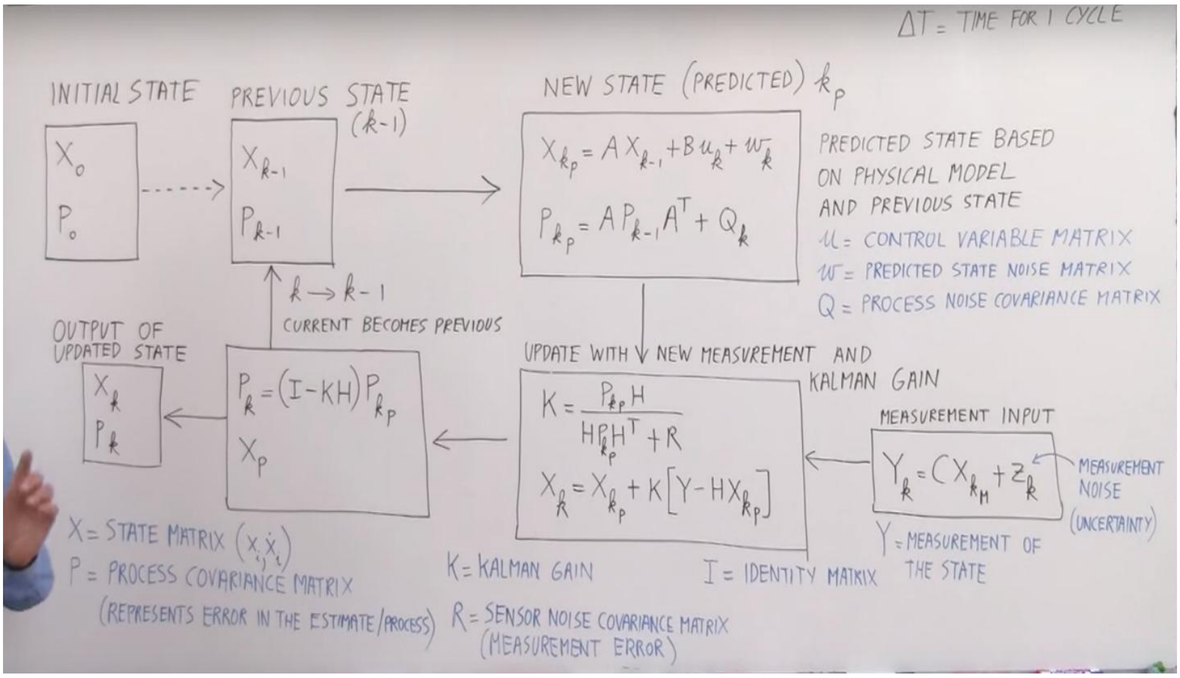
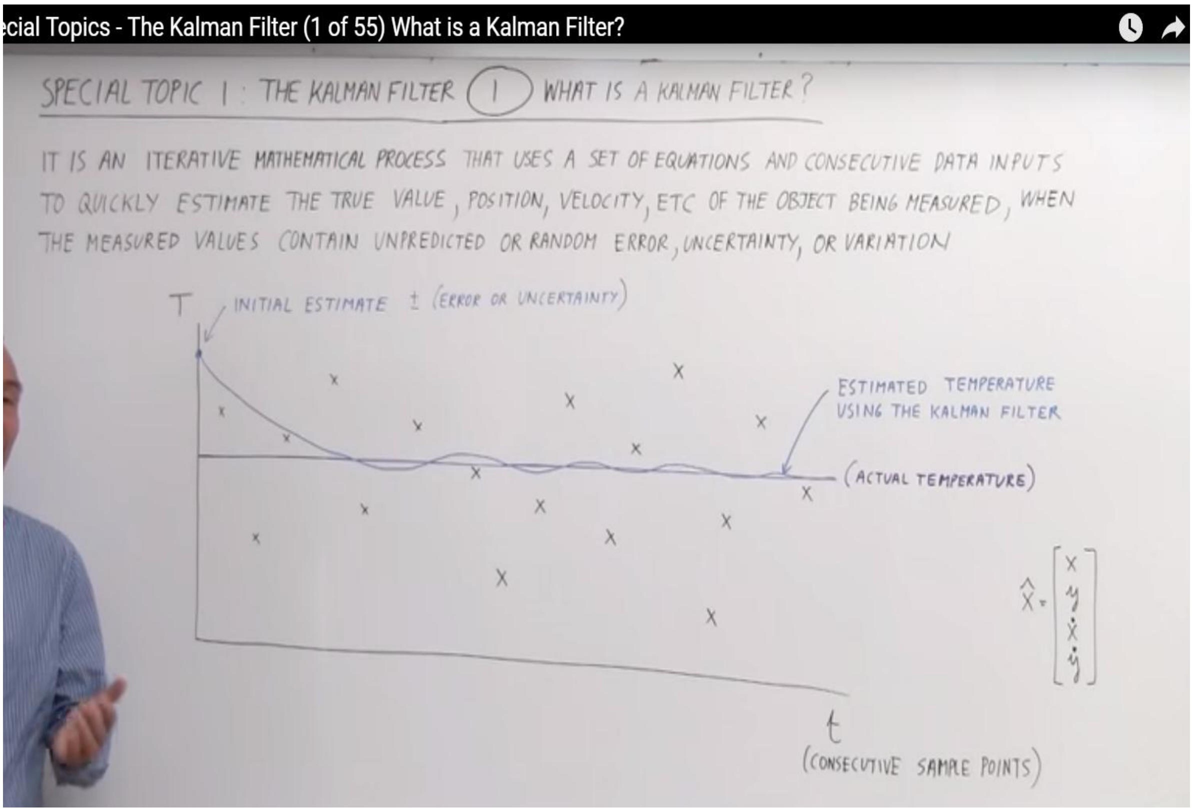
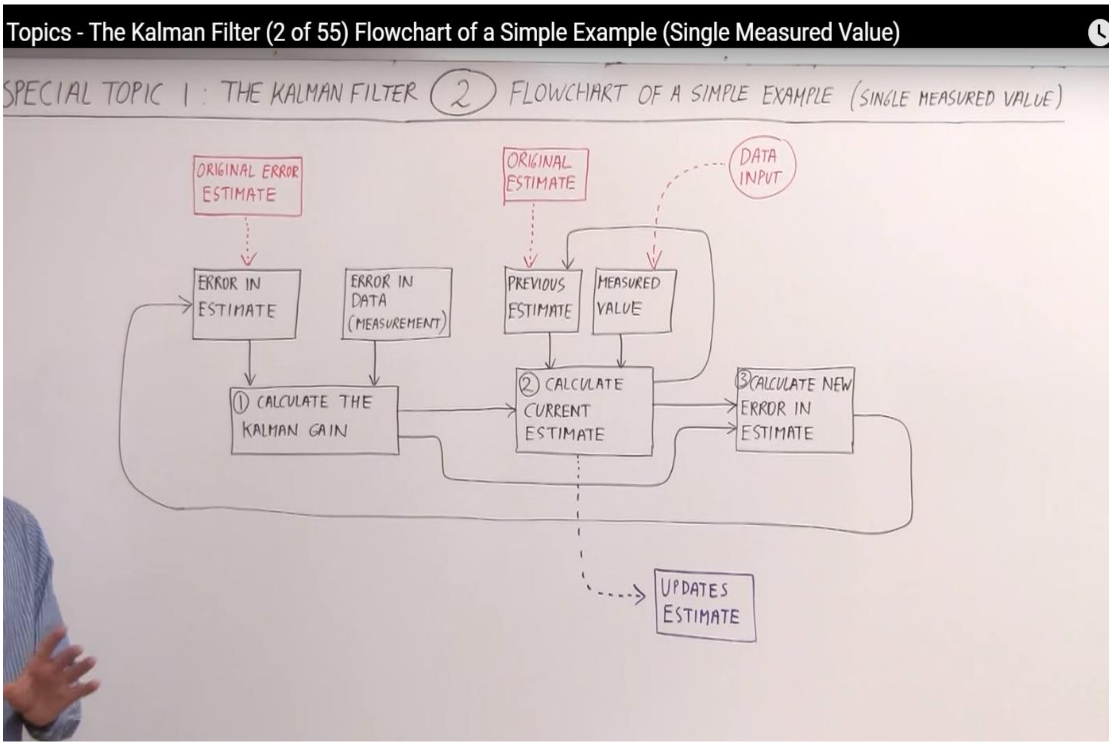
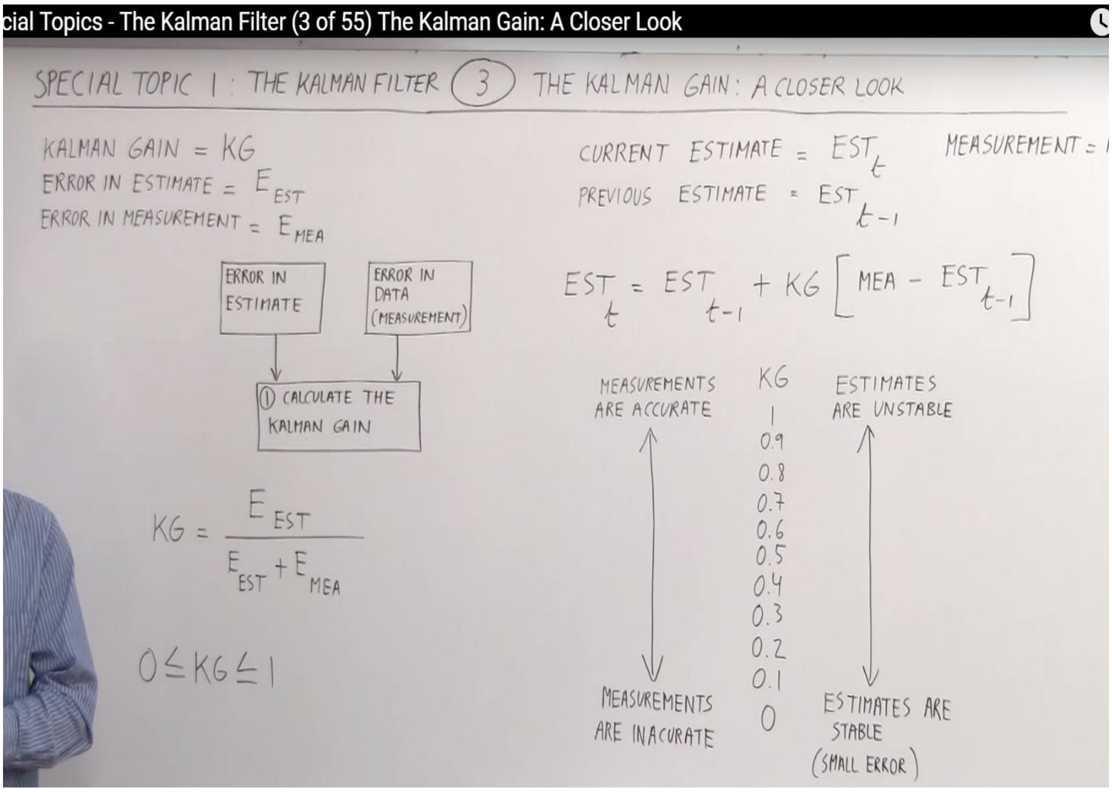
Special Topics - The Kalman Filter

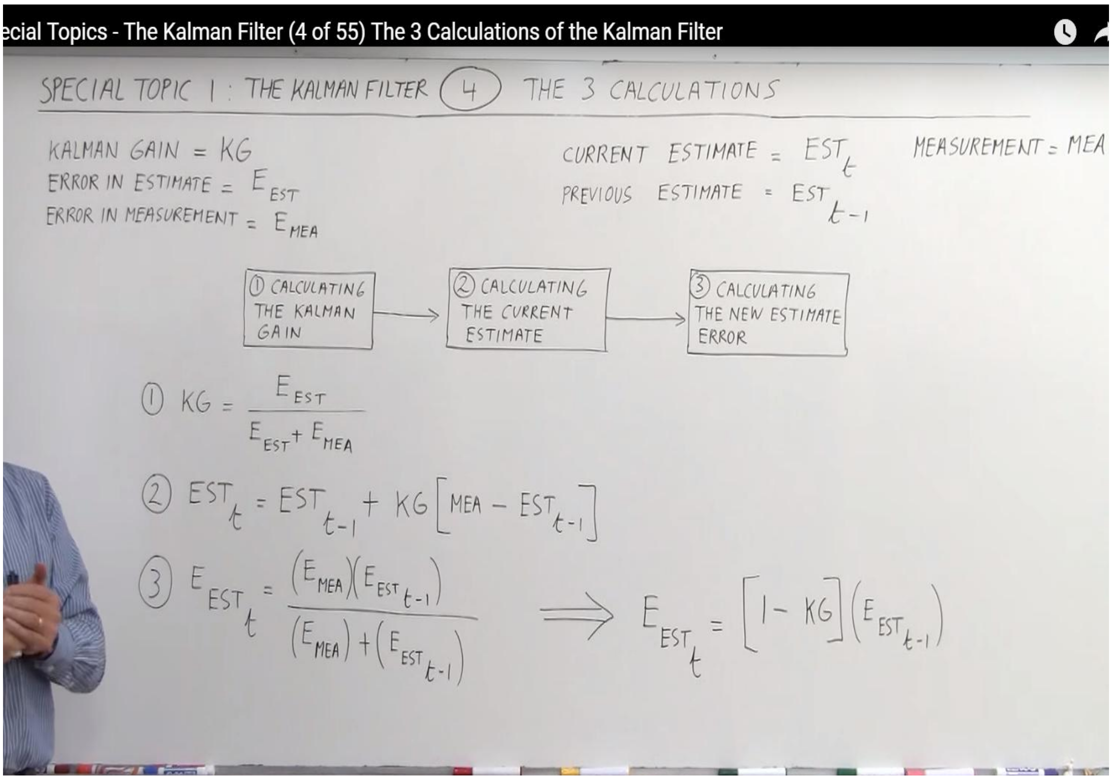
Website: https://www.youtube.com/channel/UCiGxYawhEp4QyFcX0R60YdQ

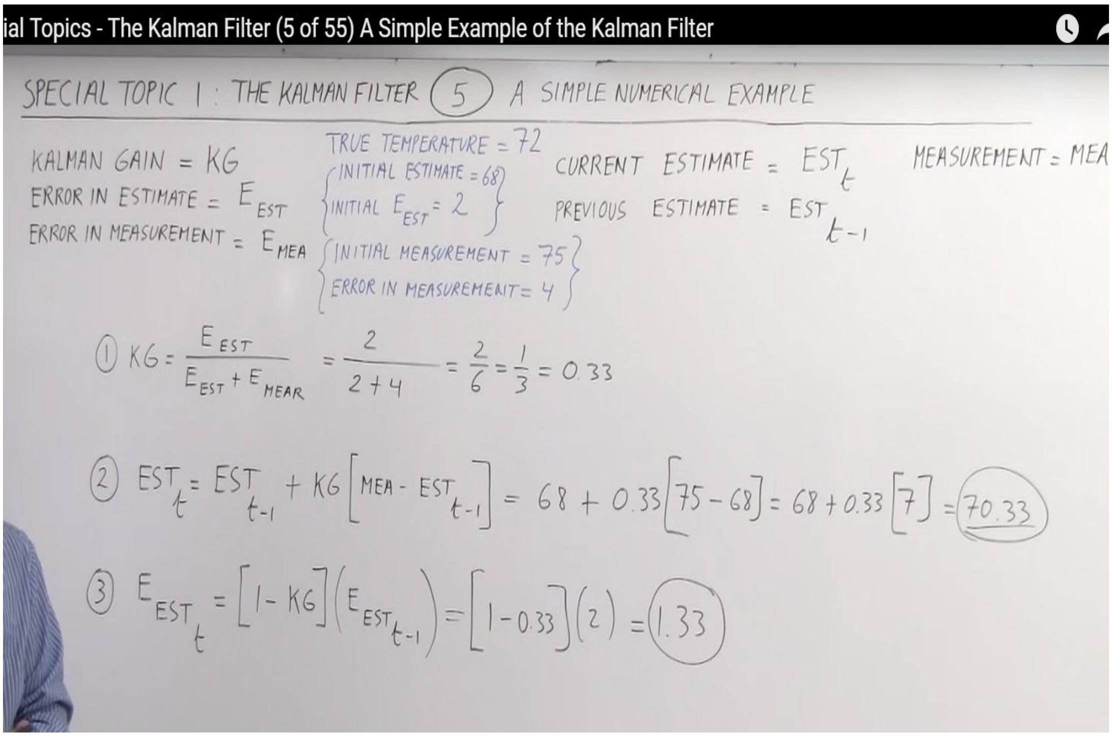


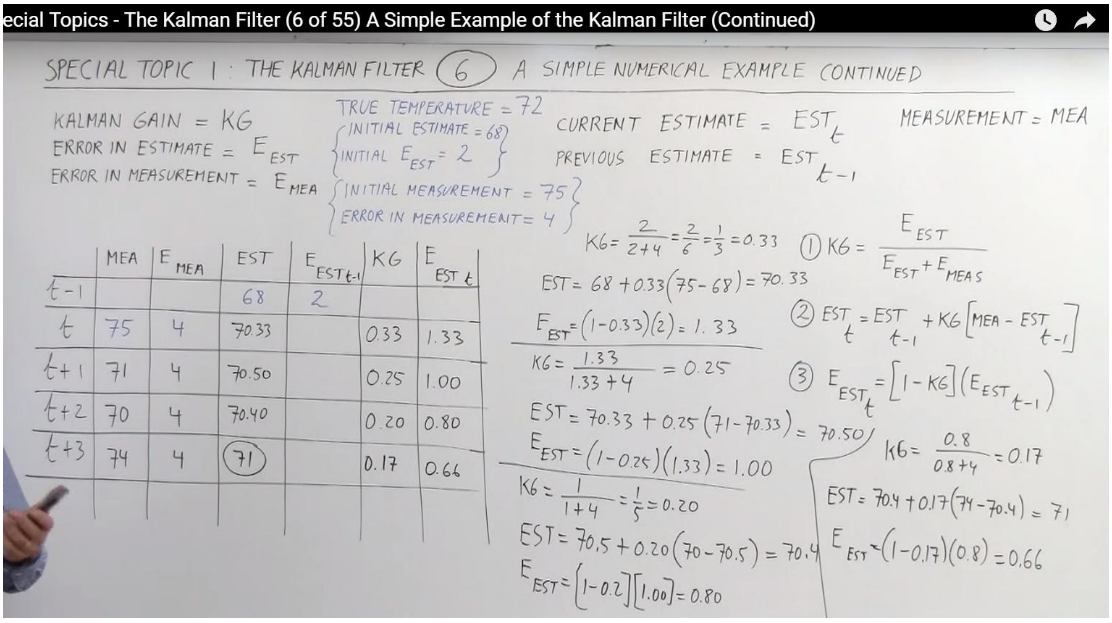


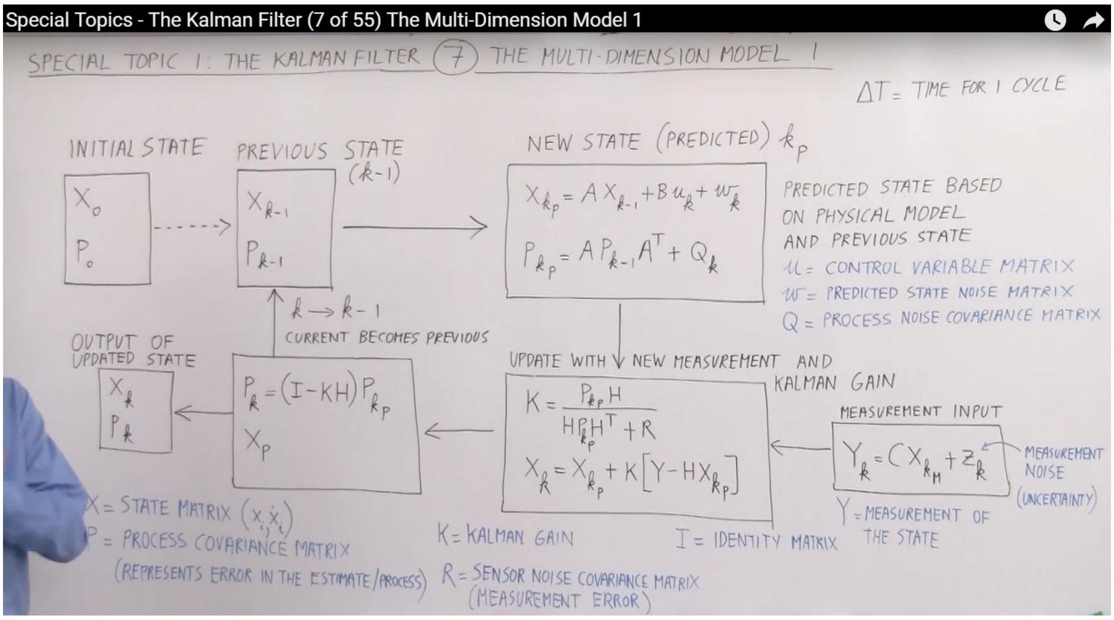

Topics - The Kalman Filter (2 of 55) Flowchart of a Simple Example (Single Measured Value)
SPECIAL TOPIC 1 : THE KALMAN FILTER (2) FLOWCHART OF A SIMPLE EXAMPLE (SINGLE MEASURED VALUE)
ORIGINAL ERROR ESTIMATE
ORIGINAL ESTIMATE
DATA INPUT
ERROR IN ESTIMATE
ERROR IN DATA (MEASUREMENT)
PREVIOUS ESTIMATE
MEASURED VALUE
(1) CALCULATE THE KALMAN GAIN
(2) CALCULATE CURRENT ESTIMATE
(3) CALCULATE NEW ERROR IN ESTIMATE
UPDATES ESTIMATE

