ACM Template

UESTC_Lasagne

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Contents

1 注意事项

输入输出格式？调试信息？初始化？算术溢出？数组大小？

左右端点范围？acos/asin/sqrt 函数定义域？精度问题？

二分答案？暴力？单调性？凸性？块状结构？函数式？对偶问题？

排序的时候注意一下是否需要记录排序前的位置！

使用 map 进行映射的时候，不要用下面这种不安全写法

```
1 if (mp.find(s) == mp.end())
2   mp[s] = mp.size()-1;//挂成狗
3
4 if (mp.find(s) == mp.end())
5 {
6   int tmp = mp.size();
7   mp[s] = tmp;//正确
8 }
```

10^6 数量级慎用后缀数组

TLE 的时候要冷静哟。。

思考的时候结合具体步骤来的话会体会到一些不同的东西

C++ 与 G++ 是很不一样的。。。

map 套字符串是很慢的。。。

栈会被记录内存。。。

浮点数最短路要注意取 $\leq$ 来判断更新。。。

注意 long long

不要相信.size()

重复利用数组时小心数组范围

先构思代码框架每当实际拍马框架变化时停手重新思考

有时候四边形不等式也是帮得上忙的 dp 优化是可以水的

结构体里面带数组会非常慢, 有时候 BFS 把数组压成数字会快很多。

```
1 void fun(int a[])
2 {
3   printf("%d\n",sizeof(a));
4 }
```

结果是 sizeof(a[0]), 如果传数组指针然后要清空的话不要用 sizeof。

sqrt 某些时候会出现 sqrt(-0.00) 的问题。

将 code::blocks 的默认终端改成 gnome-terminal

```
1 gnome-terminal -t $TITLE -x
```

最小割割集找法在残量网络中从源点出发能到的点集记为 S 原图中 S 到 S’ 的边即是最小割集

double 全局变量初始值可能不是 0

2 字符串处理

2.1 *AC 自动机

别忘记 Build

2.1.1 指针

```
const int CHAR=26;
const int TOTLEN=500000;
const int MAXLEN=1000000;
struct Vertex
{
  Vertex *fail,*next[CHAR];
  Vertex(){}
  Vertex(bool flag)//为什么要这样写?
  {
    fail=0;
    memset(next,0,sizeof(next));
  }
};
int size;
Vertex vertex[TOTLEN+1];
void init()
{
  vertex[0]=Vertex(0);
  size=1;
}
void add(Vertex *pos,int cha)
{
  vertex[size]=Vertex(0);
  pos->next[cha]=&vertex[size++];
}
void add(vector<int> s)
{
  int l=s.size();
  Vertex *pos=&vertex[0];
  for (int i=0; i<l; i++)
  {
    if (pos->next[s[i]]==NULL)
      add(pos,s[i]);
    pos=pos->next[s[i]];
  }
}
void bfs()
{
  queue<Vertex *> que;
  Vertex *u=&vertex[0];
  for (int i=0; i<CHAR; i++)
    if (u->next[i]!=NULL)
    {
```

```
44        que.push(u->next[i]);
45        u->next[i]->fail=u;
46      }
47      else
48        u->next[i]=u;
49    u->fail=NULL;
50    while (!que.empty())
51    {
52      u=que.front();
53      que.pop();
54      for (int i=0; i<CHAR; i++)
55        if (u->next[i]!=NULL)
56        {
57          que.push(u->next[i]);
58          u->next[i]->fail=u->fail->next[i];
59        }
60        else
61          u->next[i]=u->fail->next[i];
62    }
63  }
```

2.1.2 非指针

```
 1  struct Trie
 2  {
 3    int next[50][10],fail[50];
 4    bool end[50];
 5    int L,root;
 6
 7    int newNode()
 8    {
 9      for (int i = 0;i < 10;i++)
10        next[L][i] = -1;
11      end[L] = false;
12      return L++;
13    }
14
15    void Init()
16    {
17      L = 0;
18      root = newNode();
19    }
20
21    void Insert(char s[])
22    {
23      int now = root;
24      for (int i = 0;s[i] != 0;i++)
25      {
26        if (next[now][s[i]-'0'] == -1)
27          next[now][s[i]-'0'] = newNode();
28        now = next[now][s[i]-'0'];
29      }
```

```
30      end[now] = true;
31    }
32
33    void Build()
34    {
35      queue<int> Q;
36      for (int i = 0;i < 10;i++)
37        if (next[root][i] == -1)
38          next[root][i] = root;
39        else
40        {
41          fail[next[root][i]] = root;
42          Q.push(next[root][i]);
43        }
44      while (!Q.empty())
45      {
46        int now = Q.front();
47        Q.pop();
48        end[now] |= end[fail[now]];
49        for (int i = 0;i < 10;i++)
50          if (next[now][i] == -1)
51            next[now][i] = next[fail[now]][i];
52          else
53          {
54            fail[next[now][i]] = next[fail[now]][i];
55            Q.push(next[now][i]);
56          }
57      }
58    }
59  };
```

2.2 后缀数组

2.2.1 DC3

所有下标都是 0 n-1，height[0] 无意义。

```
1  //所有相关数组都要开三倍
2  const int maxn = 300010;
3  # define F(x) ((x)/3+((x)%3==1?0:tb))
4  # define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)
5  int wa[maxn * 3], wb[maxn * 3], wv[maxn * 3], ws[maxn * 3];
6  int c0(int *r, int a, int b)
7  {
8    return
9    r[a] == r[b] && r[a + 1] == r[b + 1] && r[a + 2] == r[b + 2];
10 }
11 int c12(int k, int *r, int a, int b)
12 {
13   if (k == 2)
14     return r[a] < r[b] || r[a] == r[b] && c12(1, r, a + 1, b + 1);
15   else return r[a] < r[b] || r[a] == r[b] && wv[a + 1] < wv[b + 1];
16 }
```

```
17 void sort(int *r, int *a, int *b, int n, int m)
18 {
19   int i;
20   for (i = 0; i < n; i++) wv[i] = r[a[i]];
21   for (i = 0; i < m; i++) ws[i] = 0;
22   for (i = 0; i < n; i++) ws[wv[i]]++;
23   for (i = 1; i < m; i++) ws[i] += ws[i - 1];
24   for (i = n - 1; i >= 0; i--) b[--ws[wv[i]]] = a[i];
25   return;
26 }
27 void dc3(int *r, int *sa, int n, int m)
28 {
29   int i, j, *rn = r + n;
30   int *san = sa + n, ta = 0, tb = (n + 1) / 3, tbc = 0, p;
31   r[n] = r[n + 1] = 0;
32   for (i = 0; i < n; i++) if (i % 3 != 0) wa[tbc++] = i;
33   sort(r + 2, wa, wb, tbc, m);
34   sort(r + 1, wb, wa, tbc, m);
35   sort(r, wa, wb, tbc, m);
36   for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
37     rn[F(wb[i])] = c0(r, wb[i - 1], wb[i]) ? p - 1 : p++;
38   if (p < tbc) dc3(rn, san, tbc, p);
39   else for (i = 0; i < tbc; i++) san[rn[i]] = i;
40   for (i = 0; i < tbc; i++) if (san[i] < tb) wb[ta++] = san[i] * 3;
41   if (n % 3 == 1) wb[ta++] = n - 1;
42   sort(r, wb, wa, ta, m);
43   for (i = 0; i < tbc; i++) wv[wb[i] = G(san[i])] = i;
44   for (i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
45     sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
46   for (; i < ta; p++) sa[p] = wa[i++];
47   for (; j < tbc; p++) sa[p] = wb[j++];
48 }
49 //str 和 sa 也要三倍
50 void da(int str[],int sa[],int rank[],int height[],int n,int m)
51 {
52   for (int i = n; i < n * 3; i++)
53     str[i] = 0;
54   dc3 (str , sa , n + 1 , m);
55   int i, j, k;
56   for (i = 0; i < n; i++)
57   {
58     sa[i] = sa[i + 1];
59     rank[sa[i]] = i;
60   }
61   for (i = 0, j = 0, k = 0; i < n; height[rank[i ++]] = k)
62     if (rank[i] > 0)
63       for (k ? k-- : 0 , j = sa[rank[i] - 1];
64         i + k < n && j + k < n && str[i + k] == str[j + k];
65         k++);
66 }
```

2.2.2 DA

这份似乎就没啥要注意的了。

```
const int maxn = 200010;
int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];

bool cmp(int *r,int n,int a,int b,int l)
{
  return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
}
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
  int *s = str;
  int *x=wx,*y=wy,*t,p;
  int i,j;
  for(i=0; i<m; i++)wss[i]=0;
  for(i=0; i<n; i++)wss[x[i]=s[i]]++;
  for(i=1; i<m; i++)wss[i]+=wss[i-1];
  for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
  for(j=1,p=1; p<n && j<n; j*=2,m=p)
  {
    for(i=n-j,p=0; i<n; i++)y[p++]=i;
    for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
    for(i=0; i<n; i++)wv[i]=x[y[i]];
    for(i=0; i<m; i++)wss[i]=0;
    for(i=0; i<n; i++)wss[wv[i]]++;
    for(i=1; i<m; i++)wss[i]+=wss[i-1];
    for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
    for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
      x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
  }
  for(int i=0; i<n; i++) rank[sa[i]]=i;
  for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
    if(rank[i]>0)
      for(k?k--:0,j=sa[rank[i]-1];
        i+k < n && j+k < n && str[i+k]==str[j+k];
        k++);
}
```

2.2.3 调用

注意几个数组的下标是不同的

```
char s[maxn];
int str[maxn],sa[maxn],rank[maxn],height[maxn];

int main()
{
  scanf("%s",s);
  int len = strlen(s);
  for (int i = 0;i <= len;i++)
    str[i] = s[i];
```

```
10    da(str,sa,rank,height,len,128);
11
12    for (int i = 0;i < len;i++)
13    {
14      printf("sa=␣%d,height=␣%d,s=␣%s\n",sa[i],height[i],s+sa[i]);
15    }
16    return 0;
17  }
```

2.2.4 最长公共前缀

记得不要忘记调用 lcpinit!

```
 1  int f[maxn][20];
 2  int lent[maxn];
 3  void lcpinit()
 4  {
 5    int i,j;
 6    int n = len,k = 1,l = 0;
 7    for (i = 0; i < n; i++)
 8    {
 9      f[i][0] = height[i];
10      if (i+1 > k*2)
11      {
12        k *= 2;
13        l++;
14      }
15      lent[i+1] = l;
16    }
17    for (j = 1; (1<<j)-1<n; j++)
18      for (i = 0; i+(1<<j)-1<n; i++)
19        f[i][j] = min(f[i][j-1],f[i+(1<<(j-1))][j-1]);
20  }
21  int lcp(int x,int y)
22  {
23    if (x > y)  swap(x,y);
24    if (x == y)
25      return x-sa[x];//自己和自己当然是自己的长度啦lcp
26    x++;
27    int k = lent[y-x+1];
28    return min(f[x][k],f[y-(1<<k)+1][k]);
29  }
```

2.2.5 最长公共前缀大于等于某个值的区间

```
1  void getinterv(int pos,int comlen,int& pl,int& pr)
2  {
3    int l,r,mid,cp;
4    l = 0;
5    r = pos;
6    while (l < r)
```

```
  {
    mid = l+r>>1;
    cp = lcp(mid,pos);
    if (cp < comlen)
      l = mid+1;
    else
      r = mid;
  }
  pl = l;

  l = pos;
  r = len-1;
  while (l < r)
  {
    mid = l+r+1>>1;
    cp = lcp(pos,mid);
    if (cp < comlen)
      r = mid-1;
    else
      l = mid;
  }
  pr = l;
}
```

2.3 后缀三兄弟

```
#include <cstdio>
#include <cstring>
#include <algorithm>
using namespace std;
const int CHAR = 26;
const int MAXN = 100000;
struct SAM_Node
{
  SAM_Node *fa,*next[CHAR];
  int len;
  int id,pos;
  SAM_Node() {}
  SAM_Node(int _len)
  {
    fa = 0;
    len = _len;
    memset(next,0,sizeof(next));
  }
};
SAM_Node SAM_node[MAXN * 2], *SAM_root, *SAM_last;
int SAM_size;
SAM_Node *newSAM_Node(int len)
{
  SAM_node[SAM_size] = SAM_Node(len);
  SAM_node[SAM_size].id=SAM_size;
  return &SAM_node[SAM_size++];
```

```
}
SAM_Node *newSAM_Node(SAM_Node *p)
{
  SAM_node[SAM_size] = *p;
  SAM_node[SAM_size].id=SAM_size;
  return &SAM_node[SAM_size++];
}
void SAM_init()
{
  SAM_size = 0;
  SAM_root = SAM_last = newSAM_Node(0);
  SAM_node[0].pos=0;
}
void SAM_add(int x,int len)
{
  SAM_Node *p = SAM_last, *np = newSAM_Node(p->len + 1);
  np->pos=len;
  SAM_last = np;
  for (; p && !p->next[x]; p = p->fa)
    p->next[x] = np;
  if (!p)
  {
    np->fa = SAM_root;
    return ;
  }
  SAM_Node *q = p->next[x];
  if (q->len == p->len + 1)
  {
    np->fa = q;
    return ;
  }
  SAM_Node *nq = newSAM_Node(q);
  nq->len = p->len + 1;
  q->fa = nq;
  np->fa = nq;
  for (; p && p->next[x] == q; p = p->fa)
    p->next[x] = nq;
}
void SAM_build(char *s)
{
  SAM_init();
  int l = strlen(s);
  for (int i = 0; i < l; i++)
    SAM_add(s[i] - 'a',i+1);
}

SAM_Node * SAM_add(SAM_Node *p, int x, int len)
{
  SAM_Node *np = newSAM_Node(p->len + 1);
  np->pos = len;
  SAM_last = np;
```

```
78    for (; p && !p->next[x]; p = p->fa)
79      p->next[x] = np;
80    if (!p)
81    {
82      np->fa = SAM_root;
83      return np;
84    }
85    SAM_Node *q = p->next[x];
86    if (q->len == p->len + 1)
87    {
88      np->fa = q;
89      return np;
90    }
91    SAM_Node *nq = newSAM_Node(q);
92    nq->len = p->len + 1;
93    q->fa = nq;
94    np->fa = nq;
95    for (; p && p->next[x] == q; p = p->fa)
96      p->next[x] = nq;
97    return np;
98  }
99  void SAM_build(char *s)//多串建立 注意SAM_init()的调用
100 {
101   int l = strlen(s);
102   SAM_Node *p = SAM_root;
103   for (int i = 0; i < l; i++)
104   {
105     if (!p->next[s[i] - 'a'] || !(p->next[s[i] - 'a']->len == i +
          1))
106       p=SAM_add(p,s[i] - 'a', i + 1);
107     else
108       p = p->next[s[i] - 'a'];
109   }
110 }
111
112 struct ST_Node
113 {
114   ST_Node *next[CHAR],*fa;
115   int len,pos;
116 }ST_node[MAXN*2],*ST_root;
117 int Sufpos[MAXN];
118 void ST_add(int u,int v,int chr,int len)
119 {
120   ST_node[u].next[chr]=&ST_node[v];
121   ST_node[v].len=len;
122 }
123 void init(int n)
124 {
125   for (int i=0;i<n;i++)
126   {
127     ST_node[i].pos=-1;
```

```
128     ST_node[i].fa=0;
129     memset(ST_node[i].next,0,sizeof(ST_node[i].next));
130   }
131   ST_node[0].pos=0;
132   ST_root=&ST_node[0];
133 }
134 void ST_build(char *s)
135 {
136   int n=strlen(s);
137   reverse(s,s+n);
138   SAM_build(s);
139   init(SAM_size);
140   for (int i=1;i<SAM_size;i++)
141   {
142     ST_add(SAM_node[i].fa->id,
143       SAM_node[i].id,
144       s[SAM_node[i].pos-SAM_node[i].fa->len-1]-'a',
145       SAM_node[i].len-SAM_node[i].fa->len);
146     if (SAM_node[i].pos==SAM_node[i].len)
147     {
148       Sufpos[n-SAM_node[i].pos+1]=i;
149       ST_node[i].pos=n-SAM_node[i].pos+1;
150     }
151   }
152 }
153
154 int rank[MAXN],sa[MAXN+1];
155 int height[MAXN];
156 int L;
157 void ST_dfs(ST_Node *p)
158 {
159   if (p->pos!=-1)
160     sa[L++]=p->pos;
161   for (int i=0;i<CHAR;i++)
162     if (p->next[i])
163       ST_dfs(p->next[i]);
164 }
165 char s[MAXN+1];
166 int main()
167 {
168   gets(s);
169   ST_build(s);
170   L=0;
171   ST_dfs(ST_root);
172   int n=strlen(s);
173   for (int i=0; i<n; i++)
174     sa[i]=sa[i+1]-1;
175   for (int i=0; i<n; i++)
176     rank[sa[i]]=i;
177   reverse(s,s+n);
178   for (int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
```

```
    if (rank[i])
      for (k?k--:0,j=sa[rank[i]-1]; s[i+k]==s[j+k]; k++);
}
```

2.3.1 例题

```
#include <iostream>
#include <algorithm>
#include <cstdio>
#include <cstring>
using namespace std;

const int CHAR = 26;
const int MAXN = 100000;

struct SAM_Node
{
  SAM_Node *fa,*next[CHAR];
  int len;
  int id;
  int mat[9];
  SAM_Node() {}
  SAM_Node(int _len)
  {
    fa = 0;
    len = _len;
    memset(mat,0,sizeof(mat));
    memset(next,0,sizeof(next));
  }
};
SAM_Node SAM_node[MAXN*2],*SAM_root,*SAM_last;
int SAM_size;
SAM_Node *newSAM_Node(int len)
{
  SAM_node[SAM_size] = SAM_Node(len);
  SAM_node[SAM_size].id = SAM_size;
  return &SAM_node[SAM_size++];
}
SAM_Node *newSAM_Node(SAM_Node *p)
{
  SAM_node[SAM_size] = *p;
  SAM_node[SAM_size].id = SAM_size;
  return &SAM_node[SAM_size++];
}
void SAM_init()
{
  SAM_size = 0;
  SAM_root = SAM_last = newSAM_Node(0);
}
void SAM_add(int x,int len)
{
  SAM_Node *p = SAM_last,*np = newSAM_Node(p->len+1);
```

```
47    SAM_last = np;
48    for (; p&&!p->next[x]; p=p->fa)
49      p->next[x] = np;
50    if (!p)
51    {
52      np->fa = SAM_root;
53      return;
54    }
55    SAM_Node *q = p->next[x];
56    if (q->len == p->len+1)
57    {
58      np->fa = q;
59      return;
60    }
61    SAM_Node *nq = newSAM_Node(q);
62    nq->len = p->len+1;
63    q->fa = nq;
64    np->fa = nq;
65    for (; p&&p->next[x] == q; p = p->fa)
66      p->next[x] = nq;
67  }
68  int getid(char ch)
69  {
70    return ch-'a';
71  }
72  void SAM_build(char *s)
73  {
74    SAM_init();
75    int l = strlen(s);
76    for (int i = 0; i < l; i++)
77      SAM_add(getid(s[i]),i+1);
78  }
79  char s[10][MAXN+1];
80  int ans;
81  int head[MAXN*2];
82  struct Edge
83  {
84    int to,next;
85  } edge[MAXN*2];
86  int M;
87  int n;
88  void add_edge(int u,int v)
89  {
90    edge[M].to=v;
91    edge[M].next=head[u];
92    head[u]=M++;
93  }
94  void dfs(int u)
95  {
96    for (int i=head[u]; i!=-1; i=edge[i].next)
97    {
```

```
98      int v=edge[i].to;
99      dfs(v);
100     for (int j=0; j<n-1; j++)
101     SAM_node[u].mat[j]=max(SAM_node[v].mat[j],SAM_node[u].mat[j]);
102   }
103   int tmp=SAM_node[u].len;
104   for (int i=0; i<n-1; i++)
105     tmp=min(tmp,SAM_node[u].mat[i]);
106   ans=max(ans,tmp);
107 }
108 int main()
109 {
110
111   while (scanf("%s",s[n])!=EOF)
112     n++;
113   int L=strlen(s[0]);
114   ans=M=0;
115   SAM_build(s[0]);
116   for (int j=1; j<n; j++)
117   {
118     int l=strlen(s[j]),len=0;
119     SAM_Node *p=SAM_root;
120     for (int i=0; i<l; i++)
121     {
122       if (p->next[getid(s[j][i])])
123       {
124         p=p->next[getid(s[j][i])];
125         p->mat[j-1]=max(p->mat[j-1],++len);
126       }
127       else
128       {
129         while (p && !p->next[getid(s[j][i])])
130           p=p->fa;
131         if (!p)
132         {
133           p=SAM_root;
134           len=0;
135         }
136         else
137         {
138           len=p->len+1;
139           p=p->next[getid(s[j][i])];
140         }
141         p->mat[j-1]=max(p->mat[j-1],len);
142       }
143     }
144   }
145   memset(head,-1,4*SAM_size);
146   for (int i=1; i<SAM_size; i++)
147     add_edge(SAM_node[i].fa->id,i);
148   dfs(0);
```

```
149   printf("%d\n",ans);
150   return 0;
151 }
```

LCS2

```
  1 #include <iostream>
  2 #include <algorithm>
  3 #include <cstdio>
  4 #include <cstring>
  5 using namespace std;
  6
  7 const int CHAR = 26;
  8 const int MAXN = 100000;
  9
 10 struct SAM_Node
 11 {
 12   SAM_Node *fa,*next[CHAR];
 13   int len;
 14   int id;
 15   int mat[9];
 16   SAM_Node() {}
 17   SAM_Node(int _len)
 18   {
 19     fa = 0;
 20     len = _len;
 21     memset(mat,0,sizeof(mat));
 22     memset(next,0,sizeof(next));
 23   }
 24 };
 25 SAM_Node SAM_node[MAXN*2],*SAM_root,*SAM_last;
 26 int SAM_size;
 27 SAM_Node *newSAM_Node(int len)
 28 {
 29   SAM_node[SAM_size] = SAM_Node(len);
 30   SAM_node[SAM_size].id = SAM_size;
 31   return &SAM_node[SAM_size++];
 32 }
 33 SAM_Node *newSAM_Node(SAM_Node *p)
 34 {
 35   SAM_node[SAM_size] = *p;
 36   SAM_node[SAM_size].id = SAM_size;
 37   return &SAM_node[SAM_size++];
 38 }
 39 void SAM_init()
 40 {
 41   SAM_size = 0;
 42   SAM_root = SAM_last = newSAM_Node(0);
 43 }
 44 void SAM_add(int x,int len)
 45 {
 46   SAM_Node *p = SAM_last,*np = newSAM_Node(p->len+1);
```

```
47    SAM_last = np;
48    for (; p&&!p->next[x]; p=p->fa)
49      p->next[x] = np;
50    if (!p)
51    {
52      np->fa = SAM_root;
53      return;
54    }
55    SAM_Node *q = p->next[x];
56    if (q->len == p->len+1)
57    {
58      np->fa = q;
59      return;
60    }
61    SAM_Node *nq = newSAM_Node(q);
62    nq->len = p->len+1;
63    q->fa = nq;
64    np->fa = nq;
65    for (; p&&p->next[x] == q; p = p->fa)
66      p->next[x] = nq;
67  }
68  int getid(char ch)
69  {
70    return ch-'a';
71  }
72  void SAM_build(char *s)
73  {
74    SAM_init();
75    int l = strlen(s);
76    for (int i = 0; i < l; i++)
77      SAM_add(getid(s[i]),i+1);
78  }
79  char s[MAXN+1];
80  int ans;
81  int head[MAXN*2];
82  struct Edge
83  {
84    int to,next;
85  } edge[MAXN*2];
86  int M;
87  int n;
88  void add_edge(int u,int v)
89  {
90    edge[M].to=v;
91    edge[M].next=head[u];
92    head[u]=M++;
93  }
94  void dfs(int u)
95  {
96    for (int i=head[u]; i!=-1; i=edge[i].next)
97    {
```

```
98      int v=edge[i].to;
99      dfs(v);
100     for (int j=0; j<n; j++)
101     SAM_node[u].mat[j]=max(SAM_node[v].mat[j],SAM_node[u].mat[j]);
102   }
103   int tmp=SAM_node[u].len;
104   for (int i=0; i<n; i++)
105     tmp=min(tmp,SAM_node[u].mat[i]);
106   ans=max(ans,tmp);
107 }
108 int main()
109 {
110   //freopen("in.txt","r",stdin);
111   //freopen("out.txt","w",stdout);
112   n=0;
113   gets(s);
114   SAM_build(s);
115   while (gets(s))
116   {
117     int l=strlen(s),len=0;
118     SAM_Node *p=SAM_root;
119     for (int i=0; i<l; i++)
120     {
121       if (p->next[getid(s[i])])
122       {
123         p=p->next[getid(s[i])];
124         p->mat[n]=max(p->mat[n],++len);
125       }
126       else
127       {
128         while (p && !p->next[getid(s[i])])
129           p=p->fa;
130         if (!p)
131         {
132           p=SAM_root;
133           len=0;
134         }
135         else
136         {
137           len=p->len+1;
138           p=p->next[getid(s[i])];
139         }
140         p->mat[n]=max(p->mat[n],len);
141       }
142       //printf("%d %d %d\n",i,len,p->id);
143     }
144     n++;
145   }
146   memset(head,-1,4*SAM_size);
147   for (int i=1; i<SAM_size; i++)
148     add_edge(SAM_node[i].fa->id,i);
```

```
149   dfs(0);
150   printf("%d\n",ans);
151   return 0;
152 }
```

2.4 KMP

求 A[0..i] 的一个后缀最多能匹配 B 的前缀多长。先对 B 进行自匹配然后与 A 匹配。KMP[i] 就是对应答案，p[i]+1 是 B[0..i] 的一个后缀最多能匹配 B 的前缀多长。

```
1  //自匹配过程
2  int j;
3  p [0] = j = -1;
4  for ( int i = 1; i < lb; i++)
5  {
6    while (j >= 0 && b[j + 1] != b[i]) j = p[j];
7    if (b[j + 1] == b[i]) j ++;
8    p[i] = j;
9  }
10 //下面是匹配过程
11 j = -1;
12 for ( int i = 0; i < la; i++)
13 {
14   while (j >= 0 && b[j + 1] != a[i]) j = p[j];
15   if (b[j + 1] == a[i]) j ++;
16   KMP[i] = j + 1;
17 }
```

2.5 e-KMP

求 A[i..len-1] 和 B 的最长公共前缀有多长。先对 B 进行自匹配然后与 A 匹配。eKMP[i] 就是对应答案。p[i] 是 B[i..len-1] 和 B 的最长公共前缀有多长。

```
1  //自匹配过程
2  int j = 0;
3  while (j < lb && b[j] == b[j + 1])
4    j++;
5  p[0] = lb, p[1] = j;
6  int k = 1;
7  for (int i = 2; i < lb; i++)
8  {
9    int Len = k + p[k] - 1, L = p[i - k];
10   if (L < Len - i + 1)
11     p[i] = L;
12   else
13   {
14     j = max(0, Len - i + 1);
15     while (i + j < lb && b[i + j] == b[j])
16       j++;
17     p[i] = j, k = i;
18   }
19 }
20 //下面是匹配过程
```

```
j = 0;
while (j < la && j < lb && a[j] == b[j])
  j++;
eKMP[0] = j;
k = 0;
for (int i = 1; i < la; i++)
{
  int Len = k + eKMP[k] - 1, L = p[i - k];
  if (L < Len - i + 1)
    eKMP[i] = L;
  else
  {
    j = max(0, Len - i + 1);
    while (i + j < la && j < lb && a[i + j] == b[j])
      j++;
    eKMP[i] = j, k = i;
  }
}
```

2.6 Manacher

```
const int maxn = 110000;

char Ma[maxn*2];
int Mp[maxn*2];
void Manacher(char s[],int len)
{
  int l = 0;
  Ma[l++] = '.';
  Ma[l++] = ',';
  for (int i = 0;i < len;i++)
  {
    Ma[l++] = s[i];
    Ma[l++] = ',';
  }
  Ma[l] = 0;
  int pnow = 0,pid = 0;
  for (int i = 1;i < l;i++)
  {
    if (pnow > i)
      Mp[i] = min(Mp[2*pid-i],pnow-i);
    else
      Mp[i] = 1;
    for (;Ma[i-Mp[i]] == Ma[i+Mp[i]];Mp[i]++);
    if (i+Mp[i] > pnow)
    {
      pnow = i+Mp[i];
      pid = i;
    }
  }
}
```

```
/*
abaaba
 .  ,  a  ,  b  ,  a  ,  a  ,  b  ,  a  ,
 0  1  2  1  4  1  2  7  2  1  4  1  2  1
*/
```

2.7 不同回文串

往 hash 表中插入新东西的时候就说明找到了一个新回文字串
一共 $O(n)$ 个

```
typedef unsigned int uint;

const int maxn = 110000;

char Ma[maxn*2];
int Mp[maxn*2];
void Manacher(char s[],int len)
{
  int l = 0;
  Ma[l++] = '.';
  Ma[l++] = ',';
  for (int i = 0; i < len; i++)
  {
    Ma[l++] = s[i];
    Ma[l++] = ',';
  }
  Ma[l] = 0;
  int pnow = 0,pid = 0;
  for (int i = 1; i < l; i++)
  {
    if (pnow > i)
      Mp[i] = min(Mp[2*pid-i],pnow-i);
    else
      Mp[i] = 1;
    for (; Ma[i-Mp[i]] == Ma[i+Mp[i]]; Mp[i]++);
    if (i+Mp[i] > pnow)
    {
      pnow = i+Mp[i];
      pid = i;
    }
  }
}

char s[maxn*2];
int len;
int p[maxn*2];
const int muts = 129;
uint sum[maxn];
uint mutpower[maxn];

struct hash_map
{
```

```
43    const static int mod = 300007;
44    int head[mod];
45    struct hash_tables
46    {
47      uint key1;
48      int key2;
49      int next;
50    } ele[maxn*10];
51    int N;
52    void init()
53    {
54      memset(head,-1,sizeof(head));
55      N = 0;
56    }
57    int totlen[mod];
58    void clear()
59    {
60      for (int i = 0; i < N; i++)
61        head[ele[i].key1%mod] = -1;
62      N = 0;
63    }
64    int find(uint x,int len)
65    {
66      int hashcode = x%mod;
67      for (int i = head[hashcode]; i != -1; i = ele[i].next)
68        if (ele[i].key1 == x && ele[i].key2 == len)
69          return i;
70      return -1;
71    }
72    void insert(uint x,int len)
73    {
74      int tmp = x%mod;
75      ele[N].key1 = x;
76      ele[N].key2 = len;
77      ele[N].next = head[tmp];
78      head[tmp] = N++;
79    }
80  };
81
82  hash_map hash;
83
84  uint gethashcode(int l,int r)
85  {
86    uint ret;
87    ret = sum[r];
88    if (l)
89      ret -= sum[l-1]*mutpower[r-l+1];
90    return ret;
91  }
92
93  int calc(char s[])
```

```
{
  len = strlen(s);
  Manacher(s,len);

  sum[0] = s[0];
  for (int i = 1; i < len; i++)
    sum[i] = sum[i-1]*muts+s[i];

  int res = 0;
  uint tmp;
  int nt = 0;
  hash.clear();
  //odd
  for (int i = 0; i < len; i++)
    if (Mp[i*2+2]%2 == 0)
    {
      int pl = Mp[i*2+2]/2;
      if (i+pl < nt || pl == 0) continue;
      for (int j = i-pl+1; j <= i; j++)
      {
        tmp = gethashcode(j,i);
        if (hash.find(tmp,i-j+1) != -1) break;
        hash.insert(tmp,i-j+1);
      }
      nt = i+pl;
    }
  res += hash.N;

  nt = 0;
  hash.clear();
  //even
  for (int i = 0; i < len; i++)
    if (Mp[i*2+3] > 1)
    {
      int pl = Mp[i*2+3]/2;
      if (i+pl < nt || pl == 0) continue;
      for (int j = i-pl+1; j <= i; j++)
      {
        tmp = gethashcode(j,i);
        if (hash.find(tmp,i-j+1) != -1) break;
        hash.insert(tmp,i-j+1);
      }
      nt = i+pl;
    }
  res += hash.N;
  return res;
}

int main()
{
  mutpower[0] = 1;
```

```
145   for (int i = 1; i < maxn; i++)
146     mutpower[i] = mutpower[i-1]*muts;
147   hash.init();
148
149   int totcas;
150   scanf("%d",&totcas);
151   for (int cas = 1; cas <= totcas; cas++)
152   {
153     scanf("%s",s);
154
155     printf("Case␣#%d:␣%d\n",cas,calc(s));
156   }
157   return 0;
158 }
```

2.8 * 字符串最小表示法

```
 1 int Gao(char a[],int len)
 2 {
 3   int i = 0,j = 1,k = 0;
 4   while (i < len && j < len && k < len)
 5   {
 6     int cmp = a[(j+k)%len]-a[(i+k)%len];
 7     if (cmp == 0)
 8       k++;
 9     else
10     {
11       if (cmp > 0)
12         j += k+1;
13       else
14         i += k+1;
15       if (i == j) j++;
16       k = 0;
17     }
18   }
19   return min(i,j);
20 }
```

2.9 带 * 通配符的匹配

```
 1 #include <iostream>
 2 #include <algorithm>
 3 #include <cstdio>
 4 #include <cstring>
 5 using namespace std;
 6
 7 char a[110],b[110],sp[110][110],tot,place[110];
 8 int n,la,lb,ll;
 9
10 bool check(int id,int pos)
11 {
```

```
12    for (int i = 0;sp[id][i] != 0;i++)
13      if (b[pos+i] != sp[id][i])
14        return false;
15    return true;
16  }
17
18  bool check()
19  {
20    lb = strlen(b);
21    int pre = 0;
22    for (int i = 0;i < tot;i++)
23    {
24      bool find = false;
25      for (int j = pre;j < lb;j++)
26        if (check(i,j) == true)
27        {
28          place[i] = j;
29          pre = place[i]+1;
30          find = true;
31          break;
32        }
33      if (find == false)  return false;
34    }
35    if (a[0] != '*')
36      if (place[0] != 0)
37        return false;
38    if (a[la-1] != '*')
39      if (check(tot-1,lb-ll) == false)
40        return false;
41    return true;
42  }
43
44  int main()
45  {
46    while (scanf("%s",a) != EOF)
47    {
48      tot = 0;
49      for (int i = 0;a[i] != 0;i++)
50        if (a[i] != '*')
51        {
52          int j;
53          for (j = i;a[j] != 0 && a[j] != '*';j++)
54            sp[tot][j-i] = a[j];
55          sp[tot++][j-i] = 0;
56          i = j;
57        }
58      la = strlen(a);
59      ll = strlen(sp[tot-1]);
60      scanf("%d",&n);
61      for (int i = 0;i < n;i++)
62      {
```

```
63        scanf("%s",b);
64        if (check() == true)
65          puts(b);
66      }
67    }
68    return 0;
69  }
70  /*
71  Sample Input 1
72  *.*
73  4
74  main.c
75  a.out
76  readme
77  yacc
78
79  Sample Input 2
80  *a*a*a
81  4
82  aaa
83  aaaaa
84  aaaaax
85  abababa
86
87  Sample Output 1
88  main.c
89  a.out
90
91  Sample Output 2
92  aaa
93  aaaaa
94  abababa
95  */
```

