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Reduced Differential Transform Method For Solving Time and Space Local Fractional Partial Differential Equations

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Abstract

We applied the new local fractional reduced differential transform method to obtain the solutions of some linear and non-linear partial differential equations on Cantor set. The reported results are compared with the related solutions presented in the literature and the graphs are plotted to show their behaviors. The results prove that the presented method is faster and easy to apply.

Keywords: Approximate solution, Local fractional derivative, Partial differential equations, Reduced Differential Transform Method.
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1. Introduction

Linear and non-linear fractional ordinary or partial differential equations (PDEs) models are commonly encountered in applied mathematics, physics and engineering fields [7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 21, 27, 28, 29, 30, 31, 32, 33, 35, 36, 38]. In recent years, many researches dealt the fractional differential equations due to its importance in the different kinds of applied sciences. Therefore, there are too study on solutions of fractional ordinary and PDEs. Some authors such as Poldlubny [30], Samko et al. [32], Schneider and Wyss [35], Beyer and Kemplfe [15], Mainardi [29] and Yang [38], discussed fractional order of differential equations.

The Reduced Differential Transform Method (RDTM) was first proposed by Keskin and Oturanc [23, 24, 25, 26]. This method is widely used by many researchers to study fractional and non-fractional, linear and non-linear PDEs. The method introduces a reliable and efficient process for a wide variety of

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engineering, scientific and physics applications, such as fractional and non-fractional, linear, non-linear, homogeneous and non-homogeneous PDEs [1, 2, 3, 4, 5, 6, 8, 10, 16, 19, 22, 23, 24, 25, 26, 33, 36, 43]. Recently, The local fractional derivative was introduced by Yang [37, 38]. By using this derivative, the solutions of important mathematical problems are studied [11, 20, 22, 34, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. Yang et al. in 2016 [43] proposed local fractional differential transform method (LFDTM) by using local fractional derivative (LFD) with DTM. For this method, he gave some basic theorems and also an application. Similarly, Jafari et al. in 2016 [22] introduced Local fractional reduced differential transform method (LFRDTM) by using LFD with RDTM. For this method, they gave some basic theorems and also some applications.

The main aim of this article is to present approximate analytical solutions of some linear and non-linear time and space local fractional PDEs by using LFRDTM. We discuss how to solve linear and non-linear PDEs with LFD by using RDTM.

This study is organized as follows. The basic definitions properties and theorems of LFD in Section 2 and LFRDTM in Section 3 are presented. In Section 4, the application of the new method is given. And finally, we give conclusion in Section 5.

2. Preliminaries

Definition 2.1. Let $C_\alpha(a,b)$ be a set of the non-differentiable functions with the fractal dimension $\alpha\,(\alpha\in(0,1])$. For $\psi(x)\in C_\alpha(a,b)$, the LFD operator of $\psi(x)$of order $\alpha\,(\alpha\in(0,1])$ at the $x=x_0$ is defined as follows [11, 20, 38, 39, 40, 41, 43]:

$$D^{(\alpha)}\psi(x_0)=\frac{d^\alpha\psi(x_0)}{dx^\alpha}=\lim_{x\to x_0}\frac{\Delta^\alpha\left(\psi(x)-\psi(x_0)\right)}{(x-x_0)}, \tag{2.1}$$

where

$$\Delta^\alpha\left(\psi(x)-\psi(x_0)\right)\cong\Gamma\left(1+\alpha\right)\left[\psi(x)-\psi(x_0)\right].$$

Lemma 2.2. *[38, 47, 48] Suppose that f, g are non-differentiable functions and $\alpha\in(0,1]$ is order of LFD. Then*
(i) $D^{(\alpha)}(af+bg)=a\left(D^{(\alpha)}f\right)+b\left(D^{(\alpha)}g\right)$ *for* $a,b\in\mathbb{R}$,
(ii) $D^{(\alpha)}(fg)=fD^{(\alpha)}(g)+gD^{(\alpha)}(f)$,
(iii) $D^{(\alpha)}\left(\frac{f}{g}\right)=\frac{gD^{(\alpha)}(f)-fD^{(\alpha)}(g)}{g^2}$ *provided* $g\neq0$.