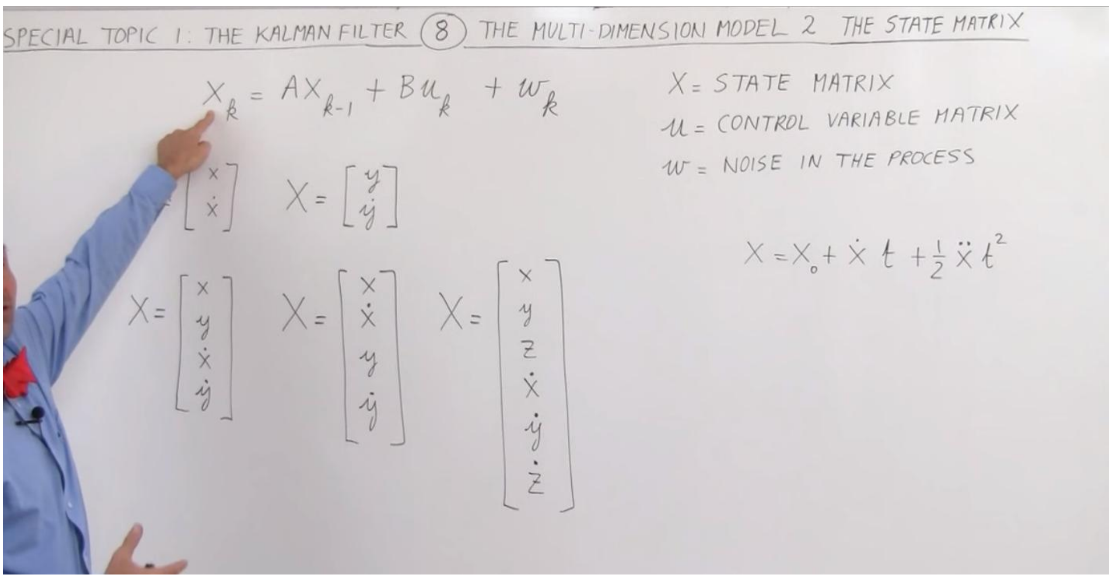

cial Topics - The Kalman Filter (3 of 55) The Kalman Gain: A Closer Look
SPECIAL TOPIC 1 : THE KALMAN FILTER (3) THE KALMAN GAIN: A CLOSER LOOK
KALMAN GAIN = KG
ERROR IN ESTIMATE = E_{EST}
ERROR IN MEASUREMENT = E_{MEA}
CURRENT ESTIMATE = EST_t
PREVIOUS ESTIMATE = EST_{t-1}
MEASUREMENT =
ERROR IN ESTIMATE
ERROR IN DATA (MEASUREMENT)
(1) CALCULATE THE KALMAN GAIN
$KG = \frac{E_{EST}}{E_{EST} + E_{MEA}}$
$0 \leq KG \leq 1$
$EST_t = EST_{t-1} + KG\,[MEA - EST_{t-1}]$
MEASUREMENTS ARE ACCURATE
KG
ESTIMATES ARE UNSTABLE
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
MEASUREMENTS ARE INACURATE
ESTIMATES ARE STABLE (SMALL ERROR)

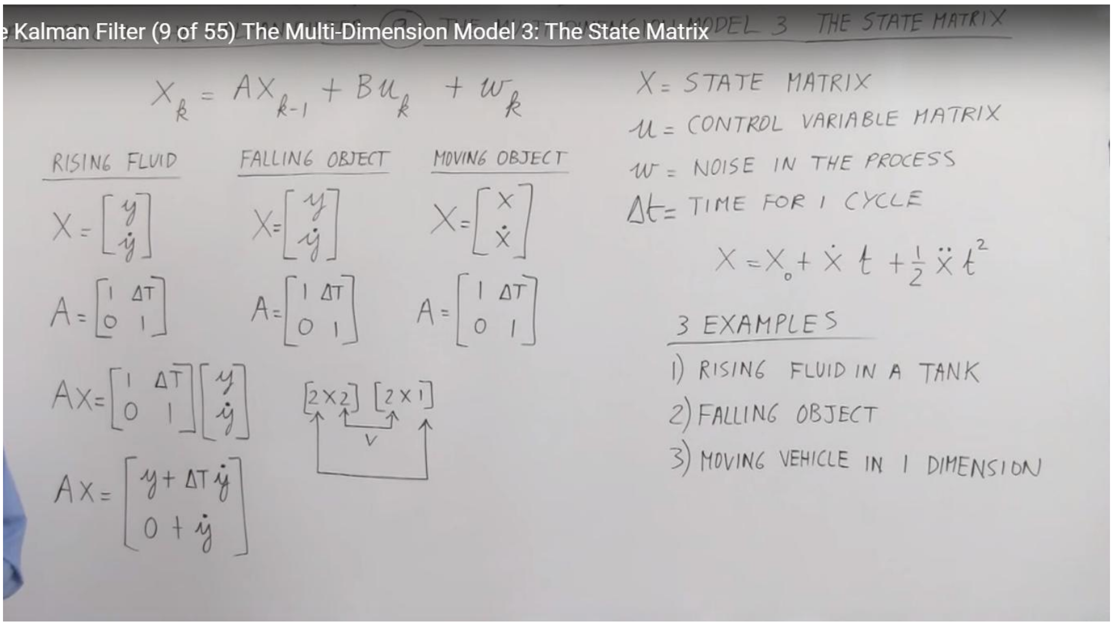

ecial Topics - The Kalman Filter (4 of 55) The 3 Calculations of the Kalman Filter
SPECIAL TOPIC 1 : THE KALMAN FILTER (4) THE 3 CALCULATIONS
KALMAN GAIN = KG
ERROR IN ESTIMATE = E_{EST}
ERROR IN MEASUREMENT = E_{MEA}
CURRENT ESTIMATE = EST_t
PREVIOUS ESTIMATE = EST_{t-1}
MEASUREMENT = MEA
(1) CALCULATING THE KALMAN GAIN
(2) CALCULATING THE CURRENT ESTIMATE
(3) CALCULATING THE NEW ESTIMATE ERROR
(1) $KG = \frac{E_{EST}}{E_{EST} + E_{MEA}}$
(2) $EST_t = EST_{t-1} + KG\left[MEA - EST_{t-1}\right]$
(3) $E_{EST_t} = \frac{(E_{MEA})(E_{EST_{t-1}})}{(E_{MEA}) + (E_{EST_{t-1}})} \Longrightarrow E_{EST_t} = \left[1 - KG\right]\left(E_{EST_{t-1}}\right)$

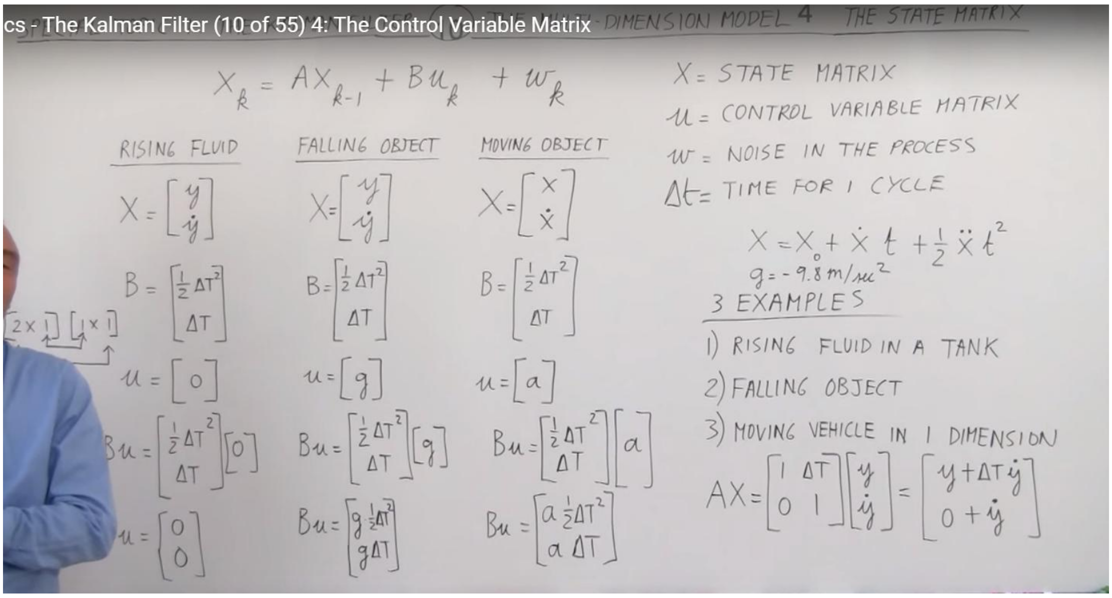

ial Topics - The Kalman Filter (5 of 55) A Simple Example of the Kalman Filter
SPECIAL TOPIC 1 : THE KALMAN FILTER (5) A SIMPLE NUMERICAL EXAMPLE
KALMAN GAIN = KG
ERROR IN ESTIMATE = E_{EST}
ERROR IN MEASUREMENT = E_{MEA}
TRUE TEMPERATURE = 72
INITIAL ESTIMATE = 68
INITIAL E_{EST} = 2
INITIAL MEASUREMENT = 75
ERROR IN MEASUREMENT = 4
CURRENT ESTIMATE = EST_t
PREVIOUS ESTIMATE = EST_{t-1}
MEASUREMENT = MEA
(1) $KG = \frac{E_{EST}}{E_{EST} + E_{MEAR}} = \frac{2}{2+4} = \frac{2}{6} = \frac{1}{3} = 0.33$
(2) $EST_t = EST_{t-1} + KG\left[MEA - EST_{t-1}\right] = 68 + 0.33\left[75 - 68\right] = 68 + 0.33\left[7\right] = 70.33$
(3) $E_{EST_t} = \left[1 - KG\right]\left(E_{EST_{t-1}}\right) = \left[1 - 0.33\right](2) = 1.33$

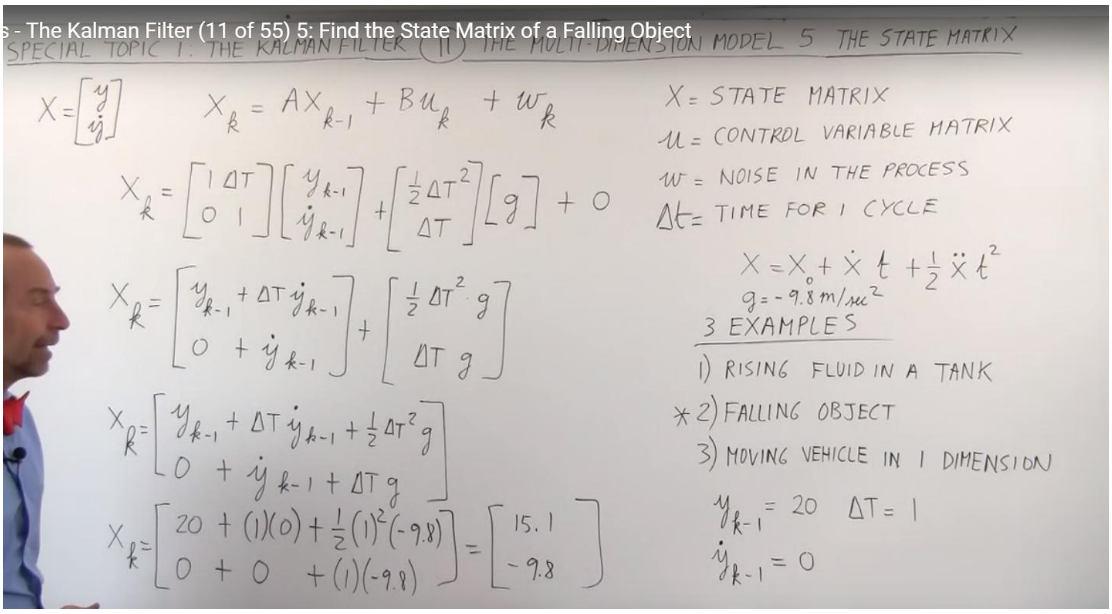

ecial Topics - The Kalman Filter (6 of 55) A Simple Example of the Kalman Filter (Continued)
SPECIAL TOPIC 1 : THE KALMAN FILTER 6 A SIMPLE NUMERICAL EXAMPLE CONTINUED
KALMAN GAIN = KG
ERROR IN ESTIMATE = E EST
ERROR IN MEASUREMENT = E MEA
TRUE TEMPERATURE = 72
INITIAL ESTIMATE = 68
INITIAL E EST = 2
INITIAL MEASUREMENT = 75
ERROR IN MEASUREMENT = 4
CURRENT ESTIMATE = EST t
PREVIOUS ESTIMATE = EST t-1
MEASUREMENT = MEA
MEA | E MEA | EST | E EST t-1 | KG | E EST t
t-1 | | | 68 | 2 | |
t | 75 | 4 | 70.33 | | 0.33 | 1.33
t+1 | 71 | 4 | 70.50 | | 0.25 | 1.00
t+2 | 70 | 4 | 70.40 | | 0.20 | 0.80
t+3 | 74 | 4 | 71 | | 0.17 | 0.66
KG = 2/(2+4) = 2/6 = 1/3 = 0.33
EST = 68 + 0.33(75 − 68) = 70.33
E EST = (1 − 0.33)(2) = 1.33
KG = 1.33/(1.33 + 4) = 0.25
EST = 70.33 + 0.25(71 − 70.33) = 70.50
E EST = (1 − 0.25)(1.33) = 1.00
KG = 1/(1+4) = 1/5 = 0.20
EST = 70.5 + 0.20(70 − 70.5) = 70.4
E EST = [1 − 0.2][1.00] = 0.80
1 KG = E EST / (E EST + E MEA)
2 EST t = EST t-1 + KG [MEA − EST t-1]
3 E EST t = [1 − KG](E EST t-1)
KG = 0.8/(0.8 + 4) = 0.17
EST = 70.4 + 0.17(74 − 70.4) = 71
E EST = (1 − 0.17)(0.8) = 0.66

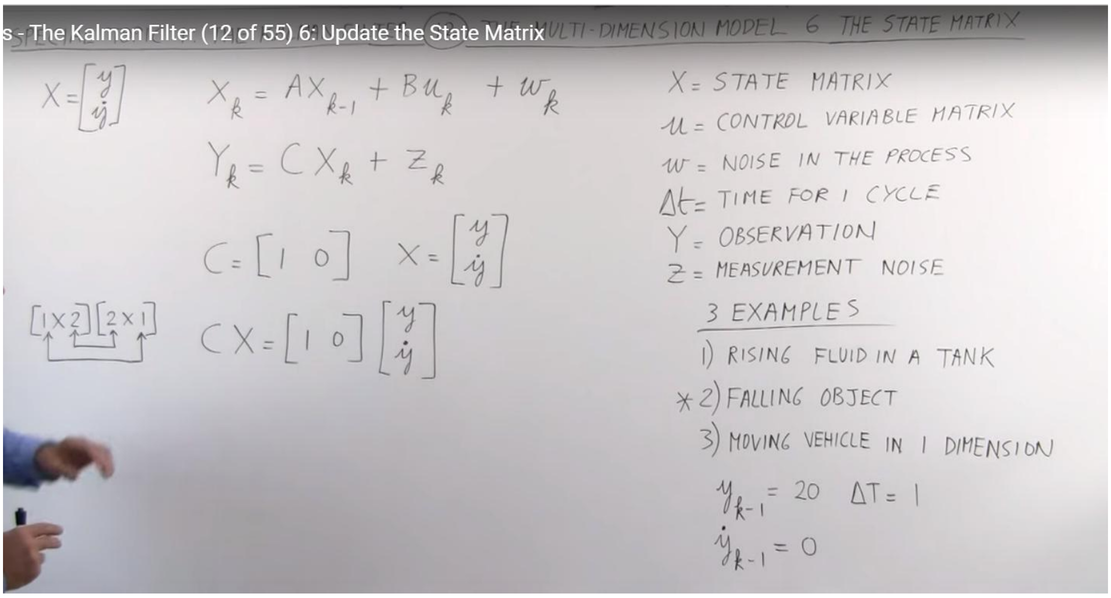

Special Topics - The Kalman Filter (7 of 55) The Multi-Dimension Model 1
SPECIAL TOPIC 1: THE KALMAN FILTER 7 THE MULTI-DIMENSION MODEL 1
ΔT = TIME FOR 1 CYCLE
INITIAL STATE
X0
P0
PREVIOUS STATE (k−1)
X k−1
P k−1
NEW STATE (PREDICTED) kp
X kp = A X k−1 + B u k + w k
P kp = A P k−1 A^T + Q k
PREDICTED STATE BASED ON PHYSICAL MODEL AND PREVIOUS STATE
u = CONTROL VARIABLE MATRIX
w = PREDICTED STATE NOISE MATRIX
Q = PROCESS NOISE COVARIANCE MATRIX
k → k−1
CURRENT BECOMES PREVIOUS
OUTPUT OF UPDATED STATE
X k
P k
P k = (I − KH) P kp
X p
UPDATE WITH NEW MEASUREMENT AND KALMAN GAIN
K = P kp H / (H P kp H^T + R)
X k = X kp + K[Y − H X kp]
MEASUREMENT INPUT
Y k = C X kM + Z k
MEASUREMENT NOISE (UNCERTAINTY)
Y = MEASUREMENT OF THE STATE
X = STATE MATRIX (x, ẋ)
P = PROCESS COVARIANCE MATRIX (REPRESENTS ERROR IN THE ESTIMATE/PROCESS)
K = KALMAN GAIN
I = IDENTITY MATRIX
R = SENSOR NOISE COVARIANCE MATRIX (MEASUREMENT ERROR)