3 数学

3.1 扩展 GCD

求 ax+by=gcd(a,b) 的一组解

```
long long ex_gcd(long long a,long long b,long long &x,long long &y)
{
  if (b)
  {
    long long ret = ex_gcd(b,a%b,x,y),tmp = x;
    x = y;
    y = tmp-(a/b)*y;
    return ret;
  }
  else
  {
    x = 1;
    y = 0;
    return a;
  }
}
```

3.2 模线性方程组

```
//有更新
int m[10],a[10];//模数m 余数a
bool solve(int &m0,int &a0,int m,int a)//模线性方程组
{
  int y,x;
  int g=ex_gcd(m0,m,x,y);
  if (abs(a-a0)%g) return 0;
  x*=(a-a0)/g;
  x%=m/g;
  a0=(x*m0+a0);
  m0*=m/g;
  a0%=m0;
  if (a0<0) a0+=m0;
  return 1;
}
int MLES()
{
  bool flag=1;
  int m0=1,a0=0;
  for (int i=0; i<n; i++)
    if (!solve(m0,a0,m[i],a[i]))
    {
      flag=0;
      break;
    }
  if (flag)
```

```
27      return a0;
28    else
29      return -1;
30  }
```

3.3 矩阵

乘法的时候将 B 数组转置一下然后 $C[i][j] = \sum A[i][k] \times B[j][k]$ 会有奇效。

```
 1  struct Matrix
 2  {
 3    int a[52][52];
 4    void clear()
 5    {
 6      memset(a,0,sizeof(a));
 7    }
 8    int det(int n)//求行列式的值模上一个数，需要预处理逆元
 9    {
10      for (int i = 0;i < n;i++)
11        for (int j = 0;j < n;j++)
12          a[i][j] = (a[i][j]%mod+mod)%mod;
13      int res = 1;
14      for (int i = 0;i < n;i++)
15      {
16        for (int j = i;j < n;j++)
17          if (a[j][i] != 0)
18          {
19            for (int k = i;k < n;k++)
20              swap(a[i][k],a[j][k]);
21            if (i != j)
22              res = (res+mod)%mod;
23            break;
24          }
25        if (a[i][i] == 0)
26        {
27          res = -1;//不存在
28          break;
29        }
30        for (int j = i+1;j < n;j++)
31        {
32          int mut = (a[j][i]*inv[a[i][i]])%mod;
33          for (int k = i;k < n;k++)
34            a[j][k] = (a[j][k]-(a[i][k]*mut)%mod+mod)%mod;
35        }
36        res = (res*a[i][i])%mod;
37      }
38      return res;
39    }
40    Matrix operator * (const Matrix &b)const
41    {
42      Matrix res;
43      for (int i = 0; i < 52; i++)
44        for (int j = 0; j < 52; j++)
```

```
45        {
46          res.a[i][j] = 0;
47          for (int k = 0; k < 52; k++)
48            res.a[i][j] += a[i][k] * b.a[k][j];
49        }
50      return res;
51    }
52    Matrix operator ^ (int y)const
53    {
54      Matrix res, x;
55      for (int i = 0; i < 52; i++)
56      {
57        for (int j = 0; j < 52; j++)
58          res.a[i][j] = 0, x.a[i][j] = a[i][j];
59        res.a[i][i] = 1;
60      }
61      for (; y; y >>= 1, x = x * x)
62        if (y & 1)
63          res = res * x;
64      return res;
65    }
66  };
```

3.4 康拓展开

```
 1  const int PermSize = 12;
 2  int factory[PermSize] =
 3  {1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800};
 4  int Cantor(int a[])
 5  {
 6    int i, j, counted;
 7    int result = 0;
 8    for (i = 0; i < PermSize; ++i)
 9    {
10      counted = 0;
11      for (j = i + 1; j < PermSize; ++j)
12        if (a[i] > a[j])
13          ++counted;
14      result = result + counted * factory[PermSize - i - 1];
15    }
16    return result;
17  }
18
19  bool h[13];
20
21  void UnCantor(int x, int res[])
22  {
23    int i,j,l,t;
24    for (i = 1;i <= 12;i++)
25      h[i] = false;
26    for (i = 1; i <= 12; i++)
```

```
  {
    t = x / factory[12 - i];
    x -= t * factory[12 - i];
    for (j = 1, l = 0; l <= t; j++)
      if (!h[j])l++;
    j--;
    h[j] = true;
    res[i - 1] = j;
  }
}
```

3.5 FFT

```
const double PI= acos(-1.0);
struct vir
{
  double re,im; //实部和虚部
  vir(double a=0,double b=0)
  {
    re=a;
    im=b;
  }
  vir operator +(const vir &b)
  {return vir(re+b.re,im+b.im);}
  vir operator -(const vir &b)
  {return vir(re-b.re, im-b.im);}
  vir operator *(const vir &b)
  {return vir(re*b.re-im*b.im , re*b.im+im*b.re);}
};
vir x1[200005],x2[200005];
void change(vir *x,int len,int loglen)
{
  int i,j,k,t;
  for(i=0;i<len;i++)
  {
    t=i;
    for(j=k=0; j<loglen; j++,t>>=1)
      k= (k<<1)|(t&1);
    if(k<i)
    {
    //  printf("%d %d\n",k,i);
      vir wt=x[k];
      x[k]=x[i];
      x[i]=wt;
    }
  }
}
void fft(vir *x,int len,int loglen)
{
  int i,j,t,s,e;
  change(x,len,loglen);
  t=1;
```

```
  for(i=0;i<loglen;i++,t<<=1)
  {
    s=0;
    e=s+t;
    while(s<len)
    {
      vir a,b,wo(cos(PI/t),sin(PI/t)),wn(1,0);
      for(j=s;j<s+t;j++)
      {
        a=x[j];
        b=x[j+t]*wn;
        x[j]=a+b;
        x[j+t]=a-b;
        wn=wn*wo;
      }
      s=e+t;
      e=s+t;
    }
  }
}
void dit_fft(vir *x,int len,int loglen)
{
  int i,j,s,e,t=1<<loglen;
  for(i=0;i<loglen;i++)
  {
    t>>=1;
    s=0;
    e=s+t;
    while(s<len)
    {
      vir a,b,wn(1,0),wo(cos(PI/t),-sin(PI/t));
      for(j=s;j<s+t;j++)
      {
        a=x[j]+x[j+t];
        b=(x[j]-x[j+t])*wn;
        x[j]=a;
        x[j+t]=b;
        wn=wn*wo;
      }
      s=e+t;
      e=s+t;
    }
  }
  change(x,len,loglen);
  for(i=0;i<len;i++)
    x[i].re/=len;
}
int main()
{
  char a[100005],b[100005];
  int i,len1,len2,len,loglen;
```

```
 91  int t,over;
 92  while(scanf("%s%s",a,b)!=EOF)
 93  {
 94    len1=strlen(a)<<1;
 95    len2=strlen(b)<<1;
 96    len=1;loglen=0;
 97    while(len<len1)
 98    {
 99      len<<=1;  loglen++;
100    }
101    while(len<len2)
102    {
103      len<<=1;  loglen++;
104    }
105    for(i=0;a[i];i++)
106    {
107      x1[i].re=a[i]-'0';
108      x1[i].im=0;
109    }
110    for(;i<len;i++)
111      x1[i].re=x1[i].im=0;
112    for(i=0;b[i];i++)
113    {
114      x2[i].re=b[i]-'0';
115      x2[i].im=0;
116    }
117    for(;i<len;i++)
118      x2[i].re=x2[i].im=0;
119    fft(x1,len,loglen);
120    fft(x2,len,loglen);
121    for(i=0;i<len;i++)
122      x1[i] = x1[i]*x2[i];
123    dit_fft(x1,len,loglen);
124    for(i=(len1+len2)/2-2,over=len=0;i>=0;i--)
125    {
126      t=(int)(x1[i].re+over+0.5);
127      a[len++]= t%10;
128      over = t/10;
129    }
130    while(over)
131    {
132      a[len++]=over%10;
133      over/=10;
134    }
135    for(len--;len>=0&&!a[len];len--);
136      if(len<0)
137      putchar('0');
138      else
139        for(;len>=0;len--)
140          putchar(a[len]+'0');
141    putchar('\n');
```

```
142   }
143   return 0;
144 }
```

3.6 爬山法计算器

注意灵活运用。
双目运算符在 calc() 中，左结合单目运算符在 P() 中，右结合单目运算符在 calc_exp 中。（但是还没遇到过。。）

```
 1 #include <iostream>
 2 #include <cstdio>
 3 #include <cstring>
 4 #include <algorithm>
 5 #include <string>
 6 using namespace std;
 7
 8 char s[100000];
 9 int n,cur;
10 const string OP = "+-*";
11
12 char next_char()
13 {
14   if (cur >= n) return EOF;
15   return s[cur];
16 }
17
18 int get_priority(char ch)
19 {
20   if (ch == '*')  return 2;
21   return 1;
22 }
23
24 int P();
25
26 int calc(int a,char op,int b)
27 {
28   if (op == '+')
29     return a+b;
30   if (op == '-')
31     return a-b;
32   if (op == '*')
33     return a*b;
34 }
35
36 int calc_exp(int p)
37 {
38   int a = P();
39   while ((OP.find(next_char()) != OP.npos) &&
40     (get_priority(next_char()) >= p))
41   {
42     char op = next_char();
```

```
43      cur++;
44      a = calc(a,op,calc_exp(get_priority(op)+1));
45    }
46    return a;
47  }
48
49  int totvar,m,var[26],varid[26];
50
51  int P()
52  {
53    if (next_char() == '-')
54    {
55      cur++;
56      return -P();
57    }
58    else if (next_char() == '+')
59    {
60      cur++;
61      return P();
62    }
63    else if (next_char() == '(')
64    {
65      cur++;
66      int res = calc_exp(0);
67      cur++;
68      return res;
69    }
70    else
71    {
72      cur++;
73      return var[varid[s[cur-1]-'a']];
74    }
75  }
76
77  int id[26],minid;
78
79  int main()
80  {
81    while (true)
82    {
83      scanf("%d%d",&totvar,&var[0]);
84      if (totvar == 0 && var[0] == 0)  break;
85      for (int i = 1;i < totvar;i++)
86        scanf("%d",&var[i]);
87      scanf("%d",&m);
88      scanf("%s",s);
89      for (int i = 0;i < 26;i++)
90        id[i] = -1;
91      minid = 0;
92      n = strlen(s);
93      for (int i = 0;i < n;i++)
```

```
94        if (s[i] >= 'a' && s[i] <= 'z')
95        {
96          if (id[s[i]-'a'] == -1)
97          {
98            id[s[i]-'a'] = minid;
99            minid++;
100         }
101         s[i] = 'a'+id[s[i]-'a'];
102       }
103     for (int i = 0;i < totvar;i++)
104       varid[i] = i;
105     int res = 0;
106     do
107     {
108       cur = 0;
109       int tmp = calc_exp(0);
110       if (tmp == m)
111       {
112         res++;
113         break;
114       }
115     }
116     while (next_permutation(varid,varid+totvar));
117     //puts(s);
118     if (res > 0)
119       puts("YES");
120     else
121       puts("NO");
122   }
123   return 0;
124 }
```

3.7 线性筛

```
1  int N;
2  bool isPrime[10001];
3  int prime[10000];
4  void getPrime(int n)
5  {
6    memset(isPrime,1,++n);
7    N=0;
8    isPrime[0]=isPrime[1]=0;
9    for (int i=2;i<n;i++)
10   {
11     if (isPrime[i])
12       prime[N++]=i;
13     for (int j=0;j<N && prime[j]*i<n;j++)
14     {
15       isPrime[i*prime[j]]=0;
16       if (i%prime[j]==0)
17         break;
```

```
    }
  }
}
```

3.8 线性规划

```
#define MAXM 20  //max num of basic varibles
#define INF 1E200

double A[MAXM+5][MAXN+MAXM+5];
double b[MAXM+5],c[MAXN+MAXM+5];
int N[MAXN+5],B[MAXM+5];
double X[MAXN+MAXM+5],V;
int n,m,R,C,nCnt,bCnt;
int v1[MAXN],v2[MAXN];

int fcmp(double a,double b)
{
  if(fabs(a-b)<1E-7) return 0;
  if(a>b) return 1;
  return -1;
}

void Pivot(int l,int e)
{
  double t=A[l][e],p=c[e];
  b[l]=b[l]/t;
  for(int i=1;i<=C;i++)
    A[l][i]/=t;
  V=V-c[e]*b[l];
  for(int i=1;i<=R;i++)
  {
    if(i==l||fcmp(A[i][e],0.0)==0)
      continue;
    t=A[i][e];
    b[i]=b[i]-t*b[l];
    for(int j=1;j<=C;j++)
      A[i][j]=A[i][j]-t*A[l][j];
  }
  for(int i=1;i<=C;i++)
    c[i]=c[i]-p*A[l][i];
  for(int i=1;i<=nCnt;i++)
  {
    if(N[i]==e)
    {
      N[i]=B[l];
      break;
    }
  }
  B[l]=e;
}

```

```
47 bool Process(double P[])
48 {
49   while(true)
50   {
51     int e=-1;
52     double mV=-INF;
53     for(int i=1;i<=nCnt;i++)
54       if(fcmp(P[N[i]],mV)==1)
55         mV=P[N[i]],e=N[i];
56
57     if(fcmp(mV,0.0)<=0) break;
58     int l=-1;
59     mV=INF;
60     for(int i=1;i<=bCnt;i++)
61     {
62       if(fcmp(A[i][e],0.0)==1)
63       {
64         double t=b[i]/A[i][e];
65         if(fcmp(mV,t)==1||(fcmp(mV,t)==0&&(l==-1||B[l]>B[i])))
66           mV=t,l=i;
67       }
68     }
69     if(l==-1) return false;
70     Pivot(l,e);
71   }
72   return true;
73 }
74
75 bool initSimplex()
76 {
77   nCnt=bCnt=0;
78   for(int i=1;i<=n;i++)
79     N[++nCnt]=i;
80   for(int i=1;i<=m;i++)
81     B[++bCnt]=i+n,A[i][n+i]=1.0;
82   R=bCnt,C=bCnt+nCnt;
83   double minV=INF;
84   int p=-1;
85   for(int i=1;i<=m;i++)
86     if(fcmp(minV,b[i])==1)
87       minV=b[i],p=i;
88   if(fcmp(minV,0.0)>=0)
89     return true;
90   N[++nCnt]=n+m+1;R++,C++;
91   for(int i=0;i<=C;i++)
92     A[R][i]=0.0;
93   for(int i=1;i<=R;i++)
94     A[i][n+m+1]=-1.0;
95   Pivot(p,n+m+1);
96   if(!Process(A[R])) return false;
97   if(fcmp(b[R],0.0)!=0)
```

```
98      return false;
99    p=-1;
100   for(int i=1;i<=bCnt&&p==-1;i++)
101     if(B[i]==n+m+1) p=i;
102   if(p!=-1)
103   {
104     for(int i=1;i<=nCnt;i++)
105     {
106       if(fcmp(A[p][N[i]],0.0)!=0)
107       {
108         Pivot(p,N[i]);
109         break;
110       }
111     }
112   }
113   bool f=false;
114   for(int i=1;i<=nCnt;i++)
115   {
116     if(N[i]==n+m+1) f=true;
117     if(f&&i+1<=nCnt)
118       N[i]=N[i+1];
119   }
120   nCnt--;
121   R--,C--;
122   return true;
123 }
124
125 //-1: no solution 1: no bound 0: has a solution -V
126 int Simplex()
127 {
128   if(!initSimplex())
129     return -1;
130   if(!Process(c))
131     return 1;
132   for(int i=1;i<=nCnt;i++)
133     X[N[i]]=0.0;
134   for(int i=1;i<=bCnt;i++)
135     X[B[i]]=b[i];
136   return 0;
137 }
138
139 int main()
140 {
141   //n = 1;m=1;
142   //V= 0.0;
143   //c[1] = 1.0;
144   //A[1][1] = 1.0;
145   //b[1] = 5.0;
146   //Simplex();
147   //printf("V = %.3f\n",V);
148
```

```
149   while(scanf("%d",&v1[1]) == 1)
150     {
151       for(int i = 2; i<=6;i++)
152         scanf("%d",&v1[i]);
153       n = 4; m = 6;
154       for(int i = 0 ; i<=m+1;i++)
155         for(int j=0;j<=n+m+2;j++)
156           A[i][j] = c[j] = 0;
157       memset(b,0,sizeof(b));
158       V = 0.0;
159       /*
160       n 为未知数个数
161       m 为约束个数
162       目标：siama(c[i]*xi)
163       约束：sigma(A[i][j]*xj) <=b[i]; j = 1 ... n
164       解存在 X 里面
165       */
166       b[1] = v1[1] ; A[1][1] = 1;A[1][4] = 1;
167       b[2] = v1[2] ; A[2][1] = 1;A[2][3] = 1;
168       b[3] = v1[3] ; A[3][3] = 1;A[3][4] = 1;
169       b[4] = v1[4] ; A[4][2] = 1;A[4][3] = 1;
170       b[5] = v1[5] ; A[5][2] = 1;A[5][4] = 1;
171       b[6] = v1[6] ; A[6][1] = 1;A[6][2] = 1;
172       c[1] = 1;c[2] = 1;c[3] = 1;c[4] = 1;
173       Simplex();
174       //printf("V = %.3f\n",V);
175       printf("%.3f␣%.3f␣%.3f␣%.3f\n",X[1],X[2],X[3],X[4]);
176
177     }
178   return 0;
179 }
```

3.9 分解质因数

3.9.1 米勒拉宾 + 分解因数

```
 1 #include<ctime>
 2 #include<iostream>
 3 #define bint long long
 4 using namespace std;
 5 const int TIME = 8;//测试次数，够了8~10
 6 int factor[100],fac_top = -1;
 7
 8 //计算两个数的gcd
 9 bint gcd(bint small,bint big)
10 {
11   while(small)
12   {
13     swap(small,big);
14     small%=big;
15   }
16   return abs(big);
```

```
}

//ret = (a*b)%n (n<2^62)
bint muti_mod(bint a,bint b,bint n)
{
  bint exp = a%n, res = 0;
  while(b)
  {
    if(b&1)
    {
      res += exp;
      if(res>n) res -= n;
    }
    exp <<= 1;
    if (exp>n) exp -= n;
    b>>=1;
  }
  return res;
}

// ret = (a^b)%n
bint mod_exp(bint a,bint p,bint m)
{
  bint exp=a%m, res=1; //
  while(p>1)
  {
    if(p&1)
      res=muti_mod(res,exp,m);
    exp = muti_mod(exp,exp,m);
    p>>=1;
  }
  return muti_mod(res,exp,m);
}

//miller-法测试素数rabin, time 测试次数
bool miller_rabin(bint n, int times)
{
  if(n==2)return 1;
  if(n<2||!(n&1))return 0;
  bint a, u=n-1, x, y;
  int t=0;
  while(u%2==0)
  {
    t++;
    u/=2;
  }
  srand(time(0));
  for(int i=0; i<times; i++)
  {
    a = rand() % (n-1) + 1;
    x = mod_exp(a, u, n);
```

```
68      for(int j=0; j<t; j++)
69      {
70        y = muti_mod(x, x, n);
71        if ( y == 1 && x != 1 && x != n-1 )
72          return false; //must not
73        x = y;
74      }
75      if( y!=1) return false;
76    }
77    return true;
78  }
79
80  bint pollard_rho(bint n,int c)//找出一个因子
81  {
82    bint x,y,d,i = 1,k = 2;
83    srand(time(0));
84    x = rand()%(n-1)+1;
85    y = x;
86    while(true)
87    {
88      i++;
89      x = (muti_mod(x,x,n) + c) % n;
90      d = gcd(y-x, n);
91      if (1 < d && d < n) return d;
92      if( y == x) return n;
93      if(i == k)
94      {
95        y = x;
96        k <<= 1;
97      }
98    }
99  }
100
101 void findFactor(bint n,int k)//二分找出所有质因子，存入factor
102 {
103   if(n==1)return;
104   if(miller_rabin(n, TIME))
105   {
106     factor[++fac_top] = n;
107     return;
108   }
109   bint p = n;
110   while(p >= n)
111     p = pollard_rho(p,k--);//值变化，防止死循环k
112   findFactor(p,k);
113   findFactor(n/p,k);
114 }
115
116 int main()
117 {
118   bint cs,n,min;
```

```
119   cin>>cs;
120   while (cs--)
121   {
122     cin>>n;
123     fac_top = min = -1;
124     if(miller_rabin(n,TIME)) cout<<"Prime"<<endl;
125     else
126     {
127       findFactor(n,107);
128       for(int i=0; i<=fac_top; i++)
129       {
130         if(min<0||factor[i]<min)
131           min = factor[i];
132       }
133       cout<<min<<endl;
134     }
135   }
136   return 0;
137 }
```

3.9.2 暴力版本

```
 1 int N;
 2 int num[30],fac[30];
 3 void getFactor(int x)
 4 {
 5   N=0;
 6   memset(num,0,sizeof(num));
 7   for (int i=0; prime[i]*prime[i]<=x && i<L; i++)
 8   {
 9     if (x%prime[i]==0)
10     {
11       while (x%prime[i]==0)
12       {
13         x/=prime[i];
14         num[N]++;
15       }
16       fac[N++]=prime[i];
17     }
18   }
19   if (x>1)
20   {
21     num[N]=1;
22     fac[N++]=x;
23   }
24 }
```

3.10 baby step giant step

3.10.1 BSGS

```
1 #define MOD 76543
2 int hs[MOD], head[MOD], next[MOD], id[MOD], top;
```

```
 3 void insert(int x, int y)
 4 {
 5   int k = x%MOD;
 6   hs[top] = x, id[top] = y, next[top] = head[k], head[k] = top++;
 7 }
 8 int find(int x)
 9 {
10   int k = x%MOD;
11   for (int i = head[k]; i; i = next[i]) if (hs[i] == x)
12     return id[i];
13   return -1;
14 }
15 int BSGS(int a, int b, int n)
16 {
17   memset(head, 0, sizeof(head));
18   top = 1;
19   if (b==1) return 0;
20   int m = sqrt(n+.0), j;
21   long long x = 1, p = 1;
22   for (int i = 0; i < m; ++i, p = p*a%n) insert(p*b%n, i);
23   for (long long i = m; ; i += m)
24   {
25     if ((j = find(x=x*p%n)) != -1) return i-j;
26     if (i > n) break;
27   }
28   return -1;
29 }
```

3.10.2 何老师的版

```
 1 //离散对数
 2 #include <cstdio>
 3 #include <cstring>
 4 #include <cmath>
 5 #include <algorithm>
 6 using namespace std;
 7 typedef long long LL;
 8 struct Hash
 9 {
10     static const int MOD = 100007;
11     static const int MaxN = 100005;
12     struct Node
13     {
14         LL k, v;//A^k = v
15         Node *nxt;
16     } buf[MaxN], *g[MaxN], *pt;
17     void init()
18     {
19         memset(g,0,sizeof(g)) ;
20         pt = buf;
21     }
22     LL find(LL v)
```

```
23      {
24          for (Node *now = g[v%MOD]; now; now = now->nxt)
25              if (now->v == v)
26                  return now->k;
27          return -1;
28      }
29      void Ins(LL k, LL v)
30      {
31          if ( find (v) != -1)return;
32          pt->k = k;
33          pt->v = v;
34          pt->nxt = g[v % MOD];
35          g[v % MOD] = pt++;
36      }
37  }hash;
38  LL gcd(LL x, LL y)
39  {
40      return y==0?x:gcd(y,x%y);
41  }
42  LL e_gcd(LL a, LL b, LL &x, LL &y)
43  {
44      if (b==0)
45      {
46          x = 1;
47          y = 0;
48          return a;
49      }
50      LL ret = e_gcd(b, a%b, y, x) ;
51      y = y - a/b*x;
52      return ret ;
53  }
54  LL Baby(LL A, LL B, LL C)//A^x = B (mod C)
55  {
56      B %= C;
57      A %= C;
58      LL x = 1%C, y;
59      for (int i = 0; i <= 64; i++)
60      {
61          if (x==B)return i;
62          x = x*A % C;
63      }
64
65      LL D = 1%C, g;
66      int cnt = 0;
67      while((g = gcd(A,C)) != 1)
68      {
69          if (B%g) return -1;
70          cnt++;
71          C /= g;
72          B /= g;
73          D = A/g * D % C;
```

```
74      }
75      hash. init () ;
76      int m = (int)sqrt(C);
77      LL Am = 1%C;
78      hash.Ins(0,Am);
79      for (int i = 1; i <= m; i++)
80      {
81          Am = Am*A % C;
82          hash. Ins ( i ,Am);
83      }
84      for (int i = 0; i <= m; i++)
85      {
86      //D*x = B (mod C), D*x + C*y = B
87          g = e_gcd(D,C,x,y);
88          x = (x*B/g%C+C)%C;
89          LL k = hash.find(x) ;
90          if (k != -1) return i*m+k+cnt;
91          D = D*Am % C;
92      }
93      return -1;
94  }
95  int main()
96  {
97      int A,B,C;
98      while(scanf("%d%d%d",&A,&C,&B) == 3 && (A+B+C))
99      {
100         if (B>=C)
101         {
102             puts("Orz,I␣' cant␣find␣D!");
103             continue;
104         }
105         LL ret = Baby(A,B,C);
106         if ( ret == -1)puts("Orz,I␣' cant␣find␣D!");
107         else printf ("%I64d\n",ret);
108     }
109     return 0;
110 }
```

3.11 原根

```
1  int getPriRoot(int p)
2  {
3    if (p==2) return 1;
4    int phi = p - 1;
5    getFactor(phi);
6    for (int g = 2; g < p; ++g)
7    {
8      bool flag=1;
9      for (int i = 0; flag && i < N; ++i)
10       if (power(g, phi/fac[i], p) == 1)
11         flag=0;
12     if (flag)
```

```
13        return g;
14    }
15 }
```

3.12 逆元

```
1  void getInv2(int x)
2  {
3    inv[1]=1;
4    for (int i=2; i<=x; i++)
5      inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
6  }
7  int getInv(int x)//为素数mod
8  {
9    return power(x,mod-2);
10 }
```

3.13 卢卡斯

卢卡斯，$num[i]$ 阶乘也

```
1  int comLucus(int n,int m,int p)
2  {
3    int ans=1;
4    for (; n && m && ans; n/=p,m/=p)
5    {
6      if (n%p>=m%p)
7        ans = ans*num[n%p]%p*getInv(num[m%p]%p)%p
8          *getInv(num[n%p-m%p])%p;
9      else
10       ans=0;
11   }
12   return ans;
13 }
```

3.14 欧拉函数

3.14.1 分解质因数

```
1  int getEuler(int x)
2  {
3    getFactor(x);
4    int ret=x;
5    for (int i=0; i<N; i++)
6      ret = ret/fac[i]*(fac[i]-1);
7    return ret;
8  }
```

3.14.2 一次预处理

```
1  void getEuler2()
2  {
3    memset(euler,0,sizeof(euler));
4    euler[1] = 1;
```

```
  for (int i = 2; i <= 3000000; i++)
  {
    if (!euler[i])
    {
      for (int j = i; j <= 3000000; j += i)
      {
        if (!euler[j])
          euler[j] = j;
        euler[j] = euler[j]/i*(i-1);
      }
    }
  }
}
```

3.15 费马降阶法

分解素数 p 为 x^2+y^2 的费马降阶法，失败返回 -1，主程序调用 $\mathrm{calcu(p,x,y)}$

```
#include <stdio.h>
#include <string.h>
#include <stdlib.h>
int p,expp,A,B,aa,ans,tt;
long long M;
long long exp(int a,int b,long long mod)
{
   long long ans=1,num=a;
   while (b!=0)
   {
       if (b&1)
       {
           ans=((ans%mod)*(num%mod))%mod;
       }
       num=((num%mod)*(num%mod))%mod;
       b>>=1;
   }
   return ans;
}
int calcu(int p,int &x,int &y)
{
      if (p%4!=1) return -1;
      else
      {
        expp=(p-1)/4;
        A,B;
        while (1)
        {
          aa=rand()%p;
          if (aa==0) continue;
          A=exp(aa,expp,p);
          ans=(((long long)A%p)*((long long)A%p))%p;
          if (ans==p-1) break;
        }
        B=1;
```

```
        M=((long long)A*(long long)A+(long long)B*(long long)B)/p;
        if (M!=1) B=p;
        while (M!=1)
        {
          if (B>A)
          {tt=A; A=B; B=tt;}
          tt=A;
          A=B;
          B=tt%B;
          M=((long long)A*(long long)A
            +(long long)B*(long long)B)/p;
        }
        if (B<=A)
        {
             x=B;
             y=A;
        }
        else
        {
         x=A;
         y=B;
        }
      }
}
int main()
{
  while (scanf("%d",&p)!=EOF)
  {
      int x,y;
      if (calcu(p,x,y)!=-1)
  }
  return 0;
}
```

3.16 自适应 simp

过了哈尔滨积分题，精度要求不高的时候可以考虑使用。
暂时我只能用这个做做类似于凸函数或者凹函数的函数。

```
double Simp(double l,double r)
{
  double h = (r-l)/2.0;
  return h*(calc(l)+4*calc((l+r)/2.0)+calc(r))/3.0;
}

double rSimp(double l,double r)
{
  double mid = (l+r)/2.0;
  if (abs((Simp(l,r)-Simp(l,mid)-Simp(mid,r)))/15 < eps)
    return Simp(l,r);
  else
    return rSimp(l,mid)+rSimp(mid,r);
```

```
14 }
```

3.17 组合数求模

模是质数

```
 1 #include<cstdio>
 2 #include<cstring>
 3 #include<iostream>
 4 using namespace std;
 5 int mod;
 6 long long num[100000];
 7 int ni[100],mi[100];
 8 int len;
 9 void init(int p)
10 {
11   mod=p;
12   num[0]=1;
13   for (int i=1; i<p; i++)
14     num[i]=i*num[i-1]%p;
15 }
16 void get(int n,int ni[],int p)
17 {
18   for (int i = 0; i < 100; i++)
19     ni[i] = 0;
20   int tlen = 0;
21   while (n != 0)
22   {
23     ni[tlen++] = n%p;
24     n /= p;
25   }
26   len = tlen;
27 }
28 long long power(long long x,long long y)
29 {
30   long long ret=1;
31   for (long long a=x%mod; y; y>>=1,a=a*a%mod)
32     if (y&1)
33       ret=ret*a%mod;
34   return ret;
35 }
36 long long getInv(long long x)//mod 为素数
37 {
38   return power(x,mod-2);
39 }
40 long long calc(int n,int m,int p)//C(n,m)%p
41 {
42   init(p);
43   long long ans=1;
44   for (; n && m && ans; n/=p,m/=p)
45   {
46     if (n%p>=m%p)
```

```
47        ans = ans*num[n%p]%p*getInv(num[m%p]%p)%p
48          *getInv(num[n%p-m%p])%p;
49      else
50        ans=0;
51    }
52    return ans;
53  }
54  int main()
55  {
56    int t;
57    scanf("%d",&t);
58    while (t--)
59    {
60      int n,m,p;
61      scanf("%d%d%d",&n,&m,&p);
62      printf("%I64d\n",calc(n+m,m,p));
63    }
64    return 0;
65  }
```

3.18 高斯消元

```
1   const double eps = 1e-8;
2
3   void Guess(int n)
4   {
5     for (int i = 0; i < n; i++)
6     {
7       for (int j = i; j < n; j++)
8         if (fabs(a[j][i]) > eps)
9         {
10          for (int k = i; k <= n; k++)
11            swap(a[i][k],a[j][k]);
12          break;
13        }
14
15      if (fabs(a[i][i]) < eps)  continue;
16
17      for (int j = 0; j < n; j++)
18        if (i != j && fabs(a[j][i]) > eps)
19        {
20          double det = a[j][i]/a[i][i];
21          for (int k = i; k <= n; k++)
22            a[j][k] -= a[i][k]*det;
23        }
24    }
25
26    for (int i = 0; i < n; i++)
27    {
28      if (fabs(a[i][i]) < eps)
29      {
```

```
30        if (fabs(a[i][n]) > eps)
31        {
32          //无解
33          puts("Fuck");
34        }
35        //否则 x_i 可以是任意解
36      }
37      else
38      {
39        a[i][n] /= a[i][i];
40        if (fabs(a[i][n]) < eps)
41          a[i][n] = 0;
42      }
43    }
44
45  }
```

3.19 整数拆分

```
 1  #include <cstdio>
 2  #include <cmath>
 3  #include <cstring>
 4  #include <map>
 5  #include <algorithm>
 6  using namespace std;
 7  bool check(int x)
 8  {
 9    for (int i=2; i*i<=x; i++)
10      if (x%i==0)
11        return 0;
12    return 1;
13  }
14  int p[100000];
15  inline int calc(int x)
16  {
17    return x*(x*3-1)/2;
18  }
19  int main()
20  {
21    p[0]=1;
22    for (int i=1;i<100000;i++)
23    {
24      for (int j=1,k=1;calc(j)<=i;j++,k*=-1)
25      {
26        p[i]+=k*p[i-calc(j)];
27        if (p[i]<0)
28          p[i]+=1000000;
29        if (p[i]>=1000000)
30          p[i]-=1000000;
31        if (calc(-j)<=i)
32          p[i]+=k*p[i-calc(-j)];
33        if (p[i]<0)
```

```
34          p[i]+=1000000;
35        if (p[i]>=1000000)
36          p[i]-=1000000;
37      }
38      if (!p[i])
39        printf("%d\n",i);
40    }
41    return 0;
42  }
```

