TRANSFORMER-BASED PHYSICS INFORMED NEURAL NETWORKS (PINN) FOR FLUID DYNAMICS

UTILIZING ATTENTION MECHANISMS FOR SIMULATING UNSTEADY FLOW OVER A CYLINDRICAL OBJECT.

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ABSTRACT

Current approaches to simulating fluid dynamics rely largely on numerical methods that are often computationally expensive and time-consuming. Recent work has explored adapting Physics-Informed Neural Networks (PINNs) [1] for approximating solutions to governing equations of fluid dynamics and other physical phenomena. In this work, we explore PINNsFormer, a Transformer-based PINN, to simulate unsteady and turbulent fluid flow over a cylindrical object.

***Index Terms*—** Deep Learning, PINNs, Attention, Navier-Stokes Equation

1. INTRODUCTION

The Navier–Stokes equations govern the motion of fluid particles as they evolve through space and time. The system consists of conservation of mass and conservation of momentum:

$$\nabla \cdot U = 0 \tag{1}$$

$$\rho \left[\frac{\partial U}{\partial t} + (U \cdot \nabla) U \right] = -\nabla p + \mu \nabla^2 U + \rho g. \tag{2}$$

These equations describe incompressible flow. Flow behavior is characterized using the Reynolds number, a dimensionless quantity describing the ratio of inertial to viscous forces.

PINNs use automatic differentiation to compute residuals of the governing equations directly during training. This avoids mesh-based discretization used in classical methods such as FEM [2]. We explore PINNsFormer [3] for the Unsteady Reynolds-Averaged Navier–Stokes (URANS) equations over a cylinder at $\mathrm{Re} = 3900$.

2. METHOD

2.1. Architecture

PINNsFormer consists of a Spatio-Temporal Mixer, an encoder–decoder Transformer module, and an output head. Inputs are projected to a higher dimension before entering the encoder. Unlike the standard Transformer [4], PINNsFormer removes decoder self-attention and uses only encoder self-attention and encoder–decoder cross-attention.

PINNsFormer also introduces the *Wavelet* activation for improved approximation of Fourier-like structures.

2.2. Dataset

Large Eddy Simulation (LES) data of flow over a circular cylinder at $\mathrm{Re} = 3900$ and $\mathrm{Re} = 280$ were used. The dataset provides velocity (u, v), pressure p, and turbulent viscosity ν_t over a 40-second sequence with time step 0.25 seconds [5]. Raw text files lacked spatial coordinates, so preprocessing aligned each sample with its (x, y) position and stored the result in HDF5 format.

2.3. Governing Equations

The non-dimensional URANS equations are imposed using PyTorch automatic differentiation.

$$\nu_t = \frac{\mu_t}{\mu}, \qquad \nu_{\text{eff}} = 1 + \nu_t. \tag{3}$$

$$\nabla \cdot \mathbf{u} = 0, \tag{4}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p^* + \frac{1}{Re}\nabla^2 \mathbf{u} + \frac{1}{Re}\nabla \cdot (\nu_t \nabla \mathbf{u}). \tag{5}$$

2.4. Loss Function

The total loss:

$$\mathcal{L} = \mathcal{L}_{\text{sup}} + \mathcal{L}_{\text{phys}}. \tag{6}$$

Supervised loss:

$$\mathcal{L}_{\text{sup}} = \frac{1}{N} \sum_{i=1}^{N} \left(\|\mathbf{U}_{\text{pred}}^{(i)} - \mathbf{U}_{\text{target}}^{(i)}\|^2 + (p_{\text{pred}}^{(i)} - p_{\text{target}}^{(i)})^2 \right). \tag{7}$$

Physics residual loss:

$$\mathcal{L}_{\text{phys}} = \frac{1}{N} \sum_{i=1}^{N} \left(f_u^{(i)2} + f_v^{(i)2} \right). \tag{8}$$

Some author footnote.