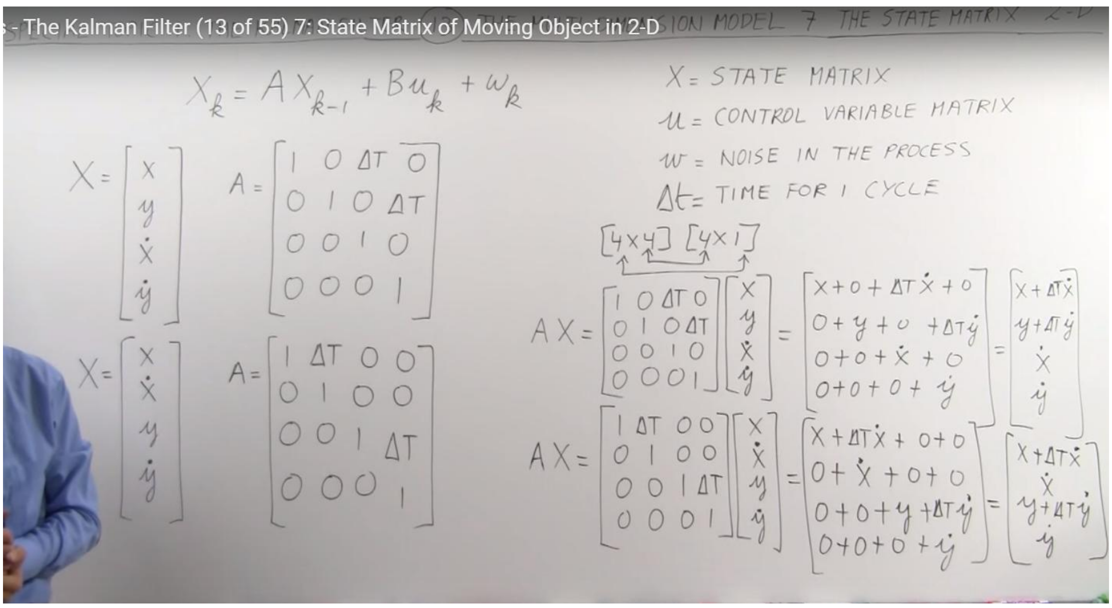

SPECIAL TOPIC 1: THE KALMAN FILTER 8 THE MULTI-DIMENSION MODEL 2 THE STATE MATRIX
X k = A X k−1 + B u k + w k
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
X = [y ẏ]
X = [x y ẋ ẏ]
X = [x ẋ y ẏ]
X = [x y z ẋ ẏ ż]
X = X0 + ẋ t + ½ ẍ t²

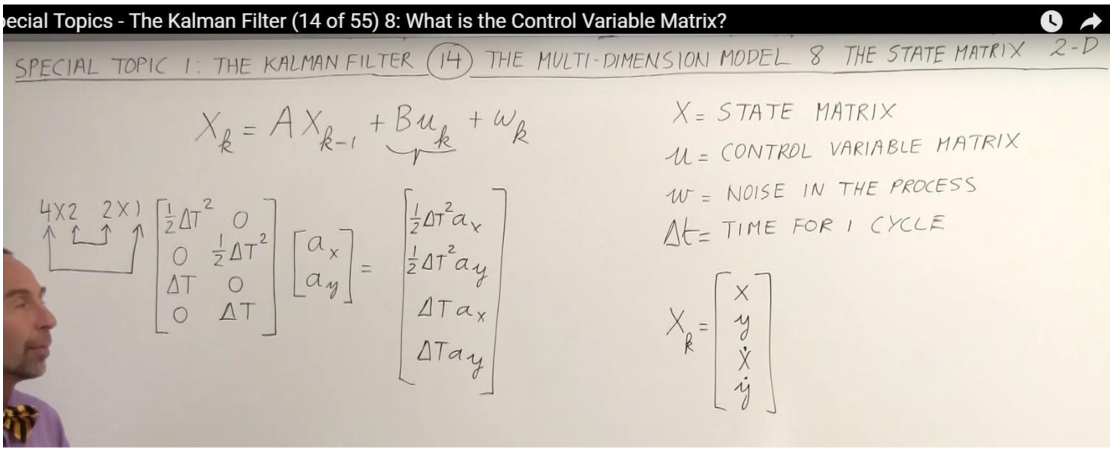

Kalman Filter (9 of 55) The Multi-Dimension Model 3: The State Matrix
THE STATE MATRIX
X_k = AX_{k-1} + Bu_k + w_k
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
RISING FLUID
FALLING OBJECT
MOVING OBJECT
X = [y ẏ]
X = [y ẏ]
X = [x ẋ]
A = [1 ΔT; 0 1]
A = [1 ΔT; 0 1]
A = [1 ΔT; 0 1]
X = X_0 + ẋ t + ½ ẍ t²
3 EXAMPLES
1) RISING FLUID IN A TANK
2) FALLING OBJECT
3) MOVING VEHICLE IN 1 DIMENSION
AX = [1 ΔT; 0 1][y ẏ]
[2×2] [2×1]
AX = [y + ΔT ẏ; 0 + ẏ]

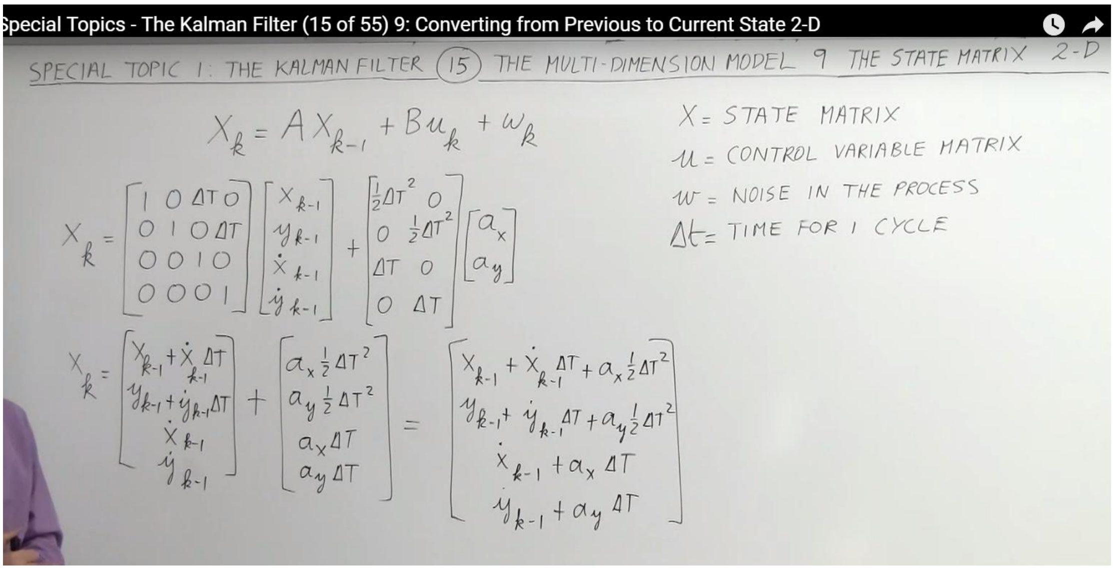

cs - The Kalman Filter (10 of 55) 4: The Control Variable Matrix
DIMENSION MODEL 4
THE STATE MATRIX
X_k = AX_{k-1} + Bu_k + w_k
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
RISING FLUID
FALLING OBJECT
MOVING OBJECT
X = [y ẏ]
X = [y ẏ]
X = [x ẋ]
X = X_0 + ẋ t + ½ ẍ t²
g = -9.8 m/s²
3 EXAMPLES
1) RISING FLUID IN A TANK
2) FALLING OBJECT
3) MOVING VEHICLE IN 1 DIMENSION
[2×1] [1×1]
B = [½ΔT²; ΔT]
B = [½ΔT²; ΔT]
B = [½ΔT²; ΔT]
u = [0]
u = [g]
u = [a]
Bu = [½ΔT²; ΔT][0]
Bu = [½ΔT²; ΔT][g]
Bu = [½ΔT²; ΔT][a]
AX = [1 ΔT; 0 1][y ẏ] = [y + ΔT ẏ; 0 + ẏ]
u = [0; 0]
Bu = [g ½ΔT²; gΔT]
Bu = [a ½ΔT²; a ΔT]

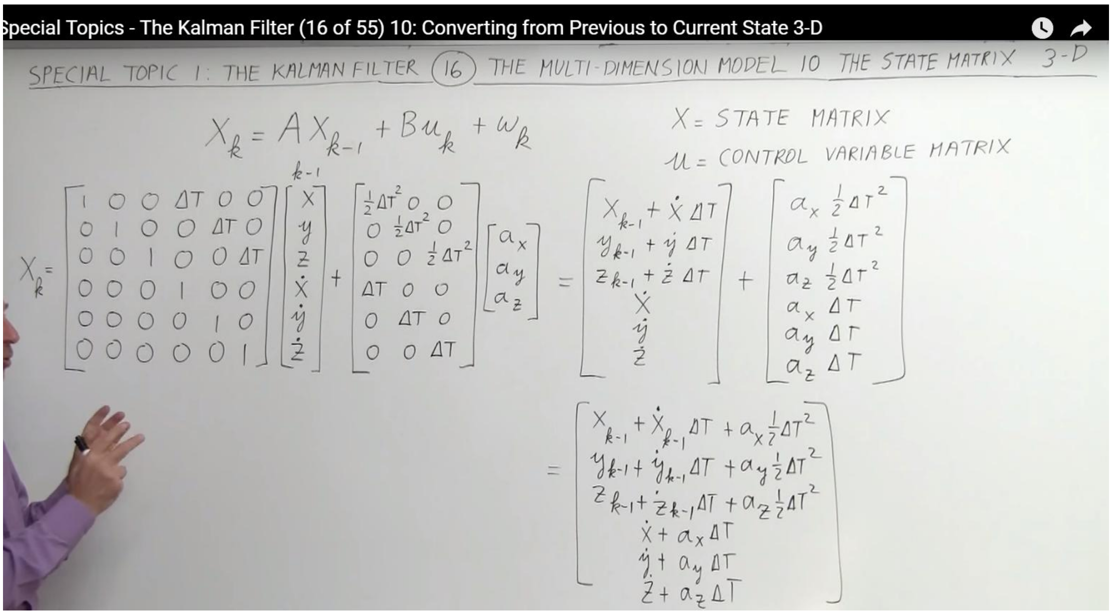

s - The Kalman Filter (11 of 55) 5: Find the State Matrix of a Falling Object
SPECIAL TOPIC 1: THE KALMAN FILTER (11) THE MULTI-DIMENSION MODEL 5 THE STATE MATRIX
$X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$
$X_k = AX_{k-1} + Bu_k + w_k$
$X_k = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix} [g] + 0$
$X_k = \begin{bmatrix} y_{k-1} + \Delta T \dot{y}_{k-1} \\ 0 + \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta T^2 \cdot g \\ \Delta T g \end{bmatrix}$
$X_k = \begin{bmatrix} y_{k-1} + \Delta T \dot{y}_{k-1} + \frac{1}{2}\Delta T^2 g \\ 0 + \dot{y}_{k-1} + \Delta T g \end{bmatrix}$
$X_k = \begin{bmatrix} 20 + (1)(0) + \frac{1}{2}(1)^2(-9.8) \\ 0 + 0 + (1)(-9.8) \end{bmatrix} = \begin{bmatrix} 15.1 \\ -9.8 \end{bmatrix}$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
$X = X_0 + \dot{X} t + \frac{1}{2} \ddot{X} t^2$
$g = -9.8\,m/sec^2$
3 EXAMPLES
1) RISING FLUID IN A TANK
* 2) FALLING OBJECT
3) MOVING VEHICLE IN 1 DIMENSION
$y_{k-1} = 20 \quad \Delta T = 1$
$\dot{y}_{k-1} = 0$

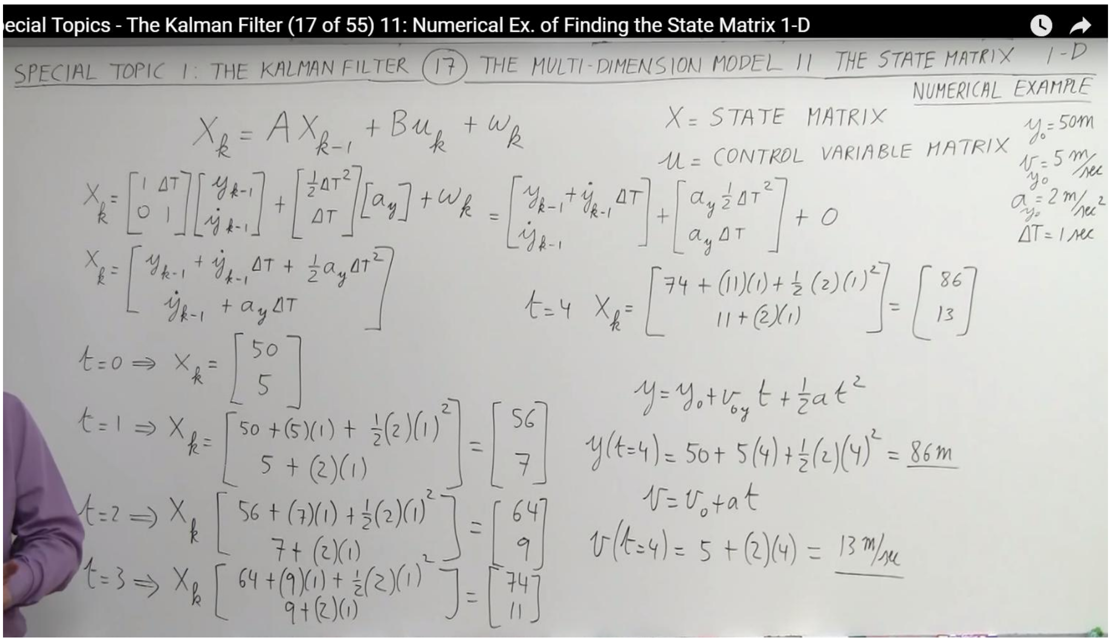

s - The Kalman Filter (12 of 55) 6: Update the State Matrix
MULTI-DIMENSION MODEL 6 THE STATE MATRIX
$X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$
$X_k = AX_{k-1} + Bu_k + w_k$
$Y_k = CX_k + Z_k$
$C = [1 \;\; 0] \quad X = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$
$[1 \times 2][2 \times 1]$
$CX = [1 \;\; 0] \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
Y = OBSERVATION
Z = MEASUREMENT NOISE
3 EXAMPLES
1) RISING FLUID IN A TANK
* 2) FALLING OBJECT
3) MOVING VEHICLE IN 1 DIMENSION
$y_{k-1} = 20 \quad \Delta T = 1$
$\dot{y}_{k-1} = 0$

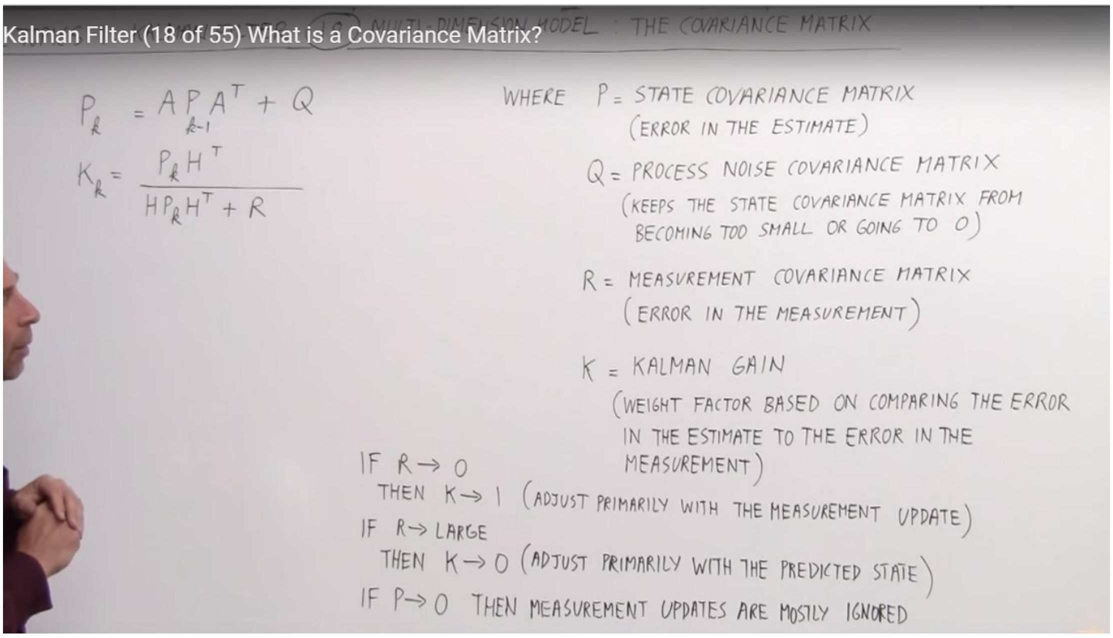

s - The Kalman Filter (13 of 55) 7: State Matrix of Moving Object in 2-D
ION MODEL 7 THE STATE MATRIX
$X_k = AX_{k-1} + Bu_k + w_k$
$X = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
$X = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} \quad A = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix}$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
$[4 \times 4][4 \times 1]$
$AX = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x + 0 + \Delta T \dot{x} + 0 \\ 0 + y + 0 + \Delta T \dot{y} \\ 0 + 0 + \dot{x} + 0 \\ 0 + 0 + 0 + \dot{y} \end{bmatrix} = \begin{bmatrix} x + \Delta T \dot{x} \\ y + \Delta T \dot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix}$
$AX = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x + \Delta T \dot{x} + 0 + 0 \\ 0 + \dot{x} + 0 + 0 \\ 0 + 0 + y + \Delta T \dot{y} \\ 0 + 0 + 0 + \dot{y} \end{bmatrix} = \begin{bmatrix} x + \Delta T \dot{x} \\ \dot{x} \\ y + \Delta T \dot{y} \\ \dot{y} \end{bmatrix}$