3.20 佩尔方程

写的不好稍微收一下

```
 1  import java.math.BigInteger;
 2  import java.util.*;
 3  public class Main
 4  {
 5    public static class Fraction
 6    {
 7      public BigInteger num,den;
 8      public Fraction()
 9      {
10        num=BigInteger.ZERO;
11        den=BigInteger.ONE;
12      }
13      public Fraction(int _num,int _den)
14      {
15        num=BigInteger.valueOf(_num);
16        den=BigInteger.valueOf(_den);
17      }
18      public Fraction(BigInteger _num,BigInteger _den)
19      {
20        num=_num;
21        den=_den;
22      }
23      public Fraction gen()
24      {
25        BigInteger g=num.gcd(den);
26        return new Fraction(num.divide(g),den.divide(g));
27      }
28      public Fraction add(Fraction x)
29      {
30        return new Fraction(x.num.multiply(den).add(num.multiply(x.
             den)),x.den.multiply(den)).gen();
31      }
32      public Fraction reciprocal()
33      {
34        return new Fraction(den,num);
35      }
36      public void out()
37      {
38        System.out.println(num+"/"+den);
```

```
39      }
40    }
41    public static BigInteger sqrt(BigInteger a)
42    {
43      BigInteger b=a;
44          while (a.compareTo(b.multiply(b))<0)
45            b=b.multiply(b).add(a).divide(b.multiply(BigInteger.
                 valueOf(2)));
46          return b;
47    }
48    public static boolean check(Fraction x,int n)
49    {
50      return x.num.multiply(x.num).add(x.den.multiply(x.den.multiply(
           BigInteger.valueOf(n))).negate()).compareTo(BigInteger.ONE)
           ==0;
51    }
52    static int p[]=new int[1000];
53    static int l;
54    public static void main(String[] args)
55      {
56      BigInteger ans=BigInteger.ZERO;
57      int idx=0;
58      for (int n=2,r=2;n<=1000;n++)
59      {
60        if (n==r*r)
61        {
62          r++;
63          continue;
64        }
65        int tmp=calc(n,0,1),a=tmp,b=n-tmp*tmp;
66        p[0]=tmp;
67        l=1;
68        while (true)
69        {
70          tmp=calc(n,a,b);
71          p[l++]=tmp;
72          a=a-tmp*b;
73          Fraction x=getFrac();
74          if (check(x,n))
75          {
76            if (ans.compareTo(x.num)<0)
77            {
78              ans=x.num;
79              idx=n;
80            }
81            break;
82          }
83          a=-a;
84          b=(n-a*a)/b;
85        }
86      }
```

```
87      System.out.println(idx);
88    }
89    private static Fraction getFrac() {
90      Fraction ret=new Fraction(p[l-1],1);
91      for (int i=l-2;i>=0;i--)
92        ret=new Fraction(p[i],1).add(ret.reciprocal());
93      return ret;
94    }
95    private static int calc(int n, int a, int b) {
96      for (long i=2;;i++)
97        if ((i*b-a)*(i*b-a)>n)
98          return (int)i-1;
99    }
100 }
```

3.21 其它公式

3.21.1 Polya

设 G 是 p 个对象的一个置换群，用 k 种颜色去染这 p 个对象，若一种染色方案在群 G 的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为：
$L=\frac{1}{|G|}\times\Sigma(k^{C(f)}), f\in G$
$C(f)$ 为循环节，$|G|$ 表示群的置换方法数

对于有 n 个位置的手镯，有 n 种旋转置换和 n 种翻转置换

对于旋转置换：
$C(f_i)=gcd(n,i)$,i 表示一次转过 i 颗宝石，$i=0$ 时 $c=n$；

对于翻转置换：

如果 n 为偶数： 则有 $\frac{n}{2}$ 个置换 $C(f)=\frac{n}{2}$，有 $\frac{n}{2}$ 个置换 $C(f)=\frac{n}{2}+1$

如果 n 为奇数： $C(f)=\frac{n}{2}+1$

3.21.2 拉格朗日插值法

已知 $y=a_0+a_1x+a_2x^2+\cdots+a_{n-1}x^{n-1}$ 曲线上的 n 个点 $(x_1,y_1),(x_2,y_2),(x_3,y_3)\cdots(x_n,y_n)$ 用拉格朗日插值法可以不求系数可知任意 x 对应的 y 值。

$$\begin{aligned}y=&y_1\frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)}\\&+y_2\frac{(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)}\\&+\cdots\\&+y_n\frac{(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})}\end{aligned}$$

特别的，如果 $x_1 \sim x_n$ 为连续自然数，那么对于下一个自然数对应的 y 值为：

$$y_{n+1} = (-1)^{n-1}C_n^0 y_1 + (-1)^{n-2}C_n^1 y_2 + \cdots + (-1)^0 C_n^{n-1} y_n$$

这个组合系数可以通过高斯消元暴出来，前提是要猜到它满足递推关系。

3.21.3 正多面体顶点着色

正四面体：$N = \frac{(n^4+11\times n^2)}{12}$
正六面体：$N = \frac{(n^8+17\times n^4+6\times n^2)}{24}$
正八面体：$N = \frac{(n^6+3\times n^4+12\times n^3+8\times n^2)}{24}$
正十二面体：$N = \frac{(n^{20}+15\times n^{10}+20\times n^8+24\times n^4)}{60}$
正二十面体：$N = \frac{(n^{12}+15\times n^6+44\times n^4)}{60}$

3.21.4 求和公式

$\sum k = \frac{n\times(n+1)}{2}$
$\sum 2k-1 = n^2$
$\sum k^2 = \frac{n\times(n+1)\times(2n+1)}{6}$
$\sum (2k-1)^2 = \frac{n\times(4n^2-1)}{3}$
$\sum k^3 = (\frac{n\times(n+1)}{2})^2$
$\sum (2k-1)^3 = n^2\times(2n^2-1)$
$\sum k^4 = \frac{n\times(n+1)\times(2n+1)\times(3n^2+3n-1)}{30}$
$\sum k^5 = \frac{n^2\times(n+1)^2\times(2n^2+2n-1)}{12}$
$\sum k\times(k+1) = \frac{n\times(n+1)\times(n+2)}{3}$
$\sum k\times(k+1)\times(k+2) = \frac{n\times(n+1)\times(n+2)\times(n+3)}{4}$
$\sum k\times(k+1)\times(k+2)\times(k+3) = \frac{n\times(n+1)\times(n+2)\times(n+3)\times(n+4)}{5}$

3.21.5 几何公式

球扇形：
全面积：$T = \pi r(2h+r_0)$，h 为球冠高，r_0 为球冠底面半径
体积：$V = \frac{2\pi r^2 h}{3}$

3.21.6 小公式

Pick 公式：$A = E\times 0.5 + I - 1$（A 是多边形面积，E 是边界上的整点，I 是多边形内部的整点）

海伦公式：$S = \sqrt{p(p-a)(p-b)(p-c)}$，其中 $p = \frac{(a+b+c)}{2}$，abc 为三角形的三条边长

求 $\binom{n}{k}$ 中素因子 P 的个数：

1. 把 n 转化为 P 进制，并记它每个位上的和为 $S1$
2. 把 $n-k$，k 做同样的处理，得到 $S2$，$S3$

则 $\binom{n}{k}$ 中素因子 P 的个数：$\frac{S2+S3-S1}{P-1}$

部分错排公式：
$n+m$ 个数中 m 个数必须错排求排列数

```
dp[i] = n*dp[i-1]+(i-1)*(dp[i-1]+dp[i-2]);
dp[0] = n!;
dp[1] = n*n!;
```

$dp[m]$ 为所求解

3.21.7 马步问题

任意步长 (p,q) 无限棋盘可达性判定

```
bool check(int dx,int dy,int p,int q)
{
  if (p < 0) p = -p;
  if (q < 0) q = -q;
  LL g = gcd(p,q);
  if (dx % g || dy % g) return false;
  dx /= g,dy /= g,p = (p / g) & 1,q = (q / g) & 1;
  return !(p == q && ((dx ^ dy) & 1));
}
```

拓展:
若可选马步可以有 N 种 (p_i,q_i), 令 $g=gcd(p_1,q_1,p_2,q_2\cdots p_N,q_N)$, 则不在 g 的整数倍点上的节点肯定不可达。坐标除 $2g$, 同时将可选马步除 g 之后放缩到 2×2 之内, 即 $(\frac{p_i}{g} \mod 2, \frac{q_i}{g} \mod 2)$。若放缩后马步中有 $(1,0)$ 或 $(0,1)$, 则全放缩后全棋盘可达, 否则只可达偶点。

$(2,1)$ 马步无限棋盘最小距离

```
int dis(int dx,int dy)
{
  if (dx < 0) dx = -dx;
  if (dy < 0) dy = -dy;
  if (dx < dy) swap(dx,dy);
  if (dx & 1)
  {
    if (dy & 1) return dis(dx+1,dy-1);
    if (dx == 1 && dy == 0) return 3;
    return dis(dx+3,dy)-1;
  }
  if (dy & 1)
  {
    if (dx == 4 && dy == 3) return 3;
    return dis(dx-2,dy-1)+1;
  }
  if (dx == 0 && dy == 0) return 0;
  if (dx == 2 && dy == 2) return 4;
  int c = (((dx-1) / 4)+1)*2;
  if (dx & 2) dy -= 2;
  if (dy <= c) return c;
```

```
22    dy -= c;
23    return c+(dy-2) / 6*2+2;
24  }
```

4 数据结构

4.1 *Splay

持续学习中。
注意节点的 size 值不一定是真实的值！如果有需要需要特别维护！

1. 旋转和 Splay 操作
2. rank 操作
3. insert 操作（。。很多题目都有）
4. del 操作（郁闷的出纳员）
5. 由数组建立 Splay
6. 前驱后继（营业额统计）
7. Pushdown Pushup 的位置
8. *。。。暂时想不起了

4.1.1 节点定义

```
const int MaxN = 50003;

struct Node
{
  int size,key;

  Node *c[2];
  Node *p;
} mem[MaxN], *cur, *nil;
```

无内存池的几个初始化函数。

```
Node *newNode(int v, Node *p)
{
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;
  return cur++;
}

void Init()
{
  cur = mem;
  nil = newNode(0, cur);
  nil->size = 0;
}
```

带内存池的几个函数。

```
int emp[MaxN], totemp;

Node *newNode(int v, Node *p)
{
  cur = mem + emp[--totemp];
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;
  return cur;
}

void Init()
{
  for (int i = 0; i < MaxN; ++i)
    emp[i] = i;
  totemp = MaxN;
  cur = mem + emp[--totemp];
  nil = newNode(0, cur);
  nil->size = 0;
}

void Recycle(Node *p)
{
  if (p == nil)   return;
  Recycle(p->c[0]), Recycle(p->c[1]);
  emp[totemp++] = p - mem;
}
```

4.1.2 维护序列

一切下标从 0 开始。

```
struct SplayTree
{
  Node *root;
  void Init()
  {
    root = nil;
  }
  void Pushup(Node *x)
  {
    if (x == nil)   return;
    Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
    x->size = x->c[0]->size + x->c[1]->size + 1;
  }
  void Pushdown(Node *x)
  {
    if (x == nil)   return;
    //do something
  }
  void Rotate(Node *x, int f)
  {
```

```
21      if (x == nil)   return;
22      Node *y = x->p;
23      y->c[f ^ 1] = x->c[f], x->p = y->p;
24      if (x->c[f] != nil)
25        x->c[f]->p = y;
26      if (y->p != nil)
27        y->p->c[y->p->c[1] == y] = x;
28      x->c[f] = y, y->p = x;
29      Pushup(y);
30    }
31    void Splay(Node *x, Node *f)
32    {
33      static Node *stack[maxn];
34      int top = 0;
35      stack[top++] = x;
36      for (Node *y = x; y != f;y = y->p)
37        stack[top++] = y->p;
38      while (top)
39        Pushdown(stack[--top]);
40
41      while (x->p != f)
42      {
43        Node *y = x->p;
44        if (y->p == f)
45          Rotate(x, x == y->c[0]);
46        else
47        {
48          int fd = y->p->c[0] == y;
49          if (y->c[fd] == x)
50            Rotate(x, fd ^ 1), Rotate(x, fd);
51          else
52            Rotate(y, fd), Rotate(x, fd);
53        }
54      }
55      Pushup(x);
56      if (f == nil)
57        root = x;
58    }
59    void Select(int k, Node *f)
60    {
61      Node *x = root;
62      Pushdown(x);
63      int tmp;
64      while ((tmp = x->c[0]->size) != k)
65      {
66        if (k < tmp)  x = x->c[0];
67        else
68          x = x->c[1], k -= tmp + 1;
69        Pushdown(x);
70      }
71      Splay(x, f);
```

```
  }
  void Select(int l, int r)
  {
    Select(l, nil), Select(r + 2, root);
  }
  Node *Make_tree(int a[], int l, int r, Node *p)
  {
    if (l > r)  return nil;
    int mid = l + r >> 1;
    Node *x = newNode(a[mid], p);
    x->c[0] = Make_tree(a, l, mid - 1, x);
    x->c[1] = Make_tree(a, mid + 1, r, x);
    Pushup(x);
    return x;
  }
  void Insert(int pos, int a[], int n)
  {
    Select(pos, nil), Select(pos + 1, root);
    root->c[1]->c[0] = Make_tree(a, 0, n - 1, root->c[1]);
    Splay(root->c[1]->c[0], nil);
  }
  void Insert(int v)
  {
    Node *x = root, *y = nil;
    //注意! 需要 pushdown, 之前只在初始化调用过这个函数所有没问题
    while (x != nil)
    {
      y = x;
      y->size++;
      x = x->c[v >= x->key];
    }
    y->c[v >= y->key] = x = newNode(v, y);
    Splay(x, nil);
  }
  void Remove(int l, int r)
  {
    Select(l, r);
    //Recycle(root->c[1]->c[0]);
    root->c[1]->c[0] = nil;
    Splay(root->c[1], nil);
  }
};
```

例题：旋转区间赋值求和求最大子序列。
注意打上懒标记后立即 Pushup。Pushup(root-c[1]-c[0]),Pushup(root-c[1]),Pushup(root);

```
  void Pushup(Node *x)
  {
    if (x == nil) return;
    Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
    x->size = x->c[0]->size+x->c[1]->size+1;

```

```
7       x->sum = x->c[0]->sum+x->c[1]->sum+x->key;
8       x->lsum = max(x->c[0]->lsum,
9         x->c[0]->sum+x->key+max(0,x->c[1]->lsum));
10      x->rsum = max(x->c[1]->rsum,
11        x->c[1]->sum+x->key+max(0,x->c[0]->rsum));
12      x->maxsum = max(max(x->c[0]->maxsum,x->c[1]->maxsum),
13        x->key+max(0,x->c[0]->rsum)+max(0,x->c[1]->lsum));
14    }
15    void Pushdown(Node *x)
16    {
17      if (x == nil) return;
18      if (x->rev)
19      {
20        x->rev = 0;
21        x->c[0]->rev ^= 1;
22        x->c[1]->rev ^= 1;
23        swap(x->c[0],x->c[1]);
24
25        swap(x->lsum,x->rsum);
26      }
27      if (x->same)
28      {
29        x->same = false;
30        x->key = x->lazy;
31        x->sum = x->key*x->size;
32        x->lsum = x->rsum = x->maxsum = max(x->key,x->sum);
33        x->c[0]->same = true, x->c[0]->lazy = x->key;
34        x->c[1]->same = true, x->c[1]->lazy = x->key;
35      }
36    }
37
38  int main()
39  {
40    int totcas;
41    scanf("%d",&totcas);
42    for (int cas = 1;cas <= totcas;cas++)
43    {
44      Init();
45      sp.Init();
46      nil->lsum = nil->rsum = nil->maxsum = -Inf;
47      sp.Insert(0);
48      sp.Insert(0);
49
50      int n,m;
51      scanf("%d%d",&n,&m);
52      for (int i = 0;i < n;i++)
53        scanf("%d",&a[i]);
54      sp.Insert(0,a,n);
55
56      for (int i = 0;i < m;i++)
57      {
```

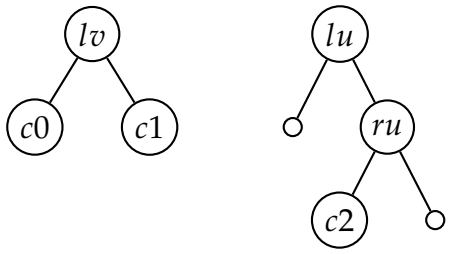
```
58        int pos,tot,c;
59        scanf("%s",buf);
60        if (strcmp(buf,"MAKE-SAME") == 0)
61        {
62          scanf("%d%d%d",&pos,&tot,&c);
63          sp.Select(pos-1,pos+tot-2);
64          sp.root->c[1]->c[0]->same = true;
65          sp.root->c[1]->c[0]->lazy = c;
66          sp.Pushup(sp.root->c[1]), sp.Pushup(sp.root);
67        }
68        else if (strcmp(buf,"INSERT") == 0)
69        {
70          scanf("%d%d",&pos,&tot);
71          for (int i = 0;i < tot;i++)
72            scanf("%d",&a[i]);
73          sp.Insert(pos,a,tot);
74        }
75        else if (strcmp(buf,"DELETE") == 0)
76        {
77          scanf("%d%d",&pos,&tot);
78          sp.Remove(pos-1,pos+tot-2);
79        }
80        else if (strcmp(buf,"REVERSE") == 0)
81        {
82          scanf("%d%d",&pos,&tot);
83          sp.Select(pos-1,pos+tot-2);
84          sp.root->c[1]->c[0]->rev ^= 1;
85          sp.Pushup(sp.root->c[1]), sp.Pushup(sp.root);
86        }
87        else if (strcmp(buf,"GET-SUM") == 0)
88        {
89          scanf("%d%d",&pos,&tot);
90          sp.Select(pos-1,pos+tot-2);
91          printf("%d\n",sp.root->c[1]->c[0]->sum);
92        }
93        else if (strcmp(buf,"MAX-SUM") == 0)
94        {
95          sp.Select(0,sp.root->size-3);
96          printf("%d\n",sp.root->c[1]->c[0]->maxsum);
97        }
98      }
99    }
100   return 0;
101 }
```

4.1.3 维护括号序列

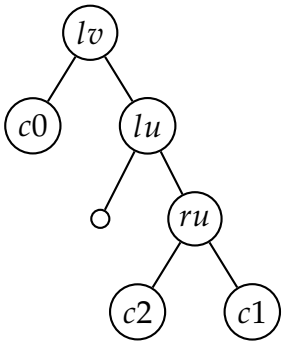
不需要哨兵。

合并操作：

先转成下面的样子：

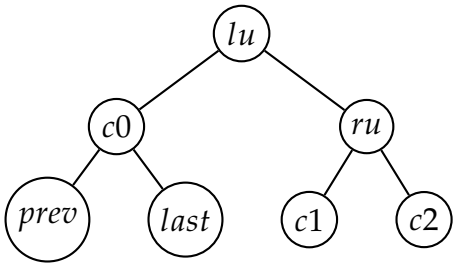


再链接成这样：

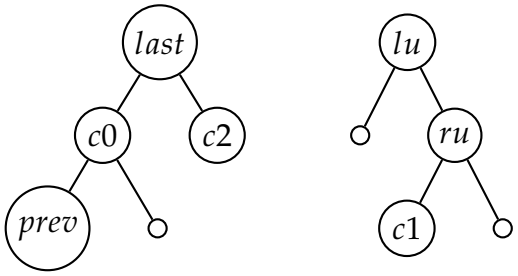


分离操作：

先把 lu 和 ru 转上去：



把 c0 和 c2 从原来的位置断开
然后接上：




```
#include <iostream>
#include <cstdio>
#include <cstring>
using namespace std;

const int maxn = 500000;
const int mod = 99990001;
struct Node
{
  int size,key;
```

```
11
12    int a,b;
13    int minid,id;
14
15    Node *c[2];
16    Node *p;
17  }mem[maxn],*cur,*nil;
18  Node *l[maxn],*r[maxn];//左括号右括号定义在前面
19
20  int emp[maxn],totemp;
21  Node *newNode(int v,Node *p)
22  {
23    cur->c[0] = cur->c[1] = nil,cur->p = p;
24    cur->size = 1;
25    cur->key = v;
26
27    cur->a = 1;
28    cur->b = 0;
29    cur->minid = cur->id = maxn;
30
31    return cur++;
32  }
33  void Init()
34  {
35    cur = mem;
36    nil = newNode(0,cur);
37    nil->size = 0;
38  }
39
40  struct SplayTree
41  {
42    Node *root;
43    void Init()
44    {
45      root = nil;
46    }
47    void Pushup(Node *x)
48    {
49      if (x == nil) return;
50      Pushdown(x);
51      Pushdown(x->c[0]);
52      Pushdown(x->c[1]);
53      x->size = x->c[0]->size+x->c[1]->size+1;
54
55      x->minid = x->id;
56      for (int i = 0;i < 2;i++)
57        if (x->c[i] != nil)
58          x->minid = min(x->minid,x->c[i]->minid);
59    }
60    void Pushdown(Node *x)
61    {
```

```
62      if (x == nil) return;
63
64      x->key = ((long long)x->key*x->a%mod+x->b)%mod;
65      for (int i = 0;i < 2;i++)
66        if (x->c[i] != nil)
67        {
68          x->c[i]->a = (long long)x->c[i]->a*x->a%mod;
69          x->c[i]->b = ((long long)x->c[i]->b*x->a%mod+x->b)%mod;
70        }
71      x->a = 1;
72      x->b = 0;
73    }
74    void Rotate(Node *x,int f)
75    {
76      if (x == nil) return;
77      Node *y = x->p;
78      y->c[f^1] = x->c[f], x->p = y->p;
79      if (x->c[f] != nil)
80        x->c[f]->p = y;
81      if (y->p != nil)
82        y->p->c[y->p->c[1] == y] = x;
83      x->c[f] = y, y->p = x;
84      Pushup(y);
85    }
86    void Splay(Node *x,Node *f)
87    {
88      static Node *stack[maxn];
89      int top = 0;
90      stack[top++] = x;
91      for (Node *y = x; y != f;y = y->p)
92        stack[top++] = y->p;
93      while (top)
94        Pushdown(stack[--top]);
95
96      while (x->p != f)
97      {
98        Node *y = x->p;
99        if (y->p == f)
100         Rotate(x,x == y->c[0]);
101       else
102       {
103         int fd = y->p->c[0] == y;
104         if (y->c[fd] == x)
105           Rotate(x, fd^1), Rotate(x,fd);
106         else
107           Rotate(y,fd), Rotate(x,fd);
108       }
109     }
110     Pushup(x);
111     if (f == nil)
112       root = x;
```

```
113   }
114   Node *Last(Node *now)
115   {
116     Splay(now,nil);
117     while (now->c[1] != nil)
118       now = now->c[1];
119     return now;
120   }
121   //把 u 接到 v 下面去, 边权为 w
122   //需要保证 u 是某棵树的根
123   void Link(int u,int v,int w)
124   {
125     Splay(l[v],nil);
126
127     Splay(l[u],nil);
128     l[u]->key = w;
129     Pushup(l[u]);
130     Splay(r[u],l[u]);
131
132     Node *c1 = l[v]->c[1];
133     l[v]->c[1] = l[u];
134     r[u]->c[1] = c1;
135     l[u]->p = l[v];
136     c1->p = r[u];
137     Pushup(r[u]);
138     Pushup(l[u]);
139     Pushup(l[v]);
140     Splay(l[u],nil);
141   }
142   //把 u 为根的子树分离开
143   int Split(int u)
144   {
145     Splay(l[u],nil);
146
147     int ret = l[u]->key;
148     Splay(r[u],l[u]);
149     Node *c0 = l[u]->c[0], *c2 = r[u]->c[1];
150
151     l[u]->key = 0;//去掉边权
152     l[u]->c[0] = r[u]->c[1] = c0->p = c2->p = nil;
153     Pushup(r[u]);
154     Pushup(l[u]);
155
156     Node *last = Last(c0);
157     Splay(last,nil);
158     last->c[1] = c2;
159     c2->p = last;
160     Pushup(last);
161
162     //对拆分后的两部份进行处理
163     Node *nu = last;
```

```
164     Node *nv = l[u];
165     if (nu->size > nv->size || (nu->size == nv->size && nu->minid >
            nv->minid))
166       swap(nu,nv);
167     nu->a = (long long)nu->a*ret%mod;
168     nu->b = (long long)nu->b*ret%mod;
169     nv->b = (nv->b+ret)%mod;
170
171     return ret;//返回原边权
172   }
173 };
174
175 SplayTree sp;
176 int n;
177 struct Edge
178 {
179   int to,next,w,id;
180 };
181 Edge edge[maxn];
182 int head[maxn],L;
183 int eid[maxn],toid[maxn];
184
185 void addedge(int u,int v,int w,int id)
186 {
187   edge[L].to = v;
188   edge[L].w = w;
189   edge[L].id = id;
190   edge[L].next = head[u];
191   head[u] = L++;
192 }
193
194 void DFS(int now,int fa)
195 {
196   for (int i = head[now];i != -1;i = edge[i].next)
197     if (edge[i].to != fa)
198     {
199       sp.Link(edge[i].to,now,edge[i].w);
200       eid[edge[i].id] = edge[i].to;
201       toid[edge[i].id] = now;
202
203       DFS(edge[i].to,now);
204     }
205 }
206
207 int main()
208 {
209   Init();
210   sp.Init();
211
212   scanf("%d",&n);
213
```

```
214   for (int i = 0;i < n;i++)
215   {
216     l[i] = newNode(0,nil);
217     r[i] = newNode(0,nil);
218     l[i]->id = r[i]->id = i;
219     l[i]->c[1] = r[i], r[i]->p = l[i];
220     sp.Pushup(l[i]);
221
222     head[i] = -1;
223   }
224   L = 0;
225
226   for (int i = 0;i < n-1;i++)
227   {
228     int u,v,w;
229     scanf("%d%d%d",&u,&v,&w);
230     u--,v--;
231
232     addedge(u,v,w,i);
233     addedge(v,u,w,i);
234   }
235
236   DFS(0,-1);
237
238   for (int i = 0;i < n-1;i++)
239   {
240     fflush(stdout);
241
242     int id;
243     scanf("%d",&id);
244     id--;
245
246     int ret = sp.Split(eid[id]);
247     printf("%d\n",ret);
248   }
249
250   return 0;
251 }
```

4.2 动态树

懒标记是否及时 Pushdown 了?
修改之后有没有及时 Pushup?

4.2.1 维护点权

查询链上的最长字段和
GetRoute 是用换根写的

```
1 const int MaxN = 110000;
```

```
 2
 3 struct Node
 4 {
 5   int size, key;
 6   bool rev;
 7
 8 //  bool same;
 9 //  int lsum, rsum, sum, maxsum, sa;
10
11   Node *c[2];
12   Node *p;
13 } mem[MaxN], *cur, *nil, *pos[MaxN];
14
15 Node *newNode(int v, Node *p)
16 {
17   cur->c[0] = cur->c[1] = nil, cur->p = p;
18   cur->size = 1;
19   cur->key = v;
20   cur->rev = false;
21
22 //  cur->same = false;
23 //  cur->sa = 0;
24 //  cur->lsum = cur->rsum = cur->maxsum = 0;
25 //  cur->sum = v;
26
27   return cur++;
28 }
29
30 void Init()
31 {
32   cur = mem;
33   nil = newNode(0, cur);
34   nil->size = 0;
35 }
36
37 struct SplayTree
38 {
39   void Pushup(Node *x)
40   {
41     if (x == nil)   return;
42     Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
43     x->size = x->c[0]->size + x->c[1]->size + 1;
44
45 //    x->sum = x->c[0]->sum + x->c[1]->sum + x->key;
46 //    x->lsum = max(x->c[0]->lsum,
47 //      x->c[0]->sum + x->key + max(0, x->c[1]->lsum));
48 //    x->rsum = max(x->c[1]->rsum,
49 //      x->c[1]->sum + x->key + max(0, x->c[0]->rsum));
50 //    x->maxsum = max(max(x->c[0]->maxsum, x->c[1]->maxsum),
51 //      x->key + max(0, x->c[0]->rsum) + max(0, x->c[1]->lsum));
52
```

```
53    }
54    void Pushdown(Node *x)
55    {
56      if (x == nil)   return;
57      if (x->rev)
58      {
59        x->rev = 0;
60        x->c[0]->rev ^= 1;
61        x->c[1]->rev ^= 1;
62        swap(x->c[0], x->c[1]);
63  //注意修改与位置有关的量
64  //      swap(x->lsum,x->rsum);
65      }
66
67  //    if (x->same)
68  //    {
69  //      x->same = false;
70  //      x->key = x->sa;
71  //      x->sum = x->sa * x->size;
72  //      x->lsum = x->rsum = x->maxsum = max(0, x->sum);
73  //      if (x->c[0] != nil)
74  //        x->c[0]->same = true, x->c[0]->sa = x->sa;
75  //      if (x->c[1] != nil)
76  //        x->c[1]->same = true, x->c[1]->sa = x->sa;
77  //    }
78    }
79    bool isRoot(Node *x)
80    {
81      return (x == nil) || (x->p->c[0] != x && x->p->c[1] != x);
82    }
83    void Rotate(Node *x, int f)
84    {
85      if (isRoot(x))   return;
86      Node *y = x->p;
87      y->c[f ^ 1] = x->c[f], x->p = y->p;
88      if (x->c[f] != nil)
89        x->c[f]->p = y;
90      if (y != nil)
91      {
92        if (y == y->p->c[1])
93          y->p->c[1] = x;
94        else if (y == y->p->c[0])
95          y->p->c[0] = x;
96      }
97      x->c[f] = y, y->p = x;
98      Pushup(y);
99    }
100   void Splay(Node *x)
101   {
102     static Node *stack[MaxN];
103     int top = 0;
```

```
104     stack[top++] = x;
105     for (Node *y = x; !isRoot(y); y = y->p)
106       stack[top++] = y->p;
107     while (top)
108       Pushdown(stack[--top]);
109
110     while (!isRoot(x))
111     {
112       Node *y = x->p;
113       if (isRoot(y))
114         Rotate(x, x == y->c[0]);
115       else
116       {
117         int fd = y->p->c[0] == y;
118         if (y->c[fd] == x)
119           Rotate(x, fd ^ 1), Rotate(x, fd);
120         else
121           Rotate(y, fd), Rotate(x, fd);
122       }
123     }
124     Pushup(x);
125   }
126   Node *Access(Node *u)
127   {
128     Node *v = nil;
129     while (u != nil)
130     {
131       Splay(u);
132       v->p = u;
133       u->c[1] = v;
134       Pushup(u);
135       u = (v = u)->p;
136       if (u == nil)
137         return v;
138     }
139   }
140   Node *LCA(Node *u, Node *v)
141   {
142     Access(u);
143     return Access(v);
144   }
145   Node *Link(Node *u, Node *v)
146   {
147     Access(u);
148     Splay(u);
149     u->rev = true;
150     u->p = v;
151   }
152   void ChangeRoot(Node *u)
153   {
154     Access(u)->rev ^= 1;
```

```
155   }
156   Node *GetRoute(Node *u, Node *v)
157   {
158     ChangeRoot(u);
159     return Access(v);
160   }
161 };
162
163 int n, m;
164 SplayTree sp;
165
166 int main(int argc, char const *argv[])
167 {
168   while (scanf("%d", &n) != EOF)
169   {
170     Init();
171     for (int i = 0; i < n; i++)
172     {
173       int v;
174       scanf("%d", &v);
175       pos[i] = newNode(v, nil);
176     }
177     for (int i = 0; i < n - 1; i++)
178     {
179       int u, v;
180       scanf("%d%d", &u, &v);
181       u--, v--;
182       sp.Link(pos[u], pos[v]);
183     }
184
185 //    scanf("%d", &m);
186 //    for (int i = 0; i < m; i++)
187 //    {
188 //      int typ, u, v, c;
189 //      scanf("%d%d%d", &typ, &u, &v);
190 //      u--, v--;
191 //      if (typ == 1)
192 //        printf("%d\n", sp.GetRoute(pos[u], pos[v])->maxsum);
193 //      else
194 //      {
195 //        scanf("%d", &c);
196 //        Node *p = sp.GetRoute(pos[u], pos[v]);
197 //        p->same = true;
198 //        p->sa = c;
199 //      }
200 //    }
201   }
202   return 0;
203 }
```

4.2.2 维护边权

刘汝佳的 Happy Painting!
查询链上边的不同颜色数量
不能换根，但是可以 Link 和 Cut

```
const int MaxN = 60000;

struct Node
{
  int size,key;

  int msk,lazy;

  Node *c[2];
  Node *p;
} mem[MaxN], *cur, *nil, *pos[MaxN];

Node *newNode(int v,Node *p)
{
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;

  cur->msk = 0;
  cur->lazy = -1;

  return cur++;
}

void Init()
{
  cur = mem;
  nil = newNode(0, cur);
  nil->size = 0;
}

struct SplayTree
{
  void Pushup(Node *x)
  {
    if (x == nil) return;
    Pushdown(x);
    Pushdown(x->c[0]);
    Pushdown(x->c[1]);
    x->size = x->c[0]->size + x->c[1]->size + 1;

    x->msk = x->c[0]->msk | x->c[1]->msk | (1<<x->key);
  }
  void Pushdown(Node *x)
  {
```

```
46      if (x == nil) return;
47
48      if (x->lazy != -1)
49      {
50        x->key = x->lazy;
51        x->msk = (1<<x->key);
52        x->c[0]->lazy = x->c[1]->lazy = x->lazy;
53        x->lazy = -1;
54      }
55    }
56    bool isRoot(Node *x)
57    {
58      return (x == nil) || (x->p->c[0] != x && x->p->c[1] != x);
59    }
60    void Rotate(Node *x, int f)
61    {
62      if (isRoot(x))  return;
63      Node *y = x->p;
64      y->c[f ^ 1] = x->c[f], x->p = y->p;
65      if (x->c[f] != nil)
66        x->c[f]->p = y;
67      if (y != nil)
68      {
69        if (y == y->p->c[1])
70          y->p->c[1] = x;
71        else if (y == y->p->c[0])
72          y->p->c[0] = x;
73      }
74      x->c[f] = y, y->p = x;
75      Pushup(y);
76    }
77    void Splay(Node *x)
78    {
79      static Node *stack[MaxN];
80      int top = 0;
81      stack[top++] = x;
82      for (Node *y = x; !isRoot(y); y = y->p)
83        stack[top++] = y->p;
84      while (top)
85        Pushdown(stack[--top]);
86
87      while (!isRoot(x))
88      {
89        Node *y = x->p;
90        if (isRoot(y))
91          Rotate(x, x == y->c[0]);
92        else
93        {
94          int fd = y->p->c[0] == y;
95          if (y->c[fd] == x)
96            Rotate(x, fd ^ 1), Rotate(x, fd);
```

```
97          else
98            Rotate(y, fd), Rotate(x, fd);
99        }
100     }
101     Pushup(x);
102   }
103   Node *Access(Node *u)
104   {
105     Node *v = nil;
106     while (u != nil)
107     {
108       Splay(u);
109       v->p = u;
110       u->c[1] = v;
111       Pushup(u);
112       u = (v = u)->p;
113       if (u == nil) return v;
114     }
115   }
116   Node *Root(Node *u)
117   {
118     Access(u);
119     Splay(u);
120     for (Pushdown(u); u->c[0] != nil; u = u->c[0])
121       Pushdown(u);
122     Splay(u);
123     return u;
124   }
125   Node *LCA(Node *u, Node *v)
126   {
127     if (Root(u) != Root(v))
128       return nil;
129     Access(u);
130     return Access(v);
131   }
132   void Cut(Node *u)
133   {
134     Access(u);
135     Splay(u);
136     u->c[0] = u->c[0]->p = nil;
137     Pushup(u);
138   }
139   void Link(Node *u, Node *v, int val)
140   {
141     Access(u);
142     Splay(u);
143     u->p = v;
144     u->key = val;
145     Pushup(u);
146   }
147 };
```

```
148
149 int cntbit(int x)
150 {
151   x = (x & 0x55555555) + ((x >> 1) & 0x55555555);
152   x = (x & 0x33333333) + ((x >> 2) & 0x33333333);
153   x = (x & 0x0F0F0F0F) + ((x >> 4) & 0x0F0F0F0F);
154   x = (x & 0x00FF00FF) + ((x >> 8) & 0x00FF00FF);
155   x = (x & 0x0000FFFF) + ((x >> 16) & 0x0000FFFF);
156   return x;
157 }
158
159 SplayTree sp;
160 int n,Q,f[MaxN];
161
162 int main(int argc, char const *argv[])
163 {
164   while (scanf("%d%d",&n,&Q) != EOF)
165   {
166     Init();
167     for (int i = 0; i < n; i++)
168     {
169       scanf("%d",&f[i]);
170       pos[i] = newNode(0, nil);
171     }
172     for (int i = 0; i < n; i++)
173     {
174       int col;
175       scanf("%d",&col);
176       if (f[i] > 0)
177         sp.Link(pos[i],pos[f[i]-1],col-1);
178     }
179     for (int q = 0; q < Q; q++)
180     {
181       int typ,x,y,c;
182       scanf("%d%d%d",&typ,&x,&y);
183       x--,y--;
184       if (typ == 3)
185       {
186         Node *lca = sp.LCA(pos[x],pos[y]);
187         if (lca == nil || x == y)
188         {
189           printf("0␣0\n");
190           continue;
191         }
192         int totedge = lca->c[1]->size;
193         int msk = lca->c[1]->msk;
194
195         if (pos[x] != lca)
196         {
197           sp.Splay(pos[x]);
198           totedge += pos[x]->size;
```

```
199           msk |= pos[x]->msk;
200         }
201
202         printf("%d %d\n",totedge,cntbit(msk));
203       }
204       else
205       {
206         scanf("%d",&c);
207         c--;
208         if (typ == 1)
209         {
210           if (x == y) continue;
211
212           Node *lca = sp.LCA(pos[x],pos[y]);
213           if (pos[x] == lca)  continue;
214
215           sp.Cut(pos[x]);
216           sp.Link(pos[x],pos[y],c);
217
218         }
219         else
220         {
221           Node *lca = sp.LCA(pos[x],pos[y]);
222
223           if (lca == nil || x == y)
224             continue;
225
226           lca->c[1]->lazy = c;
227           sp.Pushup(lca->c[1]);
228           sp.Pushup(lca);
229           if (pos[x] != lca)
230           {
231             sp.Splay(pos[x]);
232             pos[x]->lazy = c;
233             sp.Pushup(pos[x]);
234           }
235         }
236       }
237     }
238   }
239   return 0;
240 }
```