Lemma 2.3. *[38, 47, 48] Suppose that f is non-differentiable function and $\alpha\in(0,1]$ is order of LFD. Then*
(i) $D^{(\alpha)}(f(x))=0$, *for all constant functions* $f(x)=\lambda$,
(ii) $D^{(\alpha)}\left(\frac{x^{k\alpha}}{\Gamma(k\alpha+1)}\right)=\frac{x^{(k-1)\alpha}}{\Gamma((k-1)\alpha+1)}$,
(iii) $D^{(\alpha)}\left(E_\alpha(x^\alpha)\right)=E_\alpha(x^\alpha)$,
(iv) $D^{(\alpha)}\left(E_\alpha(-x^\alpha)\right)=-E_\alpha(-x^\alpha)$,
(v) $D^{(\alpha)}\left(\sin_\alpha(x^\alpha)\right)=\cos_\alpha(x^\alpha)$,
(vi) $D^{(\alpha)}\left(\cos_\alpha(x^\alpha)\right)=-\sin_\alpha(x^\alpha)$,
where $E_\alpha(x^\alpha)=\sum\limits_{k=0}^{\infty}\frac{x^{k\alpha}}{\Gamma(k\alpha+1)}$, $\sin_\alpha(x^\alpha)=\sum\limits_{k=0}^{\infty}(-1)^k\frac{x^{(2k+1)\alpha}}{\Gamma((2k+1)\alpha+1)}$ *and*
$\cos_\alpha(x^\alpha)=\sum\limits_{k=0}^{\infty}(-1)^k\frac{x^{2k\alpha}}{\Gamma(2k\alpha+1)}$.

Definition 2.4. The local fractional partial derivative (LFPD) operator of $\psi(x,t)$ of order $\alpha\,(\alpha\in(0,1])$ with respect to t at the point (x,t_0) is defined as follows [38, 43]:

$$D_t^{(\alpha)}\psi(x,t_0)=\frac{\partial^\alpha\psi(x,t_0)}{\partial t^\alpha}=\lim_{t\to t_0}\frac{\Delta^\alpha\left(\psi(x,t)-\psi(x,t_0)\right)}{(t-t_0)},$$

where

$$\Delta^{\alpha}\left(\psi(x,t)-\psi(x,t_0)\right) \cong \Gamma\left(1+\alpha\right)\left[\psi(x,t)-\psi(x,t_0)\right].$$

In view of (2.1), the LFPD operator of $\psi\left(x,t\right)$ of order $k\alpha\left(\alpha\in(0,1]\right)$ is given by [11, 20, 38, 39, 43]:

$$D_t^{(k\alpha)}\psi(x,t)=\frac{\partial^{k\alpha}\psi(x,t)}{\partial t^{k\alpha}}=\underbrace{D_t^{(\alpha)}D_t^{(\alpha)}\cdots D_t^{(\alpha)}}_{k\text{ times}}\psi(x,t).$$

3. Local Fractional Reduced Differential Transform Method

In this section, The basic definitions some properties and theorems of LFRDTM are presented as follows [22, 43]:

Lemma 3.1. *[22, 43]: **(Local fractional Taylors theorem)** Suppose that* $\frac{d^{(k+1)\alpha}}{dx^{(k+1)\alpha}}\psi\left(x\right)\in C_{\alpha}\left(a,b\right)$, *for* $a,b\in\mathbb{R}$, $k=0,1,2,...,n$ *and* $\alpha\in(0,1]$, *we have*

$$\psi\left(x\right)=\sum_{k=0}^{\infty}\frac{d^{k\alpha}}{dx^{k\alpha}}\psi\left(x_0\right)\frac{\left(x-x_0\right)^{\alpha k}}{\Gamma\left(1+k\alpha\right)}$$

where $a<x_0<x<b$, $\forall x\in(a,b)$.

