were already given in [14].

For the purpose of illustration of the methodology to the proposed method, we write the ZK( $n, n$ ) equation in the standard operator form

$$L(u(x, t)) + N(u(x, t)) = 0 \quad (7)$$

with initial condition

$$u(x, y, 0) = f(x, y) \quad (8)$$

where  $L = \frac{\partial}{\partial t}$  is a linear operator which has partial derivatives,  $N(u(x, t)) = a(u^n)_x + b(u^n)_{xxx} + k(u^n)_{yyx}$  is a nonlinear term.

According to the RDTM, we can construct the following iteration formula:

$$(k+1)U_{k+1}(x) = -N(U_k(x)) \quad (9)$$

where  $N_k = N(U_k(x))$  is the transformations of the function  $N(u(x, t))$  respectively.

From initial condition (8), we write

$$U_0(x, y) = f(x, y) \quad (10)$$

Substituting (10) into (9) and by a straight forward iterative calculations, we get the following  $U_k(x, y)$  values.

Then the inverse transformation of the set of values  $\{U_k(x, y)\}_{k=0}^n$  gives approximation solution as,

$$\tilde{u}_n(x, y, t) = \sum_{k=0}^n U_k(x, y) t^k \quad (11)$$

where  $n$  is order of approximation solution.

Therefore, the exact solution of problem is given by

$$u(x, y, t) = \lim_{n \rightarrow \infty} \tilde{u}_n(x, y, t). \quad (12)$$

## NUMERICAL APPLICATIONS

In this section, we test the RDTM for the ZK(3,3) and ZK(2,2) equations with fully nonlinear dispersion.

First we consider the following ZK(3,3) equation [18]:

$$u_t + (u^3)_x + 2(u^3)_{xxx} + 2(u^3)_{yyx} = 0 \quad (13)$$

subject to initial condition

$$u(x, y, 0) = \frac{3}{2} \lambda \sinh \frac{1}{6} (x + y) \quad (14)$$

Second we consider the following ZK(2,2) equation [18]:

$$u_t + (u^2)_x + \frac{1}{8} (u^2)_{xxx} + \frac{1}{8} (u^2)_{yyx} = 0 \quad (15)$$

subject to initial condition

$$u(x, y, 0) = -\frac{4}{3} \lambda \cosh^2 (x + y). \quad (16)$$

## RESULTS AND DISCUSSION

When analyzed the solutions of the above two examples by RDTM, the following results are obtained:

**TABLE 1.** The numerical results and absolute errors for ZK(3,3) by means of a 4-iterate RDTM solution

$\lambda$	$x$	$y$	$t$	RDTM Solution	Abs-Error-RDTM
0.00001	0.0	0.0	0.001	$-0.375000000 \times 10^{-18}$	$0.2265468750 \times 10^{-37}$
	0.0	0.5		$0.1251447262 \times 10^{-5}$	$0.2292047232 \times 10^{-37}$
	0.0	1.0		$0.2511590160 \times 10^{-5}$	$0.2373142696 \times 10^{-37}$
	0.5	0.0		$0.1251447262 \times 10^{-5}$	$0.2292047232 \times 10^{-37}$
	0.5	0.5		$0.2511590160 \times 10^{-5}$	$0.2373142096 \times 10^{-37}$
	0.5	1.0		$0.3789184752 \times 10^{-5}$	$0.2512918476 \times 10^{-37}$
	1.0	0.0		$0.2511590160 \times 10^{-5}$	$0.2373142696 \times 10^{-37}$
	1.0	0.5		$0.3789184752 \times 10^{-5}$	$0.2512918476 \times 10^{-37}$
	1.0	1.0		$0.5093108360 \times 10^{-5}$	$0.2718596318 \times 10^{-37}$

**TABLE 2.** The numerical results and absolute errors for ZK(2,2) by means of a 4-iterate RDTM solution

$\lambda$	$x$	$y$	$t$	RDTM Solution	Abs-Error-RDTM
0.00001	0.0	0.0	0.001	-0.00001333333333	$0.2932786015 \times 10^{-24}$
	0.0	0.5		-0.00001695387292	$0.2692669148 \times 10^{-20}$
	0.0	1.0		-0.00003174798469	$0.1562939098 \times 10^{-19}$
	0.5	0.0		-0.00001695387292	$0.2692669148 \times 10^{-20}$
	0.5	0.5		-0.00003174798469	$0.1562939098 \times 10^{-19}$
	0.5	1.0		-0.00007378450649	$0.1005498720 \times 10^{-18}$
	1.0	0.0		-0.00003174798469	$0.1562939098 \times 10^{-19}$
	1.0	0.5		-0.00007378450649	$0.1005498720 \times 10^{-18}$
	1.0	1.0		-0.00001887222243	$0.6999706576 \times 10^{-18}$

In this study, reduced differential transform method has been successfully applied to (2+1) dimensional ZK(3,3) and ZK(2,2) equations. Examples show that the results of the present method are in excellent agreement with those of absolute errors and the obtained solutions are shown in table.

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