4.3 可持久化线段树

区间第 k 小数，内存压缩版，POJ2014。

```
1 #include <cstdio>
2 #include <algorithm>
3 using namespace std;
4
5 const int MAXN=100000,MAXM=100000;
```

```
 6
 7 struct node
 8 {
 9   node *l,*r;
10   int sum;
11 }tree[MAXN*4+MAXM*20];
12
13 int N;
14 node *newnode()
15 {
16   tree[N].l=tree[N].r=NULL;
17   tree[N].sum=0;
18   return &tree[N++];
19 }
20 node *newnode(node *x)
21 {
22   tree[N].l=x->l;
23   tree[N].r=x->r;
24   tree[N].sum=x->sum;
25   return &tree[N++];
26 }
27 node *build(int l,int r)
28 {
29   node *x=newnode();
30   if (l<r)
31   {
32     int mid=l+r>>1;
33     x->l=build(l,mid);
34     x->r=build(mid+1,r);
35     x->sum=x->l->sum+x->r->sum;
36   }
37   else
38     x->sum=0;
39   return x;
40 }
41 node *update(node *x,int l,int r,int p,int v)
42 {
43   if (l<r)
44   {
45     int mid=l+r>>1;
46     node *nx=newnode(x);
47     if (p<=mid)
48     {
49       node *ret=update(x->l,l,mid,p,v);
50       nx->l=ret;
51     }
52     else
53     {
54       node *ret=update(x->r,mid+1,r,p,v);
55       nx->r=ret;
56     }
```

```
57      nx->sum=nx->l->sum+nx->r->sum;
58      return nx;
59    }
60    else
61    {
62      node *nx=newnode(x);
63      nx->sum+=v;
64      return nx;
65    }
66  }
67  int query(node *x1,node *x2,int l,int r,int k)
68  {
69    if (l<r)
70    {
71      int mid=l+r>>1;
72      int lsum=x2->l->sum-x1->l->sum;
73      if (lsum>=k)
74        return query(x1->l,x2->l,l,mid,k);
75      else
76        return query(x1->r,x2->r,mid+1,r,k-lsum);
77    }
78    else
79      return l;
80  }
81  char s[10];
82  node *root[MAXM+1];
83  int a[MAXN],b[MAXN];
84  int init(int n)
85  {
86    for (int i=0;i<n;i++)
87      b[i]=a[i];
88    sort(b,b+n);
89    int tn=unique(b,b+n)-b;
90    for (int i=0;i<n;i++)
91    {
92      int l=0,r=tn-1;
93      while (l<r)
94      {
95        int mid=l+r>>1;
96        if (b[mid]>=a[i])
97          r=mid;
98        else
99          l=mid+1;
100     }
101     a[i]=l;
102   }
103   return tn;
104 }
105 int main()
106 {
107   int cas=1,n;
```

```
108   while (scanf("%d",&n)!=EOF)
109   {
110     printf("Case␣%d:\n",cas++);
111     for (int i=0;i<n;i++)
112       scanf("%d",&a[i]);
113     int tn=init(n);
114     N=0;
115     root[0]=build(0,tn-1);
116     for (int i=1;i<=n;i++)
117       root[i]=update(root[i-1],0,tn-1,a[i-1],1);
118     int m;
119     scanf("%d",&m);
120     for (int i=0;i<m;i++)
121     {
122       int s,t;
123       scanf("%d%d",&s,&t);
124       printf("%d\n",b[query(root[s-1],root[t],0,tn-1,t-s+2>>1)]);
125     }
126   }
127   return 0;
128 }
```

4.4 treap 正式版

支持翻转。

```
 1 #include <cstdio>
 2 #include <cstdlib>
 3 #include <algorithm>
 4 using namespace std;
 5
 6 const int MAXN = 100000;
 7 const int MAXM = 100000;
 8 const int inf = 0x7fffffff;
 9 int a[MAXN];
10 struct Treap
11 {
12   int N;
13   Treap()
14   {
15     N = 0;
16     root = NULL;
17   }
18   void init()
19   {
20     N = 0;
21     root = NULL;
22   }
23   struct Treap_Node
24   {
25     Treap_Node *son[2];//left & right
26     int value, fix;
```

```
27      bool lazy;
28      int size;
29      Treap_Node() {}
30      Treap_Node(int _value)
31      {
32        son[0] = son[1] = NULL;
33        value = _value;
34        fix = rand() * rand();
35        lazy = 0;
36        size = 1;
37      }
38      int sonSize(bool flag)
39      {
40        if (son[flag] == NULL)
41          return 0;
42        else
43          return son[flag]->size;
44      }
45    } node[MAXN], *root, *pos[MAXN];
46    void up(Treap_Node *p)
47    {
48      p->size = p->sonSize(0) + p->sonSize(1) + 1;
49    }
50    void down(Treap_Node *p)
51    {
52      if (!p->lazy)
53        return ;
54      for (int i = 0; i < 2; i++)
55        if (p->son[i])
56          p->son[i]->lazy = !p->son[i]->lazy;
57      swap(p->son[0], p->son[1]);
58      p->lazy = 0;
59    }
60    Treap_Node *merge(Treap_Node *p, Treap_Node *q)
61    {
62      if (p == NULL)
63        return q;
64      else if (q == NULL)
65        return p;
66      if (p->fix <= q->fix)
67      {
68        down(p);
69        p->son[1] = merge(p->son[1], q);
70        up(p);
71        return p;
72      }
73      else
74      {
75        down(q);
76        q->son[0] = merge(p, q->son[0]);
77        up(q);
```

```
78        return q;
79      }
80    }
81    pair<Treap_Node *, Treap_Node *> split(Treap_Node *p, int n)
82    {
83      if (p == NULL)
84        return make_pair((Treap_Node *)NULL, (Treap_Node *)NULL);
85      if (!n)
86        return make_pair((Treap_Node *)NULL, p);
87      if (n == p->size)
88        return make_pair(p, (Treap_Node *)NULL);
89      down(p);
90      if (p->sonSize(0) >= n)
91      {
92        pair<Treap_Node *, Treap_Node *> ret = split(p->son[0], n);
93        p->son[0] = ret.second;
94        up(p);
95        return make_pair(ret.first, p);
96      }
97      else
98      {
99        pair<Treap_Node *, Treap_Node *>
100         ret = split(p->son[1], n - p->sonSize(0) - 1);
101       p->son[1] = ret.first;
102       up(p);
103       return make_pair(p, ret.second);
104     }
105   }
106   int smalls(Treap_Node *p,int value)
107   {
108     if (p==NULL)
109       return 0;
110     if (p->value<=value)
111       return 1+p->sonSize(0)+smalls(p->son[1],value);
112     else
113       return smalls(p->son[0],value);
114   }
115   void insert(int value)
116   {
117     Treap_Node *p = &node[N++];
118     *p = Treap_Node(value);
119     pair<Treap_Node *, Treap_Node *>
120       ret = split(root, smalls(root, value));
121     root = merge(merge(ret.first, p), ret.second);
122   }
123   void remove(int value)
124   {
125     pair<Treap_Node *, Treap_Node *> ret = split(root, smalls(root,
          value) - 1);
126     root = merge(ret.first, split(ret.second, 1).second);
127   }
```

```
128   Treap_Node *build(int s, int t)
129   {
130     int idx = t + s >> 1;
131     Treap_Node *p = &node[N++];
132     *p = Treap_Node(a[idx]);
133     pos[a[idx]] = p;
134     if (idx > s)
135       p = merge(build(s, idx - 1), p);
136     if (idx < t)
137       p = merge(p, build(idx + 1, t));
138     up(p);
139     return p;
140   }
141   void build(int n)
142   {
143     root = build(0, n - 1);
144   }
145   void *reverse(int s, int t)
146   {
147     pair<Treap_Node *, Treap_Node *> tmp1, tmp2;
148     tmp1 = split(root, s - 1);
149     tmp2 = split(tmp1.second, t - s + 1);
150     tmp2.first->lazy = !tmp2.first->lazy;
151     root = merge(tmp1.first, merge(tmp2.first, tmp2.second));
152   }
153 };
154 Treap treap;
155 int main()
156 {
157   treap.init();
158   int n;
159   scanf("%d", &n);
160   for (int i = 0; i < n; i++)
161     scanf("%d", &a[i]);
162   treap.build(n);
163 }
```

4.5 树链剖分

4.5.1 点权

```
 1 #include <cstdio>
 2 #include <cstring>
 3 #include <cstdlib>
 4 #include <algorithm>
 5 using namespace std;
 6 const int MAX = 12000;
 7 const int LOG = 15;
 8 const int oo = 0x3f3f3f3f;
 9 struct Edge
10 {
```

```
11      int to, w, id;
12      Edge* next;
13  } memo[MAX<<1],*cur,*g[MAX],*pree[MAX],*solid[MAX],*valid[MAX];
14  int dp[MAX][LOG], pos[MAX], lst[MAX], dep[MAX], cnt[MAX], h[MAX], K
        , n;
15  void init()
16  {
17    for (int i = 1; i <= n; i++)
18    {
19      g[i] = NULL;
20      valid[i] = NULL;
21      solid[i] = NULL;
22      pree[i] = NULL;
23    }
24    for (int i = 0; i < LOG; i++)
25    {
26      dp[1][i] = 1;
27    }
28    cur = memo;
29    K = 0;
30  }
31  void add(int u, int v, int w, int id)
32  {
33    cur->to = v;
34    cur->w = w;
35    cur->id = id;
36    cur->next = g[u];
37    g[u] = cur++;
38  }
39  void dfsLCA(int d, int u, int f)
40  {
41    dep[u] = d;
42    dp[u][0] = f;
43    cnt[u] = 1;
44    for (int i = 1; i < LOG; i++)
45    {
46      dp[u][i] = dp[dp[u][i - 1]][i - 1];
47    }
48    for (Edge* it = g[u]; it; it = it->next)
49    {
50      int v = it->to;
51      if (v != f)
52      {
53        pree[v] = it;
54        valid[it->id] = it;
55        dfsLCA(d + 1, v, u); //RE
56        cnt[u] += cnt[v];
57        if (solid[u] == NULL || cnt[solid[u]->to] < cnt[v])
58        {
59          solid[u] = it;
60        }
```

```
61      }
62    }
63  }
64  void dfsChain(int u, int head)
65  {
66    h[u] = head;
67    if (solid[u])
68    {
69      lst[pos[u] = K++] = u;
70      dfsChain(solid[u]->to, head);
71    }
72    else
73    for (Edge* it = g[u]; it; it = it->next)
74    {
75      int v = it->to;
76      if (it != solid[u] && v != dp[u][0])
77      {
78        dfsChain(v, v);
79      }
80    }
81  }
82  int getLCA(int u, int v)
83  {
84    if (dep[u] < dep[v])
85      swap(u, v);
86    for (int st = 1 << (LOG - 1), i = LOG - 1; i >= 0; i--, st >>= 1)
87    {
88      if (st <= dep[u] - dep[v])
89      {
90        u = dp[u][i];
91      }
92    }
93    if (u == v)
94      return u;
95    for (int i = LOG - 1; i >= 0; i--)
96    {
97      if (dp[u][i] != dp[v][i])
98      {
99        u = dp[u][i];
100       v = dp[v][i];
101     }
102   }
103   return dp[u][0];
104 }
105 struct Node
106 {
107     int l, r, ma, mi;
108     bool rev;
109 } seg[MAX << 2];
110 void reverse(int k)
111 {
```

```
112   seg[k].mi *= -1;
113   seg[k].ma *= -1;
114   seg[k].rev ^= 1;
115   swap(seg[k].mi, seg[k].ma);
116 }
117 void pushdown(int k)
118 {
119   if (seg[k].rev)
120   {
121     reverse(k << 1);
122     reverse(k << 1 | 1);
123     seg[k].rev = false;
124   }
125 }
126 void update(int k)
127 {
128   seg[k].mi = min(seg[k << 1].mi, seg[k << 1 | 1].mi);
129   seg[k].ma = max(seg[k << 1].ma, seg[k << 1 | 1].ma);
130 }
131 void init(int k, int l, int r)
132 {
133   seg[k].l = l;
134   seg[k].r = r;
135   seg[k].rev = false;
136   if (l == r)
137   {
138     seg[k].mi = seg[k].ma = solid[lst[l]]->w; //solid WA
139     return;
140   }
141   int mid = l + r >> 1;
142   init(k << 1, l, mid);
143   init(k << 1 | 1, mid + 1, r);
144   update(k);
145 }
146 void update(int k, int id, int v)
147 {
148   if (seg[k].l == seg[k].r)
149   {
150     seg[k].mi = seg[k].ma = solid[lst[id]]->w = v;
151     return;
152   }
153   pushdown(k);
154   int mid = seg[k].l + seg[k].r >> 1;
155   if (id <= mid)
156     update(k << 1, id, v);
157   else
158     update(k << 1 | 1, id, v);
159   update(k);
160 }
161 void reverse(int k, int l, int r)
162 {
```

```
163   if (seg[k].l > r || seg[k].r < l)
164     return;
165   if (seg[k].l >= l && seg[k].r <= r)
166   {
167     reverse(k);
168     return;
169   }
170   pushdown(k);
171   reverse(k << 1, l, r);
172   reverse(k << 1 | 1, l, r);
173   update(k);
174 }
175 int read(int k, int l, int r)
176 {
177   if (seg[k].l > r || seg[k].r < l)
178     return -oo;
179   if (seg[k].l >= l && seg[k].r <= r)
180     return seg[k].ma;
181   pushdown(k);
182   return max(read(k << 1, l, r), read(k << 1 | 1, l, r));
183 }
184 void setEdge(int id, int v)
185 {
186   Edge* it = valid[id];
187   if (h[it->to] != it->to)
188   {
189     update(1, pos[dp[it->to][0]], v);
190   }
191   else
192   {
193     it->w = v;
194   }
195 }
196 void negateLCA(int t, int u)
197 {
198   while (t != u)
199   {
200     int tmp = h[u];
201     if (dep[tmp] < dep[t])
202       tmp = t;
203     if (h[u] == u)
204     {
205       pree[u]->w *= -1;
206       u = dp[u][0];
207     }
208     else
209     {
210       reverse(1, pos[tmp], pos[dp[u][0]]);
211       u = tmp;
212     }
213   }
```

```
214 }
215 void negate(int u, int v)
216 {
217   int t = getLCA(u, v);
218   negateLCA(t, u);
219   negateLCA(t, v);
220 }
221 int maxLCA(int t, int u)
222 {
223   int ret = -oo;
224   while (t != u)
225   {
226     int tmp = h[u];
227     if (dep[tmp] < dep[t])
228       tmp = t;
229     if (h[u] == u)
230     {
231       ret = max(ret, pree[u]->w);
232       u = dp[u][0];
233     }
234     else
235     {
236       ret = max(ret, read(1, pos[tmp], pos[dp[u][0]]));
237       u = tmp;
238     }
239   }
240   return ret;
241 }
242 int query(int u, int v)
243 {
244   int t = getLCA(u, v);
245   return max(maxLCA(t, u), maxLCA(t, v));
246 }
247 int main()
248 {
249   int T;
250   int u, v, w;
251   char op[15];
252   scanf("%d", &T);
253   while (T--)
254   {
255     scanf("%d", &n);
256     init();
257     for (int i = 1; i < n; i++)
258     {
259       scanf("%d%d%d", &u, &v, &w);
260       add(u, v, w, i);
261       add(v, u, w, i);
262     }
263     dfsLCA(0, 1, 1);
264     dfsChain(1, 1);
```

```
265     init(1, 0, K - 1);
266     while (scanf("%s", op), op[0] != 'D')
267     {
268       scanf("%d%d", &u, &v);
269       if (op[0] == 'C')
270       {
271         setEdge(u, v);
272       }
273       else if (op[0] == 'N')
274       {
275         negate(u, v);
276       }
277       else
278       {
279         printf("%d\n", query(u, v));
280       }
281     }
282   }
283   return 0;
284 }
```

4.5.2 边权

```
 1 #include <cstdio>
 2 #include <iostream>
 3 #include <cstdlib>
 4 #include <algorithm>
 5 #include <cmath>
 6 #include <cstring>
 7 using namespace std;
 8 int n,m,sum,pos;
 9 int head[50005],e;
10 int s[50005],from[50005];
11 int fa[50005][20],deep[50005],num[50005];
12 int solid[50005],p[50005],fp[50005];
13 struct N
14 {
15   int l,r,mid;
16   int add,w;
17 }nod[50005*4];
18 struct M
19 {
20   int v,next;
21 }edge[100005];
22 void addedge(int u,int v)
23 {
24   edge[e].v=v;
25   edge[e].next=head[u];
26   head[u]=e++;
27
28   edge[e].v=u;
29   edge[e].next=head[v];
```

```
30    head[v]=e++;
31  }
32  void LCA(int st,int f,int d)
33  {
34    deep[st]=d;
35    fa[st][0]=f;
36    num[st]=1;
37    int i,v;
38    for(i=1;i<20;i++)
39      fa[st][i]=fa[fa[st][i-1]][i-1];
40    for(i=head[st];i!=-1;i=edge[i].next)
41    {
42      v=edge[i].v;
43      if(v!=f)
44      {
45        LCA(v,st,d+1);
46        num[st]+=num[v];
47        if(solid[st]==-1||num[v]>num[solid[st]])
48          solid[st]=v;
49      }
50    }
51  }
52  void getpos(int st,int sp)
53  {
54    from[st]=sp;
55    if(solid[st]!=-1)
56    {
57      p[st]=pos++;
58      fp[p[st]]=st;
59      getpos(solid[st],sp);
60    }
61    else
62    {
63      p[st]=pos++;
64      fp[p[st]]=st;
65      return;
66    }
67    int i,v;
68    for(i=head[st];i!=-1;i=edge[i].next)
69    {
70      v=edge[i].v;
71      if(v!=solid[st]&&v!=fa[st][0])
72        getpos(v,v);
73    }
74  }
75  int getLCA(int u,int v)
76  {
77    if(deep[u]<deep[v])
78      swap(u,v);
79    int d=1<<19,i;
80    for(i=19;i>=0;i--)
```

```
  {
    if(d<=deep[u]-deep[v])
      u=fa[u][i];
    d>>=1;
  }
  if(u==v)
    return u;
  for(i=19;i>=0;i--)
    if(fa[u][i]!=fa[v][i])
    {
      u=fa[u][i];
      v=fa[v][i];
    }
  return fa[u][0];
}
void init(int p,int l,int r)
{
  nod[p].l=l;
  nod[p].r=r;
  nod[p].mid=(l+r)>>1;
  nod[p].add=0;
  if(l==r)
    nod[p].w=s[fp[l]];
  else
  {
    init(p<<1,l,nod[p].mid);
    init(p<<1|1,nod[p].mid+1,r);
  }
}
void lazy(int p)
{
  if(nod[p].add!=0)
  {
    nod[p<<1].add+=nod[p].add;
    nod[p<<1|1].add+=nod[p].add;
    nod[p].add=0;
  }
}
void update(int p,int l,int r,int v)
{
  if(nod[p].l==l&&nod[p].r==r)
  {
    nod[p].add+=v;
    return;
  }
  lazy(p);
  if(nod[p].mid<l)
    update(p<<1|1,l,r,v);
  else if(nod[p].mid>=r)
    update(p<<1,l,r,v);
  else
```

```
132   {
133     update(p<<1,l,nod[p].mid,v);
134     update(p<<1|1,nod[p].mid+1,r,v);
135   }
136 }
137 int read(int p,int l,int r)
138 {
139   if(nod[p].l==l&&nod[p].r==r)
140     return nod[p].w+nod[p].add;
141   lazy(p);
142   if(nod[p].mid<l)
143     return read(p<<1|1,l,r);
144   else if(nod[p].mid>=r)
145     return read(p<<1,l,r);
146 }
147 void jump(int st,int ed,int val)
148 {
149   while(deep[st]>=deep[ed])
150   {
151     int tmp=from[st];
152     if(deep[tmp]<deep[ed])
153       tmp=ed;
154     update(1,p[tmp],p[st],val);
155     st=fa[tmp][0];
156   }
157 }
158 void change(int st,int ed,int val)
159 {
160   int lca=getLCA(st,ed);
161   jump(st,lca,val);
162   jump(ed,lca,val);
163   jump(lca,lca,-val);
164 }
165 int main()
166 {
167   while(scanf("%d%d%d",&n,&m,&sum)==3)
168   {
169     int i;
170     s[0]=0;pos=0;deep[0]=-1;
171     memset(fa,0,sizeof(fa));
172     for(i=1;i<=n;i++)
173     {
174       solid[i]=-1;
175       scanf("%d",&s[i]);
176     }
177     memset(head,-1,sizeof(head));
178     e=0;
179     for(i=0;i<m;i++)
180     {
181       int a,b;
182       scanf("%d%d",&a,&b);
```

```
183       addedge(a,b);
184     }
185     LCA(1,0,0);
186     getpos(1,1);
187     init(1,0,pos-1);
188     for(i=0;i<sum;i++)
189     {
190       char que[5];
191       scanf("%s",que);
192       if(que[0]!='Q')
193       {
194         int a,b,c;
195         scanf("%d%d%d",&a,&b,&c);
196         if(que[0]=='D')
197           c=-c;
198         change(a,b,c);
199       }
200       else
201       {
202         int a;
203         scanf("%d",&a);
204         printf("%d\n",read(1,p[a],p[a]));
205       }
206     }
207   }
208   return 0;
209 }
```

4.6 划分树

```
 1 int n,m;
 2 struct elem
 3 {
 4   int v,index;
 5 }a[120000];
 6 int d[30][120000];
 7 int s[30][120000];
 8
 9 bool cmp(elem a,elem b)
10 {
11   if (a.v == b.v)
12     return a.index <= b.index;
13   return a.v < b.v;
14 }
15
16 void build(int depth,int l,int r)
17 {
18   if (l == r)
19     return;
20   int mid = (l+r)/2;
21   int tl,tr;
22   tl = tr = 0;
```

```
  for (int i = l;i <= r;i++)
  {
    if (cmp(a[d[depth][i]],a[mid]))
    {
      d[depth+1][l+tl] = d[depth][i];
      tl++;
    }
    else
    {
      d[depth+1][mid+1+tr] = d[depth][i];
      tr++;
    }
    s[depth][i] = tl;
  }
  build(depth+1,l,mid);
  build(depth+1,mid+1,r);
}

int find(int depth,int dl,int dr,int fl,int fr,int k)
{
  if (fl == fr)
    return a[d[depth][fl]].v;
  int ls,rs;
  int mid = (dl+dr)/2;
  ls = (fl == dl)? 0 : s[depth][fl-1];
  rs = s[depth][fr];
  return (rs-ls < k)?
    find(depth+1,mid+1,dr,mid+fl-dl-ls+1,mid+fr-dl-rs+1,k-(rs-ls))
    : find(depth+1,dl,mid,dl+ls,dl+rs-1,k);
}

int main()
{
  while (scanf("%d%d",&n,&m) != EOF)
  {
    for (int i = 1;i <= n;i++)
    {
      scanf("%d",&a[i].v);
      a[i].index = i;
    }
    sort(a+1,a+n+1,cmp);
    for (int i = 1;i <= n;i++)
      d[0][a[i].index] = i;
    build(0,1,n);
    int l,r,k;
    for (int i = 1;i <= m;i++)
    {
      scanf("%d%d%d",&l,&r,&k);
      printf("%d\n",find(0,1,n,l,r,k));
    }
  }
```

```
74    return 0;
75  }
```

4.7 树状数组

```
 1  int read(int k)
 2  {
 3    int sum = 0;
 4    for (; k; k^=k&-k)
 5      sum+=tree[k];
 6    return sum;
 7  }
 8  void update(int k, int v)
 9  {
10    for (; k<=MaxN; k+=k&-k)
11      tree[k]+=v;
12  }
13  int find_Kth(int k)
14  {
15    int idx = 0;
16    for(int i=20; i>=0; i--)
17    {
18      idx |= 1 << i;
19      if(idx <= MaxN && tree[idx] < k)
20        k -= tree[idx];
21      else  idx ^= 1 << i;
22    }
23    return idx + 1;
24  }
```

5 图论

5.1 优先队列优化的 dijkstra

```
#include<cstdio>
#include<cstring>
#include<iostream>
#include<algorithm>
#include<queue>
#include<vector>
using namespace std;
const int MAXN=100;
const int MAXM=1000;
int N,L;
int head[MAXN];
struct edges
{
  int to,next,cost;
} edge[MAXM];
int dist[MAXN];
class states
{
public:
  int cost,id;
};
class cmp
{
public:
  bool operator ()(const states &i,const states &j)
  {
    return i.cost>j.cost;
  }
};
void init(int n)
{
  N=n;
  L=0;
  for (int i=0; i<n; i++)
    head[i]=-1;
}
void add_edge(int x,int y,int cost)
{
  edge[L].to=y;
  edge[L].cost=cost;
  edge[L].next=head[x];
  head[x]=L++;
}
int dijkstra(int s,int t)
{
  memset(dist,63,sizeof(dist));
  states u;
```

```
48    u.id=s;
49    u.cost=0;
50    dist[s]=0;
51    priority_queue<states,vector<states>,cmp> q;
52    q.push(u);
53    while (!q.empty())
54    {
55      u=q.top();
56      q.pop();
57      if (u.id==t) return dist[t];
58      if (u.cost!=dist[u.id]) continue;
59      for (int i=head[u.id]; i!=-1; i=edge[i].next)
60      {
61        states v=u;
62        v.id=edge[i].to;
63        if (dist[v.id]>dist[u.id]+edge[i].cost)
64        {
65          v.cost=dist[v.id]=dist[u.id]+edge[i].cost;
66          q.push(v);
67        }
68      }
69    }
70    return -1;
71  }
72  int main()
73  {
74    int n,m;
75    scanf("%d%d",&n,&m);
76    init(n);
77    for (int i=0; i<m; i++)
78    {
79      int x,y,z;
80      scanf("%d%d%d",&x,&y,&z);
81      add_edge(x,y,z);
82      add_edge(y,x,z);
83    }
84    int s,t;
85    scanf("%d%d",&s,&t);
86    printf("%d\n",dijkstra(s,t));
87    return 0;
88  }
```

5.2 SAP 四版

```
1  const int MAXEDGE=20400;
2  const int MAXN=400;
3  const int inf=0x3fffffff;
4  struct edges
5  {
6    int cap,to,next,flow;
7  } edge[MAXEDGE+100];
8  struct nodes
```

```
 9 {
10   int head,label,pre,cur;
11 } node[MAXN+100];
12 int L,N;
13 int gap[MAXN+100];
14 void init(int n)
15 {
16   L=0;
17   N=n;
18   for (int i=0; i<N; i++)
19     node[i].head=-1;
20 }
21 void add_edge(int x,int y,int z,int w)
22 {
23   edge[L].cap=z;
24   edge[L].flow=0;
25   edge[L].to=y;
26   edge[L].next=node[x].head;
27   node[x].head=L++;
28   edge[L].cap=w;
29   edge[L].flow=0;
30   edge[L].to=x;
31   edge[L].next=node[y].head;
32   node[y].head=L++;
33 }
34 int maxflow(int s,int t)
35 {
36   memset(gap,0,sizeof(gap));
37   gap[0]=N;
38   int u,ans=0;
39   for (int i=0; i<N; i++)
40   {
41     node[i].cur=node[i].head;
42     node[i].label=0;
43   }
44   u=s;
45   node[u].pre=-1;
46   while (node[s].label<N)
47   {
48     if (u==t)
49     {
50       int min=inf;
51       for (int i=node[u].pre; i!=-1; i=node[edge[i^1].to].pre)
52         if (min>edge[i].cap-edge[i].flow)
53           min=edge[i].cap-edge[i].flow;
54       for (int i=node[u].pre; i!=-1; i=node[edge[i^1].to].pre)
55       {
56         edge[i].flow+=min;
57         edge[i^1].flow-=min;
58       }
59       u=s;
```

```
60        ans+=min;
61        continue;
62      }
63      bool flag=false;
64      int v;
65      for (int i=node[u].cur; i!=-1; i=edge[i].next)
66      {
67        v=edge[i].to;
68        if (edge[i].cap-edge[i].flow &&
69          node[v].label+1==node[u].label)
70        {
71          flag=true;
72          node[u].cur=node[v].pre=i;
73          break;
74        }
75      }
76      if (flag)
77      {
78        u=v;
79        continue;
80      }
81      node[u].cur=node[u].head;
82      int min=N;
83      for (int i=node[u].head; i!=-1; i=edge[i].next)
84        if (edge[i].cap-edge[i].flow && node[edge[i].to].label<min)
85          min=node[edge[i].to].label;
86      gap[node[u].label]--;
87      if (!gap[node[u].label]) return ans;
88      node[u].label=min+1;
89      gap[node[u].label]++;
90      if (u!=s) u=edge[node[u].pre^1].to;
91    }
92    return ans;
93  }
```

5.3 费用流

5.3.1 三版

T 了可以改成栈。

```
1  const int MAXM=60000;
2  const int MAXN=400;
3  const int inf=0x3fffffff;
4  int L,N;
5  int K;
6  struct edges
7  {
8    int to,next,cap,flow,cost;
9  } edge[MAXM];
10 struct nodes
11 {
```

```
12    int dis,pre,head;
13    bool visit;
14  } node[MAXN];
15  void init(int n)
16  {
17    N=n;
18    L=0;
19    for (int i=0; i<N; i++)
20      node[i].head=-1;
21  }
22  void add_edge(int x,int y,int cap,int cost)
23  {
24    edge[L].to=y;
25    edge[L].cap=cap;
26    edge[L].cost=cost;
27    edge[L].flow=0;
28    edge[L].next=node[x].head;
29    node[x].head=L++;
30    edge[L].to=x;
31    edge[L].cap=0;
32    edge[L].cost=-cost;
33    edge[L].flow=0;
34    edge[L].next=node[y].head;
35    node[y].head=L++;
36  }
37  bool spfa(int s,int t)
38  {
39    queue <int> q;
40    for (int i=0; i<N; i++)
41    {
42      node[i].dis=0x3fffffff;
43      node[i].pre=-1;
44      node[i].visit=0;
45    }
46    node[s].dis=0;
47    node[s].visit=1;
48    q.push(s);
49    while (!q.empty())
50    {
51      int u=q.front();
52      node[u].visit=0;
53      for (int i=node[u].head; i!=-1; i=edge[i].next)
54      {
55        int v=edge[i].to;
56        if (edge[i].cap>edge[i].flow &&
57            node[v].dis>node[u].dis+edge[i].cost)
58        {
59          node[v].dis=node[u].dis+edge[i].cost;
60          node[v].pre=i;
61          if (!node[v].visit)
62          {
```

```
63            node[v].visit=1;
64            q.push(v);
65          }
66        }
67      }
68      q.pop();
69    }
70    if (node[t].pre==-1)
71      return 0;
72    else
73      return 1;
74  }
75  int mcmf(int s,int t,int &cost)
76  {
77    int flow=0;
78    while (spfa(s,t))
79    {
80      int max=inf;
81      for (int i=node[t].pre; i!=-1; i=node[edge[i^1].to].pre)
82      {
83        if (max>edge[i].cap-edge[i].flow)
84          max=edge[i].cap-edge[i].flow;
85      }
86      for (int i=node[t].pre; i!=-1; i=node[edge[i^1].to].pre)
87      {
88        edge[i].flow+=max;
89        edge[i^1].flow-=max;
90        cost+=edge[i].cost*max;
91      }
92      flow+=max;
93    }
94    return flow;
95  }
```