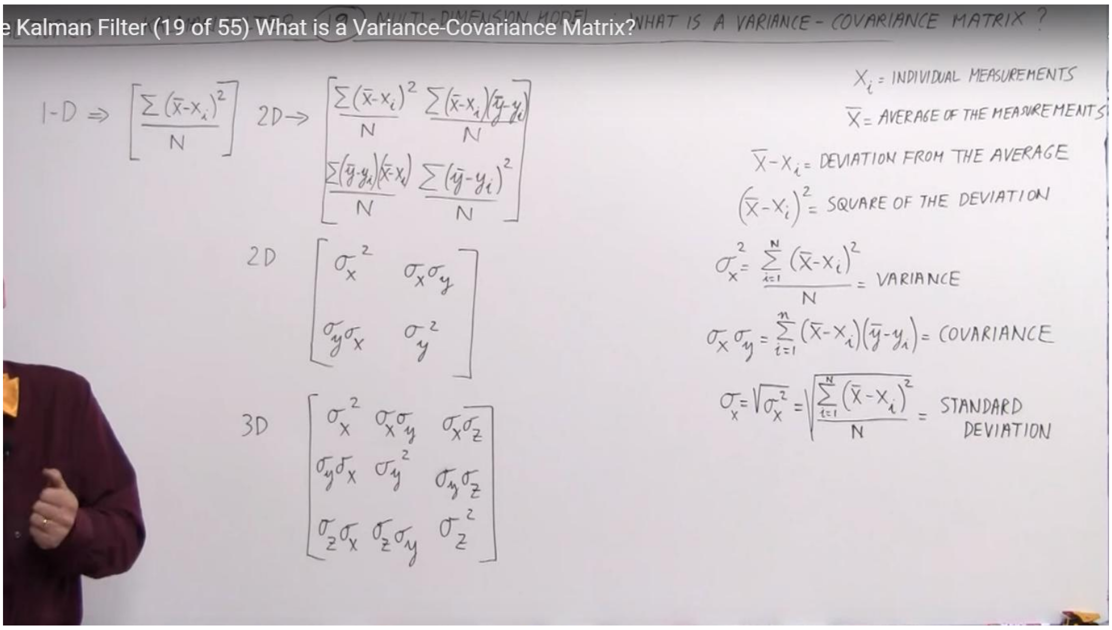

ecial Topics - The Kalman Filter (14 of 55) 8: What is the Control Variable Matrix?
SPECIAL TOPIC 1: THE KALMAN FILTER (14) THE MULTI-DIMENSION MODEL 8 THE STATE MATRIX 2-D
$X_k = AX_{k-1} + Bu_k + w_k$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
4X2 2X1
$\begin{bmatrix} \frac{1}{2}\Delta T^2 & 0 \\ 0 & \frac{1}{2}\Delta T^2 \\ \Delta T & 0 \\ 0 & \Delta T \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Delta T^2 a_x \\ \frac{1}{2}\Delta T^2 a_y \\ \Delta T a_x \\ \Delta T a_y \end{bmatrix}$
$X_k = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$

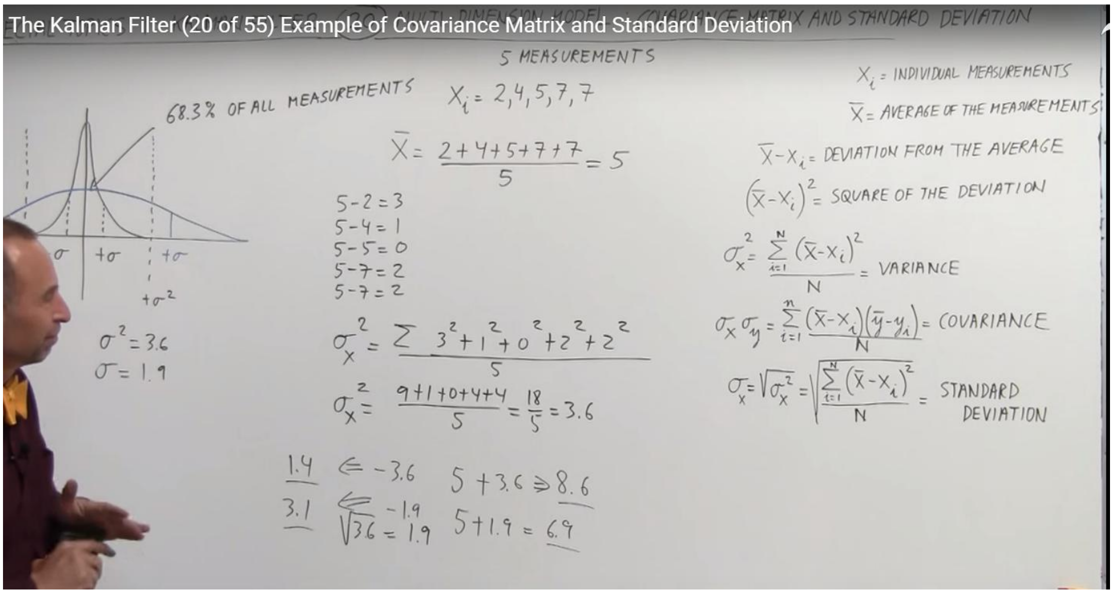

Special Topics - The Kalman Filter (15 of 55) 9: Converting from Previous to Current State 2-D
SPECIAL TOPIC 1: THE KALMAN FILTER (15) THE MULTI-DIMENSION MODEL 9 THE STATE MATRIX 2-D
$X_k = AX_{k-1} + Bu_k + w_k$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
w = NOISE IN THE PROCESS
Δt = TIME FOR 1 CYCLE
$X_k = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \dot{x}_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta T^2 & 0 \\ 0 & \frac{1}{2}\Delta T^2 \\ \Delta T & 0 \\ 0 & \Delta T \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$
$X_k = \begin{bmatrix} x_{k-1} + \dot{x}_{k-1}\Delta T \\ y_{k-1} + \dot{y}_{k-1}\Delta T \\ \dot{x}_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} a_x \frac{1}{2}\Delta T^2 \\ a_y \frac{1}{2}\Delta T^2 \\ a_x \Delta T \\ a_y \Delta T \end{bmatrix} = \begin{bmatrix} x_{k-1} + \dot{x}_{k-1}\Delta T + a_x \frac{1}{2}\Delta T^2 \\ y_{k-1} + \dot{y}_{k-1}\Delta T + a_y \frac{1}{2}\Delta T^2 \\ \dot{x}_{k-1} + a_x \Delta T \\ \dot{y}_{k-1} + a_y \Delta T \end{bmatrix}$

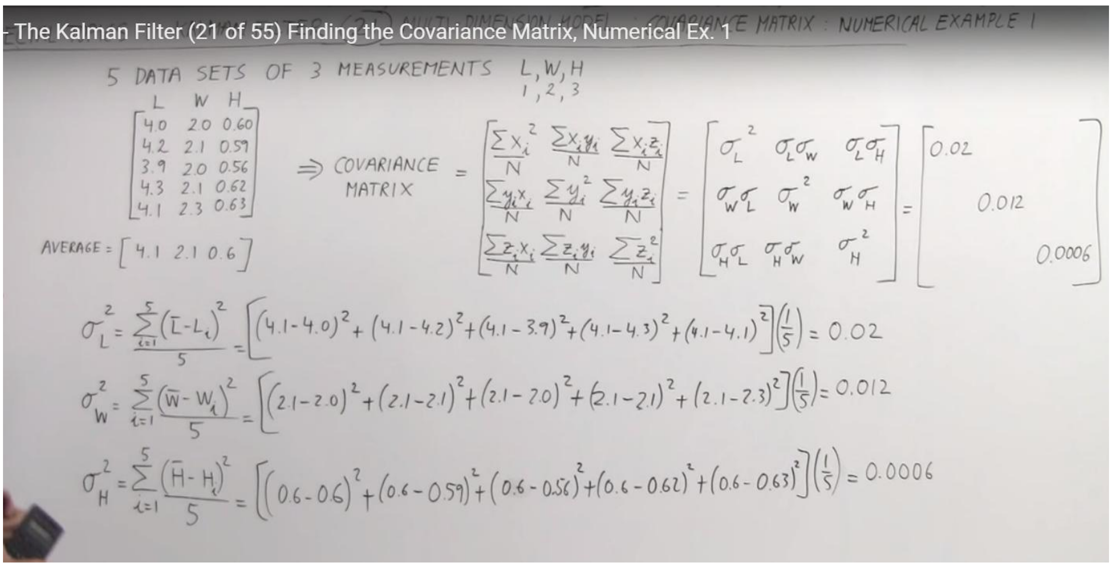

Special Topics - The Kalman Filter (16 of 55) 10: Converting from Previous to Current State 3-D
SPECIAL TOPIC 1: THE KALMAN FILTER (16) THE MULTI-DIMENSION MODEL 10 THE STATE MATRIX 3-D
$X_k = AX_{k-1} + Bu_k + w_k$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
$X_k = \begin{bmatrix} 1 & 0 & 0 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta T & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_{k-1} + \begin{bmatrix} \frac{1}{2}\Delta T^2 & 0 & 0 \\ 0 & \frac{1}{2}\Delta T^2 & 0 \\ 0 & 0 & \frac{1}{2}\Delta T^2 \\ \Delta T & 0 & 0 \\ 0 & \Delta T & 0 \\ 0 & 0 & \Delta T \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} x_{k-1} + \dot{x}\Delta T \\ y_{k-1} + \dot{y}\Delta T \\ z_{k-1} + \dot{z}\Delta T \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} a_x \frac{1}{2}\Delta T^2 \\ a_y \frac{1}{2}\Delta T^2 \\ a_z \frac{1}{2}\Delta T^2 \\ a_x \Delta T \\ a_y \Delta T \\ a_z \Delta T \end{bmatrix}$
$= \begin{bmatrix} x_{k-1} + \dot{x}_{k-1}\Delta T + a_x \frac{1}{2}\Delta T^2 \\ y_{k-1} + \dot{y}_{k-1}\Delta T + a_y \frac{1}{2}\Delta T^2 \\ z_{k-1} + \dot{z}_{k-1}\Delta T + a_z \frac{1}{2}\Delta T^2 \\ \dot{x} + a_x \Delta T \\ \dot{y} + a_y \Delta T \\ \dot{z} + a_z \Delta T \end{bmatrix}$

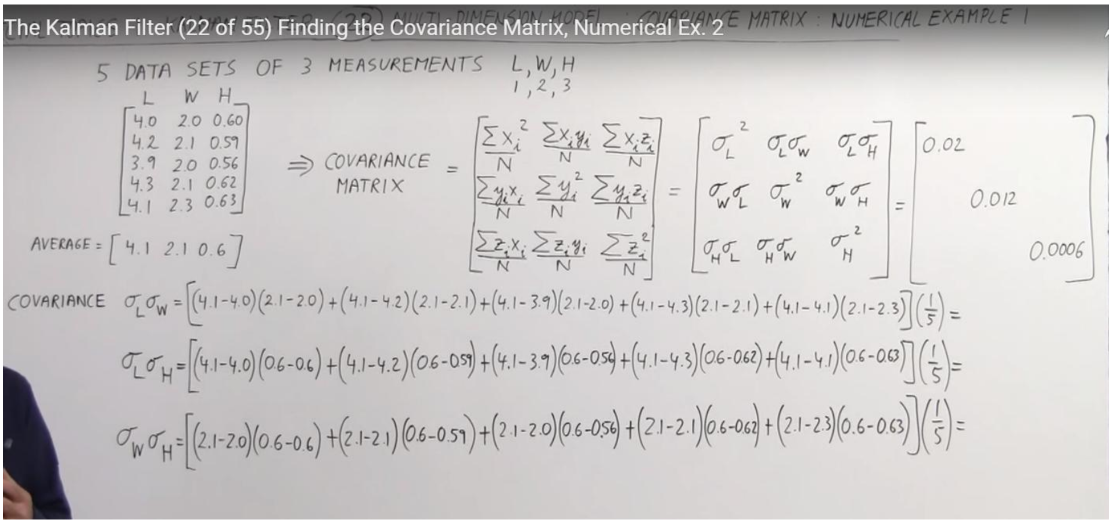

ecial Topics - The Kalman Filter (17 of 55) 11: Numerical Ex. of Finding the State Matrix 1-D
SPECIAL TOPIC I: THE KALMAN FILTER (17) THE MULTI-DIMENSION MODEL II THE STATE MATRIX 1-D
NUMERICAL EXAMPLE
$X_k = AX_{k-1} + Bu_k + w_k$
X = STATE MATRIX
u = CONTROL VARIABLE MATRIX
$y_0 = 50m$
$v_{y_0} = 5 m/sec$
$a_{y_0} = 2 m/sec^2$
$\Delta T = 1 sec$
$X_k = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta T^2 \\ \Delta T \end{bmatrix}[a_y] + w_k = \begin{bmatrix} y_{k-1} + \dot{y}_{k-1}\Delta T \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} a_y \frac{1}{2}\Delta T^2 \\ a_y \Delta T \end{bmatrix} + 0$
$X_k = \begin{bmatrix} y_{k-1} + \dot{y}_{k-1}\Delta T + \frac{1}{2}a_y \Delta T^2 \\ \dot{y}_{k-1} + a_y \Delta T \end{bmatrix}$
$t=0 \Rightarrow X_k = \begin{bmatrix} 50 \\ 5 \end{bmatrix}$
$t=1 \Rightarrow X_k = \begin{bmatrix} 50 + (5)(1) + \frac{1}{2}(2)(1)^2 \\ 5 + (2)(1) \end{bmatrix} = \begin{bmatrix} 56 \\ 7 \end{bmatrix}$
$t=2 \Rightarrow X_k \begin{bmatrix} 56 + (7)(1) + \frac{1}{2}(2)(1)^2 \\ 7 + (2)(1) \end{bmatrix} = \begin{bmatrix} 64 \\ 9 \end{bmatrix}$
$t=3 \Rightarrow X_k \begin{bmatrix} 64 + (9)(1) + \frac{1}{2}(2)(1)^2 \\ 9 + (2)(1) \end{bmatrix} = \begin{bmatrix} 74 \\ 11 \end{bmatrix}$
$t=4 \quad X_k = \begin{bmatrix} 74 + (11)(1) + \frac{1}{2}(2)(1)^2 \\ 11 + (2)(1) \end{bmatrix} = \begin{bmatrix} 86 \\ 13 \end{bmatrix}$
$y = y_0 + v_{0y} t + \frac{1}{2}at^2$
$y(t=4) = 50 + 5(4) + \frac{1}{2}(2)(4)^2 = 86m$
$v = v_0 + at$
$v(t=4) = 5 + (2)(4) = 13 m/sec$

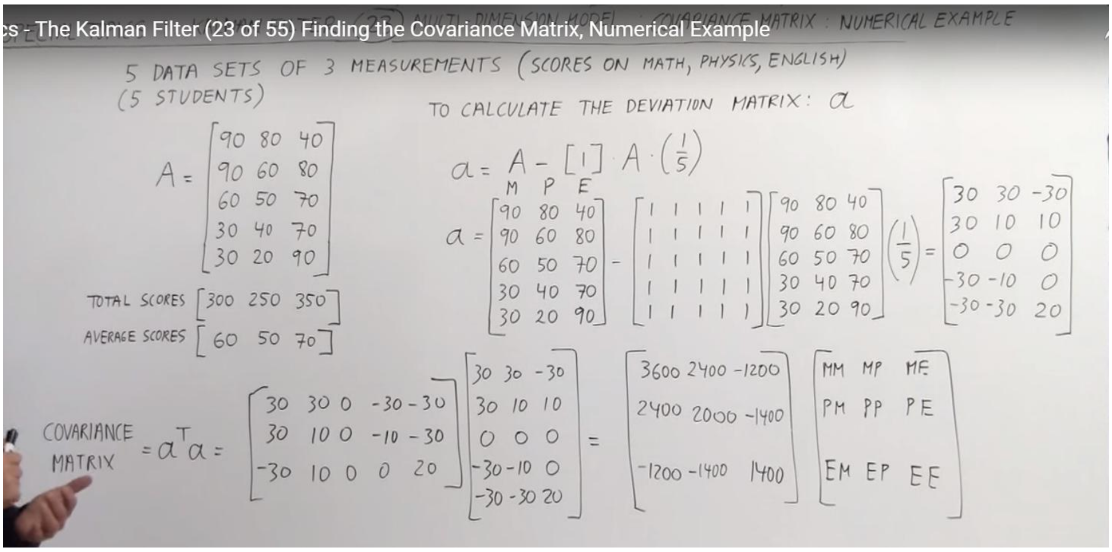

Kalman Filter (18 of 55) What is a Covariance Matrix?
MULTI-DIMENSION MODEL : THE COVARIANCE MATRIX
$P_k = AP_{k-1}A^T + Q$
$K_k = \frac{P_k H^T}{HP_k H^T + R}$
WHERE P = STATE COVARIANCE MATRIX
(ERROR IN THE ESTIMATE)
Q = PROCESS NOISE COVARIANCE MATRIX
(KEEPS THE STATE COVARIANCE MATRIX FROM BECOMING TOO SMALL OR GOING TO 0)
R = MEASUREMENT COVARIANCE MATRIX
(ERROR IN THE MEASUREMENT)
K = KALMAN GAIN
(WEIGHT FACTOR BASED ON COMPARING THE ERROR IN THE ESTIMATE TO THE ERROR IN THE MEASUREMENT)
IF R → 0
THEN K → 1 (ADJUST PRIMARILY WITH THE MEASUREMENT UPDATE)
IF R → LARGE
THEN K → 0 (ADJUST PRIMARILY WITH THE PREDICTED STATE)
IF P → 0 THEN MEASUREMENT UPDATES ARE MOSTLY IGNORED

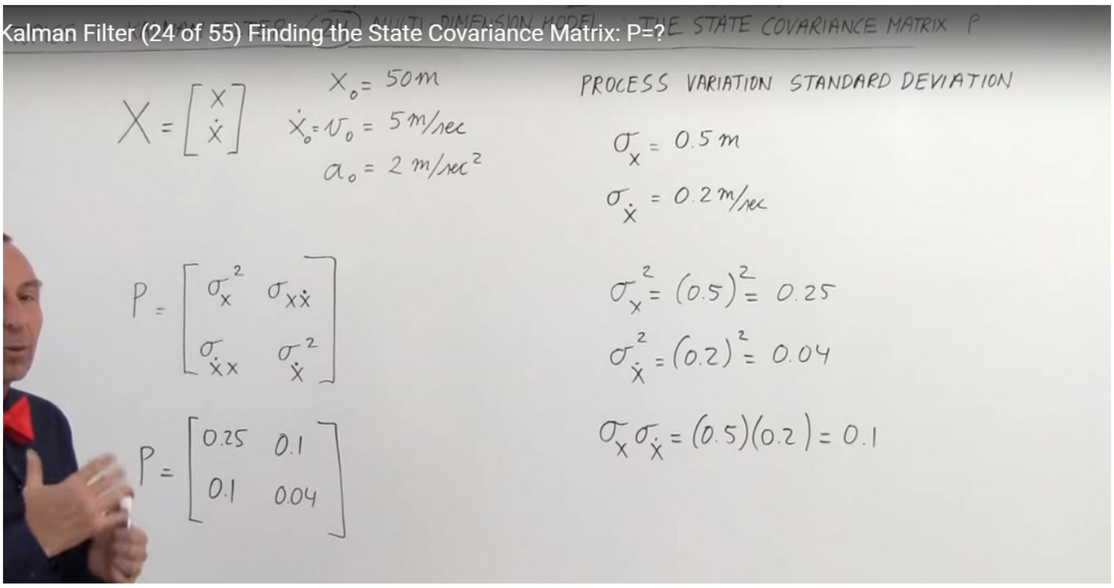

The Kalman Filter (19 of 55) What is a Variance-Covariance Matrix?
WHAT IS A VARIANCE - COVARIANCE MATRIX ?
1-D ⇒ [Σ(x̄-xᵢ)² / N]
2D →
2D
3D
Xᵢ = INDIVIDUAL MEASUREMENTS
X̄ = AVERAGE OF THE MEASUREMENTS
X̄-Xᵢ = DEVIATION FROM THE AVERAGE
(X̄-Xᵢ)² = SQUARE OF THE DEVIATION
= VARIANCE
= COVARIANCE
= STANDARD DEVIATION