Definition 3.2. The LFRDT $\Psi_k(x)$ of the function $\psi(x,t)$ is defined as [22, 43]:

$$\Psi_k(x)=\frac{1}{\Gamma(1+k\alpha)}\left[\frac{\partial^{k\alpha}\psi(x,t)}{\partial t^{k\alpha}}\right]_{t=t_0}$$

where $k=0,1,2,\ldots,n$ and $\alpha\in(0,1]$.

Definition 3.3. The LFRDT of $\Psi_k(x)$ is defined by the following formula [22, 43]:

$$\psi(x,t)=\sum_{k=0}^{\infty}\Psi_k(x)(t-t_0)^{k\alpha}$$

where $\alpha\in(0,1]$.

Using Definition 3.2 and Definition 3.3, the fundamental mathematical operations of the LFRDTM [22] are deduced in Table 1.

4. Numerical Consideration

In this section we will illustrate the LFRDTM technique by four examples. These examples give exact answer in the sense of exact solutions. This approach shows the accurate evaluation of the analytical technique and the examination of the LFD on the behavior of the solutions. All operations are calculated by software MAPLE.

Example 4.1. Let us first consider the following linear time and space local fractional equation on Cantor set [43]:

$$\frac{\partial^{\alpha}}{\partial t^{\alpha}}\psi\left(x,t\right)-\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\psi\left(x,t\right)=0\ ,\ 0\ \alpha\leqslant 1 \tag{4.1}$$

with initial condition (IC)

$$\psi(x,0)=E_{\alpha}(x^{\alpha}). \tag{4.2}$$

The exact solutions of (4.1) is

$$\psi(x,t)=E_{\alpha}\left((x+t)^{\alpha}\right).$$

Table 1: Basic operations of the LFRDTM

Original function	Local transformed function
$\psi(x,t)$	$\Psi_k(x) = \frac{1}{\Gamma(1+k\alpha)}\left[\frac{\partial^{k\alpha}\psi(x,t)}{\partial t^{k\alpha}}\right]_{t=t_0}$
$\psi(x,t) = a\pi(x,t) \pm b\phi(x,t)$	$\Psi_k(x) = a\Pi_k(x) \pm b\Phi_k(x)$
$\psi(x,t) = \pi(x,t)\,\varphi(x,t)$	$\Psi_k(x) = \sum_{s=0}^{k} \Pi_s(x)\,\Phi_{k-s}(x)$
$\psi(x,t) = \frac{\partial^{n\alpha}}{\partial t^{n\alpha}}\pi(x,t)$	$\Psi_k(x) = \frac{\Gamma(1+k\alpha+n\alpha)}{\Gamma(k\alpha+1)}\Pi_{k+n}(x)$
$\psi(x,t) = \frac{(x-x_0)^{m\alpha}}{\Gamma(1+m\alpha)}\frac{(t-t_0)^{n\beta}}{\Gamma(1+n\beta)}$	$\Psi_k(x) = \frac{x^{m\alpha}}{\Gamma(1+m\alpha)}\delta_\alpha(k-n)$
$\psi(x,t) = E_\alpha((a(x-x_0))^\alpha)E_\beta((b(t-t_0))^\beta)$	$\Psi(k) = E_\alpha((a(x-x_0))^\alpha)\,\frac{a^{k\alpha}}{\Gamma(1+k\alpha)}$

If we take the local LFRDT of (4.1), we get the following iteration formula:

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}\Psi_{k+1}(x) = \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\Psi_k(x) \tag{4.3}$$

where Ψ_k is the transformed function. From the IC (4.2) we write

$$\Psi_0(x) = E_\alpha(x^\alpha). \tag{4.4}$$

Substituting (4.4) into (4.3), and iterative steps, we obtain the following $\Psi_k(x)$ values

$$\Psi_1(x) = \frac{E_\alpha(x^\alpha)}{\Gamma(\alpha+1)},\ \Psi_2(x) = \frac{E_\alpha(x^\alpha)}{\Gamma(2\alpha+1)},\ \cdots,\ \Psi_n(x) = \frac{E_\alpha(x^\alpha)}{\Gamma(n\alpha+1)},\ \cdots \tag{4.5}$$

From (4.5), we find the LFRDTM solution of equation (4.1) as

$$\tilde{\psi}_n(x,t) = \sum_{k=0}^{n} E_\alpha(x^\alpha)\frac{t^{\alpha k}}{\Gamma(k\alpha+1)}.$$

Hence $\psi(x,t)$ is

$$\psi(x,t) = \lim_{n\to\infty}\tilde{\psi}_n(x,t) = E_\alpha(x^\alpha)E_\alpha(t^\alpha).$$

This finding is same as result given in [43], also it is the exact solution. The graph of this solution is given in Fig. 1 for $\alpha = \frac{\ln 2}{\ln 3}$.

Example 4.2. Let us consider linear time and space local fractional equation on Cantor set [48]:

$$\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\psi(x,t) - \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\psi(x,t) = 0\ ,\ 0\ \alpha \leqslant 1 \tag{4.6}$$

subject to ICs

$$\psi(x,0) = -E_\alpha(x^\alpha)\ \ \text{and}\ \ \frac{\partial^\alpha}{\partial t^\alpha}\psi(x,0) = 0 \tag{4.7}$$

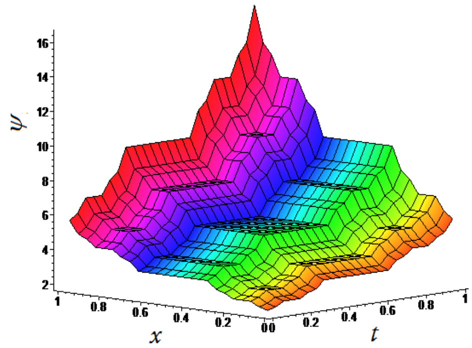


Figure 1: The solution for (4.1) equation of non-differentiable type. ($\alpha = \ln 2/\ln 3$).

The exact solutions of (4.6) is

$$\psi(x,t) = -E_\alpha(x^\alpha)\cos_\alpha(t^\alpha).$$

If we take the LFRDT of (4.6), we get the following iteration formula:

$$\frac{\Gamma(k\alpha+2\alpha+1)}{\Gamma(k\alpha+1)}\Psi_{k+2}(x) = -\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\Psi_k(x) \tag{4.8}$$

where Ψ_k is the transformed function. From the ICs (4.7) we write

$$\Psi_0(x) = -E_\alpha(x^\alpha) \;\; \text{and} \;\; \Psi_1(x) = 0 \tag{4.9}$$

Substituting (4.9) into (4.8), and by iterative steps, we obtain the following $\Psi_k(x)$ values

$$\Psi_k(x) = \begin{cases} (-1)^{i+1}\dfrac{E_\alpha(x^\alpha)}{\Gamma(2i\alpha+1)} & \text{if} \;\; k = 2i \\ 0 & \text{if} \;\; k = 2i+1 \end{cases} \tag{4.10}$$

From (4.10), we find the LFRDTM solution of equation (4.6) as

$$\tilde{\psi}_n(x,t) = \sum_{k=0}^{n}(-1)^{k+1}\frac{E_\alpha(x^\alpha)}{\Gamma(2k\alpha+1)}t^{2k\alpha}.$$

Hence $\psi(x,t)$ is

$$\psi(x,t) = \lim_{n\to\infty}\tilde{\psi}_n(x,t) = -E_\alpha(x^\alpha)\cos_\alpha(t^\alpha).$$