5.3.2 dijkstra 加改点堆

```
 1  #include <cstdio>
 2  #include <cstring>
 3  #include <algorithm>
 4  #include <queue>
 5  #include <stack>
 6  using namespace std;
 7  int N, L;
 8  int head[2003];
 9  struct Edge
10  {
11      int to, next, flow, cost;
12  } edge[2000 * 1999 / 2 + 2000 * 3 << 1];
13  int h[2003], dis[2003], pre[2003];
14  struct Heap
15  {
```

```
16      int value[2004], id[2004];
17      int pos[2003];
18      int size;
19      void init()
20      {
21          size = 1;
22      }
23      void swap2(int p, int q)
24      {
25          swap(value[p], value[q]);
26          swap(id[p], id[q]);
27          pos[id[p]] = p;
28          pos[id[q]] = q;
29      }
30      void push_up(int p)
31      {
32          while (p > 1 && value[p / 2] > value[p])
33          {
34              swap2(p, p / 2);
35              p /= 2;
36          }
37      }
38      void push_down(int p)
39      {
40          while (p * 2 < size)
41          {
42              int best = p;
43              if (p * 2 < size && value[p] > value[p * 2])
44                  best = p * 2;
45              if (p * 2 + 1 < size && value[best] > value[p * 2 + 1])
46                  best = p * 2 + 1;
47              if (p == best)
48                  break;
49              swap2(p, best);
50              p = best;
51          }
52      }
53      void push(int _value, int _id)
54      {
55          value[size] = _value;
56          id[size] = _id;
57          pos[_id] = size;
58          push_up(size++);
59      }
60      int top()
61      {
62          return id[1];
63      }
64      void pop()
65      {
66          value[1] = value[size - 1];
```

```
67          id[1] = id[--size];
68          pos[id[1]] = 1;
69          push_down(1);
70      }
71      void update(int _value, int _id)
72      {
73          int p = pos[_id];
74          value[p] = _value;
75          push_up(p);
76      }
77  } heap;
78  bool inque[2003];
79  void init(int n)
80  {
81      N = n;
82      L = 0;
83      memset(head, -1, 4 * n);
84  }
85  void add_edge(int u, int v, int flow, int cost)
86  {
87      edge[L].to = v;
88      edge[L].flow = flow;
89      edge[L].cost = cost;
90      edge[L].next = head[u];
91      head[u] = L++;
92      edge[L].to = u;
93      edge[L].flow = 0;
94      edge[L].cost = -cost;
95      edge[L].next = head[v];
96      head[v] = L++;
97  }
98  void spfa(int s)
99  {
100     memset(dis, 63, 4 * N);
101     memset(inque, 0, N);
102     memset(pre, -1, 4 * N);
103     dis[s] = 0;
104     stack <int> que;
105     que.push(s);
106     while (!que.empty())
107     {
108         int u = que.top();
109         inque[u] = 0;
110         que.pop();
111         for (int i = head[u]; i != -1; i = edge[i].next)
112             if (edge[i].flow)
113             {
114                 int v = edge[i].to;
115                 if (dis[v] > dis[u] + edge[i].cost)
116                 {
117                     dis[v] = dis[u] + edge[i].cost;
```

```
118                     pre[v] = i;
119                     if (!inque[v])
120                     {
121                         inque[v] = 1;
122                         que.push(v);
123                     }
124                 }
125             }
126     }
127 }
128 void dijkstra(int s)
129 {
130     for (int i = 0; i < N; ++i)
131         h[i] += dis[i];
132     memset(dis, 63, 4 * N);
133     memset(pre, -1, 4 * N);
134     memset(inque, 0, N);
135     dis[s] = 0;
136     inque[s] = 1;
137     heap.init();
138     heap.push(0, s);
139     while (heap.size > 1)
140     {
141         int u = heap.top();
142         heap.pop();
143         for (int i = head[u]; i != -1; i = edge[i].next)
144             if (edge[i].flow)
145             {
146                 int v = edge[i].to;
147                 if (dis[v] > dis[u] + edge[i].cost + h[u] - h[v])
148                 {
149                     dis[v] = dis[u] + edge[i].cost + h[u] - h[v];
150                     pre[v] = i;
151                     if (!inque[v])
152                     {
153                         heap.push(dis[v], v);
154                         inque[v] = 1;
155                     }
156                     else
157                         heap.update(dis[v], v);
158                 }
159             }
160     }
161 }
162 int mcmf(int s, int t, int &cost)
163 {
164     int flow = 0;
165     memset(h, 0, 4 * N);
166     for (spfa(s); pre[t] != -1; dijkstra(s))
167   {
168     int maxs = edge[pre[t]].flow;
```

```
169     for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to])
170       maxs = min(maxs, edge[i].flow);
171         for (int i = pre[t]; i != -1; i = pre[edge[i ^ 1].to])
172         {
173             edge[i].flow -= maxs;
174             edge[i ^ 1].flow += maxs;
175             cost += edge[i].cost * maxs;
176         }
177     flow += maxs;
178   }
179     return flow;
180 }
181 int a[1000];
182 int main()
183 {
184     int t;
185     scanf("%d", &t);
186     for (int cas = 1; cas <= t; ++cas)
187     {
188         int n;
189         scanf("%d", &n);
190         init(n * 2 + 3);
191         for (int i = 0; i < n; ++i)
192         {
193             int x;
194             scanf("%d", &x);
195             a[i] = x;
196             add_edge(i * 2, i * 2 + 1, 1, -1);
197             add_edge(i * 2 + 1, n * 2 + 1, 1, 0);
198             add_edge(n * 2 + 2, i * 2, 1, 0);
199         }
200         for (int i = 0; i < n; ++i)
201             for (int j = i + 1; j < n; ++j)
202                 if (a[i] < a[j])
203                     add_edge(i * 2 + 1, j * 2, 1, 0);
204         add_edge(n * 2, n * 2 + 2, 2, 0);
205         int ans = 0;
206         mcmf(n * 2, n * 2 + 1, ans);
207         printf("Case␣#%d:␣%d\n", cas, -ans);
208     }
209     return 0;
210 }
```

5.4 网络单纯形

返回 pair(流量，费用)

```
1 #include <iostream>
2 #include <cstdio>
3 #include <cstring>
4 #include <algorithm>
5 using namespace std;
6
```

```
 7 const int N = 1100;
 8 const int M = 50000*2*5+2;
 9 struct Node
10 {
11   int depth,pot,id;
12   Node *parent,*thread,*prev,*last;
13 };
14 struct Arc
15 {
16   int tail,head,u,c;
17 };
18 Arc arc[M];
19 int cnt_arc;
20 Node nnode[N],*rec[N];
21 int cr;
22 const int inf = 0x22222222;
23 void dfs(Node *n)
24 {
25   cr=0;
26   rec[cr++]=n;
27   while(cr)
28   {
29     Node *nn=n->thread;
30     nn->prev=n;
31     n=nn;
32     int d=n->depth;
33     if(d<=rec[cr-1]->depth)
34     {
35       Node *l=rec[cr-1];
36       while(cr&&d<=rec[cr-1]->depth)
37       {
38         rec[--cr]->last=l;
39       }
40     }
41     if(cr) rec[cr++]=n;
42   }
43 }
44 Node*pivot(Node*n1,Node*n2,Node*n3,Node*n4)
45 {
46   dfs(n1);
47   Node *n1l=n1->last,*n3l,*n3p,*end;
48   if(n1l->thread==n2)
49   {
50     n2->parent=n1;
51     dfs(n3);
52     return n3->last->thread;
53   }
54   if(n1l->thread!=n3)
55   {
56     dfs(n4);
57     n3p=n3->prev;
```

```
58      n3l=n3->last;
59      n3p->thread=n3l->thread;
60      if(n3l==n1l)
61        n3l->thread=n1->thread,n1->thread=n2;
62      else
63        n3l->thread=n1l->thread,n1l->thread=n2;
64    }
65    else
66    {
67      dfs(n3);
68      n3l=n3->last;
69      n1l->thread=n2;
70    }
71    end=n3l->thread;
72    Node *n2l=n2->last,*n2lp=NULL,*p2=n1,*pp=NULL;
73    int pid;
74    int first=0;
75    while(n2!=n3)
76    {
77      n2l=n2->last;
78      Node *xp=pp;
79      if(n2l!=n2lp) xp=n2l,n2lp=n2l;
80      pp=n2->prev;
81      pp->thread=xp->thread;
82      xp->thread=n2->parent;
83      Node *n2n=n2->parent;
84      int ppid=n2->id;
85      if(first) n2->id=pid^1;
86      pid=ppid;
87      first++;
88      n2->parent=p2;
89      p2=n2;
90      n2=n2n;
91    }
92    if(first) n2->id=pid^1;
93    n2->parent=p2;
94    return end;
95  }
96  void update(Node *n1,Node *n2)
97  {
98    n1=n1->thread;
99    while(n1!=n2)
100   {
101     Node *n1p=n1->parent;
102     n1->depth=n1p->depth+1;
103     n1->pot = n1p->pot - arc[n1->id].c;
104     n1=n1->thread;
105   }
106 }
107 void pivot(int id)
108 {
```

```
109   int tail=arc[id].tail,head=arc[id].head;
110   int u=arc[id].u,c=arc[id].c;
111   Node *uu=nnode+tail,*vv=nnode+head;
112   Node *mu=0,*mv=0;
113   int mn_f = u;
114   while(uu!=vv)
115   {
116     int nid;
117     if(uu->depth>vv->depth)
118       nid=uu->id,uu=uu->parent;
119     else nid=vv->id^1,vv=vv->parent;
120     if(arc[nid].u<mn_f) mn_f=arc[nid].u;
121   }
122   uu=nnode+tail,vv=nnode+head;
123   arc[id].u-=mn_f;
124   arc[id^1].u+=mn_f;
125   while(uu!=vv)
126   {
127     int nid;
128     if(uu->depth>vv->depth)
129     {
130       nid=uu->id;
131       arc[nid].u-=mn_f;
132       arc[nid^1].u+=mn_f;
133       if(arc[nid].u==0&&mu==0)
134         mu=uu,mv=0;
135       uu=uu->parent;
136     }
137     else
138     {
139       nid=vv->id^1;
140       arc[nid].u-=mn_f;
141       arc[nid^1].u+=mn_f;
142       if(arc[nid].u==0&&mu==0)
143         mv=vv;
144       vv=vv->parent;
145     }
146   }
147   if(arc[id].u)
148   {
149     if(mv)
150     {
151       int n3=mv-nnode,n4=mv->parent-nnode;
152       Node *nx=pivot(nnode+tail,nnode+head,mv,mv->parent);
153       nnode[head].id=id;
154       update(nnode+tail,nx);
155     }
156     else
157     {
158       int n3=mu-nnode,n4=mu->parent-nnode;
159       Node *nx=pivot(nnode+head,nnode+tail,mu,mu->parent);
```

```
160       nnode[tail].id=id^1;
161       update(nnode+head,nx);
162     }
163   }
164 }
165 struct List
166 {
167   int v;
168   List *R;
169 };
170 List lst[M],*cl,*hd;
171 pair<int,int> cost(int s,int t,int cnt_node)
172 {
173   int tmp_arc=cnt_arc;
174   int i;
175   int sid=-1;
176   nnode[cnt_node].depth=0;
177   nnode[cnt_node].parent=nnode+cnt_node;
178   nnode[cnt_node].pot=0;
179   nnode[cnt_node].thread=nnode;
180   for(i=0; i<cnt_node; i++)
181   {
182     nnode[i].depth=1;
183     nnode[i].parent=nnode+cnt_node;
184     nnode[i].thread=nnode+(i+1);
185     nnode[i].id=tmp_arc;
186     int pot=-inf,u=0,tot=0;
187     if(i==s)
188       pot=-pot,tot=inf,u=inf,sid=tmp_arc+1;
189     if(i==t) tot=inf;
190     nnode[i].pot=pot;
191     arc[tmp_arc].tail=cnt_node;
192     arc[tmp_arc].head=i;
193     arc[tmp_arc].u=u;
194     arc[tmp_arc].c=-pot;
195     tmp_arc++;
196     arc[tmp_arc].tail=i;
197     arc[tmp_arc].head=cnt_node;
198     arc[tmp_arc].u=tot-u;
199     arc[tmp_arc].c=pot;
200     tmp_arc++;
201   }
202   while(1)
203   {
204     bool fd=false;
205     int i,tot=0;
206     hd=cl=lst;
207     cl->R=cl++;
208     int tail,head,c,cc;
209     const int MAXQ = cnt_node/4+5;
210     const int MINQ = cnt_node/10+2;
```

```
211     for(i=0; i<cnt_arc; i++)
212     {
213       if(arc[i].u==0) continue;
214       tail=arc[i].tail;
215       head=arc[i].head,c=arc[i].c;
216       cc=c-nnode[tail].pot+nnode[head].pot;
217       if(cc<0)
218       {
219         fd=true;
220         cl->v=i;
221         cl->R=hd->R;
222         hd->R=cl;
223         cl++;
224         tot++;
225         if(tot>=MAXQ)
226         {
227           while(tot>MINQ)
228           {
229             List *mj=NULL,*mjp,*it=hd->R,*itp=hd;
230             int mp=0;
231             while(it!=hd)
232             {
233               int id=it->v;
234               tail=arc[id].tail,head=arc[id].head,c=arc[id].c;
235               cc=c-nnode[tail].pot+nnode[head].pot;
236               if(cc>=0)
237               {
238                 itp->R=it->R;
239                 tot--;
240               }
241               else if(cc<mp)
242                 mp=cc,mj=it,mjp=itp;
243               if(itp->R==it) itp=it;
244               it=it->R;
245             }
246             if(mj==NULL) break;
247             pivot(mj->v);
248             mjp->R=mj->R;
249             tot--;
250           }
251         }
252       }
253     }
254     while(tot)
255     {
256       List *it=hd->R,*itp=hd;
257       while(it!=hd)
258       {
259         int id=it->v;
260         tail=arc[id].tail,head=arc[id].head;
261         c=arc[id].c;
```

```
262         cc=c-nnode[tail].pot+nnode[head].pot;
263         if(cc<0) pivot(id);
264         itp->R=it->R;
265         tot--;
266         it=it->R;
267       }
268     }
269     if(!fd) break;
270   }
271   int sum=0;
272   for(i=0; i<cnt_arc; i+=2)
273   {
274     sum+=arc[i].c*arc[i^1].u;
275   }
276   return pair<int,int>(arc[sid].u,sum);
277 }
278 void add_arc(int tail,int head,int u,int c)
279 {
280   arc[cnt_arc].tail=tail;
281   arc[cnt_arc].head=head;
282   arc[cnt_arc].u=u;
283   arc[cnt_arc].c=c;
284   cnt_arc++;
285 }
286
287 void init()
288 {
289   cnt_arc = 0;
290 }
291
292 void add_edge(int x,int y,int cap,int cost)
293 {
294   add_arc(x,y,cap,cost);
295   add_arc(y,x,0,-cost);
296 }
297
298 int main()
299 {
300   int n,m,k;
301   while (scanf("%d%d%d",&n,&m,&k)!=EOF)
302   {
303     init();
304     add_edge(0,1,k,0);
305     for (int i=0;i<m;i++)
306     {
307       int u,v,a,c;
308       scanf("%d%d%d%d",&u,&v,&a,&c);
309       for (int j=0;j<c;j++)
310         add_edge(u,v,1,a*(j*2+1));
311     }
312     pair<int,int> ret = cost(0,n,n+1);
```

```
313     if (ret.first == k)
314       printf("%d\n",ret.second);
315     else
316       puts("-1");
317   }
318   return 0;
319 }
```

5.5 匈牙利

5.5.1 新版, 隐式图可解

```
 1 bool check(int u)
 2 {
 3   for (int i=head[u]; i!=-1; i=edge[i].next)
 4   {
 5     int v=edge[i].to;
 6     if (matc[v]==u) continue;
 7     if (!use[v])
 8     {
 9       use[v]=1;
10       if (matc[v]==-1 || check(matc[v]))
11       {
12         matc[v]=u;
13         matc[u]=v;
14         return 1;
15       }
16     }
17   }
18   return 0;
19 }
20 int match()
21 {
22   int ret=0;
23   memset(matc,-1,sizeof(matc));
24   for (int u=0; u<N; u++)
25   {
26     if (matc[u]!=-1) continue;
27     memset(use,0,sizeof(use));
28     if (check(u))
29       ret++;
30   }
31   return ret;
32 }
```

5.5.2 邻接矩阵

```
1 bool check(int u)
2 {
3   for (int v=0; v<N; v++)
4     if (am[u][v] && !use[v])
5     {
```

```
      use[v]=1;
      if (pre[v]==-1 || check(pre[v]))
      {
        pre[v]=u;
        return 1;
      }
    }
  return 0;
}
int match()
{
  int ret=0;
  memset(pre,-1,sizeof(pre));
  for (int u=0; u<N; u++)
  {
    memset(use,0,sizeof(use));
    if (check(u))
      ret++;
  }
  return ret;
}
```

5.5.3 邻接表

```
bool check(int u)
{
  for (int i=head[u]; i!=-1; i=edge[i].next)
  {
    int v=edge[i].to;
    if (!use[v])
    {
      use[v]=1;
      if (pre[v]==-1 || check(pre[v]))
      {
        pre[v]=u;
        return 1;
      }
    }
  }
  return 0;
}
int match()
{
  int ret=0;
  memset(pre,-1,sizeof(pre));
  for (int u=1; u<=N; u++)
  {
    memset(use,0,sizeof(use));
    if (check(u))
      ret++;
  }
  return ret;
```

```
}
```

5.6 一般图最大加权匹配

注意 G 初始化

```
#define N 229
int G[N][N];
int cnt_node;
int dist[N];
int rec[N],cr,M[N],P[N];
bool vst[N];
const int inf = 0x3f3f3f3f;
bool spfa(int u)
{
   rec[cr++]=u;
   if(vst[u]) return true;
   vst[u]=true;
   int v;
   for(v=0; v<cnt_node; v++)
   {
      if(v!=u&&M[u]!=v&&!vst[v])
      {
        int w=M[v];
        if(dist[w]<dist[u]+G[u][v]-G[v][w])
        {
           dist[w]=dist[u]+G[u][v]-G[v][w];
           if(spfa(w))
           {
              return true;
           }
        }
      }
   }
   cr--;
   vst[u]=false;
   return false;
}
int match()
{
   int i;
   for(i=0; i<cnt_node; i++) P[i]=i;
   for(i=0; i<cnt_node; i+=2) M[i]=i+1,M[i+1]=i;
   int cnt=0;
   while(1)
   {
      memset(dist,0,sizeof(dist));
      cr=0;
      int i;
      bool fd=false;
      memset(vst,0,sizeof(vst));
      for(i=0; i<cnt_node; i++)
```

```
47        {
48          if(spfa(P[i]))
49          {
50             fd=true;
51             int j;
52             int nx=M[rec[cr-1]];
53             for(j=cr-2; rec[j]!=rec[cr-1]; j--)
54             {
55                M[nx]=rec[j];
56                int tmp=nx;
57                nx=M[rec[j]];
58                M[rec[j]]=tmp;
59             }
60             M[nx]=rec[j];
61             M[rec[j]]=nx;
62             break;
63          }
64        }
65        if(!fd)
66        {
67          cnt++;
68          if(cnt>=3) break;
69          random_shuffle(P,P+cnt_node);
70        }
71     }
72     int sum=0;
73     for(i=0; i<cnt_node; i++)
74     {
75        int v=M[i];
76        if(i<v)
77        {
78          sum+=G[i][v];
79        }
80     }
81     return sum;
82  }
```

5.7 一般图匹配带花树

```
 1  const int MaxN = 222;
 2  int N;
 3  bool Graph[MaxN+1][MaxN+1];
 4  int Match[MaxN+1];
 5  bool InQueue[MaxN+1],InPath[MaxN+1],InBlossom[MaxN+1];
 6  int Head,Tail;
 7  int Queue[MaxN+1];
 8  int Start,Finish;
 9  int NewBase;
10  int Father[MaxN+1],Base[MaxN+1];
11  int Count;
12  void CreateGraph()
```

```
13 {
14   int u,v;
15   memset(Graph,false,sizeof(Graph));
16   scanf("%d",&N);
17   while (scanf("%d%d",&u,&v) != EOF)
18     Graph[u][v] = Graph[v][u] = true;
19 }
20 void Push(int u)
21 {
22   Queue[Tail] = u;
23   Tail++;
24   InQueue[u] = true;
25 }
26 int Pop()
27 {
28   int res = Queue[Head];
29   Head++;
30   return res;
31 }
32 int FindCommonAncestor(int u,int v)
33 {
34   memset(InPath,false,sizeof(InPath));
35   while (true)
36   {
37     u = Base[u];
38     InPath[u] = true;
39     if (u == Start) break;
40     u = Father[Match[u]];
41   }
42   while (true)
43   {
44     v = Base[v];
45     if (InPath[v]) break;
46     v = Father[Match[v]];
47   }
48   return v;
49 }
50 void ResetTrace(int u)
51 {
52   int v;
53   while (Base[u] != NewBase)
54   {
55     v = Match[u];
56     InBlossom[Base[u]] = InBlossom[Base[v]] = true;
57     u = Father[v];
58     if (Base[u] != NewBase) Father[u] = v;
59   }
60 }
61 void BlossomContract(int u,int v)
62 {
63   NewBase = FindCommonAncestor(u,v);
```

```
  memset(InBlossom,false,sizeof(InBlossom));
  ResetTrace(u);
  ResetTrace(v);
  if (Base[u] != NewBase) Father[u] = v;
  if (Base[v] != NewBase) Father[v] = u;
  for (int tu = 1; tu <= N; tu++)
    if (InBlossom[Base[tu]])
    {
      Base[tu] = NewBase;
      if (!InQueue[tu]) Push(tu);
    }
}
void FindAugmentingPath()
{
  memset(InQueue,false,sizeof(InQueue));
  memset(Father,0,sizeof(Father));
  for (int i = 1; i <= N; i++)
    Base[i] = i;
  Head = Tail = 1;
  Push(Start);
  Finish = 0;
  while (Head < Tail)
  {
    int u = Pop();
    for (int v = 1; v <= N; v++)
      if (Graph[u][v] && (Base[u] != Base[v]) && (Match[u] != v))
      {
        if ((v == Start) ||
          ((Match[v] > 0) && (Father[Match[v]] > 0)))
          BlossomContract(u,v);
        else if (Father[v] == 0)
        {
          Father[v] = u;
          if (Match[v] > 0)
            Push(Match[v]);
          else
          {
            Finish = v;
            return;
          }
        }
      }
  }
}
void AugmentPath()
{
  int u,v,w;
  u = Finish;
  while (u > 0)
  {
    v = Father[u];
```

```
115     w = Match[v];
116     Match[v] = u;
117     Match[u] = v;
118     u = w;
119   }
120 }
121 void Edmonds()
122 {
123   memset(Match,0,sizeof(Match));
124   for (int u = 1; u <= N; u++)
125     if (Match[u] == 0)
126     {
127       Start = u;
128       FindAugmentingPath();
129       if (Finish > 0) AugmentPath();
130     }
131 }
132 void PrintMatch()
133 {
134   for (int u = 1; u <= N; u++)
135     if (Match[u] > 0)
136       Count++;
137   printf("%d\n",Count);
138   for (int u = 1; u <= N; u++)
139     if (u < Match[u])
140       printf("%d␣%d\n",u,Match[u]);
141 }
142 int main()
143 {
144   CreateGraph();
145   Edmonds();
146   PrintMatch();
147 }
```

5.8 KM

5.8.1 最大加权匹配

```
1  bool visx[N],visy[N];//x,y 中的点是否被访问
2  int lx[N],ly[N];//x,y 中的点的标号
3  int matchy[N];//y 中各点匹配状态
4  int map[N][N];//二分图描述 [x][y]
5  bool find(int x)
6  {
7    visx[x]=true;
8    int t;
9    for (int y=0;y<ycnt;y++)
10   {
11     if (!visy[y])
12     {
13       t=lx[x]+ly[y]-map[x][y];
14       if (t==0)
```

```
      {
        visy[y]=true;
        if (matchy[y]==-1 || find(matchy[y]))
        {
          matchy[y]=x;
          return true;
        }
      }
      else if (lack>t) lack=t;
    }
  }
  return false;
}
void KM()
{
  memset(lx,0,sizeof(lx));
  memset(ly,0,sizeof(ly));
  memset(matchy,-1,sizeof(matchy));
  for (int i=0;i<xcnt;i++)
    for (int j=0;j<ycnt;j++)
      if (map[i][j]>lx[i])
        lx[i]=map[i][j];
  for (int x=0;x<xcnt;x++)
  {
    while (true)
    {
      memset(visx,false,sizeof(visx));
      memset(visy,false,sizeof(visy));
      lack=INFI;
      if (find(x)) break;
      for (int i=0;i<xcnt;i++)
      {
        if (visx[i]) lx[i]-=lack;
        if (visy[i]) ly[i]+=lack;
      }
    }
  }
  int cost=0;
  for (int i=0;i<ycnt;i++)
    cost+=map[matchy[i]][i];
}
```

5.8.2 自认为正确的 Kuhn_Munkras

未验证

```
#include<cstdio>
#include<cstring>
#include<algorithm>
using namespace std;
const int MAXN=100;
const int inf=0x3f3f3f3f;
bool visitx[MAXN],visity[MAXN];
```

```
 8 int labx[MAXN],laby[MAXN],matx[MAXN],maty[MAXN],slack[MAXN];
 9 int ma[MAXN][MAXN];
10 bool check(int x,int n)
11 {
12   visitx[x]=1;
13   for (int i=0; i<n; i++)
14     if (!visity[i])
15       if (labx[x]+laby[i]==ma[x][i])
16       {
17         visity[i]=1;
18         if (maty[i]==-1 || check(maty[i],n))
19         {
20           matx[x]=i;
21           maty[i]=x;
22           return 1;
23         }
24       }
25       else
26         slack[i]=min(slack[i],labx[x]+laby[i]-ma[x][i]);
27
28   return 0;
29 }
30 void maintain(int n)
31 {
32   int diff=inf;
33   for (int i=0; i<n; i++)
34     if (!visity[i])
35       diff=min(diff,slack[i]);
36   for (int i=0; i<n; i++)
37   {
38     if (visitx[i])
39       labx[i]-=diff;
40     if (visity[i])
41       laby[i]+=diff;
42     else
43       slack[i]-=diff;
44   }
45 }
46 int Kuhn_Munkras(int n)
47 {
48   for (int i=0; i<n; i++)
49   {
50     labx[i]=-inf;
51     for (int j=0; j<n; j++)
52       labx[i]=max(labx[i],ma[i][j]);
53   }
54   memset(laby,0,4*n);
55   memset(matx,-1,4*n);
56   memset(maty,-1,4*n);
57   for (int i=0; i<n; i++)
58   {
```

```
59      memset(visitx,0,n);
60      memset(visity,0,n);
61      memset(slack,63,4*n);
62      while (!check(i,n))
63      {
64        maintain(n);
65        memset(visitx,0,n);
66        memset(visity,0,n);
67      }
68    }
69    int ret=0;
70    for (int i=0;i<n;i++)
71      ret+=labx[i]+laby[i];
72    return ret;
73  }
74  int main()
75  {
76    int n,m;
77    scanf("%d%d",&m,&n);
78    for (int i=m; i<n; i++)
79      for (int j=0; j<n; j++)
80        ma[i][j]=0;
81    for (int i=0; i<m; i++)
82      for (int j=0; j<n; j++)
83        scanf("%d",&ma[i][j]);
84    printf("%d\n",Kuhn_Munkras(n));
85    printf("%d",matx[0]+1);
86    for (int i=1;i<m;i++)
87      printf("␣%d",matx[i]+1);
88    puts("");
89    return 0;
90  }
```

5.9 * 二维平面图的最大流

待整理

```
 1  #include <iostream>
 2  #include <algorithm>
 3  #include <cstdio>
 4  #include <cstring>
 5  #include <vector>
 6  #include <cmath>
 7  #include <map>
 8  #include <queue>
 9  using namespace std;
10
11  const int maxn = 100100;
12  const int inf = 0x3f3f3f3f;
13  struct Point
14  {
```

```
15    int x,y,id;
16    double theta;
17    Point() {}
18    Point(int _x,int _y)
19    {
20      x = _x;
21      y = _y;
22    }
23    Point(Point _s,Point _e,int _id)
24    {
25      id = _id;
26      x = _s.x-_e.x;
27      y = _s.y-_e.y;
28      theta = atan2(y,x);
29    }
30    bool operator < (const Point &b)const
31    {
32      return theta < b.theta;
33    }
34  };
35
36  map<pair<int,int>,int > idmap;
37  struct Edge
38  {
39    int from,to,next,cap,near,mark;
40  };
41  Edge edge[maxn*2];
42  int head[maxn],L;
43  int cntd[maxn];
44  void addedge(int u,int v,int cap)
45  {
46    cntd[u]++;
47    cntd[v]++;
48    idmap[make_pair(u,v)] = L;
49    edge[L].from = u;
50    edge[L].to = v;
51    edge[L].cap = cap;
52    edge[L].next = head[u];
53    edge[L].mark = -1;
54    head[u] = L++;
55  }
56
57  int rtp[maxn];
58  Point p[maxn],tp[maxn];
59  int n,m,S,T;
60  int vid;
61
62  struct Edge2
63  {
64    int to,next,dis;
65  } edge2[maxn*2];
```

```
66  int head2[maxn],L2;
67
68  void addedge2(int u,int v,int dis)
69  {
70    edge2[L2].to = v;
71    edge2[L2].dis = dis;
72    edge2[L2].next = head2[u];
73    head2[u] = L2++;
74  }
75
76  int dist[maxn];
77  bool inq[maxn];
78  int SPFA(int s,int t)
79  {
80    queue<int> Q;
81    memset(inq,false,sizeof(inq));
82    memset(dist,63,sizeof(dist));
83    Q.push(s);
84    dist[s] = 0;
85    while (!Q.empty())
86    {
87      int now = Q.front();
88      Q.pop();
89      for (int i = head2[now]; i != -1; i = edge2[i].next)
90        if (dist[edge2[i].to] > dist[now]+edge2[i].dis)
91        {
92          dist[edge2[i].to] = dist[now]+edge2[i].dis;
93          if (inq[edge2[i].to] == false)
94          {
95            inq[edge2[i].to] = true;
96            Q.push(edge2[i].to);
97          }
98        }
99      inq[now] = false;
100   }
101   return dist[t];
102 }
103
104 int main()
105 {
106   int totcas;
107   scanf("%d",&totcas);
108   for (int cas = 1; cas <= totcas; cas++)
109   {
110     idmap.clear();
111     L = 0;
112     scanf("%d%d",&n,&m);
113     S = T = 0;
114     for (int i = 0; i < n; i++)
115     {
116       head[i] = -1;
```

```
117       scanf("%d%d",&p[i].x,&p[i].y);
118       if (p[S].x > p[i].x)
119         S = i;
120       if (p[T].x < p[i].x)
121         T = i;
122       cntd[i] = 0;
123     }
124     //源汇中间加入一个特殊节点
125     head[n] = -1;
126     n ++;
127     addedge(S,n-1,inf);
128     addedge(n-1,S,inf);
129     addedge(T,n-1,inf);
130     addedge(n-1,T,inf);
131
132     for (int i = 0; i < m; i++)
133     {
134       int u,v,cap;
135       scanf("%d%d%d",&u,&v,&cap);
136       u--;
137       v--;
138       addedge(u,v,cap);
139       addedge(v,u,cap);
140     }
141
142     for (int i = 0; i < n; i++)
143     {
144       int tot = 0;
145       //源点汇点连到特殊点的方向需要特别考虑一下
146       if (i == S)
147         tp[tot++] = Point(Point(0,0),Point(-1,0),n-1);
148       else if (i == T)
149         tp[tot++] = Point(Point(0,0),Point(1,0),n-1);
150       else if (i == n-1)
151       {
152         tp[tot++] = Point(Point(0,0),Point(1,0),S);
153         tp[tot++] = Point(Point(0,0),Point(-1,0),T);
154       }
155       if (i < n-1)
156       {
157         for (int j = head[i]; j != -1; j = edge[j].next)
158         {
159           if (i == S && edge[j].to == n-1)  continue;
160           if (i == T && edge[j].to == n-1)  continue;
161           tp[tot++] = Point(p[i],p[edge[j].to],edge[j].to);
162         }
163       }
164       sort(tp,tp+tot);
165       for (int j = 0; j < tot; j++)
166         rtp[tp[j].id] = j;
167       for (int j = head[i]; j != -1; j = edge[j].next)
```

```
168         edge[j].near = tp[(rtp[edge[j].to]+1)%tot].id;
169     }
170
171     vid = 0;
172     for (int i = 0;i < L;i++)
173       if (edge[i].mark == -1)
174       {
175         int now = edge[i].from;
176         int eid = i;
177         int to  = edge[i].to;
178         while (true)
179         {
180           edge[eid].mark = vid;
181           eid ^= 1;
182           now = to;
183           to = edge[eid].near;
184           eid = idmap[make_pair(now,to)];
185
186           if (now == edge[i].from)  break;
187         }
188         vid++;
189       }
190
191     L2 = 0;
192     for (int i = 0; i < vid; i++)
193       head2[i] = -1;
194     for (int i = 0; i < L; i++)
195       addedge2(edge[i].mark,edge[i^1].mark,edge[i].cap);
196     printf("%d\n",SPFA(edge[0].mark,edge[1].mark));
197   }
198   return 0;
199 }
```

5.10 强联通

```
 1 int dfsnum[2000];
 2 int low[2000];
 3 int stack[2000];
 4 int top;
 5 int ans;
 6 int an;
 7 int be[2000];
 8 int flag[2000];
 9 void dfs(int x)
10 {
11   dfsnum[x] = low[x] = ans++;
12   stack[++top] = x;
13   flag[x] = 1;
14   for (int i = head[x];i != -1;i = edge[i].next)
15   {
16     int y = edge[i].to;
```

```
17      if (dfsnum[y] == -1)
18      {
19        dfs(y);
20        low[x] = min(low[x],low[y]);
21      }
22      else if (flag[y] == 1)
23        low[x] = min(low[x],dfsnum[y]);
24    }
25    if (dfsnum[x] == low[x])
26    {
27      while (stack[top] != x)
28      {
29        flag[stack[top]] = 0;
30        be[stack[top]] = an;
31        top--;
32      }
33      flag[x] = 0;
34      be[x] = an++;
35      top--;
36    }
37  }
```

调用：

```
1  void SC()
2  {
3    memset(dfsnum,-1,sizeof(dfsnum));
4    memset(flag,0,sizeof(flag));
5    top = 0;
6    an = 0;
7    ans = 0;
8    for (int i = 0;i < n;i++)
9      if (dfsnum[i] == -1)
10       dfs(i);
11 }
```

5.11 最大团以及相关知识

独立集： 独立集是指图的顶点集的一个子集, 该子集的导出子图不含边. 如果一个独立集不是任何一个独立集的子集, 那么称这个独立集是一个极大独立集. 一个图中包含顶点数目最多的独立集称为最大独立集。最大独立集一定是极大独立集，但是极大独立集不一定是最大的独立集。

支配集： 与独立集相对应的就是支配集，支配集也是图顶点集的一个子集，设 S 是图 G 的一个支配集，则对于图中的任意一个顶点 u，要么属于集合 s, 要么与 s 中的顶点相邻。在 s 中除去任何元素后 s 不再是支配集，则支配集 s 是极小支配集。称 G 的所有支配集中顶点个数最少的支配集为最小支配集，最小支配集中的顶点个数成为支配数。