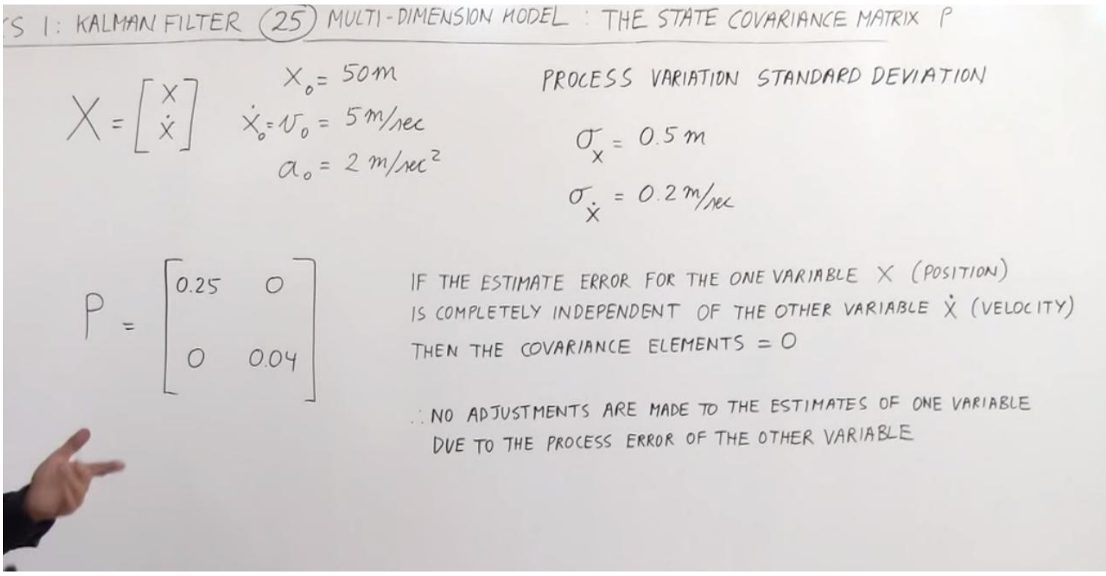

The Kalman Filter (20 of 55) Example of Covariance Matrix and Standard Deviation
COVARIANCE MATRIX AND STANDARD DEVIATION
5 MEASUREMENTS
68.3% OF ALL MEASUREMENTS
Xᵢ = 2,4,5,7,7
X̄ = (2+4+5+7+7)/5 = 5
5-2=3
5-4=1
5-5=0
5-7=2
5-7=2
σ² = 3.6
σ = 1.9
σx² = (9+1+0+4+4)/5 = 18/5 = 3.6
√3.6 = 1.9
5+3.6 ⇒ 8.6
5+1.9 = 6.9
Xᵢ = INDIVIDUAL MEASUREMENTS
X̄ = AVERAGE OF THE MEASUREMENTS
X̄-Xᵢ = DEVIATION FROM THE AVERAGE
(X̄-Xᵢ)² = SQUARE OF THE DEVIATION
= VARIANCE
= COVARIANCE
= STANDARD DEVIATION

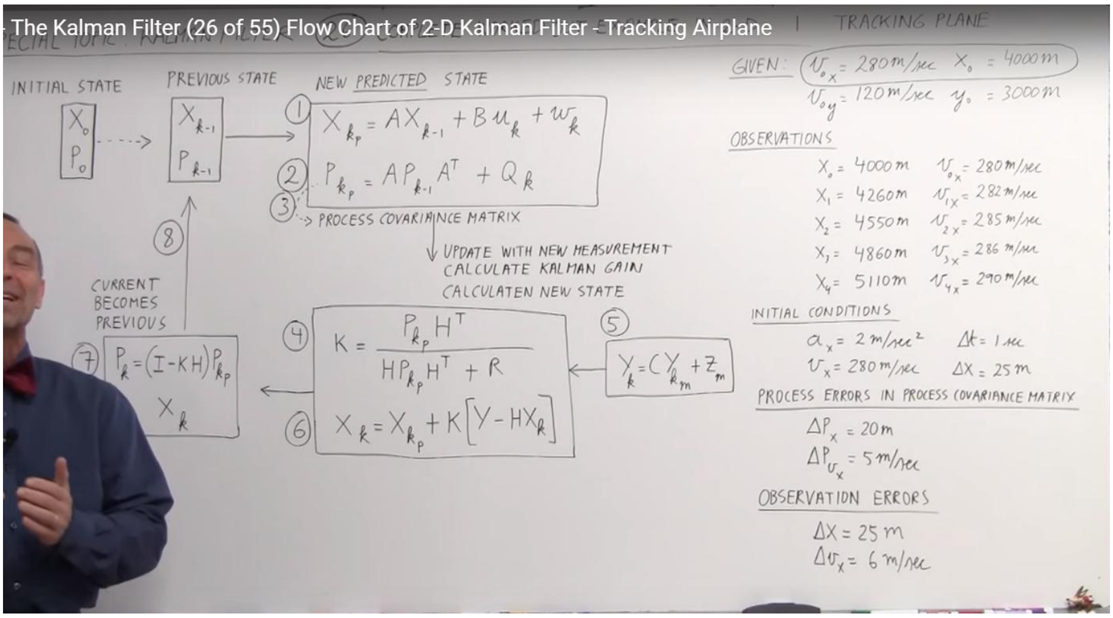

The Kalman Filter (21 of 55) Finding the Covariance Matrix, Numerical Ex. 1
COVARIANCE MATRIX : NUMERICAL EXAMPLE 1
5 DATA SETS OF 3 MEASUREMENTS L,W,H
1,2,3
L W H
4.0 2.0 0.60
4.2 2.1 0.59
3.9 2.0 0.56
4.3 2.1 0.62
4.1 2.3 0.63
AVERAGE = [4.1 2.1 0.6]
⇒ COVARIANCE MATRIX =
0.02
0.012
0.0006
σL² = [(4.1-4.0)² + (4.1-4.2)² + (4.1-3.9)² + (4.1-4.3)² + (4.1-4.1)²](1/5) = 0.02
σW² = [(2.1-2.0)² + (2.1-2.1)² + (2.1-2.0)² + (2.1-2.1)² + (2.1-2.3)²](1/5) = 0.012
σH² = [(0.6-0.6)² + (0.6-0.59)² + (0.6-0.56)² + (0.6-0.62)² + (0.6-0.63)²](1/5) = 0.0006

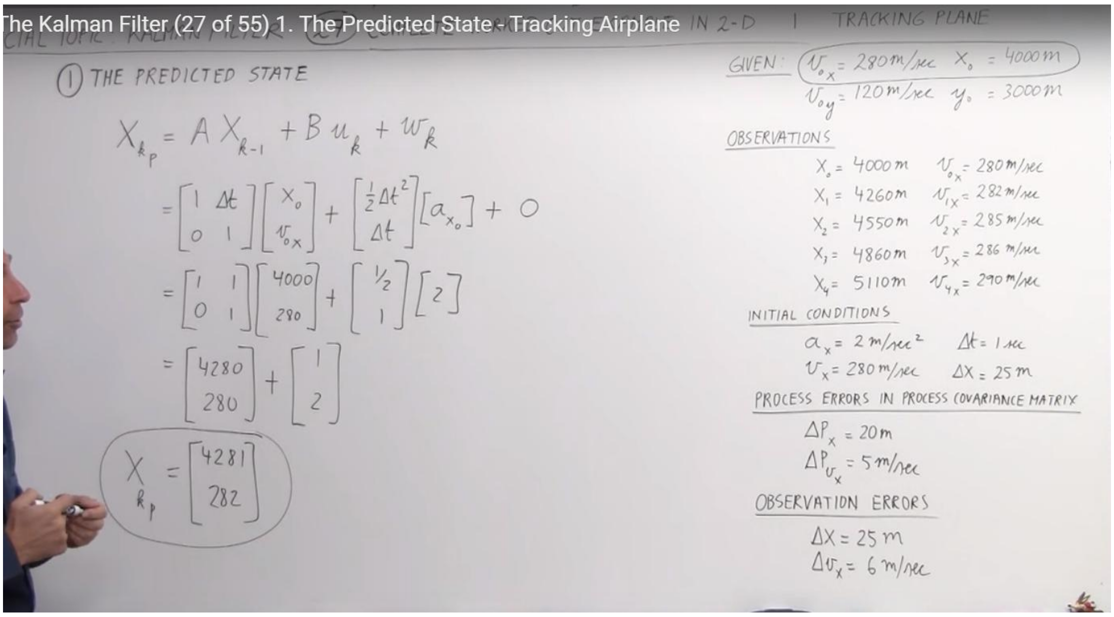

The Kalman Filter (22 of 55) Finding the Covariance Matrix, Numerical Ex. 2
COVARIANCE MATRIX : NUMERICAL EXAMPLE 1
5 DATA SETS OF 3 MEASUREMENTS L, W, H
1, 2, 3
L W H
4.0 2.0 0.60
4.2 2.1 0.59
3.9 2.0 0.56
4.3 2.1 0.62
4.1 2.3 0.63
⇒ COVARIANCE MATRIX =
0.02
0.012
0.0006
AVERAGE = [4.1 2.1 0.6]
COVARIANCE

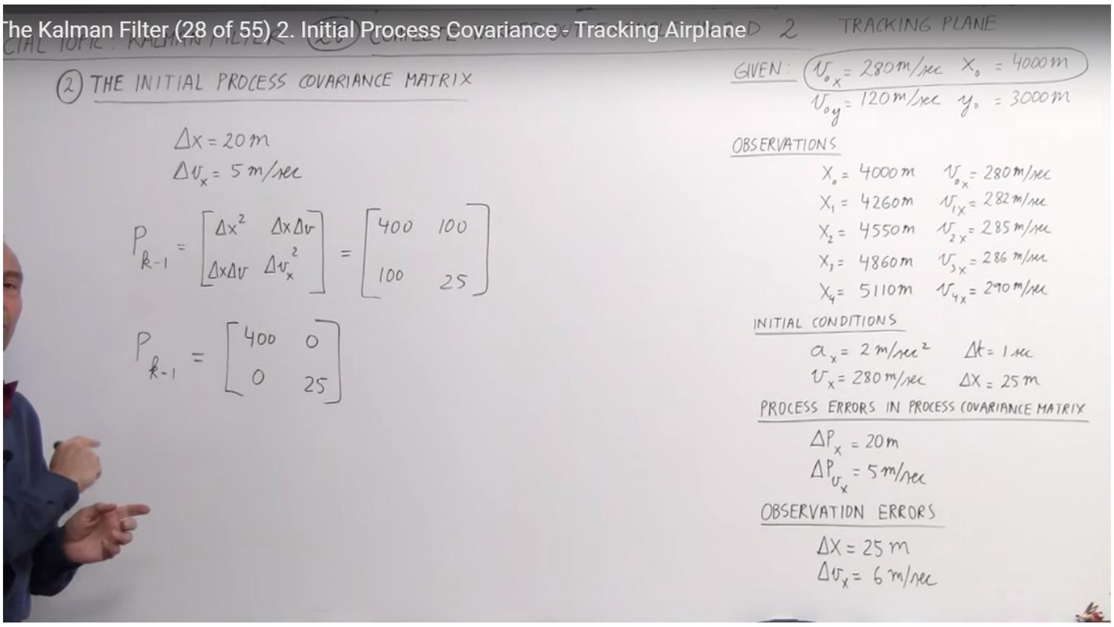

The Kalman Filter (23 of 55) Finding the Covariance Matrix, Numerical Example
COVARIANCE MATRIX : NUMERICAL EXAMPLE
5 DATA SETS OF 3 MEASUREMENTS (SCORES ON MATH, PHYSICS, ENGLISH)
(5 STUDENTS)
TO CALCULATE THE DEVIATION MATRIX: α
M P E
TOTAL SCORES [300 250 350]
AVERAGE SCORES [60 50 70]
COVARIANCE MATRIX
MM MP ME
PM PP PE
EM EP EE

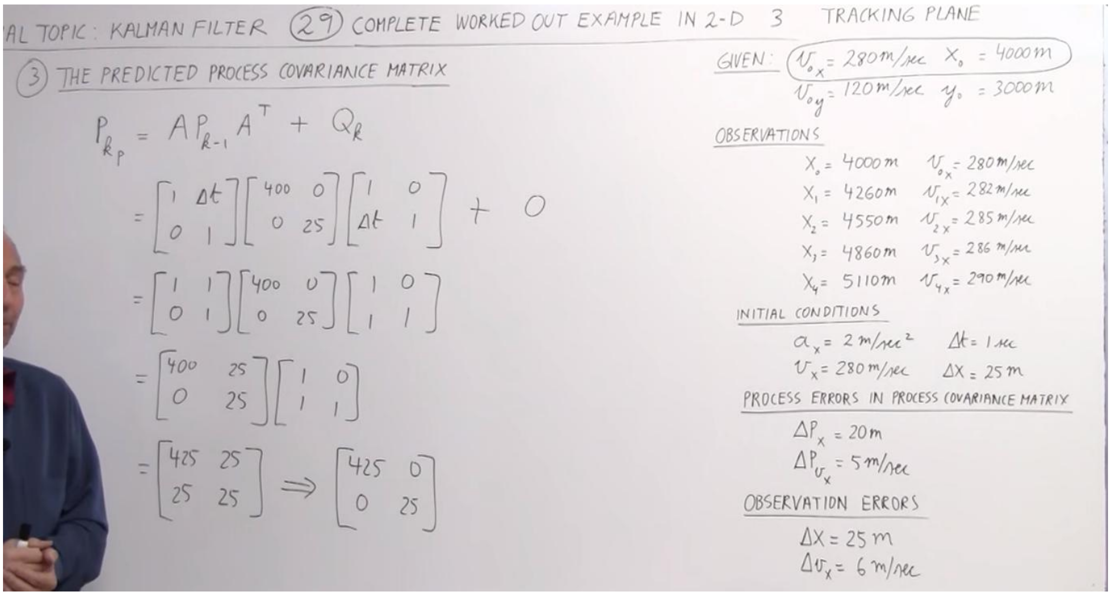

Kalman Filter (24 of 55) Finding the State Covariance Matrix: P=?
THE STATE COVARIANCE MATRIX P
X₀ = 50m
Ẋ₀ = V₀ = 5 m/sec
a₀ = 2 m/sec²
PROCESS VARIATION STANDARD DEVIATION
σx = 0.5 m
σẋ = 0.2 m/sec
σx² = (0.5)² = 0.25
σẋ² = (0.2)² = 0.04
σx σẋ = (0.5)(0.2) = 0.1
P = [0.25 0.1; 0.1 0.04]

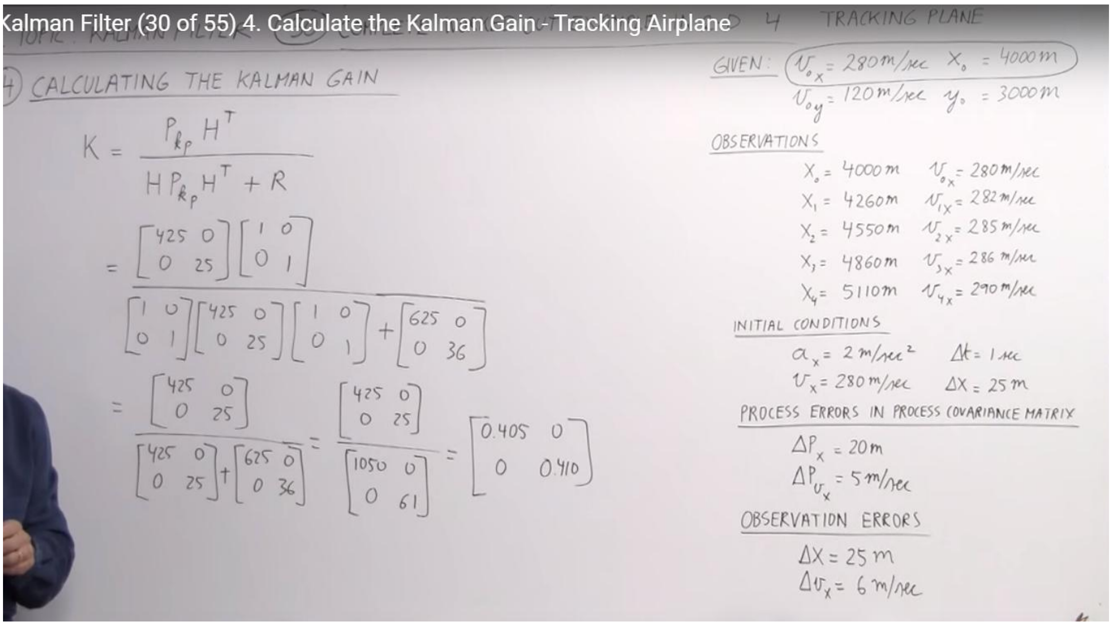

'S 1: KALMAN FILTER (25) MULTI-DIMENSION MODEL : THE STATE COVARIANCE MATRIX P
X = [x ẋ]
x₀ = 50m
ẋ₀ = v₀ = 5 m/sec
a₀ = 2 m/sec²
PROCESS VARIATION STANDARD DEVIATION
σx = 0.5 m
σẋ = 0.2 m/sec
P = [0.25 0; 0 0.04]
IF THE ESTIMATE ERROR FOR THE ONE VARIABLE X (POSITION)
IS COMPLETELY INDEPENDENT OF THE OTHER VARIABLE Ẋ (VELOCITY)
THEN THE COVARIANCE ELEMENTS = 0
∴ NO ADJUSTMENTS ARE MADE TO THE ESTIMATES OF ONE VARIABLE
DUE TO THE PROCESS ERROR OF THE OTHER VARIABLE