This finding is the exact solution. The graph of this solution is given in Fig. 2 for $\alpha = \frac{\ln 2}{\ln 3}$

Example 4.3. Let us consider non-linear time and space local fractional equation on Cantor set [48]:

$$\frac{\partial^\alpha}{\partial t^\alpha}\psi(x,t) - \psi(x,t)\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\psi(x,t) + \psi(x,t)\frac{\partial^\alpha}{\partial x^\alpha}\psi(x,t) = 0\,,\; 0\;\alpha \leqslant 1 \tag{4.11}$$

subject to IC

$$\psi(x,0) = E_\alpha(x^\alpha). \tag{4.12}$$

The exact solutions of (4.11) is

$$\psi(x,t) = E_\alpha(x^\alpha).$$

If we take the LFRDT of (4.11), we get the following iteration formula:

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}\Psi_{k+2}(x) = \sum_{r=0}^{k}\Psi_{k-r}(x)\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\Psi_r(x) - \sum_{r=0}^{k}\Psi_{k-r}(x)\frac{\partial^\alpha}{\partial x^\alpha}\Psi_r(x) \tag{4.13}$$

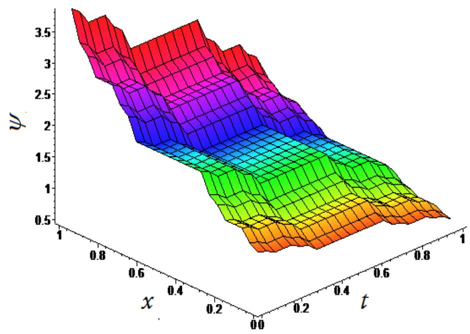


Figure 2: The solution for (4.6) equation of non-differentiable type. ($\alpha = \ln 2/\ln 3$).

where Ψ_k is the transformed function. From the IC (4.12) we write

$$\Psi_0(x) = E_\alpha\left(x^\alpha\right) \tag{4.14}$$

Substituting (4.14) into (4.13), and by iterative steps, we obtain the following $\Psi_k(x)$ values

$$\Psi_k(x) = 0, \quad \text{for} \;\; k = 1, 2, 3, \ldots \tag{4.15}$$

From (4.15), we find the LFRDTM solution of equation (4.11) as

$$\psi(x,t) = \tilde{\psi}_n(x,t) = \sum_{k=0}^{n} \Psi_k(x) t^{k\alpha} (-1)^{k+1} = E_\alpha\left(x^\alpha\right).$$

This finding is same as result given in [48], also it is the exact solution. The graph of this solution is given in Fig. 3 for $\alpha = \frac{\ln 2}{\ln 3}$

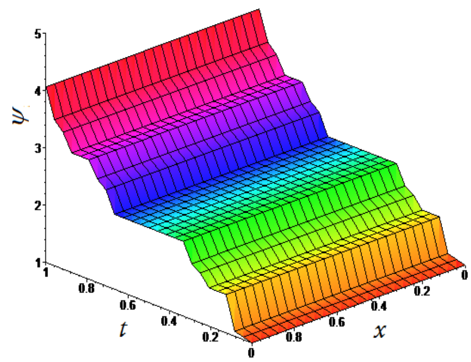


Figure 3: The solution for (4.11) equation of non-differentiable type. ($\alpha = \ln 2/\ln 3$).