最小点的覆盖： 最小点的覆盖也是图的顶点集的一个子集，如果我们选中一个点，则称这个点将以他为端点的所有边都覆盖了。将图中所有的边都覆盖所用顶点数最少，这个集合就是最小的点的覆盖。

最大团： 图 G 的顶点的子集，设 D 是最大团，则 D 中任意两点相邻。若 u，v 是最大团，则 u,v 有边相连，其补图 u,v 没有边相连，所以图 G 的最大团 $=$ 其补图的最大独立集。

给定无向图 $G=(V,E)$，如果 U 属于 V，并且对于任意 u,v 包含于 U 有 $<u,v>$ 包含于 E，则称 U 是 G 的完全子图，G 的完全子图 U 是 G 的团，当且仅当 U 不包含在 G 的更大的完全子图中，G 的最大团是指 G 中所含顶点数目最多的团。如果 U 属于 V，并且对于任意 u,v 包含于 U 有 $<u,v>$ 不包含于 E，则称 U 是 G 的空子图，G 的空子图 U 是 G 的独立集，当且仅当 U 不包含在 G 的更大的独立集，G 的最大团是指 G 中所含顶点数目最多的独立集。

一些性质： 最大独立集 + 最小覆盖集 $=V$，最大团 $=$ 补图的最大独立集，最小覆盖集 $=$ 最大匹配

```
#include <cstdio>
bool am[100][100];
int ans;
int c[100];
int U[100][100];
int n;
bool dfs(int rest,int num)
{
  if (!rest)
  {
    if (num>=ans)
      return 1;
    else
      return 0;
  }
  int pre=-1;
  for (int i=0;i<rest && rest-i+num>=ans;i++)
  {
    int idx=U[num][i];
    if (num+c[idx]<ans)
      return 0;
    int nrest=0;
    for (int j=i+1; j<rest; j++)
      if (am[idx][U[num][j]])
        U[num+1][nrest++]=U[num][j];
    if (dfs(nrest,num+1))
      return 1;
  }
  return 0;
}
int main()
{
  while (scanf("%d",&n),n)
  {
    for (int i=0;i<n;i++)
      for (int j=0;j<n;j++)
        scanf("%d",&am[i][j]);
    ans=0;
    for (int i=n-1; i>=0; i--)
    {
      int rest=0;
      for (int j=i+1; j<n; j++)
```

```
43          if (am[i][j])
44            U[0][rest++]=j;
45        ans+=dfs(rest,0);
46        c[i]=ans;
47      }
48      printf("%d\n",ans);
49    }
50    return 0;
51  }
```

5.12 双连通分量

标号从 0 起

```
 1  #include<cstdio>
 2  #include<cstring>
 3  #include<stack>
 4  #include<queue>
 5  #include<algorithm>
 6  using namespace std;
 7  const int MAXN=100000*2;
 8  const int MAXM=200000;
 9  struct edges
10  {
11    int to,next;
12    bool cut,visit;
13  } edge[MAXM<<1];
14  int head[MAXN],low[MAXN],dpt[MAXN],L;
15  bool visit[MAXN],cut[MAXN];
16  void init(int n)
17  {
18    L=0;
19    memset(head,-1,4*n);
20    memset(visit,0,n);
21  }
22  void add_edge(int u,int v)
23  {
24    edge[L].cut=edge[L].visit=0;
25    edge[L].to=v;
26    edge[L].next=head[u];
27    head[u]=L++;
28  }
29  int idx;
30  stack<int> st;
31  int bcc[MAXM];
32  void dfs(int u,int fu,int deg)
33  {
34    cut[u]=0;
35    visit[u]=1;
36    low[u]=dpt[u]=deg;
37    int tot=0;
38    for (int i=head[u]; i!=-1; i=edge[i].next)
```

```
39    {
40      int v=edge[i].to;
41      if (edge[i].visit)
42        continue;
43      st.push(i/2);
44      edge[i].visit=edge[i^1].visit=1;
45      if (visit[v])
46      {
47        low[u]=dpt[v]>low[u]?low[u]:dpt[v];
48        continue;
49      }
50      dfs(v,u,deg+1);
51      edge[i].cut=edge[i^1].cut=(low[v]>dpt[u] || edge[i].cut);
52      if (u!=fu) cut[u]=low[v]>=dpt[u]?1:cut[u];
53      if (low[v]>=dpt[u] || u==fu)
54      {
55        while (st.top()!=i/2)
56        {
57          int x=st.top()*2,y=st.top()*2+1;
58          bcc[st.top()]=idx;
59          st.pop();
60        }
61        bcc[i/2]=idx++;
62        st.pop();
63      }
64      low[u]=low[v]>low[u]?low[u]:low[v];
65      tot++;
66    }
67    if (u==fu && tot>1) cut[u]=1;
68  }
69  int main()
70  {
71    int n,m;
72    while (scanf("%d%d",&n,&m)!=EOF)
73    {
74      init(n);
75      for (int i=0; i<m; i++)
76      {
77        int u,v;
78        scanf("%d%d",&u,&v);
79        add_edge(u,v);
80        add_edge(v,u);
81      }
82      idx=0;
83      for (int i=0; i<n; i++)
84        if (!visit[i])
85          dfs(i,i,0);
86    }
87    return 0;
88  }
```

5.13 割点与桥

```
#include<cstdio>
#include<cstring>
const int MAXN=10000;
struct edges
{
  int to,next;
  bool cut,visit;
  int from;
} edge[MAXN-1<<1];
int head[MAXN],low[MAXN],dfn[MAXN],L;
bool visit[MAXN],cut[MAXN];
void init(int n)
{
  L=0;
  memset(head,-1,4*n);
  memset(cut,0,4*n);
  memset(visit,0,4*n);
}
void add_edge(int u,int v)
{
  edge[L].from=u;
  edge[L].cut=edge[L].visit=0;
  edge[L].to=v;
  edge[L].next=head[u];
  head[u]=L++;
}
int idx;
void dfs(int u,int fu)
{
  visit[u]=1;
  low[u]=dfn[u]=idx++;
  int tot=0;
  for (int i=head[u]; i!=-1; i=edge[i].next)
  {
    int v=edge[i].to;
    if (edge[i].visit)
      continue;
    edge[i].visit=edge[i^1].visit=1;
    if (visit[v])
    {
      low[u]=dfn[v]>low[u]?low[u]:dfn[v];
      continue;
    }
    dfs(v,u);
    edge[i].cut=edge[i^1].cut=low[v]>dfn[u] || edge[i].cut;
    if (u!=fu) cut[u]=low[v]>=dfn[u]?1:cut[u];
    low[u]=low[v]>low[u]?low[u]:low[v];
    tot++;
  }
```

```
50    if (u==fu && tot>1) cut[u]=1;
51  }
52  int main()
53  {
54    int t;
55    scanf("%d",&t);
56    while (t--)
57    {
58      int n,m;
59      scanf("%d%d",&n,&m);
60      init(n);
61      for (int i=0; i<m; i++)
62      {
63        int u,v;
64        scanf("%d%d",&u,&v);
65        add_edge(--u,--v);
66        add_edge(v,u);
67      }
68      for (int i=0; i<n; i++)
69        if (!visit[i])
70        {
71          idx=0;
72          dfs(i,i);
73        }
74    }
75    return 0;
76  }
```

5.14 LCA

在线 LCA，bfs

```
1   #include<cstdio>
2   #include<cstring>
3   #include<queue>
4   using namespace std;
5   const int NSIZE = 50000;
6   const int DEG = 20;
7   struct trees
8   {
9
10    int fa[DEG];
11    int head,deg;
12  } tree[NSIZE];
13  struct edges
14  {
15    int to , next;
16  } edge[NSIZE];
17  struct states
18  {
19    int u,fu,deg;
20  };
21  int L;
```

```
22 void add_edge(int x, int y)
23 {
24   edge[L].to = y;
25   edge[L].next = tree[x].head;
26   tree[x].head = L++;
27 }
28 int Root;
29 void BFS(int s)
30 {
31   queue<states> que;
32   states st;
33   st.deg=0;
34   st.fu=st.u=s;
35   que.push(st);
36   while(!que.empty())
37   {
38     states st=que.front();
39     que.pop();
40     tree[st.u].deg = st.deg;
41     tree[st.u].fa[0] = st.fu;
42     for (int i=1;i<DEG;i++)
43       tree[st.u].fa[i]=s;
44     for (int tmp=st.fu,num=1;tree[tmp].deg;tmp=tree[st.u].fa[num
       ++])
45       tree[st.u].fa[num]=tree[tmp].fa[num-1];
46     for(int i = tree[st.u].head ; i != -1; i = edge[i].next)
47     {
48       int v = edge[i].to;
49       if (v == st.fu) continue;
50       states nst;
51       nst.u=v;
52       nst.fu=st.u;
53       nst.deg=st.deg+1;
54       que.push(nst);
55     }
56   }
57 }
58 int LCA(int x, int y)
59 {
60   if(tree[x].deg > tree[y].deg) swap(x,y);
61   int hx=tree[x].deg,hy=tree[y].deg;
62   int tx=x,ty=y;
63   for (int det=hy-hx,i=0; det; det>>=1,i++)
64     if (det&1)
65       ty=tree[ty].fa[i];
66   if(tx == ty) return tx;
67   for (int i=DEG-1; i>=0; i--)
68   {
69     if(tree[tx].fa[i] == tree[ty].fa[i])
70       continue;
71     tx = tree[tx].fa[i];
```

```
72      ty = tree[ty].fa[i];
73    }
74    return tree[tx].fa[0];
75  }
76  int main()
77  {
78    int t;
79    scanf("%d",&t);
80    while(t--)
81    {
82      int n;
83      scanf("%d",&n);
84      L = 0;
85      for(int i = 0 ; i < n ; i++)
86        tree[i].head = -1;
87      for(int i = 0 ; i < n-1 ; i++)
88      {
89        int a,b;
90        scanf("%d%d",&a ,&b);
91        add_edge(a-1,b-1);
92        add_edge(b-1,a-1);
93      }
94      Root=0;
95      BFS(Root);
96      int a,b;
97      scanf("%d%d",&a,&b);
98      int lca=LCA(a-1,b-1)+1;
99      printf("%d\n",lca);
100   }
101   return 0;
102 }
```

5.15 最优比例生成树

```
1  #include<stdio.h>
2  #include<string.h>
3  #include<math.h>
4  struct
5  {
6    int x,y;
7    double z;
8  } node[1100];
9  struct
10 {
11   double l,c;
12 } map[1100][1100];
13 int n,l,f[1100],pre[1100];
14 double dis[1100];
15 double mst(double x)
16 {
17   int i,j,tmp;
```

```
18    double min,s=0,t=0;
19    memset(f,0,sizeof(f));
20    f[1]=1;
21    for (i=2; i<=n; i++)
22    {
23      dis[i]=map[1][i].c-map[1][i].l*x;
24      pre[i]=1;
25    }
26    for (i=1; i<n; i++)
27    {
28      min=1e10;
29      for (j=1; j<=n; j++)
30        if (!f[j] && min>dis[j])
31        {
32          min=dis[j];
33          tmp=j;
34        }
35      f[tmp]=1;
36      t+=map[pre[tmp]][tmp].l;
37      s+=map[pre[tmp]][tmp].c;
38      for (j=1; j<=n; j++)
39        if (!f[j] && map[tmp][j].c-map[tmp][j].l*x<dis[j])
40        {
41          dis[j]=map[tmp][j].c-map[tmp][j].l*x;
42          pre[j]=tmp;
43        }
44    }
45    return s/t;
46  }
47  int main()
48  {
49    int i,j;
50    double a,b;
51    scanf("%d",&n);
52    while (n)
53    {
54      for (i=1; i<=n; i++)
55        scanf("%d%d%lf",&node[i].x,&node[i].y,&node[i].z);
56      for (i=1; i<=n; i++)
57        for (j=i+1; j<=n; j++)
58        {
59          map[j][i].l=map[i][j].l=sqrt(1.0*(node[i].x-node[j].x)*(
              node[i].x-node[j].x)+(node[i].y-node[j].y)*(node[i].y-
              node[j].y));
60          map[j][i].c=map[i][j].c=fabs(node[i].z-node[j].z);
61        }
62      a=0,b=mst(a);
63      while (fabs(b-a)>1e-8)
64      {
65        a=b;
66        b=mst(a);
```

```
67      }
68      printf("%.3f\n",b);
69      scanf("%d",&n);
70    }
71  }
```

5.16 生成树计数

根据邻接矩阵构造 Laplacian matrix。

```
1  Matrix laplacian;
2  laplacian.clear();
3  for (int i = 0;i < n;i++)
4    for (int j = 0;j < n;j++)
5    if (i != j && G[i][j])
6    {
7      laplacian.a[i][j] = -1;
8      laplacian.a[i][i]++;
9    }
10 printf("%d\n",laplacian.det(n-1));
```

5.17 全局最小割

```
1  #include <iostream>
2  using namespace std;
3  const int maxn=510;
4  int map[maxn][maxn];
5  int n;
6  void contract(int x,int y)
7  {
8    int i,j;
9    for (i=0; i<n; i++)
10     if (i!=x) map[x][i]+=map[y][i],map[i][x]+=map[i][y];
11   for (i=y+1; i<n; i++) for (j=0; j<n; j++)
12     {
13       map[i-1][j]=map[i][j];
14       map[j][i-1]=map[j][i];
15     }
16   n--;
17 }
18 int w[maxn],c[maxn];
19 int sx,tx;
20 int mincut()
21 {
22   int i,j,k,t;
23   memset(c,0,sizeof(c));
24   c[0]=1;
25   for (i=0; i<n; i++) w[i]=map[0][i];
26   for (i=1; i+1<n; i++)
27   {
28     t=k=-1;
29     for (j=0; j<n; j++) if (c[j]==0&&w[j]>k)
```

```
30          k=w[t=j];
31      c[sx=t]=1;
32      for (j=0; j<n; j++) w[j]+=map[t][j];
33    }
34    for (i=0; i<n; i++) if (c[i]==0) return w[tx=i];
35  }
36  int main()
37  {
38    int i,j,k,m;
39    while (scanf("%d%d",&n,&m)!=EOF)
40    {
41      memset(map,0,sizeof(map));
42      while (m--)
43      {
44        scanf("%d%d%d",&i,&j,&k);
45        map[i][j]+=k;
46        map[j][i]+=k;
47      }
48      int mint=999999999;
49      while (n>1)
50      {
51        k=mincut();
52        if (k<mint) mint=k;
53        contract(sx,tx);
54      }
55      printf("%d\n",mint);
56    }
57    return 0;
58  }
```

5.18 欧拉路

5.18.1 有向图

```
1  void solve(int x)
2  {
3    int i;
4    if (!match[x])
5    {
6      path[++l]=x;
7      return ;
8    }
9    for (i=1; i<=n; i++)
10     if (b[x][i])
11     {
12       b[x][i]--;
13       match[x]--;
14       solve(i);
15     }
16   path[++l]=x;
17 }
```

5.18.2 无向图

```
void solve(int x)
{
  int i;
  if (!match[x])
  {
    path[++l]=x;
    return ;
  }
  for (i=1; i<=n; i++)
    if (b[x][i])
    {
      b[x][i]--;
      b[i][x]--;
      match[x]--;
      match[i]--;
      solve(i);
    }
  path[++l]=x;
}
```

5.18.3 混合图

zju1992

```
int in[MAXN+100],out[MAXN+100];
int main()
{
  int t;
  scanf("%d",&t);
  while (t--)
  {
    int n,m;
    scanf("%d%d",&n,&m);
    N=n+2;L=-1;
    for (int i=0;i<N;i++)
      head[i]=-1;
    memset(in,0,sizeof(in));
    memset(out,0,sizeof(out));

    for (int i=0;i<m;i++)
    {
      int x,y,z;
      scanf("%d%d%d",&x,&y,&z);
      in[y]++;out[x]++;
      if (!z)
        add_edge(x,y,1);
    }
    int flag=1;
    for (int i=1;i<=n;i++)
    {
      if (in[i]-out[i]>0)
```

```
28          add_edge(i,n+1,(in[i]-out[i])/2);
29        else
30        if (out[i]-in[i]>0)
31          add_edge(0,i,(out[i]-in[i])/2);
32        //printf("%d %d %d\n",i,out[i],in[i]);
33        if ((in[i]+out[i])&1)
34        {
35          flag=0;
36          break;
37        }
38      }
39      maxflow(0,n+1);
40      for (int i=head[0];i!=-1;i=edge[i].next)
41        if (edge[i].cap>0 && edge[i].cap>edge[i].flow)
42        {
43          flag=0;
44          break;
45        }
46      if (flag)
47        puts("possible");
48      else
49        puts("impossible");
50    }
51    return 0;
52  }
```

5.19 K 短路

```
1  #include<cstdio>
2  #include<cstring>
3  #include<queue>
4  using namespace std;
5  int K;
6  class states
7  {
8  public:
9    int cost,id;
10 };
11 int dist[1000];
12 class cmp
13 {
14 public:
15   bool operator ()(const states &i,const states &j)
16   {
17     return i.cost>j.cost;
18   }
19 };
20 class cmp2
21 {
22 public:
23   bool operator ()(const states &i,const states &j)
```

```
24    {
25      return i.cost+dist[i.id]>j.cost+dist[j.id];
26    }
27  };
28  struct edges
29  {
30    int to,next,cost;
31  } edger[100000],edge[100000];
32  int headr[1000],head[1000],Lr,L;
33  void dijkstra(int s)
34  {
35    states u;
36    u.id=s;
37    u.cost=0;
38    dist[s]=0;
39    priority_queue<states,vector<states>,cmp> q;
40    q.push(u);
41    while (!q.empty())
42    {
43      u=q.top();
44      q.pop();
45      if (u.cost!=dist[u.id]) continue;
46      for (int i=headr[u.id]; i!=-1; i=edger[i].next)
47      {
48        states v=u;
49        v.id=edger[i].to;
50        if (dist[v.id]>dist[u.id]+edger[i].cost)
51        {
52          v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
53          q.push(v);
54        }
55      }
56    }
57  }
58  int num[1000];
59  void init(int n)
60  {
61    Lr=L=0;
62    memset(head,-1,4*n);
63    memset(headr,-1,4*n);
64    memset(dist,63,4*n);
65    memset(num,0,4*n);
66  }
67  void add_edge(int u,int v,int x)
68  {
69    edge[L].to=v;
70    edge[L].cost=x;
71    edge[L].next=head[u];
72    head[u]=L++;
73    edger[Lr].to=u;
74    edger[Lr].cost=x;
```

```
  edger[Lr].next=headr[v];
  headr[v]=Lr++;
}
int a_star(int s,int t)
{
  if (dist[s]==0x3f3f3f3f)
    return -1;
  priority_queue<states,vector<states>,cmp2> q;
  states tmp;
  tmp.id=s;
  tmp.cost=0;
  q.push(tmp);
  while (!q.empty())
  {
    states u=q.top();
    q.pop();
    num[u.id]++;
    if (num[t]==K)
      return u.cost;
    for (int i=head[u.id]; i!=-1; i=edge[i].next)
    {
      int v=edge[i].to;
      tmp.id=v;
      tmp.cost=u.cost+edge[i].cost;
      q.push(tmp);
    }
  }
  return -1;
}
int main()
{
  int n,m;
  scanf("%d%d",&n,&m);
  init(n);
  for (int i=0; i<m; i++)
  {
    int u,v,x;
    scanf("%d%d%d",&u,&v,&x);
    add_edge(u-1,v-1,x);
  }
  int s,t;
  scanf("%d%d%d",&s,&t,&K);
  if (s==t)
    K++;
  dijkstra(t-1);
  printf("%d\n",a_star(s-1,t-1));
}
```

5.20 稳定婚姻

假定有 n 个男生和 个女生，理想的拍拖状态就是对于每对情侣 (a,b)，找不到另一对情侣 (c,d) 使得 c 更喜欢 b,b 也更喜欢 c，同理，对 a 来说也没有 (e,f) 使得 a 更喜欢 e 而 e 更喜

欢 a, 当然最后会有一些人落单。这样子一个状态可以称为理想拍拖状态，它也有一个专业的名词叫稳定婚姻。
求解这个问题可以用一个专有的算法，延迟认可算法，其核心就是让每个男生按自己喜欢的顺序逐个向女生表白，例如 leokan 向一个女生求爱，这个过程中，若这个女生没有男朋友，那么这个女生就暂时成为 leokan 的女朋友，或这个女生喜欢她现有男朋友的程度没有喜欢 leokan 高，这个女生也暂时成为 leokan 的女朋友，而她原有的男朋友则再将就找下一个次喜欢的女生来当女朋友。

```
#include<string.h>
#include<stdio.h>
#define N 1050
int boy[N][N];
int girl[N][N];
int ans[N];
int cur[N];
int n;
void getMarry(int g)
{
  for (int i=ans[g]+1;i<n;i++)
  {
    int b=girl[g][i]-1;
    if (cur[b]<0)
    {
      ans[g]=i;
      cur[b]=g;
      return;
    }
    int og=cur[b];
    if (boy[b][og] > boy[b][g])
    {
      cur[b]=g;
      ans[g]=i;
      getMarry(og);
      return;
    }
  }
};
int main()
{
  int t,a;
  scanf("%d",&t);
  while(t--)
  {
    memset(girl,0,sizeof(girl));
    memset(boy,0,sizeof(boy));
    scanf("%d",&n);
    for (int i=0;i<n;i++)
      for (int j=0;j<n;j++)
        scanf("%d",&girl[i][j]);
    for (int i=0;i<n;i++)
      for (int j=0;j<n;j++)
      {
```

```
45          scanf("%d",&a);
46          boy[i][a-1]=j;
47        }
48      memset(cur,0xff,sizeof(cur));
49      memset(ans,0xff,sizeof(ans));
50      for (int i=0;i<n;i++)
51        getMarry(i);
52      for (int i=0;i<n;i++)
53        printf("%d\n",girl[i][ans[i]]);
54    }
55    return 0;
56  }
```

5.21 最小树形图

```
1  const int inf = 19921005;
2  int n,m,u,v,cost,dis[1001][1001],L;
3
4  void init(int n)
5  {
6    L = 0;
7    for (int i = 0; i < n; i++)
8      for (int j = 0; j < n; j++)
9        dis[i][j] = inf;
10 }
11
12 struct Edge
13 {
14   int u,v,cost;
15 };
16
17 Edge e[1001*1001];
18
19 int pre[1001],id[1001],visit[1001],in[1001];
20
21 int zhuliu(int root,int n,int m,Edge e[])
22 {
23   int res = 0,u,v;
24   while (true)
25   {
26     for (int i = 0; i < n; i++)
27       in[i] = inf;
28     for (int i = 0; i < m; i++)
29       if (e[i].u != e[i].v && e[i].cost < in[e[i].v])
30       {
31         pre[e[i].v] = e[i].u;
32         in[e[i].v] = e[i].cost;
33       }
34     for (int i = 0; i < n; i++)
35       if (i != root)
36         if (in[i] == inf)   return -1;
```

```
37      int tn = 0;
38      memset(id,-1,sizeof(id));
39      memset(visit,-1,sizeof(visit));
40      in[root] = 0;
41      for (int i = 0; i < n; i++)
42      {
43        res += in[i];
44        v = i;
45        while (visit[v] != i && id[v] == -1 && v != root)
46        {
47          visit[v] = i;
48          v = pre[v];
49        }
50        if(v != root && id[v] == -1)
51        {
52          for(int u = pre[v] ; u != v ; u = pre[u])
53            id[u] = tn;
54          id[v] = tn++;
55        }
56      }
57      if(tn == 0) break;
58      for (int i = 0; i < n; i++)
59        if (id[i] == -1)
60          id[i] = tn++;
61      for (int i = 0; i < m;)
62      {
63        int v = e[i].v;
64        e[i].u = id[e[i].u];
65        e[i].v = id[e[i].v];
66        if (e[i].u != e[i].v)
67          e[i++].cost -= in[v];
68        else
69          swap(e[i],e[--m]);
70      }
71      n = tn;
72      root = id[root];
73    }
74    return res;
75  }
76
77  int main()
78  {
79    freopen("in.txt","r",stdin);
80    while (scanf("%d%d",&n,&m) != EOF)
81    {
82      init(n);
83      for (int i = 0; i < m; i++)
84      {
85        scanf("%d%d%d",&u,&v,&cost);
86        if (u == v) continue;
87        dis[u][v] = min(dis[u][v],cost);
```

```
88      }
89      L = 0;
90      for (int i = 0; i < n; i++)
91        for (int j = 0; j < n; j++)
92          if (dis[i][j] != inf)
93          {
94            e[L].u = i;
95            e[L].v = j;
96            e[L++].cost = dis[i][j];
97          }
98      printf("%d\n",zhuliu(0,n,L,e));
99    }
100   return 0;
101 }
```

6 计算几何

6.1 注意事项

如果用整数小心越界（多次乘法?）
如果用浮点数判断的时候一定要用 eps!

6.2 基本函数

6.2.1 Point 定义

```
struct Point
{
  double x, y;
  Point() {}
  Point(double _x, double _y)
  {
    x = _x, y = _y;
  }
  Point operator -(const Point &b)const
  {
    return Point(x-b.x, y-b.y);
  }
  double operator *(const Point &b)const
  {
    return x*b.y-y*b.x;
  }
  double operator &(const Point &b)const
  {
    return x*b.x+y*b.y;
  }
  void transXY(double B)
  {
    double tx = x,ty = y;
    x = tx*cos(B)-ty*sin(B);
    y = tx*sin(B)+ty*cos(B);
  }
};
```

6.2.2 Line 定义

```
struct Line
{
  Point s, e;
  double k;
  Line() {}
  Line(Point _s, Point _e)
  {
    s = _s, e = _e;
    k = atan2(e.y-s.y, e.x-s.x);
  }
```

```
11    Point operator &(const Line &b)const
12    {
13      Point res = s;
14      //注意：有些题目可能会有直线相交或者重合情况
15      //可以把返回值改成 pair<Point,int> 来返回两直线的状态。
16      double t = ((s-b.s)*(b.s-b.e))/((s-e)*(b.s-b.e));
17      res.x += (e.x-s.x)*t;
18      res.y += (e.y-s.y)*t;
19      return res;
20    }
21  };
```

6.2.3 距离：点到直线距离

result: 点到直线最近点

```
1  Point NPT(Point P, Line L)
2  {
3    Point result;
4    double a, b, t;
5
6    a = L.e.x-L.s.x;
7    b = L.e.y-L.s.y;
8    t = ((P.x-L.s.x)*a+(P.y-L.s.y)*b)/(a*a+b*b);
9
10   result.x = L.s.x+a*t;
11   result.y = L.s.y+b*t;
12   return dist(P, result);
13 }
```

6.2.4 距离：点到线段距离

res：点到线段最近点

```
1  Point NearestPointToLineSeg(Point P, Line L)
2  {
3    Point result;
4    double a, b, t;
5
6    a = L.e.x-L.s.x;
7    b = L.e.y-L.s.y;
8    t = ( (P.x-L.s.x)*a+(P.y-L.s.y)*b )/(a*a+b*b);
9
10   if (t >= 0 && t <= 1)
11   {
12     result.x = L.s.x+a*t;
13     result.y = L.s.y+b*t;
14   }
15   else
16   {
17     if (dist(P,L.s) < dist(P,L.e))
18       result = L.s;
19     else
20       result = L.e;
```

```
21    }
22    return result;
23  }
```

旧版

```
1  double CalcDis(Point a,Point s,Point e) //点到线段距离
2  {
3    if (sgn((e-s)*(a-s)) < 0 || sgn((s-e)*(a-e)) < 0)
4      return min(dist(a,s),dist(a,e));
5    return abs(((s-a)*(e-a))/dist(s-e));
6  }
```

6.2.5 面积：多边形

点按逆时针排序。

```
1  double CalcArea(Point p[], int n)
2  {
3    double res = 0;
4    for (int i = 0; i < n; i++)
5      res += (p[i]*p[(i+1) % n])/2;
6    return res;
7  }
```

6.2.6 判断：线段相交

```
1  bool inter(Line l1,Line l2)
2  {
3    return
4    max(l1.s.x,l1.e.x) >= min(l2.s.x,l2.e.x) &&
5    max(l2.s.x,l2.e.x) >= min(l1.s.x,l1.e.x) &&
6    max(l1.s.y,l1.e.y) >= min(l2.s.y,l2.e.y) &&
7    max(l2.s.y,l2.e.y) >= min(l1.s.y,l1.e.y) &&
8    sgn((l2.s-l1.s)*(l1.e-l1.s))*sgn((l2.e-l1.s)*(l1.e-l1.s)) <= 0 &&
9    sgn((l1.s-l2.s)*(l2.e-l2.s))*sgn((l1.e-l2.s)*(l2.e-l2.s)) <= 0;
10 }
```

6.2.7 判断：点在线段上

```
1  bool OnSeg(Line a,Point b)
2  {
3    return ((a.s-b)*(a.e-b) == 0 &&
4        (b.x-a.s.x)*(b.x-a.e.x) <= 0 &&
5        (b.y-a.s.y)*(b.y-a.e.y) <= 0);
6  }
```

6.2.8 判断：点在多边形内

凸包且按逆时针排序

```
1  bool inPoly(Point a,Point p[],int n)
2  {
3    for (int i = 0;i < n;i++)
4      if ((p[i]-a)*(p[(i+1)%n]-a) < 0)
```

```
5       return false;
6   return true;
7 }
```

射线法, 多边形可以是凸的或凹的
poly 的顶点数目要大于等于 3
返回值为：
0 – 点在 poly 内
1 – 点在 poly 边界上
2 – 点在 poly 外

```
1  int inPoly(Point p,Point poly[], int n)
2  {
3    int i, count;
4    Line ray, side;
5
6    count = 0;
7    ray.s = p;
8    ray.e.y  = p.y;
9    ray.e.x  = -1;//-INF, 注意取值防止越界!
10
11   for (i = 0; i < n; i++)
12   {
13     side.s = poly[i];
14     side.e = poly[(i+1)%n];
15
16     if(OnSeg(p, side))
17       return 1;
18
19     // 如果平行轴则不作考虑sidex
20     if (side.s.y == side.e.y)
21       continue;
22
23     if (OnSeg(side.s, ray))
24     {
25       if (side.s.y > side.e.y) count++;
26     }
27     else if (OnSeg(side.e, ray))
28     {
29       if (side.e.y > side.s.y) count++;
30     }
31     else if (inter(ray, side))
32     {
33       count++;
34     }
35   }
36   return ((count % 2 == 1) ? 0 : 2);
37 }
```

6.2.9 判断：两凸包相交

需要考虑这几个：一个凸包的点在另外一个图包内（包括边界）；一个凸包的某条边与另一个凸包某条边相交；如果凸包可能退化成点线还需要判断点在线段上和点和点重合。

6.2.10 排序：叉积极角排序

```
bool cmp(const Point& a,const Point& b)
{
  if (a.y*b.y <= 0)
  {
    if (a.y > 0 || b.y > 0) return a.y < b.y;
    if (a.y == 0 && b.y == 0) return a.x < b.x;
  }
  return a*b > 0;
}
```

6.3 三维几何

6.3.1 Point 定义

```
struct Point3D
{
  double x,y,z;
  Point3D() {}
  Point3D(double _x,double _y,double _z)
  {
    x = _x;
    y = _y;
    z = _z;
  }
  Point3D operator -(const Point3D& b)const
  {
    return Point3D(x-b.x,y-b.y,z-b.z);
  }
  Point3D operator *(const Point3D& b)const
  {
    return Point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
  }
  double operator &(const Point3D& b)const
  {
    return x*b.x+y*b.y+z*b.z;
  }
};
//模
double Norm(Point3D p)
{
  return sqrt(p&p);
}
//绕单位向量 V 旋转 θ 角度
Point3D Trans(Point3D pa,Point3D V,double theta)
{
  double s = sin(theta);
  double c = cos(theta);
  double x,y,z;
  x = V.x;
  y = V.y;
```

```
37    z = V.z;
38    Point3D pp =
39    Point3D(
40      (x*x*(1-c)+c)*pa.x+(x*y*(1-c)-z*s)*pa.y+(x*z*(1-c)+y*s)*pa.z,
41      (y*x*(1-c)+z*s)*pa.x+(y*y*(1-c)+c)*pa.y+(y*z*(1-c)-x*s)*pa.z,
42      (x*z*(1-c)-y*s)*pa.x+(y*z*(1-c)+x*s)*pa.y+(z*z*(1-c)+c)*pa.z);
43    return pp;
44  }
```

6.3.2 经度纬度转换

直角坐标系与极坐标系转换：

$$\begin{cases} x = r \times sin\theta \times cos\varphi \\ y = r \times sin\theta \times sin\varphi \\ z = r \times cos\theta \end{cases} \quad \begin{cases} r = \sqrt{x \times 2 + y \times 2 + z \times 2} \\ \varphi = arctan(\frac{y}{x}) \\ \theta = arccos(\frac{z}{r}) \end{cases} \quad r \in [0, +\infty), \varphi \in [0, 2\pi], \theta \in [0, \pi]$$

经度维度转换（$lat1 \in (-\frac{\pi}{2}, \frac{\pi}{2}), lng1 \in (-\pi, \pi)$）

```
1  Point3D getPoint3D(double lat,double lng,double r)
2  {
3    lat += pi/2;
4    lng += pi;
5    return
6      Point3D(r*sin(lat)*cos(lng),r*sin(lat)*sin(lng),r*cos(lat));
7  }
```

6.3.3 判断：直线相交

```
1  bool LineIntersect(Line3D L1, Line3D L2)
2  {
3    Point3D s = L1.s-L1.e;
4    Point3D e = L2.s-L2.e;
5    Point3D p  = s*e;
6    if (ZERO(p)) return false;    //是否平行
7    p = (L2.s-L1.e)*(L1.s-L1.e);
8    return ZERO(p&L2.e);     //是否共面
9  }
```

6.3.4 判断：线段相交

需要先判断是否在一个平面上：

```
1  bool inter(Point a,Point b,Point c,Point d)
2  {
3    Point ret = (a-b)*(c-d);
4    Point t1 = (b-a)*(c-a);
5    Point t2 = (b-a)*(d-a);
6    Point t3 = (d-c)*(a-c);
7    Point t4 = (d-c)*(b-c);
8    return sgn(t1&ret)*sgn(t2&ret) < 0 &&
9           sgn(t3&ret)*sgn(t4&ret) < 0;
10 }
```

6.3.5 判断：三维向量是否为 0

```
inline bool ZERO(Point3D p)
{
  return (ZERO(p.x) && ZERO(p.y) && ZERO(p.z));
}
```

6.3.6 判断：点在直线上

```
bool OnLine(Point3D p, Line3D L)
{
  return ZERO((p-L.s)*(L.e-L.s));
}
```

6.3.7 判断：点在线段上

```
bool OnSeg(Point3D p, Line3D L)
{
  return (ZERO((L.s-p)*(L.e-p)) &&
    EQ(Norm(p-L.s)+Norm(p-L.e),Norm(L.e-L.s)));
}
```