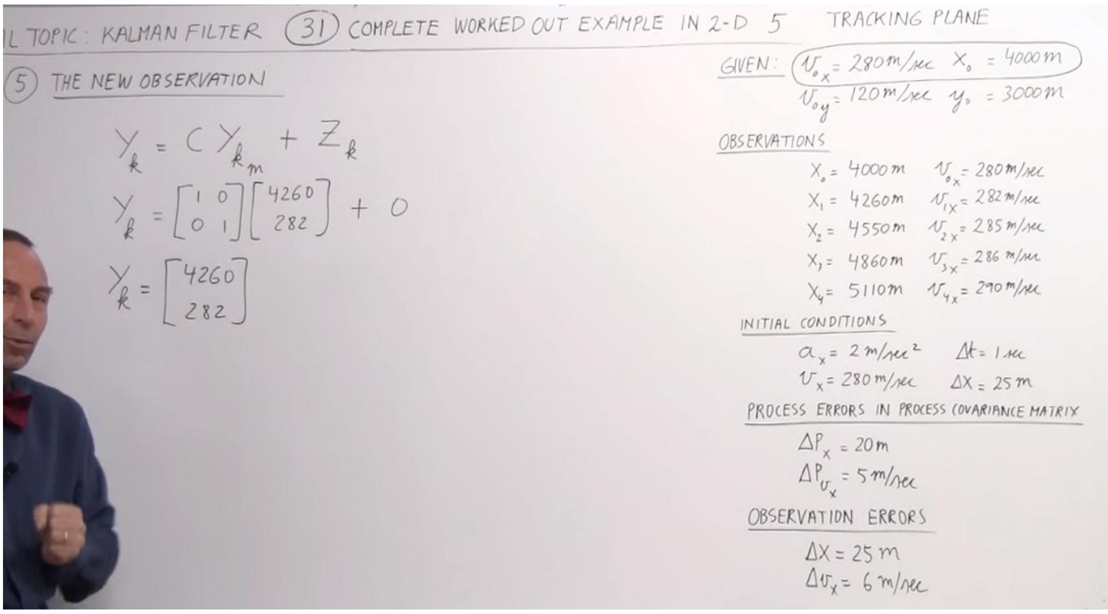

The Kalman Filter (26 of 55) Flow Chart of 2-D Kalman Filter - Tracking Airplane
TRACKING PLANE
INITIAL STATE
PREVIOUS STATE
NEW PREDICTED STATE
1 X_kp = A X_k-1 + B u_k + w_k
2 P_kp = A P_k-1 A^T + Q_k
3 PROCESS COVARIANCE MATRIX
UPDATE WITH NEW MEASUREMENT
CALCULATE KALMAN GAIN
CALCULATEN NEW STATE
8
CURRENT BECOMES PREVIOUS
4 K = P_kp H^T / (H P_kp H^T + R)
5 Y_k = C Y_km + Z_m
6 X_k = X_kp + K[Y - H X_k]
7 P_k = (I - KH) P_kp
X_k
GIVEN: v0x = 280 m/sec X0 = 4000 m
v0y = 120 m/sec y0 = 3000 m
OBSERVATIONS
X0 = 4000 m v0x = 280 m/sec
X1 = 4260 m v1x = 282 m/sec
X2 = 4550 m v2x = 285 m/sec
X3 = 4860 m v3x = 286 m/sec
X4 = 5110 m v4x = 290 m/sec
INITIAL CONDITIONS
ax = 2 m/sec² Δt = 1 sec
vx = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔPx = 20 m
ΔPvx = 5 m/sec
OBSERVATION ERRORS
ΔX = 25 m
Δvx = 6 m/sec

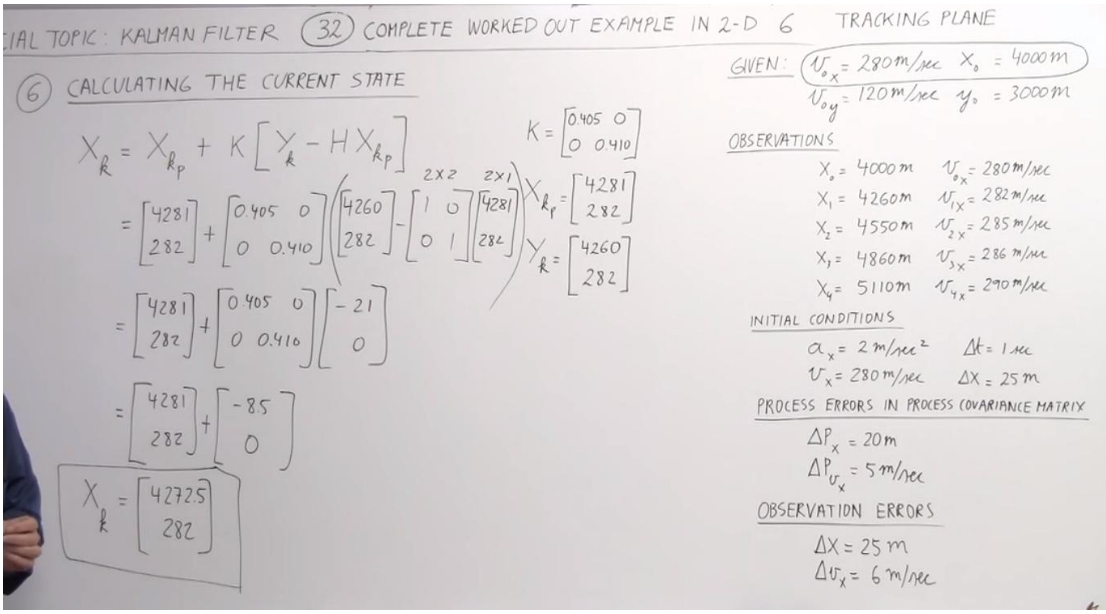

The Kalman Filter (27 of 55) 1. The Predicted State - Tracking Airplane
TRACKING PLANE
1 THE PREDICTED STATE
X_kp = A X_k-1 + B u_k + w_k
= [1 Δt; 0 1][X0; v0x] + [½Δt²; Δt][a_x0] + 0
= [1 1; 0 1][4000; 280] + [½; 1][2]
= [4280; 280] + [1; 2]
X_kp = [4281; 282]
GIVEN: v0x = 280 m/sec X0 = 4000 m
v0y = 120 m/sec y0 = 3000 m
OBSERVATIONS
X0 = 4000 m v0x = 280 m/sec
X1 = 4260 m v1x = 282 m/sec
X2 = 4550 m v2x = 285 m/sec
X3 = 4860 m v3x = 286 m/sec
X4 = 5110 m v4x = 290 m/sec
INITIAL CONDITIONS
ax = 2 m/sec² Δt = 1 sec
vx = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔPx = 20 m
ΔPvx = 5 m/sec
OBSERVATION ERRORS
ΔX = 25 m
Δvx = 6 m/sec

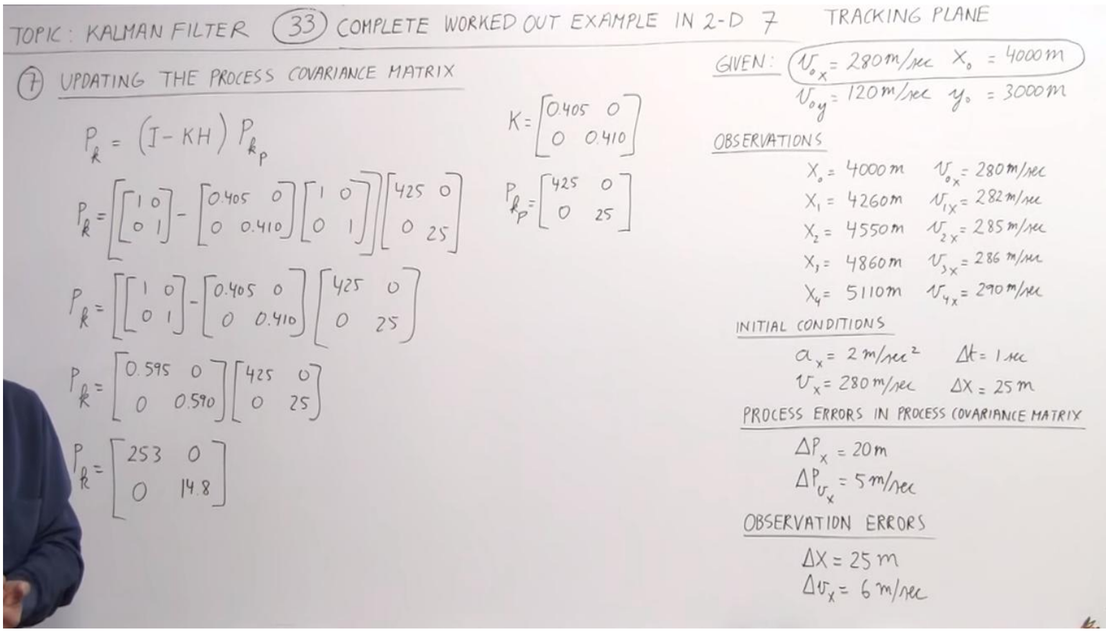

The Kalman Filter (28 of 55) 2. Initial Process Covariance - Tracking Airplane
2 TRACKING PLANE
(2) THE INITIAL PROCESS COVARIANCE MATRIX
$\Delta x = 20m$
$\Delta v_x = 5 m/sec$
$P_{k-1} = \begin{bmatrix} \Delta x^2 & \Delta x \Delta v \\ \Delta x \Delta v & \Delta v_x^2 \end{bmatrix} = \begin{bmatrix} 400 & 100 \\ 100 & 25 \end{bmatrix}$
$P_{k-1} = \begin{bmatrix} 400 & 0 \\ 0 & 25 \end{bmatrix}$
GIVEN: $v_{0x} = 280 m/sec$ $X_0 = 4000m$
$v_{0y} = 120 m/sec$ $y_0 = 3000m$
OBSERVATIONS
$X_0 = 4000m$ $v_{0x} = 280 m/sec$
$X_1 = 4260m$ $v_{1x} = 282 m/sec$
$X_2 = 4550m$ $v_{2x} = 285 m/sec$
$X_3 = 4860m$ $v_{3x} = 286 m/sec$
$X_4 = 5110m$ $v_{4x} = 290 m/sec$
INITIAL CONDITIONS
$a_x = 2 m/sec^2$ $\Delta t = 1 sec$
$v_x = 280 m/sec$ $\Delta X = 25m$
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20m$
$\Delta P_{v_x} = 5 m/sec$
OBSERVATION ERRORS
$\Delta X = 25m$
$\Delta v_x = 6 m/sec$

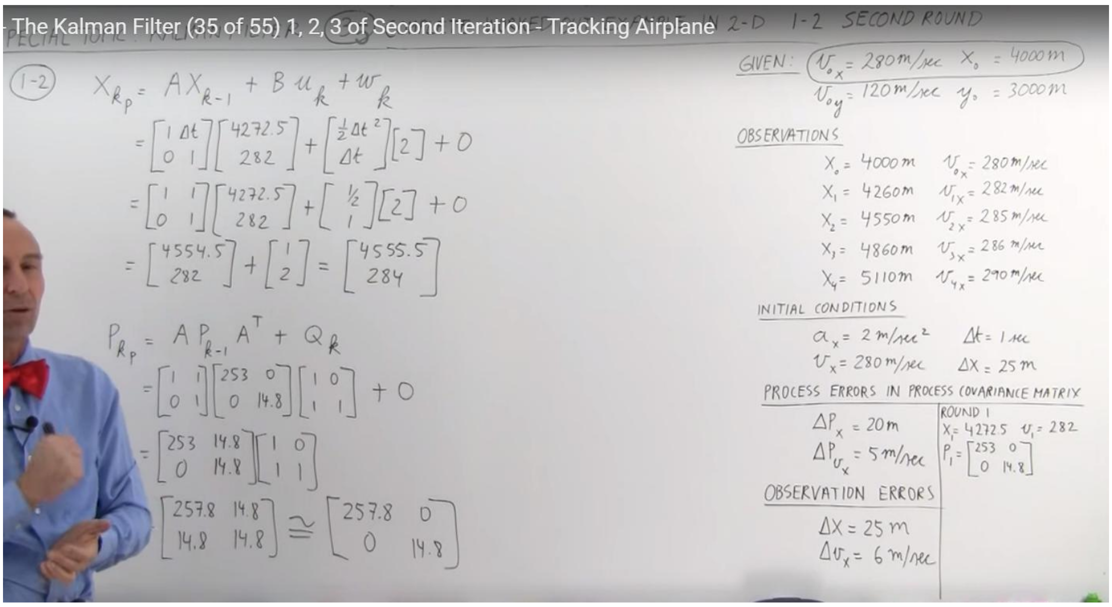

AL TOPIC: KALMAN FILTER (29) COMPLETE WORKED OUT EXAMPLE IN 2-D 3 TRACKING PLANE
(3) THE PREDICTED PROCESS COVARIANCE MATRIX
$P_{k_p} = A P_{k-1} A^T + Q_k$
$= \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 400 & 0 \\ 0 & 25 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \Delta t & 1 \end{bmatrix} + 0$
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 400 & 0 \\ 0 & 25 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 400 & 25 \\ 0 & 25 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 425 & 25 \\ 25 & 25 \end{bmatrix} \Rightarrow \begin{bmatrix} 425 & 0 \\ 0 & 25 \end{bmatrix}$
GIVEN: $v_{0x} = 280 m/sec$ $X_0 = 4000m$
$v_{0y} = 120 m/sec$ $y_0 = 3000m$
OBSERVATIONS
$X_0 = 4000m$ $v_{0x} = 280 m/sec$
$X_1 = 4260m$ $v_{1x} = 282 m/sec$
$X_2 = 4550m$ $v_{2x} = 285 m/sec$
$X_3 = 4860m$ $v_{3x} = 286 m/sec$
$X_4 = 5110m$ $v_{4x} = 290 m/sec$
INITIAL CONDITIONS
$a_x = 2 m/sec^2$ $\Delta t = 1 sec$
$v_x = 280 m/sec$ $\Delta X = 25m$
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20m$
$\Delta P_{v_x} = 5 m/sec$
OBSERVATION ERRORS
$\Delta X = 25m$
$\Delta v_x = 6 m/sec$

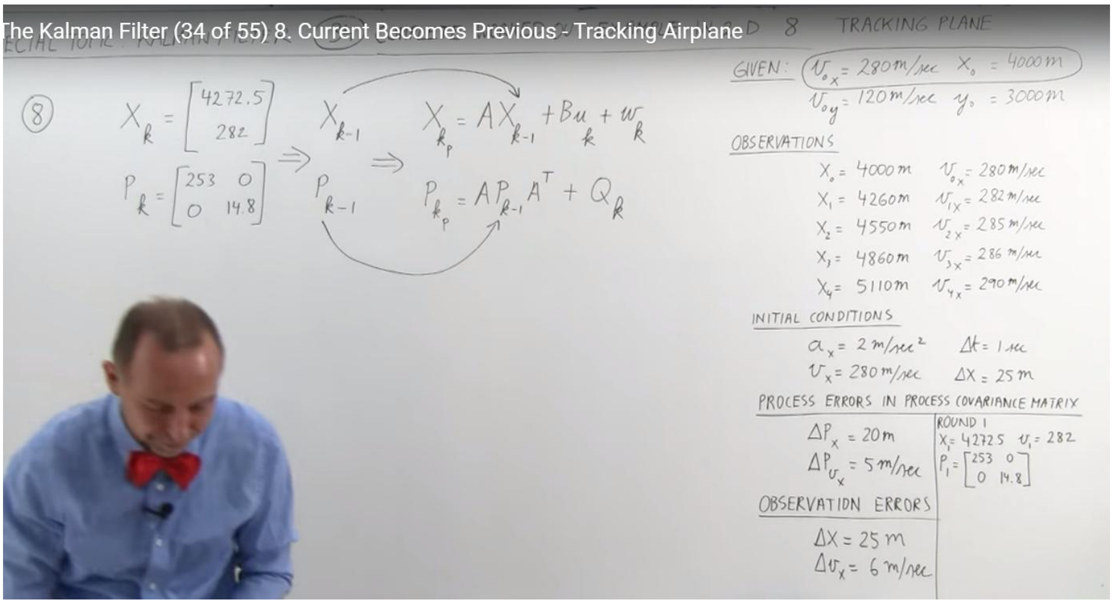

Kalman Filter (30 of 55) 4. Calculate the Kalman Gain - Tracking Airplane
4 TRACKING PLANE
4 CALCULATING THE KALMAN GAIN
K = P_kp H^T / (H P_kp H^T + R)
= [425 0; 0 25][1 0; 0 1] / ([1 0; 0 1][425 0; 0 25][1 0; 0 1] + [625 0; 0 36])
= [425 0; 0 25] / ([425 0; 0 25] + [625 0; 0 36]) = [425 0; 0 25] / [1050 0; 0 61] = [0.405 0; 0 0.410]
GIVEN: V_0x = 280 m/sec X_0 = 4000 m
V_0y = 120 m/sec y_0 = 3000 m
OBSERVATIONS
X_0 = 4000 m V_0x = 280 m/sec
X_1 = 4260 m V_1x = 282 m/sec
X_2 = 4550 m V_2x = 285 m/sec
X_3 = 4860 m V_3x = 286 m/sec
X_4 = 5110 m V_4x = 290 m/sec
INITIAL CONDITIONS
a_x = 2 m/sec² Δt = 1 sec
V_x = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔP_x = 20 m
ΔP_vx = 5 m/sec
OBSERVATION ERRORS
ΔX = 25 m
Δv_x = 6 m/sec

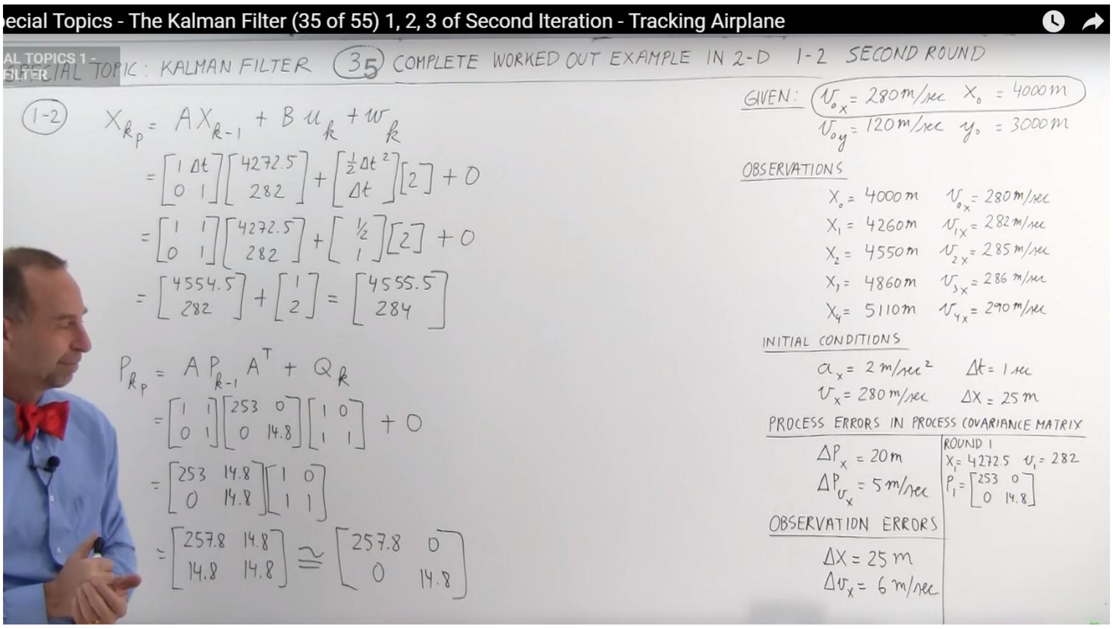

L TOPIC: KALMAN FILTER 31 COMPLETE WORKED OUT EXAMPLE IN 2-D 5 TRACKING PLANE
5 THE NEW OBSERVATION
Y_k = C Y_km + Z_k
Y_k = [1 0; 0 1][4260; 282] + 0
Y_k = [4260; 282]
GIVEN: V_0x = 280 m/sec X_0 = 4000 m
V_0y = 120 m/sec y_0 = 3000 m
OBSERVATIONS
X_0 = 4000 m V_0x = 280 m/sec
X_1 = 4260 m V_1x = 282 m/sec
X_2 = 4550 m V_2x = 285 m/sec
X_3 = 4860 m V_3x = 286 m/sec
X_4 = 5110 m V_4x = 290 m/sec
INITIAL CONDITIONS
a_x = 2 m/sec² Δt = 1 sec
V_x = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔP_x = 20 m
ΔP_vx = 5 m/sec
OBSERVATION ERRORS
ΔX = 25 m
Δv_x = 6 m/sec