Example 4.4. Let us consider non-linear time and space local fractional equation on Cantor set [48]:

$$\frac{\partial^\alpha}{\partial t^\alpha}\psi\left(x,t\right) - \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\psi\left(x,t\right) - \psi\left(x,t\right)\frac{\partial^\alpha}{\partial x^\alpha}\psi\left(x,t\right) = 0\ ,\ 0\ \alpha \leqslant 1 \tag{4.16}$$

subject to IC

$$\psi(x,0) = \frac{x^\alpha}{\Gamma\left(\alpha+1\right)}. \tag{4.17}$$

If we take the LFRDT of (4.16), we get the following iteration formula:

$$\frac{\Gamma(k\alpha+\alpha+1)}{\Gamma(k\alpha+1)}\Psi_{k+2}(x)=\frac{\partial^{2\alpha}}{\partial x^{2\alpha}}\Psi_k(x)+\sum_{r=0}^{k}\Psi_{k-r}(x)\frac{\partial^{\alpha}}{\partial x^{\alpha}}\Psi_r(x) \tag{4.18}$$

where Ψ_k is the transformed function. From the IC (4.17) we write

$$\Psi_0(x)=\frac{x^{\alpha}}{\Gamma(\alpha+1)} \tag{4.19}$$

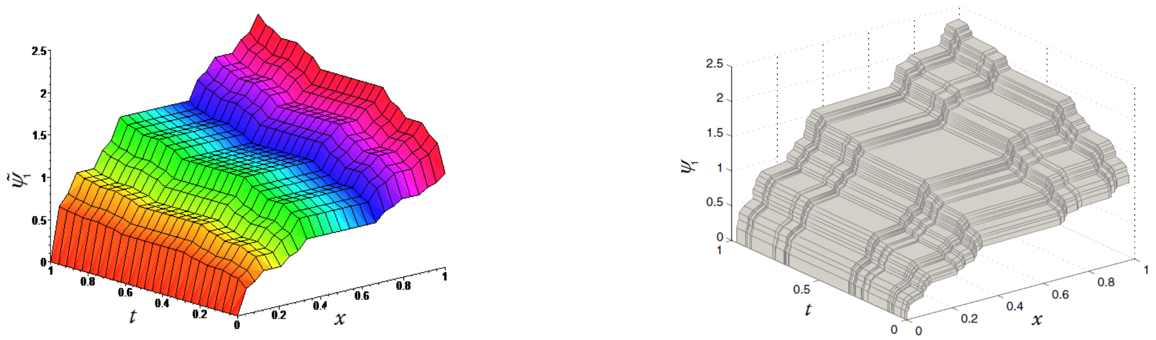
Substituting (4.19) into (4.18), and by iterative steps, we obtain the following $\Psi_k(x)$ values

$$\begin{aligned}\Psi_1(x) &= \frac{1}{\Gamma(\alpha+1)}\frac{x^{\alpha}}{\Gamma(\alpha+1)},\ \Psi_2(x)=\frac{1}{\Gamma(2\alpha+1)}\frac{x^{\alpha}}{\Gamma(\alpha+1)},\\ \Psi_3(x) &= \frac{\left((\Gamma(\alpha+1))^2+\Gamma(2\alpha+1)\right)}{(\Gamma(\alpha+1))^2\Gamma(3\alpha+1)}\frac{x^{\alpha}}{\Gamma(\alpha+1)},\ \cdots\end{aligned} \tag{4.20}$$

From (4.20), we find the LFRDTM solution of equation (4.16) as

$$\tilde{\psi}_n(x,t)=\sum_{k=0}^{n}\Psi_k(x)t^{k\alpha}.$$

Now this approximate solution is compared with the variational iteration method (VIM) result in [48]. The approaches of $\tilde{\psi}_1$, $\tilde{\psi}_2$ and $\tilde{\psi}_4$ are plotted in Fig.4, Fig.5 and Fig.6 respectively.



(a) (b)

Figure 4: Comparison of approximate solution (a) with VIM solution (b) in [48] for (3.16) equation. ($\alpha=\ln 2/\ln 3$).