6.3.8 距离：点到直线

```
double Distance(Point3D p, Line3D L)
{
  return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
}
```

6.3.9 夹角

返回值是 $[0,\pi]$ 之间的弧度

```
double Inclination(Line3D L1, Line3D L2)
{
  Point3D u = L1.e - L1.s;
  Point3D v = L2.e - L2.s;
  return acos( (u & v) / (Norm(u)*Norm(v)) );
}
```

6.4 圆

6.4.1 面积：两圆相交

圆不可包含

```
double dis(int x,int y)
{
  return sqrt((double)(x*x+y*y));
}
double area(int x1,int y1,int x2,int y2,double r1,double r2)
{
  double s=dis(x2-x1,y2-y1);
  if(r1+r2<s) return 0;
  else if(r2-r1>s) return PI*r1*r1;
  else if(r1-r2>s) return PI*r2*r2;
```

```
11    double q1=acos((r1*r1+s*s-r2*r2)/(2*r1*s));
12    double q2=acos((r2*r2+s*s-r1*r1)/(2*r2*s));
13    return (r1*r1*q1+r2*r2*q2-r1*s*sin(q1));
14  }
```

6.4.2 三角形外接圆

```
 1  void CircumscribedCircle()
 2  {
 3    for (int i = 0; i < 3; i++)
 4      scanf("%lf%lf",&p[i].x,&p[i].y);
 5    tp = Point((p[0].x+p[1].x)/2,(p[0].y+p[1].y)/2);
 6    l[0] = Line(tp,Point(tp.x-(p[1].y-p[0].y),tp.y+(p[1].x-p[0].x)));
 7    tp = Point((p[0].x+p[2].x)/2,(p[0].y+p[2].y)/2);
 8    l[1] = Line(tp,Point(tp.x-(p[2].y-p[0].y),tp.y+(p[2].x-p[0].x)));
 9    tp = LineToLine(l[0],l[1]);
10    r = Point(tp,p[0]).Length();
11    printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
12  }
```

6.4.3 三角形内切圆

```
 1  void InscribedCircle()
 2  {
 3    for (int i = 0; i < 3; i++)
 4      scanf("%lf%lf",&p[i].x,&p[i].y);
 5    if (xmult(Point(p[0],p[1]),Point(p[0],p[2])) < 0)
 6      swap(p[1],p[2]);
 7    for (int i = 0; i < 3; i++)
 8      len[i] = Point(p[i],p[(i+1)%3]).Length();
 9    tr = (len[0]+len[1]+len[2])/2;
10    r = sqrt((tr-len[0])*(tr-len[1])*(tr-len[2])/tr);
11    for (int i = 0; i < 2; i++)
12    {
13      v = Point(p[i],p[i+1]);
14      tv = Point(-v.y,v.x);
15      tr = tv.Length();
16      tv = Point(tv.x*r/tr,tv.y*r/tr);
17      tp = Point(p[i].x+tv.x,p[i].y+tv.y);
18      l[i].s = tp;
19      tp = Point(p[i+1].x+tv.x,p[i+1].y+tv.y);
20      l[i].e = tp;
21    }
22    tp = LineToLine(l[0],l[1]);
23    printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
24  }
```

6.4.4 点对圆的两个切点

```
1  void calc_qie(Point poi,Point o,double r,Point &result1,Point &
      result2)
2  {
```

```
  double line = sqrt((poi.x-o.x)*(poi.x-o.x)+(poi.y-o.y)*(poi.y-o.y
     ));
  double angle = acos(r/line);
  Point unitvector,lin;
  lin.x = poi.x-o.x;
  lin.y = poi.y-o.y;
  unitvector.x = lin.x/sqrt(lin.x*lin.x+lin.y*lin.y)*r;
  unitvector.y = lin.y/sqrt(lin.x*lin.x+lin.y*lin.y)*r;
  result1 = unitvector.Rotate(-angle);
  result2 = unitvector.Rotate(angle);
  result1.x += o.x;
  result1.y += o.y;
  result2.x += o.x;
  result2.y += o.y;
}
```

6.4.5 两圆公切点

```
void Gao()
{
  tn = 0;
  Point a,b,vab;
  double tab,tt,dis,theta;
  for (int i = 0; i < tc; i++)
    for (int j = 0; j < tc; j++)
      if (i != j)
      {
        a = c[i];
        b = c[j];
        vab = Point(a,b);
        tab = atan2(vab.y,vab.x);
        dis = sqrt(vab.x*vab.x+vab.y*vab.y);
        if (b.r > a.r)
          tt = asin((b.r-a.r)/dis);
        else
          tt = -asin((a.r-b.r)/dis);
        theta = tab+pi/2+tt;
        tp[tn++] = Point(a.x+a.r*cos(theta),a.y+a.r*sin(theta));
        tp[tn++] = Point(b.x+b.r*cos(theta),b.y+b.r*sin(theta));
      }
}
```

6.4.6 两圆交点

```
lab = Point(p[j].x-p[i].x,p[j].y-p[i].y);
AB = lab.Length();
AC = cr[i];
BC = cr[j];

if (cmp(AB+AC,BC) <= 0) continue;//包含
if (cmp(AB+BC,AC) <= 0) continue;
if (cmp(AB,AC+BC) > 0)  continue;//相离

```

```
theta = atan2(lab.y,lab.x);
fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
a0 = theta-fai;
if (cmp(a0,-pi) < 0)  a0 += 2*pi;
a1 = theta+fai;
if (cmp(a1,pi) > 0) a1 -= 2*pi;
//答案
xp[totp++] = Point(p[i].x+cr[i]*cos(a0),p[i].y+cr[i]*sin(a0));
xp[totp++] = Point(p[i].x+cr[i]*cos(a1),p[i].y+cr[i]*sin(a1));
```

6.5 三角形相关

费马点： 在 $\triangle ABC$ 内求一点 P, 使 $PA+PB+PC$ 之值为最小的点。当三角形有一个内角大于或等于 120 的时候，费马点就是该内角的顶点
若没有，则费马点就是使得该点至三角形三顶点的连线两两夹角为 120 度的点.

等角共轭点： 对于三角形内任意一点 P, 过 A 做直线 L_1 与 AP 关于角 A 的角平分线对称, 同样过 B,C 分别做 L_2,L_3. 这三条直线交于 P_1, 则 P_1 是 P 的等角共轭点.
重心的等角共轭点到三边距离的平方和最小的点.

6.6 矩阵

6.6.1 基本矩阵

按向量 $\overrightarrow{(x,y,z)}$ 平移：

$$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

按比例 (x,y,z) 缩放：

$$\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

绕单位向量 $\overrightarrow{(x,y,z)}$ 旋转 $angle$ 角度：

$$\begin{pmatrix} x^2\times(1-c)+c & x\times y\times(1-c)-z\times s & x\times z\times(1-c)+y\times s & 0 \\ y\times x\times(1-c)+z\times s & y^2\times(1-c)+c & y\times z\times(1-c)-x\times s & 0 \\ x\times z\times(1-c)-y\times s & y\times z\times(1-c)+x\times s & z^2\times(1-c)+c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{cases} s=sin(angle) \\ c=cos(angle) \end{cases}$$

以上矩阵变换都把点当作列向量，旋转角度的正负由右手定则决定

6.6.2 刘汝佳的几何教室

```
const double pi = acos(-1.0);

int n,m,q;
struct Point
{
  double a,b,c,d;
};
Point p[50000],f[50000];

double a,b,c,theta,mt[4][4],tmp[4][4],tmt[4][4],rmt[4][8];
char com[20];

void TRANSLATE()
{
  memset(tmt,0,sizeof(tmt));
  tmt[0][0] = tmt[1][1] = tmt[2][2] = tmt[3][3] = 1;
  tmt[3][0] = a;
  tmt[3][1] = b;
  tmt[3][2] = c;
  memset(tmp,0,sizeof(tmp));
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      for (int k = 0; k < 4; k++)
        tmp[i][j] += mt[i][k]*tmt[k][j];
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      mt[i][j] = tmp[i][j];
}

void ROTATE()
{
  theta = -theta*pi/180;
  memset(tmt,0,sizeof(tmt));
  tmt[3][3] = 1;
  tmt[0][0] = cos(theta)+(1-cos(theta))*a*a;
  tmt[1][0] = (1-cos(theta))*a*b+c*sin(theta);
  tmt[2][0] = (1-cos(theta))*a*c-b*sin(theta);
  tmt[0][1] = (1-cos(theta))*a*b-c*sin(theta);
  tmt[1][1] = cos(theta)+(1-cos(theta))*b*b;
  tmt[2][1] = (1-cos(theta))*b*c+a*sin(theta);
  tmt[0][2] = (1-cos(theta))*a*c+b*sin(theta);
  tmt[1][2] = (1-cos(theta))*b*c-a*sin(theta);
  tmt[2][2] = cos(theta)+(1-cos(theta))*c*c;
  memset(tmp,0,sizeof(tmp));
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      for (int k = 0; k < 4; k++)
        tmp[i][j] += mt[i][k]*tmt[k][j];
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      mt[i][j] = tmp[i][j];
```

```
52 }
53
54 void SCALE()
55 {
56   memset(tmt,0,sizeof(tmt));
57   tmt[0][0] = a;
58   tmt[1][1] = b;
59   tmt[2][2] = c;
60   tmt[3][3] = 1;
61   memset(tmp,0,sizeof(tmp));
62   for (int i = 0; i < 4; i++)
63     for (int j = 0; j < 4; j++)
64       for (int k = 0; k < 4; k++)
65         tmp[i][j] += mt[i][k]*tmt[k][j];
66   for (int i = 0; i < 4; i++)
67     for (int j = 0; j < 4; j++)
68       mt[i][j] = tmp[i][j];
69 }
70
71 void solvep(Point p)
72 {
73   memset(tmt,0,sizeof(tmt));
74   tmt[0][0] = p.a;
75   tmt[0][1] = p.b;
76   tmt[0][2] = p.c;
77   tmt[0][3] = 1;
78   memset(tmp,0,sizeof(tmp));
79   for (int i = 0; i < 1; i++)
80     for (int j = 0; j < 4; j++)
81       for (int k = 0; k < 4; k++)
82         tmp[i][j] += tmt[i][k]*mt[k][j];
83   printf("%.2f␣%.2f␣%.2f\n",tmp[0][0],tmp[0][1],tmp[0][2]);
84 }
85
86 void solvef(Point f)
87 {
88   memset(tmt,0,sizeof(tmt));
89   tmt[0][0] = f.a;
90   tmt[1][0] = f.b;
91   tmt[2][0] = f.c;
92   tmt[3][0] = 0;
93   memset(tmp,0,sizeof(tmp));
94   for (int i = 0;i < 4;i++)
95     for (int j = 0;j < 1;j++)
96       for (int k = 0;k < 4;k++)
97         tmp[i][j] += mt[i][k]*tmt[k][j];
98   tmp[3][0] += f.d;
99   double kk = tmp[0][0]*tmp[0][0]+tmp[1][0]*tmp[1][0]+tmp[2][0]*tmp
       [2][0];
100  kk = sqrt(1/kk);
101  for (int i = 0;i < 4;i++)
```

```
102     printf("%.2f␣",tmp[i][0]*kk);
103   printf("\n");
104 }
105
106 void solvermt()
107 {
108   memset(rmt,0,sizeof(rmt));
109   for (int i = 0;i < 4;i++)
110     for (int j = 0;j < 4;j++)
111       rmt[i][j] = mt[i][j];
112   rmt[0][4] = rmt[1][5] = rmt[2][6] = rmt[3][7] = 1;
113   for (int i = 0;i < 4;i++)
114   {
115     for (int j = i;j < 4;j++)
116       if (fabs(rmt[j][i]) > 1e-8)
117       {
118         for (int k = i;k < 8;k++)
119           swap(rmt[i][k],rmt[j][k]);
120         break;
121       }
122     double tt = rmt[i][i];
123     for (int j = i;j < 8;j++)
124       rmt[i][j] /= tt;
125     for (int j = 0;j < 4;j++)
126       if (i != j)
127       {
128         tt = rmt[j][i];
129         for (int k = i;k < 8;k++)
130           rmt[j][k] -= rmt[i][k]*tt;
131       }
132   }
133   for (int i = 0;i < 4;i++)
134     for (int j = 0;j < 4;j++)
135       mt[i][j] = rmt[i][4+j];
136 }
137
138 int main()
139 {
140   scanf("%d%d%d",&n,&m,&q);
141   for (int i = 0; i < n; i++)
142     scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].c);
143   for (int i = 0; i < m; i++)
144     scanf("%lf%lf%lf%lf",&f[i].a,&f[i].b,&f[i].c,&f[i].d);
145   memset(mt,0,sizeof(mt));
146   mt[0][0] = mt[1][1] = mt[2][2] = mt[3][3] = 1;
147   for (int i = 0; i < q; i++)
148   {
149     scanf("%s",com);
150     if (strcmp(com,"TRANSLATE") == 0)
151     {
152       scanf("%lf%lf%lf",&a,&b,&c);
```

```
153       TRANSLATE();
154     }
155     else if (strcmp(com,"ROTATE") == 0)
156     {
157       scanf("%lf%lf%lf%lf",&a,&b,&c,&theta);
158       ROTATE();
159     }
160     else if (strcmp(com,"SCALE") == 0)
161     {
162       scanf("%lf%lf%lf",&a,&b,&c);
163       SCALE();
164     }
165   }
166   //处理点
167   for (int i = 0; i < n; i++)
168     solvep(p[i]);
169   //处理面
170   solvermt();
171   for (int i = 0; i < m; i++)
172     solvef(f[i]);
173   return 0;
174 }
```

6.7 重心

```
 1 Point CenterOfPolygon(Point poly[],int n)
 2 {
 3   Point p, p0, p1, p2, p3;
 4   double m, m0;
 5   p1 = poly[0];
 6   p2 = poly[1];
 7   p.x = p.y = m = 0;
 8   for (int i = 2; i < n; i++)
 9   {
10     p3 = poly[i];
11     p0.x = (p1.x + p2.x + p3.x) / 3.0;
12     p0.y = (p1.y + p2.y + p3.y) / 3.0;
13     m0 = p1.x*p2.y+p2.x*p3.y+p3.x*p1.y-p1.y*p2.x-p2.y*p3.x-p3.y*p1.
          x;
14     if (cmp(m + m0,0.0) == 0)
15     m0 += eps;
16     p.x = (m * p.x + m0 * p0.x) / (m + m0);
17     p.y = (m * p.y + m0 * p0.y) / (m + m0);
18     m = m + m0;
19     p2 = p3;
20   }
21   return p;
22 }
```

6.8 KD 树

查找某个点距离最近的点，基本思想是每次分治把点分成两部分，建议按照坐标规模决定是垂直划分还是水平划分，查找时先往分到的那一部分查找，然后根据当前最优答案决定是否

去另一个区间查找。

```
bool Div[MaxN];
void BuildKD(int deep,int l, int r, Point p[])\\记得备份一下 P
{
  if (l > r) return;
  int mid = l + r >> 1;
  int minX, minY, maxX, maxY;
  minX = min_element(p + l, p + r + 1, cmpX)->x;
  minY = min_element(p + l, p + r + 1, cmpY)->y;
  maxX = max_element(p + l, p + r + 1, cmpX)->x;
  maxY = max_element(p + l, p + r + 1, cmpY)->y;
  Div[mid] = (maxX - minX >= maxY - minY);
  nth_element(p + l, p + mid, p + r + 1, Div[mid] ? cmpX : cmpY);
  BuildKD(l, mid - 1, p);
  BuildKD(mid + 1, r, p);
}

long long res;
void Find(int l, int r, Point a, Point p[])\\查找
{
  if (l > r)  return;
  int mid = l + r >> 1;
  long long dist = dist2(a, p[mid]);
  if (dist > 0)//如果有重点不能这样判断
    res = min(res, dist);
  long long d = Div[mid] ? (a.x - p[mid].x) : (a.y - p[mid].y);
  int l1, l2, r1, r2;
  l1 = l, l2 = mid + 1;
  r1 = mid - 1, r2 = r;
  if (d > 0)
    swap(l1, l2), swap(r1, r2);
  Find(l1, r1, a, p);
  if (d * d < res)
    Find(l2, r2, a, p);
}
```

6.8.1 例题

查询一个点为中心的给定正方形内所有点并删除（2012 金华网赛 A）

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
#include <cmath>
#include <queue>
using namespace std;

const int MaxN = 100000;
struct Point
```

```
11 {
12   int x,y,r;
13   int id;
14   bool del;
15 };
16
17 int cmpTyp;
18 bool cmp(const Point& a,const Point& b)
19 {
20   if (cmpTyp == 0)
21     return a.x < b.x;
22   else
23     return a.y < b.y;
24 }
25
26 int cnt[MaxN];
27 bool Div[MaxN];
28 int minX[MaxN],minY[MaxN],maxX[MaxN],maxY[MaxN];
29 void BuildKD(int l,int r,Point p[])
30 {
31   if (l > r)  return;
32   int mid = l+r>>1;
33   cmpTyp = 0;
34   minX[mid] = min_element(p+l,p+r+1,cmp)->x;
35   maxX[mid] = max_element(p+l,p+r+1,cmp)->x;
36   cmpTyp = 1;
37   minY[mid] = min_element(p+l,p+r+1,cmp)->y;
38   maxY[mid] = max_element(p+l,p+r+1,cmp)->y;
39
40   cnt[mid] = r-l+1;
41   cmpTyp = Div[mid] = (maxX[mid]-minX[mid] < maxY[mid]-minY[mid]);
42   nth_element(p+l,p+mid,p+r+1,cmp);
43   BuildKD(l,mid-1,p);
44   BuildKD(mid+1,r,p);
45 }
46
47 queue<int> Q;
48 int Find(int l,int r,Point a,Point p[])
49 {
50   if (l > r)  return 0;
51   int mid = l+r>>1;
52   if (cnt[mid] == 0)  return 0;
53
54   if (maxX[mid] < a.x-a.r ||
55       minX[mid] > a.x+a.r ||
56       maxY[mid] < a.y-a.r ||
57       minY[mid] > a.y+a.r)
58     return 0;
59
60   int totdel = 0;
61
```

```
62    if (p[mid].del == false)
63      if (abs(p[mid].x-a.x) <= a.r && abs(p[mid].y-a.y) <= a.r)
64      {
65        p[mid].del = true;
66        Q.push(p[mid].id);
67        totdel++;
68      }
69
70    totdel += Find(l,mid-1,a,p);
71    totdel += Find(mid+1,r,a,p);
72
73    cnt[mid] -= totdel;
74
75    return totdel;
76  }
77
78  Point p[MaxN],tp[MaxN];
79  int n;
80
81  int main()
82  {
83    int cas = 1;
84    while (true)
85    {
86      scanf("%d",&n);
87      if (n == 0) break;
88
89      for (int i = 0;i < n;i++)
90      {
91        p[i].id = i;
92        int tx,ty;
93        scanf("%d%d%d",&tx,&ty,&p[i].r);
94        p[i].x = tx-ty;
95        p[i].y = tx+ty;
96        p[i].del = false;
97        tp[i] = p[i];
98      }
99      BuildKD(0,n-1,tp);
100
101     printf("Case␣#%d:\n",cas++);
102     int q;
103     scanf("%d",&q);
104     for (int i = 0;i < q;i++)
105     {
106       int id;
107       scanf("%d",&id);
108       int res = 0;
109       id--;
110       Q.push(id);
111       while (!Q.empty())
112       {
```

```
113         int now = Q.front();
114         Q.pop();
115         if (p[now].del == true) continue;
116         p[now].del = true;
117         res += Find(0,n-1,p[now],tp);
118       }
119       printf("%d\n",res);
120     }
121   }
122   return 0;
123 }
```

6.9 半平面交

直线左边代表有效区域。

```
 1 bool HPIcmp(Line a, Line b)
 2 {
 3   if (fabs(a.k - b.k) > eps)  return a.k < b.k;
 4   return ((a.s - b.s) * (b.e-b.s)) < 0;
 5 }
 6
 7 Line Q[100];
 8 void HPI(Line line[], int n, Point res[], int &resn)
 9 {
10   int tot = n;
11   sort(line, line + n, HPIcmp);
12   tot = 1;
13   for (int i = 1; i < n; i++)
14     if (fabs(line[i].k - line[i - 1].k) > eps)
15       line[tot++] = line[i];
16   int head = 0, tail = 1;
17   Q[0] = line[0];
18   Q[1] = line[1];
19   resn = 0;
20   for (int i = 2; i < tot; i++)
21   {
22     if (fabs((Q[tail].e-Q[tail].s)*(Q[tail - 1].e-Q[tail - 1].s)) <
         eps ||
23         fabs((Q[head].e-Q[head].s)*(Q[head + 1].e-Q[head + 1].s)) <
             eps)
24       return;
25     while (head < tail && (((Q[tail]&Q[tail - 1]) - line[i].s) * (
        line[i].e-line[i].s)) > eps)
26       tail--;
27     while (head < tail && (((Q[head]&Q[head + 1]) - line[i].s) * (
        line[i].e-line[i].s)) > eps)
28       head++;
29     Q[++tail] = line[i];
30   }
31   while (head < tail && (((Q[tail]&Q[tail - 1]) - Q[head].s) * (Q[
      head].e-Q[head].s)) > eps)
```

```
32      tail--;
33    while (head < tail && (((Q[head]&Q[head + 1]) - Q[tail].s) * (Q[
          tail].e-Q[tail].s)) > eps)
34      head++;
35    if (tail <= head + 1) return;
36    for (int i = head; i < tail; i++)
37      res[resn++] = Q[i] & Q[i + 1];
38    if (head < tail + 1)
39      res[resn++] = Q[head] & Q[tail];
40  }
```

6.10 凸包

得到的凸包按照逆时针方向排序。

```
1  //判断是否是共点或者共线用
2  bool conPoint(Point p[],int n)
3  {
4    for (int i = 1;i < n;i++)
5      if (p[i].x != p[0].x || p[i].y != p[0].y)
6        return false;
7    return true;
8  }
9  bool conLine(Point p[],int n)
10 {
11   for (int i = 2;i < n;i++)
12     if ((p[i]-p[0])*(p[1]-p[0]) != 0)
13       return false;
14   return true;
15 }
16
17 bool GScmp(Point a, Point b)
18 {
19   if (fabs(a.x - b.x) < eps)
20     return a.y < b.y - eps;
21   return a.x < b.x - eps;
22 }
23
24 void GS(Point p[],int n,Point res[],int &resn)
25 {
26   resn = 0;
27   int top = 0;
28   sort(p,p+n,GScmp);
29
30   if (conPoint(p,n))
31   {
32     res[resn++] = p[0];
33     return;
34   }
35   if (conLine(p,n))
36   {
37     res[resn++] = p[0];
```

```
38      res[resn++] = p[n-1];
39      return;
40    }
41
42    for (int i = 0;i < n;)
43      if (resn < 2 ||
44        (res[resn-1]-res[resn-2])*(p[i]-res[resn-1]) > 0)
45        res[resn++] = p[i++];
46      else
47        --resn;
48    top = resn-1;
49    for (int i = n-2;i >= 0;)
50      if (resn < top+2 ||
51        res[resn-1]-res[resn-2])*(p[i]-res[resn-1]) > 0)
52        res[resn++] = p[i--];
53      else
54        --resn;
55    resn--;
56  }
```

6.11 直线与凸包求交点

复杂度 $O(\log n)$。
需要先预处理几个东西。

```
1  //二分 [la,lb] 这段区间那条边与 line 相交
2  int Gao(int la,int lb,Line line)
3  {
4    if (la > lb)
5      lb += n;
6    int l = la,r = lb,mid;
7    while (l < r)
8    {
9      mid = l+r+1>>1;
10     if (cmp((line.e-line.s)*(p[la]-line.s),0)*cmp((line.e-line.s)*(
         p[mid]-line.s),0) >= 0)
11       l = mid;
12     else
13       r = mid-1;
14   }
15   return l%n;
16 }
17 //求 l 与凸包的交点
18
19 //先调用 Gettheta 预处理出凸包每条边的斜率，然后处理成升序排列
20 double theta[maxn];
21
22 void Gettheta()
23 {
24   for (int i = 0;i < n;i++)
25   {
```

```
26      Point v = p[(i+1)%n]-p[i];
27      theta[i] = atan2(v.y,v.x);
28    }
29    for (int i = 1;i < n;i++)
30      if (theta[i-1] > theta[i]+eps)
31        theta[i] += 2*pi;
32  }
33
34  double Calc(Line l)
35  {
36    double tnow;
37    Point v = l.e-l.s;
38    tnow = atan2(v.y,v.x);
39    if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
40    int pl = lower_bound(theta,theta+n,tnow)-theta;
41    tnow = atan2(-v.y,-v.x);
42    if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
43    int pr = lower_bound(theta,theta+n,tnow)-theta;
44    //pl 和 pr 是在 l 方向上距离最远的点对
45    pl = pl%n;
46    pr = pr%n;
47
48    if (cmp(v*(p[pl]-l.s),0)*cmp(v*(p[pr]-l.s),0) >= 0)
49      return 0.0;
50
51    int xa = Gao(pl,pr,l);
52    int xb = Gao(pr,pl,l);
53
54    if (xa > xb)  swap(xa,xb);
55    //与 [xa,xa+1] 和 [xb,xb+1] 这两条线段相交
56
57    if (cmp(v*(p[xa+1]-p[xa]),0) == 0)  return 0.0;
58    if (cmp(v*(p[xb+1]-p[xb]),0) == 0)  return 0.0;
59
60    Point pa,pb;
61    pa = Line(p[xa],p[xa+1])&l;
62    pb = Line(p[xb],p[xb+1])&l;
63    //题目：求直线切凸包得到的两部分的面积
64    double area0 = sum[xb]-sum[xa+1]+(pa*p[xa+1])/2.0+(p[xb]*pb)
        /2.0+(pb*pa)/2.0;
65    double area1 = sum[xa+n]-sum[xb+1]+(pb*p[xb+1])/2.0+(p[xa]*pa)
        /2.0+(pa*pb)/2.0;
66
67    return min(area0,area1);
68  }
```

6.12 点对凸包的两切点

过了 sgu500 的前七组数据，用前需谨慎，虽然我不认为这个有问题。

```
1  double theta[MaxN];
2  void Gettheta(Point p[],int n)
```

```
 3 {
 4   for (int i = 0;i < n;i++)
 5   {
 6     Point v = p[(i+1)%n]-p[i];
 7     theta[i] = atan2(v.y,v.x);
 8   }
 9   for (int i = 1;i < n;i++)
10     if (theta[i-1] > theta[i]+eps)
11       theta[i] += 2*pi;
12 }
13 int cmp(double a,double b)
14 {
15   if (fabs(a-b) < eps)  return 0;
16   if (a < b)  return -1;
17   return 1;
18 }
19 int Gao(int la,int lb,Line line,Point p[],int n)
20 {
21   if (la > lb)
22     lb += n;
23   int l = la,r = lb,mid;
24   while (l < r)
25   {
26     mid = l+r+1>>1;
27     if (cmp((line.e-line.s)*(p[la%n]-line.s),0)*cmp((line.e-line.s)
          *(p[mid%n]-line.s),0) >= 0)
28       l = mid;
29     else
30       r = mid-1;
31   }
32   return l%n;
33 }
34 int Gao(int la,int lb,int dir,Point s,Point p[],int n)
35 {
36   if (la > lb)
37     lb += n;
38   if (la == lb) return la;
39
40   int l = la+1,r = lb,mid;
41
42   while (l < r)
43   {
44     mid = l+r+1>>1;
45
46     int ret = cmp((p[mid%n]-s)*(p[(mid-1)%n]-s),0);
47     if (dir*ret < 0)
48       l = mid;
49     else if (dir*ret > 0)
50       r = mid-1;
51     else
52     {
```

```
53        if (dir == 1)
54          l = mid;
55        else
56          r = mid-1;
57      }
58    }
59
60    int ret = cmp((p[l%n]-s)*(p[(l-1)%n]-s),0);
61    if (dir*ret < 0)
62      return l%n;
63    else if (dir*ret > 0)
64      return (l-1)%n;
65    else
66    {
67      if (dir == 1)
68        return l%n;
69      else
70        return (l-1)%n;
71    }
72  }
73  //Gettheta(p,n) first!
74  //返回 S 对于 p[] 的两个切点 p[pl],p[pr]
75  void Calc(Point s,Point p[],int n,int& pl,int& pr)
76  {
77    Line l = Line(s,p[0]);
78    Point v = l.e-l.s;
79    double tnow = atan2(v.y,v.x);
80    if (tnow < theta[0]-eps)  tnow += 2*pi;
81    int tpl = lower_bound(theta,theta+n,tnow)-theta;
82    tnow = atan2(-v.y,-v.x);
83    if (tnow < theta[0]-eps)  tnow += 2*pi;
84    int tpr = lower_bound(theta,theta+n,tnow)-theta;
85
86    pl = tpl = tpl%n;
87    pr = tpr = tpr%n;
88
89    int px = Gao(pr,pl,l,p,n);
90    //printf("pr = %d -> px = %d\n",tpr,px);
91    //printf("px = %d -> pl = %d\n",px,tpl);
92    //pr -> px
93    //px -> pl
94
95    pl = Gao(tpr,px,1,s,p,n);
96    pr = Gao(px,tpl,-1,s,p,n);
97
98  }
```

6.13 三维凸包

暴力写法

```
1  #define eps 1e-7
```

```
 2 #define MAXV 505
 3
 4 struct pt
 5 {
 6   double x, y, z;
 7   pt() {}
 8   pt(double _x, double _y, double _z): x(_x), y(_y), z(_z) {}
 9   pt operator – (const pt p1)
10   {
11     return pt(x – p1.x, y – p1.y, z – p1.z);
12   }
13   pt operator * (pt p)
14   {
15     return pt(y*p.z–z*p.y, z*p.x–x*p.z, x*p.y–y*p.x);
16   }
17   double operator ^ (pt p)
18   {
19     return x*p.x+y*p.y+z*p.z;
20   }
21 };
22 struct _3DCH
23 {
24   struct fac
25   {
26     int a, b, c;
27     bool ok;
28   };
29   int n;
30   pt P[MAXV];
31   int cnt;
32   fac F[MAXV*8];
33   int to[MAXV][MAXV];
34   double vlen(pt a)
35   {
36     return sqrt(a.x*a.x+a.y*a.y+a.z*a.z);
37   }
38   double area(pt a, pt b, pt c)
39   {
40     return vlen((b–a)*(c–a));
41   }
42   double volume(pt a, pt b, pt c, pt d)
43   {
44     return (b–a)*(c–a)^(d–a);
45   }
46   double ptof(pt &p, fac &f)
47   {
48     pt m = P[f.b]–P[f.a], n = P[f.c]–P[f.a], t = p–P[f.a];
49     return (m * n) ^ t;
50   }
51   void deal(int p, int a, int b)
52   {
```

```
53      int f = to[a][b];
54      fac add;
55      if (F[f].ok)
56      {
57        if (ptof(P[p], F[f]) > eps)
58          dfs(p, f);
59        else
60        {
61          add.a = b, add.b = a, add.c = p, add.ok = 1;
62          to[p][b] = to[a][p] = to[b][a] = cnt;
63          F[cnt++] = add;
64        }
65      }
66    }
67    void dfs(int p, int cur)
68    {
69      F[cur].ok = 0;
70      deal(p, F[cur].b, F[cur].a);
71      deal(p, F[cur].c, F[cur].b);
72      deal(p, F[cur].a, F[cur].c);
73    }
74    bool same(int s, int t)
75    {
76      pt &a = P[F[s].a], &b = P[F[s].b], &c = P[F[s].c];
77      return fabs(volume(a, b, c, P[F[t].a])) < eps && fabs(volume(a,
            b, c,
78          P[F[t].b])) < eps && fabs(volume(a, b, c, P[F[t].c])) < eps
              ;
79    }
80    void construct()
81    {
82      cnt = 0;
83      if (n < 4)
84        return;
85      bool sb = 1;
86      for (int i = 1; i < n; i++)
87      {
88        if (vlen(P[0] - P[i]) > eps)
89        {
90          swap(P[1], P[i]);
91          sb = 0;
92          break;
93        }
94      }
95      if (sb)return;
96      sb = 1;
97      for (int i = 2; i < n; i++)
98      {
99        if (vlen((P[0] - P[1]) * (P[1] - P[i])) > eps)
100       {
101         swap(P[2], P[i]);
```

```
102         sb = 0;
103         break;
104       }
105     }
106     if (sb)return;
107     sb = 1;
108     for (int i = 3; i < n; i++)
109     {
110       if (fabs((P[0] - P[1]) * (P[1] - P[2]) ^ (P[0] - P[i])) > eps
            )
111       {
112         swap(P[3], P[i]);
113         sb = 0;
114         break;
115       }
116     }
117     if (sb)return;
118     fac add;
119     for (int i = 0; i < 4; i++)
120     {
121       add.a = (i+1)%4, add.b = (i+2)%4, add.c = (i+3)%4, add.ok =
            1;
122       if (ptof(P[i], add) > 0)
123         swap(add.b, add.c);
124       to[add.a][add.b] = to[add.b][add.c] = to[add.c][add.a] = cnt;
125       F[cnt++] = add;
126     }
127     for (int i = 4; i < n; i++)
128     {
129       for (int j = 0; j < cnt; j++)
130       {
131         if (F[j].ok && ptof(P[i], F[j]) > eps)
132         {
133           dfs(i, j);
134           break;
135         }
136       }
137     }
138     int tmp = cnt;
139     cnt = 0;
140     for (int i = 0; i < tmp; i++)
141     {
142       if (F[i].ok)
143       {
144         F[cnt++] = F[i];
145       }
146     }
147   }
148 //表面积
149   double area()
150   {
```

```
151     double ret = 0.0;
152     for (int i = 0; i < cnt; i++)
153     {
154       ret += area(P[F[i].a], P[F[i].b], P[F[i].c]);
155     }
156     return ret / 2.0;
157   }
158 //体积
159   double volume()
160   {
161     pt O(0, 0, 0);
162     double ret = 0.0;
163     for (int i = 0; i < cnt; i++)
164     {
165       ret += volume(O, P[F[i].a], P[F[i].b], P[F[i].c]);
166     }
167     return fabs(ret / 6.0);
168   }
169 //表面三角形数
170   int facetCnt_tri()
171   {
172     return cnt;
173   }
174 //表面多边形数
175   int facetCnt()
176   {
177     int ans = 0;
178     for (int i = 0; i < cnt; i++)
179     {
180       bool nb = 1;
181       for (int j = 0; j < i; j++)
182       {
183         if (same(i, j))
184         {
185           nb = 0;
186           break;
187         }
188       }
189       ans += nb;
190     }
191     return ans;
192   }
193
194   pt Fc[MAXV*8];
195   double V[MAXV*8];
196   pt Center()//重心
197   {
198     pt O(0,0,0);
199     for (int i = 0; i < cnt; i++)
200     {
201       Fc[i].x = (O.x+P[F[i].a].x+P[F[i].b].x+P[F[i].c].x)/4.0;
```