Diffusion terms:

$$\mathcal{D}_u = \nu_{\text{eff}}(u_{xx} + u_{yy}) + (\nu_{\text{eff}})_x u_x + (\nu_{\text{eff}})_y u_y, \tag{9}$$
$$\mathcal{D}_v = \nu_{\text{eff}}(v_{xx} + v_{yy}) + (\nu_{\text{eff}})_x v_x + (\nu_{\text{eff}})_y v_y. \tag{10}$$

Velocity from stream function:

$$u = \psi_y, \qquad v = -\psi_x. \tag{11}$$

3. RESULTS

3.1. Training Behavior

Figure 1 shows the supervised and physics loss components decreasing smoothly during training, indicating stable convergence.

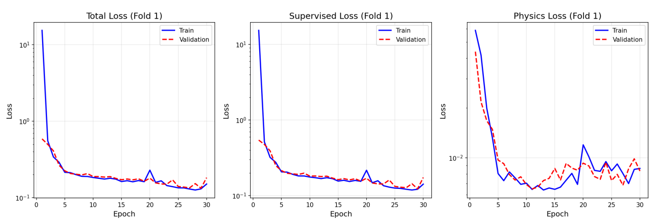

Fig. 1. Training and validation loss curves for PINNsFormer.

3.2. Vorticity Field Reconstruction

The vorticity field,

$$\omega = v_x - u_y,$$

highlights wake structure, vortex shedding, and coherent patterns. Figure 2 compares model-predicted vorticity with LES reference data.

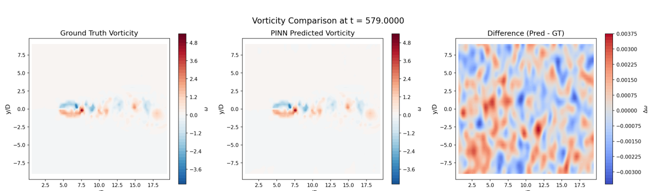

Fig. 2. Predicted vs. ground-truth vorticity fields (representative time slice).

3.3. Velocity Probe Time-Series

Velocity probes at $(x/D, y/D) = (1.0, 0.3)$ and $(2.0, 0.0)$ measure temporal accuracy. Figure 3 shows that PINNsFormer accurately follows both phase and amplitude of vortex-shedding oscillations.

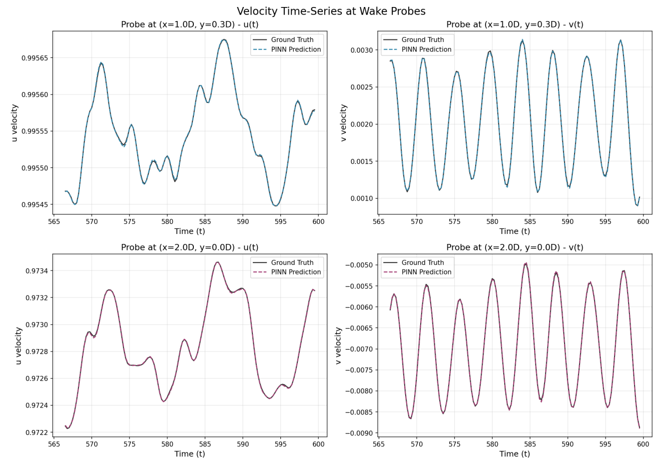

Fig. 3. Predicted vs. ground-truth velocity probe time-series.

4. ILLUSTRATIONS, GRAPHS, AND PHOTOGRAPHS

5. FOOTNOTES

6. ACKNOWLEDGMENTS

7. REFERENCES

[1] Maziar Raissi, Paris Perdikaris, and George E Karniadakis, "Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations," *Journal of Computational Physics*, vol. 378, pp. 686–707, 2019.

[2] C.D. Jones, A.B. Smith, and E.F. Roberts, "Paper title," in *Proceedings Title*. IEEE, 2003, vol. II, pp. 803–806.

[3] Leo Zhao, Xueying Ding, and B. Aditya Prakash, "PINNsFormer: A Transformer-Based Framework For Physics-Informed Neural Networks," *arXiv preprint arXiv:2307.11833*, 2023.

[4] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Lukasz Kaiser, and Illia Polosukhin, "Attention is all you need," in *Advances in neural information processing systems*, 2017, vol. 30.

[5] Laurence WALLIAN, Pierre JACQUET, and Valentin RESSEGUIER, "OpenFOAM data for a circular cylinder wake flow (Re=3900)," 2025.