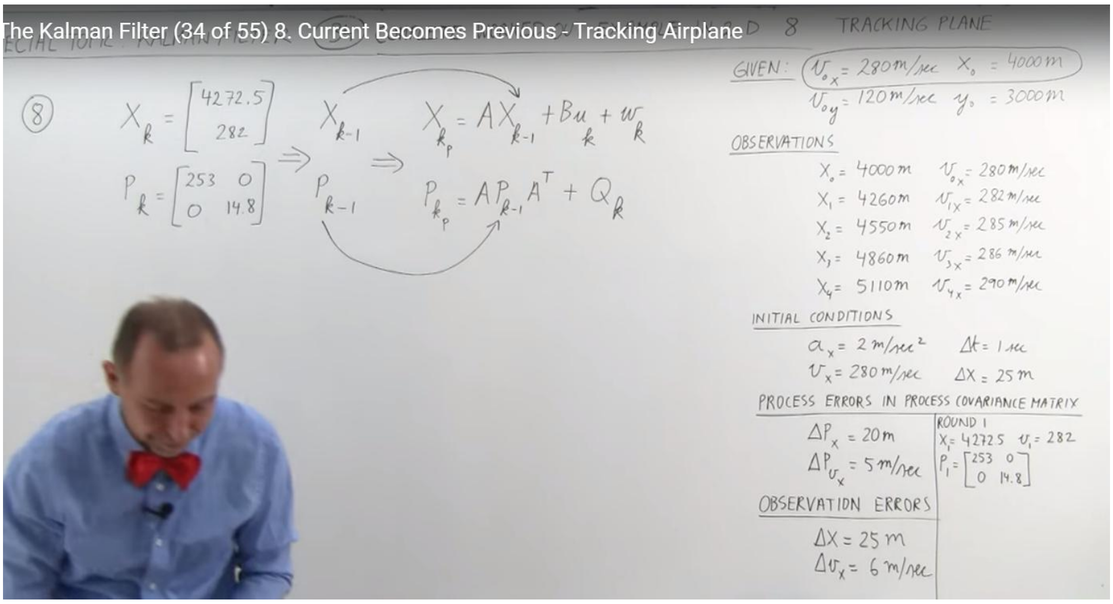

IAL TOPIC: KALMAN FILTER 32 COMPLETE WORKED OUT EXAMPLE IN 2-D 6 TRACKING PLANE
6 CALCULATING THE CURRENT STATE
$X_k = X_{k_p} + K\left[Y_k - H X_{k_p}\right]$
$K = \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}$
$X_{k_p} = \begin{bmatrix} 4281 \\ 282 \end{bmatrix}$ $Y_k = \begin{bmatrix} 4260 \\ 282 \end{bmatrix}$
2x2 2x1
$= \begin{bmatrix} 4281 \\ 282 \end{bmatrix} + \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}\left(\begin{bmatrix} 4260 \\ 282 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4281 \\ 282 \end{bmatrix}\right)$
$= \begin{bmatrix} 4281 \\ 282 \end{bmatrix} + \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}\begin{bmatrix} -21 \\ 0 \end{bmatrix}$
$= \begin{bmatrix} 4281 \\ 282 \end{bmatrix} + \begin{bmatrix} -8.5 \\ 0 \end{bmatrix}$
$\boxed{X_k = \begin{bmatrix} 4272.5 \\ 282 \end{bmatrix}}$
GIVEN: $v_{0x} = 280$ m/sec $X_0 = 4000$ m
$v_{0y} = 120$ m/sec $y_0 = 3000$ m
OBSERVATIONS
$X_0 = 4000$ m $v_{0x} = 280$ m/sec
$X_1 = 4260$ m $v_{1x} = 282$ m/sec
$X_2 = 4550$ m $v_{2x} = 285$ m/sec
$X_3 = 4860$ m $v_{3x} = 286$ m/sec
$X_4 = 5110$ m $v_{4x} = 290$ m/sec
INITIAL CONDITIONS
$a_x = 2$ m/sec^2 $\Delta t = 1$ sec
$v_x = 280$ m/sec $\Delta X = 25$ m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20$ m
$\Delta P_{v_x} = 5$ m/sec
OBSERVATION ERRORS
$\Delta X = 25$ m
$\Delta v_x = 6$ m/sec

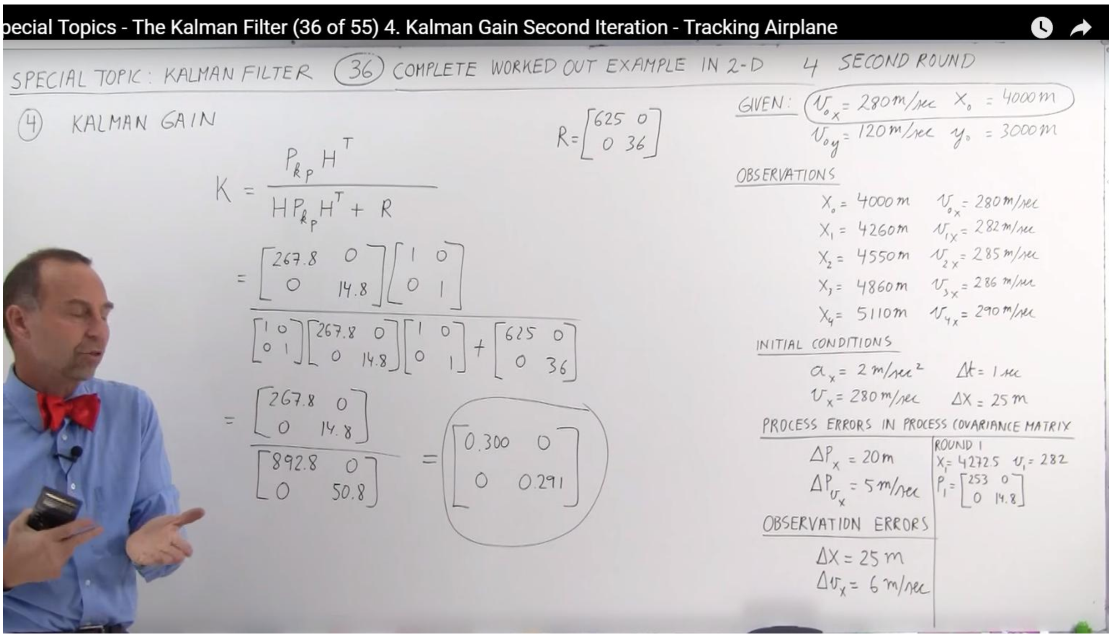

TOPIC: KALMAN FILTER 33 COMPLETE WORKED OUT EXAMPLE IN 2-D 7 TRACKING PLANE
7 UPDATING THE PROCESS COVARIANCE MATRIX
$P_k = (I - KH) P_{k_p}$
$K = \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}$
$P_{k_p} = \begin{bmatrix} 425 & 0 \\ 0 & 25 \end{bmatrix}$
$P_k = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]\begin{bmatrix} 425 & 0 \\ 0 & 25 \end{bmatrix}$
$P_k = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.405 & 0 \\ 0 & 0.410 \end{bmatrix}\right]\begin{bmatrix} 425 & 0 \\ 0 & 25 \end{bmatrix}$
$P_k = \begin{bmatrix} 0.595 & 0 \\ 0 & 0.590 \end{bmatrix}\begin{bmatrix} 425 & 0 \\ 0 & 25 \end{bmatrix}$
$P_k = \begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}$
GIVEN: $v_{0x} = 280$ m/sec $X_0 = 4000$ m
$v_{0y} = 120$ m/sec $y_0 = 3000$ m
OBSERVATIONS
$X_0 = 4000$ m $v_{0x} = 280$ m/sec
$X_1 = 4260$ m $v_{1x} = 282$ m/sec
$X_2 = 4550$ m $v_{2x} = 285$ m/sec
$X_3 = 4860$ m $v_{3x} = 286$ m/sec
$X_4 = 5110$ m $v_{4x} = 290$ m/sec
INITIAL CONDITIONS
$a_x = 2$ m/sec^2 $\Delta t = 1$ sec
$v_x = 280$ m/sec $\Delta X = 25$ m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20$ m
$\Delta P_{v_x} = 5$ m/sec
OBSERVATION ERRORS
$\Delta X = 25$ m
$\Delta v_x = 6$ m/sec

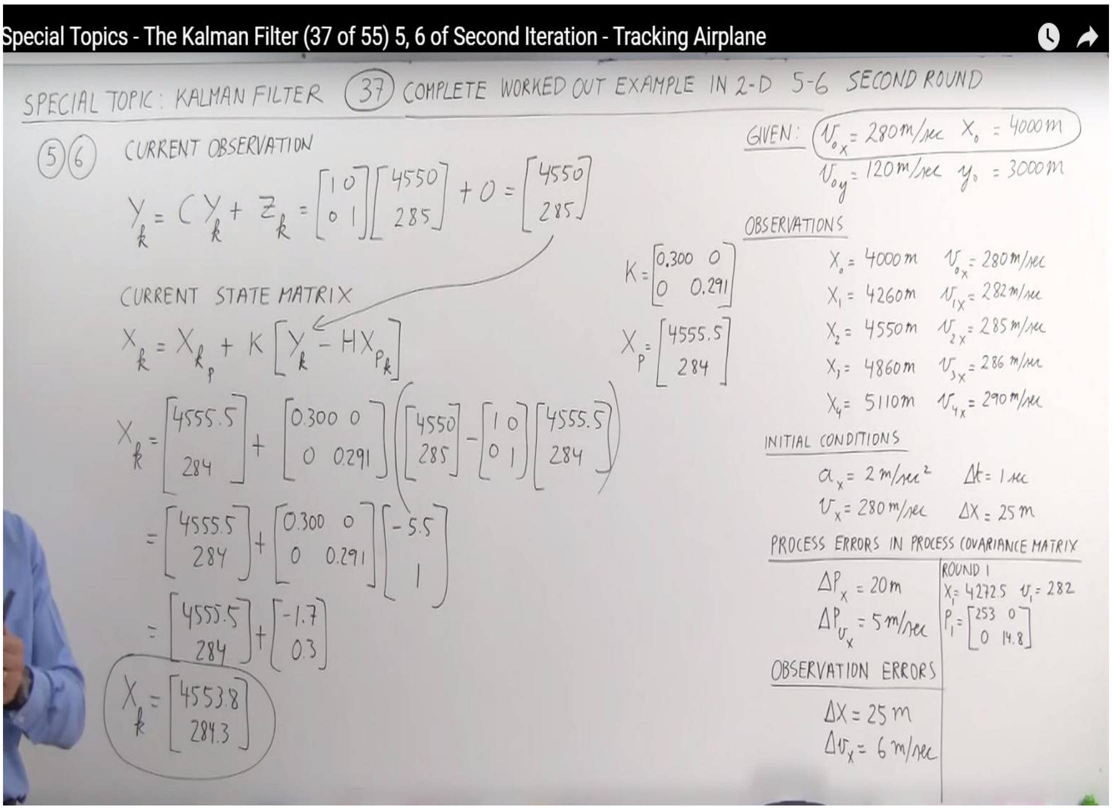

The Kalman Filter (35 of 55) 1, 2, 3 of Second Iteration- Tracking Airplane
2-D 1-2 SECOND ROUND
1-2 $X_{k_p} = A X_{k-1} + B u_k + w_k$
$= \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4272.5 \\ 282 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix}\begin{bmatrix} 2 \end{bmatrix} + 0$
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4272.5 \\ 282 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}\begin{bmatrix} 2 \end{bmatrix} + 0$
$= \begin{bmatrix} 4554.5 \\ 282 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4555.5 \\ 284 \end{bmatrix}$
$P_{k_p} = A P_{k-1} A^T + Q_k$
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 0$
$= \begin{bmatrix} 253 & 14.8 \\ 0 & 14.8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
$\begin{bmatrix} 257.8 & 14.8 \\ 14.8 & 14.8 \end{bmatrix} \cong \begin{bmatrix} 257.8 & 0 \\ 0 & 14.8 \end{bmatrix}$
GIVEN: $v_{0x} = 280$ m/sec $X_0 = 4000$ m
$v_{0y} = 120$ m/sec $y_0 = 3000$ m
OBSERVATIONS
$X_0 = 4000$ m $v_{0x} = 280$ m/sec
$X_1 = 4260$ m $v_{1x} = 282$ m/sec
$X_2 = 4550$ m $v_{2x} = 285$ m/sec
$X_3 = 4860$ m $v_{3x} = 286$ m/sec
$X_4 = 5110$ m $v_{4x} = 290$ m/sec
INITIAL CONDITIONS
$a_x = 2$ m/sec^2 $\Delta t = 1$ sec
$v_x = 280$ m/sec $\Delta X = 25$ m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20$ m
$\Delta P_{v_x} = 5$ m/sec
ROUND 1
$X_1 = 4272.5$ $v_1 = 282$
$P_1 = \begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}$
OBSERVATION ERRORS
$\Delta X = 25$ m
$\Delta v_x = 6$ m/sec

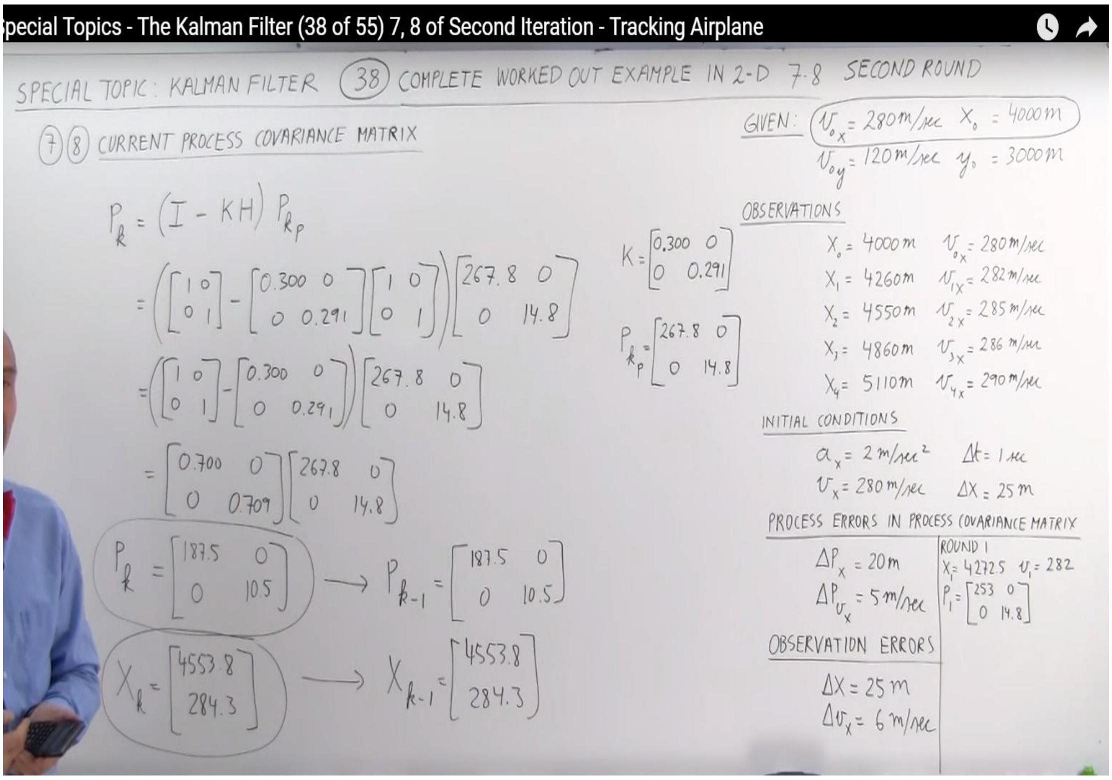

The Kalman Filter (34 of 55) 8. Current Becomes Previous - Tracking Airplane
8 TRACKING PLANE
GIVEN: v_{0x} = 280 m/sec X_0 = 4000 m
v_{0y} = 120 m/sec y_0 = 3000 m
OBSERVATIONS
X_0 = 4000 m v_{0x} = 280 m/sec
X_1 = 4260 m v_{1x} = 282 m/sec
X_2 = 4550 m v_{2x} = 285 m/sec
X_3 = 4860 m v_{3x} = 286 m/sec
X_4 = 5110 m v_{4x} = 290 m/sec
INITIAL CONDITIONS
a_x = 2 m/sec² Δt = 1 sec
v_x = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔP_x = 20 m
ΔP_{v_x} = 5 m/sec
ROUND 1
X_1 = 4272.5 v_1 = 282
OBSERVATION ERRORS
ΔX = 25 m
Δv_x = 6 m/sec
8
$X_k = \begin{bmatrix} 4272.5 \\ 282 \end{bmatrix} \Rightarrow X_{k-1} \Rightarrow X_{k_p} = AX_{k-1} + Bu_k + w_k$
$P_k = \begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix} \Rightarrow P_{k-1} \Rightarrow P_{k_p} = AP_{k-1}A^T + Q_k$

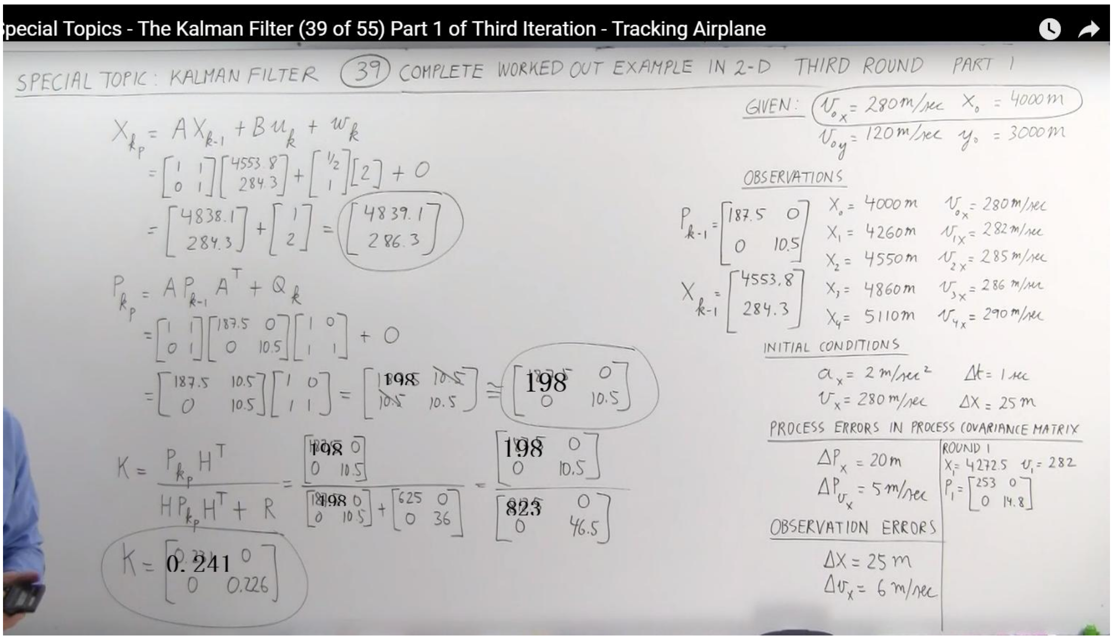

Special Topics - The Kalman Filter (35 of 55) 1, 2, 3 of Second Iteration - Tracking Airplane
SPECIAL TOPIC: KALMAN FILTER 35 COMPLETE WORKED OUT EXAMPLE IN 2-D 1-2 SECOND ROUND
1-2
$X_{k_p} = AX_{k-1} + Bu_k + w_k$
$= \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4272.5 \\ 282 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix}[2] + 0$
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4272.5 \\ 282 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix}[2] + 0$
$= \begin{bmatrix} 4554.5 \\ 282 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4555.5 \\ 284 \end{bmatrix}$
$P_{k_p} = AP_{k-1}A^T + Q_k$
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 0$
$= \begin{bmatrix} 253 & 14.8 \\ 0 & 14.8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$
$= \begin{bmatrix} 257.8 & 14.8 \\ 14.8 & 14.8 \end{bmatrix} \cong \begin{bmatrix} 257.8 & 0 \\ 0 & 14.8 \end{bmatrix}$