5. Conclusion

In this study, Local fractional reduced differential transform method (LFRDTM) has been used in linear and non-linear, time and space local fractional PDEs on Cantor set. Then, this new method is applied to four different time and space local PDEs with non-differentiable initial values. In the first three examples, the LFRDTM results with non-differentiable terms are same as the exact solutions with non-differentiable terms. For these examples, 3D graphs of solutions are plotted in Figs.1–3 to show the behavior of the methods respectively. Also, in the last example, our approximate solution with non-differentiable terms is compared with the result obtained by VIM in [48]. For this example, the comparisons are plotted in 3D graphs of solutions in Figs. 4–6. The results show that it is easier to make calculations with LFRDTM, because it doesn't include integrals like local fractional VIM, ADM and HAM. In addition, LFRDTM technique does not require any discretization, linearization or small perturbations and therefore it reduces significantly the numerical computation. Hence, it can be said that LFRDTM is very powerful and easy applicable mathematical tool for linear and non-linear PDEs with non-differentiable terms on Cantor set.

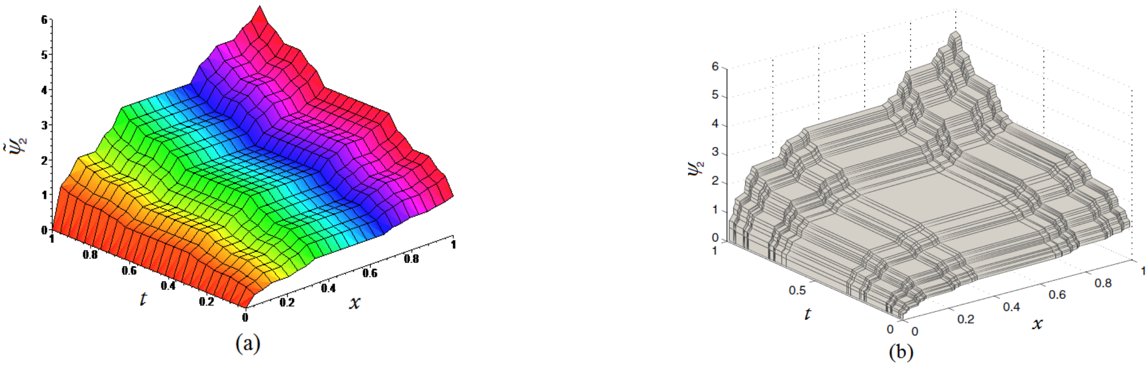


Figure 5: Comparison of approximate solution (a) with VIM solution (b) in [48] for (3.16) equation. ($\alpha = \ln 2/\ln 3$).

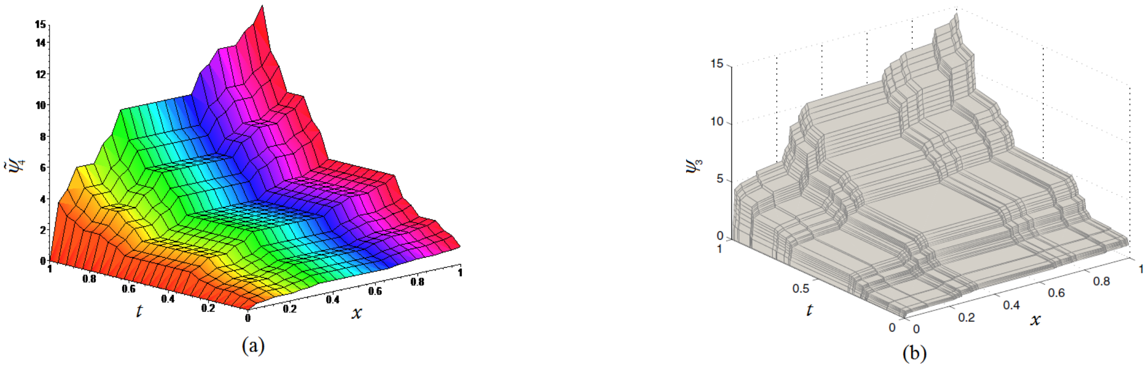


Figure 6: Comparison of approximate solution (a) with VIM solution (b) in [48] for (3.16) equation. ($\alpha = \ln 2/\ln 3$).

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