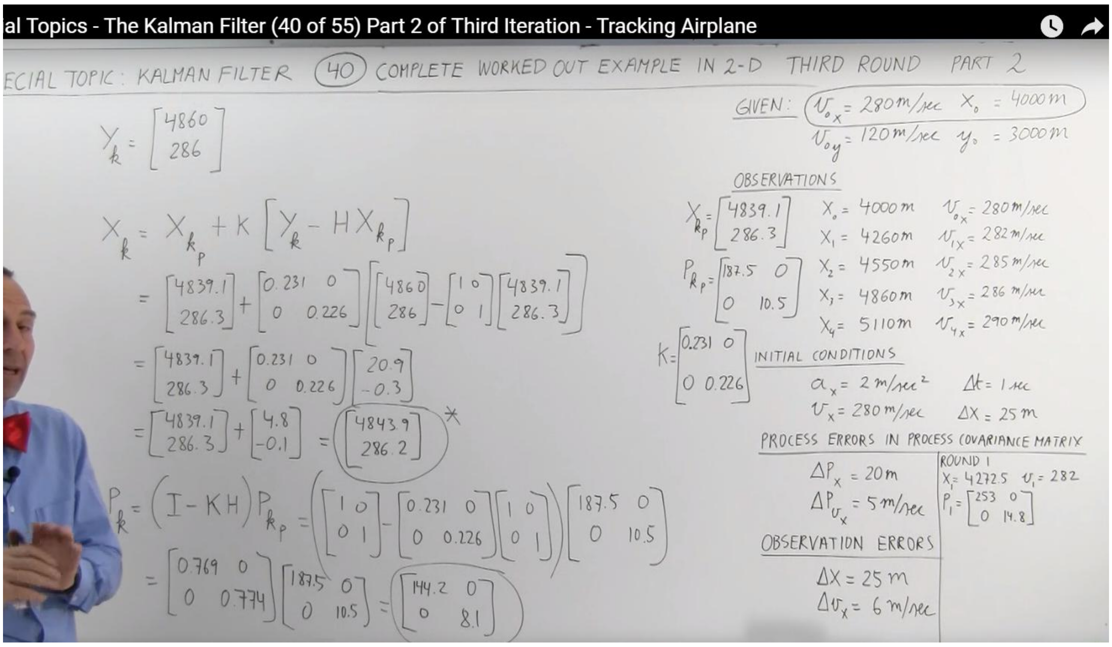

The Kalman Filter (34 of 55) 8. Current Becomes Previous - Tracking Airplane
8 TRACKING PLANE
GIVEN: V0x = 280 m/sec X0 = 4000 m
V0y = 120 m/sec y0 = 3000 m
OBSERVATIONS
X0 = 4000 m V0x = 280 m/sec
X1 = 4260 m V1x = 282 m/sec
X2 = 4550 m V2x = 285 m/sec
X3 = 4860 m V3x = 286 m/sec
X4 = 5110 m V4x = 290 m/sec
INITIAL CONDITIONS
ax = 2 m/sec² Δt = 1 sec
Vx = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔPx = 20 m
ΔPvx = 5 m/sec
ROUND 1
X1 = 4272.5 V1 = 282
OBSERVATION ERRORS
ΔX = 25 m
ΔVx = 6 m/sec

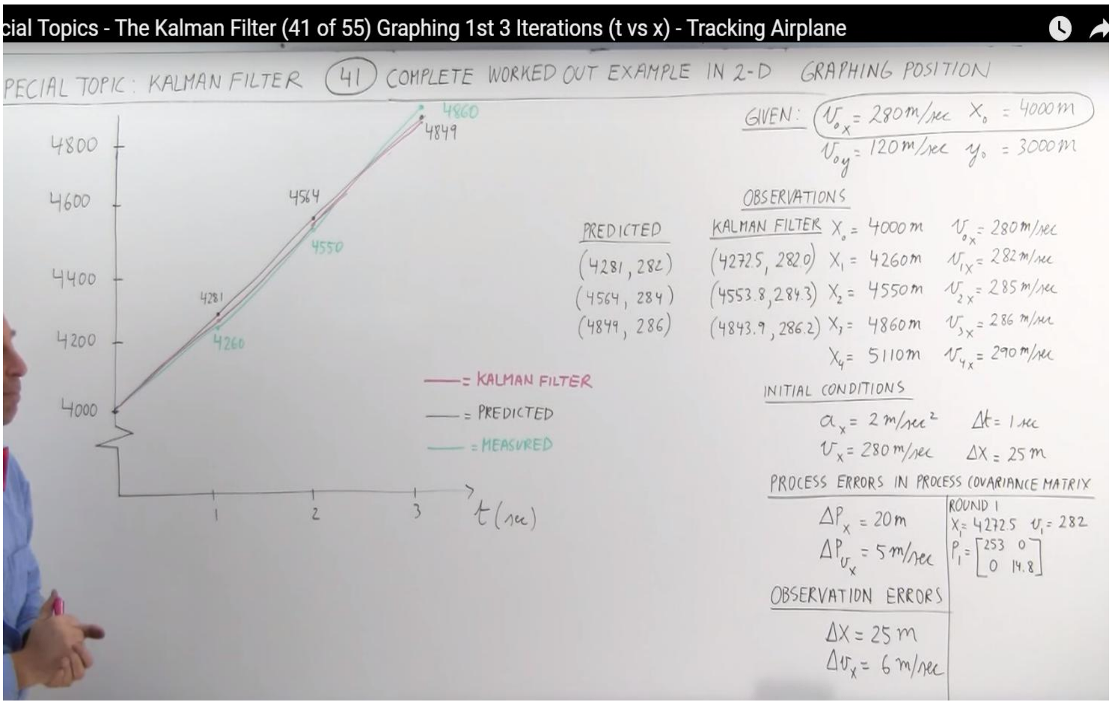

Special Topics - The Kalman Filter (36 of 55) 4. Kalman Gain Second Iteration - Tracking Airplane
SPECIAL TOPIC: KALMAN FILTER 36 COMPLETE WORKED OUT EXAMPLE IN 2-D 4 SECOND ROUND
4 KALMAN GAIN
GIVEN: V0x = 280 m/sec X0 = 4000 m
V0y = 120 m/sec y0 = 3000 m
OBSERVATIONS
X0 = 4000 m V0x = 280 m/sec
X1 = 4260 m V1x = 282 m/sec
X2 = 4550 m V2x = 285 m/sec
X3 = 4860 m V3x = 286 m/sec
X4 = 5110 m V4x = 290 m/sec
INITIAL CONDITIONS
ax = 2 m/sec² Δt = 1 sec
Vx = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔPx = 20 m
ΔPvx = 5 m/sec
ROUND 1
X1 = 4272.5 V1 = 282
OBSERVATION ERRORS
ΔX = 25 m
ΔVx = 6 m/sec


Special Topics - The Kalman Filter (37 of 55) 5, 6 of Second Iteration - Tracking Airplane
SPECIAL TOPIC: KALMAN FILTER 37 COMPLETE WORKED OUT EXAMPLE IN 2-D 5-6 SECOND ROUND
5 6 CURRENT OBSERVATION
$Y_k = C Y_k + Z_k = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}4550\\285\end{bmatrix} + 0 = \begin{bmatrix}4550\\285\end{bmatrix}$
CURRENT STATE MATRIX
$X_k = X_{k_p} + K[Y_k - HX_{p_k}]$
$X_k = \begin{bmatrix}4555.5\\284\end{bmatrix} + \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}\left(\begin{bmatrix}4550\\285\end{bmatrix} - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}4555.5\\284\end{bmatrix}\right)$
$= \begin{bmatrix}4555.5\\284\end{bmatrix} + \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}\begin{bmatrix}-5.5\\1\end{bmatrix}$
$= \begin{bmatrix}4555.5\\284\end{bmatrix} + \begin{bmatrix}-1.7\\0.3\end{bmatrix}$
$X_k = \begin{bmatrix}4553.8\\284.3\end{bmatrix}$
$K = \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}$
$X_p = \begin{bmatrix}4555.5\\284\end{bmatrix}$
GIVEN: $v_{0x} = 280$ m/sec $X_0 = 4000$ m
$v_{0y} = 120$ m/sec $y_0 = 3000$ m
OBSERVATIONS
$X_0 = 4000$ m $v_{0x} = 280$ m/sec
$X_1 = 4260$ m $v_{1x} = 282$ m/sec
$X_2 = 4550$ m $v_{2x} = 285$ m/sec
$X_3 = 4860$ m $v_{3x} = 286$ m/sec
$X_4 = 5110$ m $v_{4x} = 290$ m/sec
INITIAL CONDITIONS
$a_x = 2$ m/sec^2 $\Delta t = 1$ sec
$v_x = 280$ m/sec $\Delta X = 25$ m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20$ m
$\Delta P_{v_x} = 5$ m/sec
ROUND 1
$X_1 = 4272.5$ $v_1 = 282$
$P_1 = \begin{bmatrix}253 & 0\\0 & 14.8\end{bmatrix}$
OBSERVATION ERRORS
$\Delta X = 25$ m
$\Delta v_x = 6$ m/sec

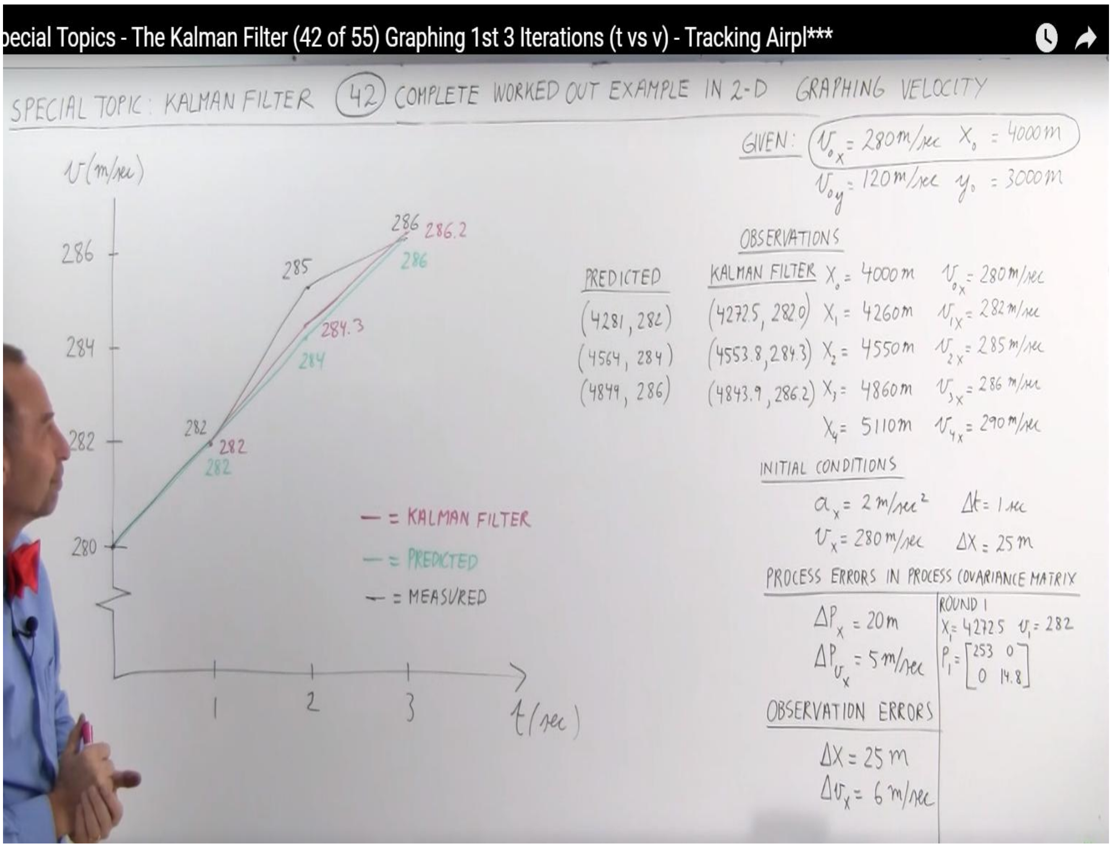

Special Topics - The Kalman Filter (38 of 55) 7, 8 of Second Iteration - Tracking Airplane
SPECIAL TOPIC: KALMAN FILTER 38 COMPLETE WORKED OUT EXAMPLE IN 2-D 7-8 SECOND ROUND
7 8 CURRENT PROCESS COVARIANCE MATRIX
$P_k = (I - KH) P_{k_p}$
$= \left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\begin{bmatrix}267.8 & 0\\0 & 14.8\end{bmatrix}$
$= \left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}\right)\begin{bmatrix}267.8 & 0\\0 & 14.8\end{bmatrix}$
$= \begin{bmatrix}0.700 & 0\\0 & 0.709\end{bmatrix}\begin{bmatrix}267.8 & 0\\0 & 14.8\end{bmatrix}$
$P_k = \begin{bmatrix}187.5 & 0\\0 & 10.5\end{bmatrix} \longrightarrow P_{k-1} = \begin{bmatrix}187.5 & 0\\0 & 10.5\end{bmatrix}$
$X_k = \begin{bmatrix}4553.8\\284.3\end{bmatrix} \longrightarrow X_{k-1} = \begin{bmatrix}4553.8\\284.3\end{bmatrix}$
$K = \begin{bmatrix}0.300 & 0\\0 & 0.291\end{bmatrix}$
$P_{k_p} = \begin{bmatrix}267.8 & 0\\0 & 14.8\end{bmatrix}$
GIVEN: $v_{0x} = 280$ m/sec $X_0 = 4000$ m
$v_{0y} = 120$ m/sec $y_0 = 3000$ m
OBSERVATIONS
$X_0 = 4000$ m $v_{0x} = 280$ m/sec
$X_1 = 4260$ m $v_{1x} = 282$ m/sec
$X_2 = 4550$ m $v_{2x} = 285$ m/sec
$X_3 = 4860$ m $v_{3x} = 286$ m/sec
$X_4 = 5110$ m $v_{4x} = 290$ m/sec
INITIAL CONDITIONS
$a_x = 2$ m/sec^2 $\Delta t = 1$ sec
$v_x = 280$ m/sec $\Delta X = 25$ m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
$\Delta P_x = 20$ m
$\Delta P_{v_x} = 5$ m/sec
ROUND 1
$X_1 = 4272.5$ $v_1 = 282$
$P_1 = \begin{bmatrix}253 & 0\\0 & 14.8\end{bmatrix}$
OBSERVATION ERRORS
$\Delta X = 25$ m
$\Delta v_x = 6$ m/sec


pecial Topics - The Kalman Filter (39 of 55) Part 1 of Third Iteration - Tracking Airplane
SPECIAL TOPIC: KALMAN FILTER
39 COMPLETE WORKED OUT EXAMPLE IN 2-D THIRD ROUND PART 1
GIVEN:
OBSERVATIONS
INITIAL CONDITIONS
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ROUND 1
OBSERVATION ERRORS
K = 0.241


al Topics - The Kalman Filter (40 of 55) Part 2 of Third Iteration - Tracking Airplane
ECIAL TOPIC: KALMAN FILTER
40 COMPLETE WORKED OUT EXAMPLE IN 2-D THIRD ROUND PART 2
GIVEN:
OBSERVATIONS
INITIAL CONDITIONS
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ROUND 1
OBSERVATION ERRORS



cial Topics - The Kalman Filter (41 of 55) Graphing 1st 3 Iterations (t vs x) - Tracking Airplane
PECIAL TOPIC: KALMAN FILTER 41 COMPLETE WORKED OUT EXAMPLE IN 2-D GRAPHING POSITION
GIVEN: v_{0x} = 280 m/sec X_0 = 4000 m
v_{0y} = 120 m/sec y_0 = 3000 m
4800
4600
4400
4200
4000
4860
4849
4564
4550
4281
4260
1
2
3
t (sec)
= KALMAN FILTER
= PREDICTED
= MEASURED
PREDICTED
(4281, 282)
(4564, 284)
(4849, 286)
OBSERVATIONS
KALMAN FILTER
(4272.5, 282.0)
(4553.8, 284.3)
(4843.9, 286.2)
X_0 = 4000 m v_{0x} = 280 m/sec
X_1 = 4260 m v_{1x} = 282 m/sec
X_2 = 4550 m v_{2x} = 285 m/sec
X_3 = 4860 m v_{3x} = 286 m/sec
X_4 = 5110 m v_{4x} = 290 m/sec
INITIAL CONDITIONS
a_x = 2 m/sec² Δt = 1 sec
v_x = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔP_x = 20 m
ΔP_{v_x} = 5 m/sec
ROUND 1
X_1 = 4272.5 v_1 = 282
$P_1 = \begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}$
OBSERVATION ERRORS
ΔX = 25 m
Δv_x = 6 m/sec



pecial Topics - The Kalman Filter (42 of 55) Graphing 1st 3 Iterations (t vs v) - Tracking Airpl***
SPECIAL TOPIC: KALMAN FILTER 42 COMPLETE WORKED OUT EXAMPLE IN 2-D GRAPHING VELOCITY
GIVEN: v_{0x} = 280 m/sec X_0 = 4000 m
v_{0y} = 120 m/sec y_0 = 3000 m
v (m/sec)
286
284
282
280
286
286.2
286
285
284.3
284
282
282
282
1
2
3
t (sec)
= KALMAN FILTER
= PREDICTED
= MEASURED
PREDICTED
(4281, 282)
(4564, 284)
(4849, 286)
OBSERVATIONS
KALMAN FILTER
(4272.5, 282.0)
(4553.8, 284.3)
(4843.9, 286.2)
X_0 = 4000 m v_{0x} = 280 m/sec
X_1 = 4260 m v_{1x} = 282 m/sec
X_2 = 4550 m v_{2x} = 285 m/sec
X_3 = 4860 m v_{3x} = 286 m/sec
X_4 = 5110 m v_{4x} = 290 m/sec
INITIAL CONDITIONS
a_x = 2 m/sec² Δt = 1 sec
v_x = 280 m/sec ΔX = 25 m
PROCESS ERRORS IN PROCESS COVARIANCE MATRIX
ΔP_x = 20 m
ΔP_{v_x} = 5 m/sec
ROUND 1
X_1 = 4272.5 v_1 = 282
$P_1 = \begin{bmatrix} 253 & 0 \\ 0 & 14.8 \end{bmatrix}$
OBSERVATION ERRORS
ΔX = 25 m
Δv_x = 6 m/sec