```
202       Fc[i].y = (O.y+P[F[i].a].y+P[F[i].b].y+P[F[i].c].y)/4.0;
203       Fc[i].z = (O.z+P[F[i].a].z+P[F[i].b].z+P[F[i].c].z)/4.0;
204       V[i] = volume(O,P[F[i].a],P[F[i].b],P[F[i].c]);
205     }
206     pt res = Fc[0],tmp;
207     double m = V[0];
208     for (int i = 1; i < cnt; i++)
209     {
210       if (fabs(m+V[i]) < eps)
211         V[i] += eps;
212       tmp.x = (m*res.x+V[i]*Fc[i].x)/(m+V[i]);
213       tmp.y = (m*res.y+V[i]*Fc[i].y)/(m+V[i]);
214       tmp.z = (m*res.z+V[i]*Fc[i].z)/(m+V[i]);
215       m += V[i];
216       res = tmp;
217     }
218     return res;
219   }
220 };
221
222 _3DCH hull;
223
224 int main()
225 {
226   while (scanf("%d",&hull.n) != EOF)
227   {
228     for (int i = 0; i < hull.n; i++)
229       scanf("%lf%lf%lf",&hull.P[i].x,&hull.P[i].y,&hull.P[i].z);
230     hull.construct();
231   }
232   return 0;
233 }
```

6.14 旋转卡壳

“对踵”

6.14.1 单个凸包

```
 1 void solve(Point p[],int n)
 2 {
 3   Point v;
 4   int cur = 1;
 5   for (int i = 0;i < n;i++)
 6   {
 7     v = p[i]-p[(i+1)%n];
 8     while (v*(p[(cur+1)%n]-p[cur]) < 0)
 9       cur = (cur+1)%n;
10     //p[cur] -> p[i]
11     //p[cur] -> p[i+1]
```

```
12      //p[cur] -> (p[i],p[i+1])
13    }
14  }
```

6.14.2 两个凸包

注意初始点的选取，代码只是个示例。
有时候答案需要取 solve(p0,n,p1,m) 和 solve(p1,m,p0,n) 的最优值。
何老鱼说我的是错的。。

```
 1  void solve(Point p0[],int n,Point p1[],int m)
 2  {
 3    Point v;
 4    int cur = 0;
 5    for (int i = 0;i < n;i++)
 6    {
 7      v = p0[i]-p0[(i+1)%n];
 8      while (v*(p1[(cur+1)%m]-p1[cur]) < 0)
 9        cur = (cur+1)%m;
10      //p1[cur] -> p0[i]
11      //p1[cur] -> p0[i+1]
12      //p1[cur] -> (p0[i],p0[i+1])
13    }
14  }
```

6.14.3 外接矩形

```
 1  void solve()
 2  {
 3    resa = resb = 1e100;
 4    double dis1,dis2;
 5    Point xp[4];
 6    Line l[4];
 7    int a,b,c,d;
 8    int sa,sb,sc,sd;
 9    a = b = c = d = 0;
10    sa = sb = sc = sd = 0;
11    Point va,vb,vc,vd;
12    for (a = 0; a < n; a++)
13    {
14      va = Point(p[a],p[(a+1)%n]);
15      vc = Point(-va.x,-va.y);
16      vb = Point(-va.y,va.x);
17      vd = Point(-vb.x,-vb.y);
18      if (sb < sa)
19      {
20        b = a;
21        sb = sa;
22      }
23      while (xmult(vb,Point(p[b],p[(b+1)%n])) < 0)
24      {
25        b = (b+1)%n;
```

```
26        sb++;
27      }
28      if (sc < sb)
29      {
30        c = b;
31        sc = sb;
32      }
33      while (xmult(vc,Point(p[c],p[(c+1)%n])) < 0)
34      {
35        c = (c+1)%n;
36        sc++;
37      }
38      if (sd < sc)
39      {
40        d = c;
41        sd = sc;
42      }
43      while (xmult(vd,Point(p[d],p[(d+1)%n])) < 0)
44      {
45        d = (d+1)%n;
46        sd++;
47      }
48
49      //卡在 p[a],p[b],p[c],p[d] 上
50      sa++;
51    }
52  }
```

6.15 三角形内点个数

6.15.1 无三点共线

```
1  Point p[1000], tp[2000], base;
2
3  bool cmp(const Point &a, const Point &b)
4  {
5    return a.theta < b.theta;
6  }
7
8  int cnt[1000][1000];
9  int cntleft[1000][1000];
10 int n, m;
11
12 int calc(int a, int b, int c)
13 {
14   Point p1 = p[b] - p[a], p2 = p[c] - p[a];
15   if (atan2(p1.y, p1.x) > atan2(p2.y, p2.x))
16     swap(b, c);
17   if ((p[b] - p[a]) * (p[c] - p[a]) > 0)
18     return cnt[a][c] - cnt[a][b] - 1;
19   else
20     return n - 3 - (cnt[a][c] - cnt[a][b] - 1);
```

```
21 }
22
23 int main(int argc, char const *argv[])
24 {
25   int totcas;
26   scanf("%d", &totcas);
27   for (int cas = 1; cas <= totcas; ++cas)
28   {
29     scanf("%d", &n);
30     for (int i = 0; i < n; ++i)
31     {
32       scanf("%lld%lld", &p[i].x, &p[i].y);
33       p[i].id = i;
34     }
35     for (int i = 0; i < n; ++i)
36     {
37       m = 0;
38       base = p[i];
39       for (int j = 0; j < n; ++j)
40         if (i != j)
41         {
42           tp[m] = p[j];
43           Point v = tp[m]-base;
44           tp[m++].theta = atan2(v.y,v.x);
45         }
46
47       sort(tp, tp + m, cmp);
48       for (int j = 0; j < m; ++j)
49         tp[m + j] = tp[j];
50
51       //calc cnt
52       for (int j = 0; j < m; ++j)
53         cnt[i][tp[j].id] = j;
54
55       //calc cntleft
56       for (int j = 0, k = 0, tot = 0; j < m; ++j)
57       {
58         while (k == j || (k < j + m && (tp[j] - base) * (tp[k] -
              base) > 0))
59           k++, tot++;
60         cntleft[i][tp[j].id] = --tot;
61       }
62     }
63
64     printf("Case␣%d:\n", cas);
65     int q;
66     scanf("%d", &q);
67     for (int i = 0; i < q; ++i)
68     {
69       int x, y, z;
70       scanf("%d%d%d", &x, &y, &z);
```

```
      if ((p[z] - p[x]) * (p[y] - p[x]) > 0)
        swap(y, z);
      int res = cntleft[x][z] + cntleft[z][y] + cntleft[y][x];
      res += calc(x, y, z) + calc(y, z, x) + calc(z, x, y);
      res -= 2 * (n - 3);
      printf("%d\n", res);
    }
  }
  return 0;
}
```

6.15.2 有三点共线且点有类别之分

```
int n,n0,n1,m;
Point p[3000], tp[3000], base;

bool cmp(const Point &a, const Point &b)
{
  if ((a-base)*(b-base) == 0)
  {
    return (a-base).getMol() < (b-base).getMol();
  }
  return a.theta < b.theta;
}

int cnt[100][100];
int cntleft[100][100];

int calc(int a,int b,int c)
{
  Point p1 = p[b]-p[a],p2 = p[c]-p[a];
  if (atan2(1.0*p1.y,1.0*p1.x) > atan2(1.0*p2.y,1.0*p2.x))
    swap(b,c);
  int res = cnt[a][c]-cnt[a][b];
  if ((p[b]-p[a])*(p[c]-p[a]) > 0)
    return res;
  else
    return n1-res;
}

int main()
{
  int cas = 0;
  while (scanf("%d%d",&n0,&n1) != EOF)
  {
    n = n1+n0;
    for (int i = 0; i < n; i++)
    {
      scanf("%I64d%I64d",&p[i].x,&p[i].y);
      p[i].id = i;
    }
    for (int i = 0; i < n0; ++i)
```

```
40      {
41        m = 0;
42        base = p[i];
43        for (int j = 0; j < n; ++j)
44          if (i != j)
45          {
46            tp[m] = p[j];
47            Point v = tp[m]-base;
48            tp[m++].theta = atan2(1.0*v.y,1.0*v.x);
49          }
50
51        sort(tp, tp + m, cmp);
52        for (int j = 0; j < m; ++j)
53          tp[m + j] = tp[j];
54
55        for (int j = 0,tot = 0; j < m; ++j)
56        {
57          if (tp[j].id < n0)
58            cnt[i][tp[j].id] = tot;
59          else
60            tot++;
61        }
62
63        for (int j = 0, k = 0, tot = 0; j < m; ++j)
64        {
65          while (k == j || (k < j + m && (tp[j] - base) * (tp[k] -
               base) > 0))
66          {
67            if (tp[k].id >= n0)
68              tot++;
69            k++;
70          }
71          if (tp[j].id >= n0)
72            tot--;
73          else
74            cntleft[i][tp[j].id] = tot;
75        }
76      }
77
78      int ans = 0;
79      for (int i = 0; i < n0; i++)
80        for (int j = i+1; j < n0; j++)
81          for (int k = j+1; k < n0; k++)
82          {
83            int x = i,y = j,z = k;
84
85            if ((p[z] - p[x]) * (p[y] - p[x]) > 0)
86              swap(y, z);
87            int res = cntleft[x][z] + cntleft[z][y] + cntleft[y][x];
88
89            res += calc(x, y, z) + calc(y, z, x) + calc(z, x, y);
```

```
90
91            res -= 2 * n1;
92
93            //printf("%d %d %d %d\n",x,y,z,res);
94
95            if (res%2 == 1)
96              ans++;
97          }
98      printf("Case␣%d:␣%d\n",++cas,ans);
99    }
100   return 0;
101 }
```

6.16 最近点对

6.16.1 类快排算法

```
1  double calc_dis(Point &a ,Point &b) {
2    return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
3  }
4  //别忘了排序
5  bool operator<(const Point &a ,const Point &b) {
6    if(a.y != b.y) return a.x < b.x;
7    return a.x < b.x;
8  }
9  double Gao(int l ,int r ,Point pnts[]) {
10   double ret = inf;
11   if(l == r) return ret;
12   if(l+1 ==r) {
13     ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
14     return ret;
15   }
16   if(l+2 ==r) {
17     ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
18     ret = min(calc_dis(pnts[l],pnts[l+2]) ,ret);
19     ret = min(calc_dis(pnts[l+1],pnts[l+2]) ,ret);
20     return ret;
21   }
22
23   int mid = l+r>>1;
24   ret = min (ret ,Gao(l ,mid,pnts));
25   ret = min (ret , Gao(mid+1, r,pnts));
26
27   for(int c = l ; c<=r; c++)
28     for(int d = c+1; d <=c+7 && d<=r; d++) {
29       ret = min(ret , calc_dis(pnts[c],pnts[d]));
30     }
31   return ret;
32 }
```

6.16.2 随机增量法

```
1  #include <iostream>
```

```
 2 #include <cstdio>
 3 #include <cstring>
 4 #include <map>
 5 #include <vector>
 6 #include <cmath>
 7 #include <algorithm>
 8 #define Point pair<double,double>
 9 using namespace std;
10
11 const int step[9][2] =
      {{-1,-1},{-1,0},{-1,1},{0,-1},{0,0},{0,1},{1,-1},{1,0},{1,1}};
12 int n,x,y,nx,ny;
13 map<pair<int,int>,vector<Point > > g;
14 vector<Point > tmp;
15 Point p[20000];
16 double tx,ty,ans,nowans;
17 vector<Point >::iterator it,op,ed;
18 pair<int,int> gird;
19 bool flag;
20
21 double Dis(Point p0,Point p1)
22 {
23   return sqrt((p0.first-p1.first)*(p0.first-p1.first)+
24         (p0.second-p1.second)*(p0.second-p1.second));
25 }
26
27 double CalcDis(Point p0,Point p1,Point p2)
28 {
29   return Dis(p0,p1)+Dis(p0,p2)+Dis(p1,p2);
30 }
31
32 void build(int n,double w)
33 {
34   g.clear();
35   for (int i = 0;i < n;i++)
36     g[make_pair((int)floor(p[i].first/w),(int)floor(p[i].second/w))
         ].push_back(p[i]);
37 }
38
39 int main()
40 {
41   int t;
42   scanf("%d",&t);
43   for (int ft = 1;ft <= t;ft++)
44   {
45     scanf("%d",&n);
46     for (int i = 0;i < n;i++)
47     {
48       scanf("%lf%lf",&tx,&ty);
49       p[i] = make_pair(tx,ty);
50     }
```

```
51      random_shuffle(p,p+n);
52      ans = CalcDis(p[0],p[1],p[2]);
53      build(3,ans/2.0);
54      for (int i = 3;i < n;i++)
55      {
56        x = (int)floor(2.0*p[i].first/ans);
57        y = (int)floor(2.0*p[i].second/ans);
58        tmp.clear();
59        for (int k = 0;k < 9;k++)
60        {
61          nx = x+step[k][0];
62          ny = y+step[k][1];
63          gird = make_pair(nx,ny);
64          if (g.find(gird) != g.end())
65          {
66            op = g[gird].begin();
67            ed = g[gird].end();
68            for (it = op;it != ed;it++)
69              tmp.push_back(*it);
70          }
71        }
72        flag = false;
73        for (int j = 0;j < tmp.size();j++)
74          for (int k = j+1;k < tmp.size();k++)
75          {
76            nowans = CalcDis(p[i],tmp[j],tmp[k]);
77            if (nowans < ans)
78            {
79              ans = nowans;
80              flag = true;
81            }
82          }
83        if (flag == true)
84          build(i+1,ans/2.0);
85        else
86          g[make_pair((int)floor(2.0*p[i].first/ans),(int)floor(2.0*p
              [i].second/ans))].push_back(p[i]);
87      }
88      printf("%.3f\n",ans);
89    }
90  }
```

6.17 多圆面积并

6.17.1 去重

有时候可能需要去掉不需要的圆

```
1  for (int i = 0; i < n; i++)
2  {
3    scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
4    del[i] = false;
5  }
```

```
for (int i = 0; i < n; i++)
  if (del[i] == false)
  {
    if (c[i].r == 0.0)  del[i] = true;
    for (int j = 0; j < n; j++)
      if (i != j)
        if (del[j] == false)
          if (cmp(Point(c[i].c,c[j].c).Len()+c[i].r,c[j].r) <= 0)
            del[i] = true;
  }
tn = n;
n = 0;
for (int i = 0; i < tn; i++)
  if (del[i] == false)
    c[n++] = c[i];
```

6.17.2 圆并

$ans[i]$ 表示被覆盖 i 次的面积

```
const double pi = acos(-1.0);
const double eps = 1e-8;
struct Point
{
  double x,y;
  Point(){}
  Point(double _x,double _y)
    {
      x = _x;
      y = _y;
    }
  double Length()
    {
      return sqrt(x*x+y*y);
    }
};
struct Circle
{
  Point c;
  double r;
};
struct Event
{
  double tim;
  int typ;
  Event(){}
  Event(double _tim,int _typ)
    {
      tim = _tim;
      typ = _typ;
    }
};
```

```
33
34 int cmp(const double& a,const double& b)
35 {
36   if (fabs(a-b) < eps)  return 0;
37   if (a < b)  return -1;
38   return 1;
39 }
40
41 bool Eventcmp(const Event& a,const Event& b)
42 {
43   return cmp(a.tim,b.tim) < 0;
44 }
45
46 double Area(double theta,double r)
47 {
48   return 0.5*r*r*(theta-sin(theta));
49 }
50
51 double xmult(Point a,Point b)
52 {
53   return a.x*b.y-a.y*b.x;
54 }
55
56 int n,cur,tote;
57 Circle c[1000];
58 double ans[1001],pre[1001],AB,AC,BC,theta,fai,a0,a1;
59 Event e[4000];
60 Point lab;
61
62 int main()
63 {
64   while (scanf("%d",&n) != EOF)
65   {
66     for (int i = 0;i < n;i++)
67       scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
68     for (int i = 1;i <= n;i++)
69       ans[i] = 0.0;
70     for (int i = 0;i < n;i++)
71     {
72       tote = 0;
73       e[tote++] = Event(-pi,1);
74       e[tote++] = Event(pi,-1);
75       for (int j = 0;j < n;j++)
76         if (j != i)
77         {
78           lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i].c.y);
79           AB = lab.Length();
80           AC = c[i].r;
81           BC = c[j].r;
82           if (cmp(AB+AC,BC) <= 0)
83           {
```

```
84              e[tote++] = Event(-pi,1);
85              e[tote++] = Event(pi,-1);
86              continue;
87            }
88            if (cmp(AB+BC,AC) <= 0) continue;
89            if (cmp(AB,AC+BC) > 0)  continue;
90            theta = atan2(lab.y,lab.x);
91            fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
92            a0 = theta-fai;
93            if (cmp(a0,-pi) < 0)  a0 += 2*pi;
94            a1 = theta+fai;
95            if (cmp(a1,pi) > 0)  a1 -= 2*pi;
96            if (cmp(a0,a1) > 0)
97            {
98              e[tote++] = Event(a0,1);
99              e[tote++] = Event(pi,-1);
100             e[tote++] = Event(-pi,1);
101             e[tote++] = Event(a1,-1);
102           }
103           else
104           {
105             e[tote++] = Event(a0,1);
106             e[tote++] = Event(a1,-1);
107           }
108         }
109       sort(e,e+tote,Eventcmp);
110       cur = 0;
111       for (int j = 0;j < tote;j++)
112       {
113         if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0)
114         {
115           ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
116           ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos(pre[cur]),c[i
                ].c.y+c[i].r*sin(pre[cur])),
117                     Point(c[i].c.x+c[i].r*cos(e[j].tim),c[i].c.y+c[
                        i].r*sin(e[j].tim)))/2.0;
118         }
119         cur += e[j].typ;
120         pre[cur] = e[j].tim;
121       }
122     }
123     for (int i = 1;i < n;i++)
124       ans[i] -= ans[i+1];
125     for (int i = 1;i <= n;i++)
126       printf("[%d] = %.3f\n",i,ans[i]);
127   }
128   return 0;
129 }
```

6.18 一个圆与多边形面积交

```
bool InCircle(Point a,double r)
{
  return cmp(a.x*a.x+a.y*a.y,r*r) <= 0;
  //这里判断的时候 EPS 一定不要太小!!
}

double CalcArea(Point a,Point b,double r)
{
  Point p[4];
  int tot = 0;
  p[tot++] = a;

  Point tv = Point(a,b);
  Line tmp = Line(Point(0,0),Point(tv.y,-tv.x));
  Point near = LineToLine(Line(a,b),tmp);
  if (cmp(near.x*near.x+near.y*near.y,r*r) <= 0)
  {
    double A,B,C;
    A = near.x*near.x+near.y*near.y;
    C = r;
    B = C*C-A;
    double tvl = tv.x*tv.x+tv.y*tv.y;
    double tmp = sqrt(B/tvl); //这样做只用一次开根
    p[tot] = Point(near.x+tmp*tv.x,near.y+tmp*tv.y);
    if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
    p[tot] = Point(near.x-tmp*tv.x,near.y-tmp*tv.y);
    if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
  }
  if (tot == 3)
  {
    if (cmp(Point(p[0],p[1]).Length(),Point(p[0],p[2]).Length()) >
       0)
      swap(p[1],p[2]);
  }
  p[tot++] = b;

  double res = 0.0,theta,a0,a1,sgn;
  for (int i = 0;i < tot-1;i++)
  {
    if (InCircle(p[i],r) == true && InCircle(p[i+1],r) == true)
    {
      res += 0.5*xmult(p[i],p[i+1]);
    }
    else
    {
      a0 = atan2(p[i+1].y,p[i+1].x);
      a1 = atan2(p[i].y,p[i].x);
      if (a0 < a1)  a0 += 2*pi;
      theta = a0-a1;
      if (cmp(theta,pi) >= 0) theta = 2*pi-theta;
      sgn = xmult(p[i],p[i+1])/2.0;
```

```
51        if (cmp(sgn,0) < 0) theta = -theta;
52        res += 0.5*r*r*theta;
53      }
54    }
55    return res;
56  }
```

调用

```
1  area2 = 0.0;
2  for (int i = 0;i < resn;i++) //遍历每条边，按照逆时针
3    area2 += CalcArea(p[i],p[(i+1)%resn],r);
```

6.19 精度问题

6.19.1 浮点数为啥会有精度问题

浮点数 (以 C/C++ 为准)，一般用的较多的是 float、double。

	占字节数	数值范围	十进制精度位数
float	4	$-3.4e-38 \sim 3.4e38$	$6 \sim 7$
double	8	$-1.7e-308 \sim 1.7e308$	$14 \sim 15$

如果内存不是很紧张或者精度要求不是很低，一般选用 double。14 位的精度 (是有效数字位，不是小数点后的位数) 通常够用了。注意，问题来了，数据精度位数达到了 14 位，但有些浮点运算的结果精度并达不到这么高，可能准确的结果只有 $10 \sim 12$ 位左右。那低几位呢？自然就是不可预料的数字了。这给我们带来这样的问题：即使是理论上相同的值，由于是经过不同的运算过程得到的，他们在低几位有可能 (一般来说都是) 是不同的。这种现象看似没太大的影响，却会一种运算产生致命的影响：==。恩，就是判断相等。注意，C/C++ 中浮点数的 == 需要完全一样才能返回 true。

6.19.2 eps

eps 缩写自 epsilon，表示一个小量，但这个小量又要确保远大于浮点运算结果的不确定量。eps 最常见的取值是 $1e-8$ 左右。引入 eps 后，我们判断两浮点数 a、b 相等的方式如下:

```
1  int sgn(double a){return a < -eps ? -1 : a < eps ? 0 : 1;}
```

这样，我们才能把相差非常近的浮点数判为相等；同时把确实相差较大 (差值大于 eps) 的数判为不相等。
养成好习惯，尽量不要再对浮点数做 == 判断。

6.19.3 eps 带来的函数越界

如果 sqrt(a), asin(a), acos(a) 中的 a 是你自己算出来并传进来的，那就得小心了。
如果 a 本来应该是 0 的，由于浮点误差，可能实际是一个绝对值很小的负数 (比如 $-1e-12$)，这样 sqrt(a) 应得 0 的，直接因 a 不在定义域而出错。
类似地，如果 a 本来应该是 ± 1, 则 asin(a)、acos(a) 也有可能出错。
因此，对于此种函数，必需事先对 a 进行校正。

6.19.4 输出陷阱 I

现在考虑一种情况，题目要求输出保留两位小数。有个 case 的正确答案的精确值是 0.005, 按理应该输出 0.01，但你的结果可能是 0.005000000001(恭喜)，也有可能是 0.004999999999(悲剧)，如果按照 `printf("%.2lf", a)` 输出，那你的遭遇将和括号里的字相同。
解决办法是，如果 a 为正，则输出 $a + eps$, 否则输出 $a - eps$

6.19.5 输出陷阱 II

ICPC 题目输出有个不成文的规定 (有时也成文)，不要输出：-0.000
那我们首先要弄清，什么时候按 `printf("%.3lf", a)` 输出会出现这个结果。
直接给出结果好了：$a \in (-0.000499999\cdots, -0.000\cdots 1)$
所以，如果你发现 a 落在这个范围内，请直接输出 0.000。更保险的做法是用 `sprintf` 直接判断输出结果是不是 -0.000 再予处理。

6.19.6 范围越界

请注意，虽然 double 可以表示的数的范围很大，却不是不穷大，上面说过最大是 $1e308$。所以有些时候你得小心了，比如做连乘的时候，必要的时候要换成对数的和。

6.19.7 关于 set

经观察,set 不是通过 $==$ 来判断相等的，是通过 $<$ 来进行的，具体说来，只要 $a < b$ 和 $b < a$ 都不成立，就认为 a 和 b 相等，可以发现，如果将小于定义成：

```
bool operator < (const Dat dat)const{return val < dat.val - eps;}
```

就可以解决问题了。(基本类型不能重载运算符，所以封装了下)

6.19.8 输入值波动过大

这种情况不常见，不过可以帮助你更熟悉 eps。假如一道题输入说，给一个浮点数 a, $1e-20 < a < 1e20$。那你还敢用 $1e-8$ 做 eps 么？合理的做法是把 eps 按照输入规模缩放到合适大小。

6.19.9 一些建议

容易产生较大浮点误差的函数有 asin、acos。欢迎尽量使用 atan2。
另外，如果数据明确说明是整数，而且范围不大的话，使用 int 或者 long long 代替 double 都是极佳选择，因为就不存在浮点误差了

7 搜索

7.1 Dancing Links

7.1.1 估价函数

```
int h()
{
  bool vis[100];
  memset(vis,false,sizeof(vis));
  int i,j,k,res=0,mi,col;
  while(1)
  {
    mi=inf;
    for(i=R[head]; i!=head&&i<=2*n; i=R[i])
      if(mi>nk[i]&&!vis[i])
      {
        mi=nk[i];
        col=i;
      }
    if(mi==inf)
      break;
    res++;
    vis[col]=true;
    for(j=D[col]; j!=col; j=D[j])
      for(k=R[j]; k!=j; k=R[k])
      {
        if(C[k]>2*n)
          continue;
        vis[C[k]]=true;
      }
  }
  return res;
}
```

7.1.2 DLX

```
void remove1(int col)
{
  int i,j;
  L[R[col]]=L[col];
  R[L[col]]=R[col];
  for(i=D[col];i!=col;i=D[i])
  {
    L[R[i]]=L[i];
    R[L[i]]=R[i];
  }
}
void remove2(int col)
{
  int i,j;
  L[R[col]]=L[col];
```

```
  R[L[col]]=R[col];
  for(i=D[col];i!=col;i=D[i])
  {
    for(j=R[i];j!=i;j=R[j])
    {
      U[D[j]]=U[j];
      D[U[j]]=D[j];
      --nk[C[j]];
    }
  }
}
void resume1(int col)
{
  int i,j;
  for(i=U[col];i!=col;i=U[i])
  {
    L[R[i]]=i;
    R[L[i]]=i;
  }
  L[R[col]]=col;
  R[L[col]]=col;
}
void resume2(int col)
{
  int i,j;
  for(i=U[col];i!=col;i=U[i])
  {
    for(j=L[i];j!=i;j=L[j])
    {
      ++nk[C[j]];
      U[D[j]]=j;
      D[U[j]]=j;
    }
  }
  L[R[col]]=col;
  R[L[col]]=col;
}
int h()
{
  bool vis[100];
  memset(vis,false,sizeof(vis));
  int i,j,k,res=0,mi,col;
  while(1)
  {
    mi=inf;
    for(i=R[head];i!=head&&i<=2*n;i=R[i])
      if(mi>nk[i]&&!vis[i])
      {
        mi=nk[i];
        col=i;
      }
```

```
67      if(mi==inf)
68        break;
69      res++;vis[col]=true;
70      for(j=D[col];j!=col;j=D[j])
71        for(k=R[j];k!=j;k=R[k])
72        {
73          if(C[k]>2*n)
74            continue;
75          vis[C[k]]=true;
76        }
77    }
78    return res;
79  }
80  bool DLX(int d,int deep)
81  {
82    if(d+h()>deep) return false;
83    if(R[head]==head||R[head]>2*n)
84      return true;
85    if(d>=deep)
86      return false;
87    int col,ma=inf;
88    int i,j;
89    for(i=R[head];i!=head&&i<=2*n;i=R[i])
90      if(nk[i]<ma)
91      {
92        col=i;
93        ma=nk[i];
94      }
95    remove1(col);
96    for(i=D[col];i!=col;i=D[i])
97    {
98      int flag=1;
99      for(j=R[i];;j=R[j])
100     {
101       if(j==R[i]&&!flag)
102         break;
103       U[D[j]]=U[j];
104       D[U[j]]=D[j];
105       if(C[j]>2*n)
106         remove2(C[j]);
107       else
108         remove1(C[j]);
109       flag=0;
110     }
111     if(DLX(d+1,deep))
112       return true;
113     flag=1;
114     for(j=L[i];;j=L[j])
115     {
116       if(j==L[i]&&!flag)
117         break;
```

```
118       if(C[j]>2*n)
119         resume2(C[j]);
120       else
121         resume1(C[j]);
122       U[D[j]]=j;
123       D[U[j]]=j;
124       flag=0;
125     }
126   }
127   resume1(col);
128   return false;
129 }
```

8 动态规划

8.1 斜率优化

```
#include<cstdio>
#include<algorithm>
using namespace std;
int a[1000],sum[1001],dp[1000][1000];
int deque[1000];
const int inf=0x7fffffff;
int N,s,t;
int calc(int i,int l,int j)//决策值计算
{
  return dp[j][l-1]-(sum[i]-sum[j])*(sum[N]-sum[i]);
}
bool check(int i,int l)//尾端判断
{
  int k1=deque[t-1],k2=deque[t-2];
  return (long long)(dp[k1][l]-dp[k2][l])*(sum[i]-sum[k1])>(long
     long)(dp[i][l]-dp[k1][l])*(sum[k1]-sum[k2]);
}
int main()
{
  int n,m;
  while (scanf("%d%d",&n,&m),n)
  {
    for (int i=0; i<n; i++)
      scanf("%d",&a[i]);
    N=n;
    sum[0]=0;
    for (int i=0; i<n; i++)
      sum[i+1]=sum[i]+a[i];
    dp[0][0]=0;
    for (int i=0; i<n; i++)
      for (int j=i+1; j<n; j++)
        dp[0][0]+=a[i]*a[j];
    for (int i=1; i<n; i++)
      dp[i][0]=inf;
    for (int i=1; i<n; i++)
    {
      dp[i][1]=inf;
      for (int j=0; j<i; j++)
        dp[i][1]=min(dp[i][1],calc(i,1,j));
    }
    for (int l=2; l<=m; l++)
    {
      s=t=0;//双端队列清空
      for (int i=l; i<n; i++)
      {
        while (t-s>1 && check(i-1,l-1)) t--;
        deque[t++]=i-1;//决策加入
```

```
47          while (t-s>1 && calc(i,l,deque[s])>calc(i,l,deque[s+1])) s
                ++;
48          dp[i][l]=calc(i,l,deque[s]);
49        }
50      }
51      int ans=0x7fffffff;
52      for (int i=m; i<n; i++)
53        ans=min(ans,dp[i][m]);
54      printf("%d\n",ans);
55    }
56    return 0;
57  }
```

8.2 RMQ 二版

```
1  void init()
2  {
3    int i,j;
4    int n=N,k=1,l=0;
5    for (i=0; i<n; i++)
6    {
7      f[i][0]=ele[i].num;
8      if (i+1>k*2)
9      {
10       k*=2;
11       l++;
12     }
13     lent[i+1]=l;
14   }
15   for (j=1; (1<<j)-1<n; j++)
16     for (i=0; i+(1<<j)-1<n; i++)
17       f[i][j]=max(f[i][j-1],f[i+(1<<(j-1))][j-1]);
18 }
19 int fint(int x,int y)
20 {
21   int k=lent[y-x+1];
22   return max(f[x][k],f[y-(1<<k)+1][k]);
23 }
```

8.3 二维 LIS

```
1  #include<cstdio>
2  #include<map>
3  using namespace std;
4  map<int,int> mp[100001];
5  bool check(int idx,int x,int y)
6  {
7    if (!idx) return 1;
8    if (mp[idx].begin()->first>=x) return 0;
9    map<int,int> ::iterator it=mp[idx].lower_bound(x);
10   it--;
11   if (it->second<y) return 1;
```

```
12    else return 0;
13  }
14  int main()
15  {
16    int n;
17    scanf("%d",&n);
18    int l=0,r=0;
19    for (int i=0;i<n;i++)
20    {
21      int x,y;
22      scanf("%d%d",&x,&y);
23      int tl=l,tr=r;
24      while (tl<tr)
25      {
26        int mid=(tl+tr+1)/2;
27        if (check(mid,x,y))
28          tl=mid;
29        else
30          tr=mid-1;
31      }
32      if (tl==r) r++;
33      int idx=tl+1;
34      map<int,int> ::iterator itl=mp[idx].lower_bound(x),itr=itl;
35      while (itr!=mp[idx].end() && itr->second>y) itr++;
36      if (mp[idx].find(x)!=mp[idx].end())
37        y=min(y,mp[idx][x]);
38      if (itl!=itr) mp[idx].erase(itl,itr);
39      if (mp[idx].find(x)==mp[idx].end() || mp[idx][x]>y)
40        mp[idx][x]=y;
41    }
42    printf("%d\n",r);
43    return 0;
44  }
```

8.4 插头 DP

Tower Defence 独立插头 + 构造解
构造解的时候保存的是在 hash_map 的 ele 数组的下标位置
没想清楚千万别去写

```
1  int bit[12];
2
3  inline int getbit(long long sta,int pos)
4  {
5    return sta/bit[pos]%bit[1];
6  }
7
8  inline long long setbit(long long sta,int pos,int val)
9  {
10   return sta/bit[pos+1]*bit[pos+1]+val*bit[pos]+sta%bit[pos];
11 }
12
```

```
13 int n,m,mp[30][10];
14 char buf[30][10];
15 hash_map dp[2];
16 bool flag;
17 int key,val,upd,l,u,res,msk,cov,now,pr,resnow,resmsk,pru;
18 int w[15],s[15],top;
19 int pre[210][10007],preuse[210][10007];
20
21 void decode(int msk,int& key,int& cov)
22 {
23   int tmp;
24   key = cov = 0;
25   for (int i = 0; i < m+1; i++)
26   {
27     tmp = getbit(msk,i);
28     if (tmp > 0)
29     {
30       key = setbit(key,i,tmp-1);
31       cov = setbit(cov,i,1);
32     }
33   }
34 }
35
36 int encode(int key,int cov)
37 {
38   int res = 0,tmp;
39   for (int i = 0; i < m+1; i++)
40   {
41     tmp = getbit(cov,i);
42     if (tmp > 0)
43     {
44       tmp = getbit(key,i);
45       res = setbit(res,i,tmp+1);
46     }
47   }
48   return res;
49 }
50
51 void update(int a,int key,int cov,int val)
52 {
53   int msk = encode(key,cov);
54   int pos;
55   if (dp[a][msk] < val)
56   {
57     dp[a][msk] = val;
58     pos = dp[a].fint(msk);
59     pre[now][pos] = pr;
60     preuse[now][pos] = pru;
61   }
62 }
63
```

```
64 int count3(int sta)
65 {
66   int res = 0;
67   for (int i = 0; i < m+1; i++)
68     if (getbit(sta,i) == 3)
69       res++;
70   return res;
71 }
72
73 void expand(int sta)
74 {
75   top = 0;
76   for (int i = 0; i < m+1; i++)
77     if (getbit(sta,i) == 1)
78       s[top++] = i;
79     else if (getbit(sta,i) == 2)
80     {
81       w[s[top-1]] = i;
82       w[i] = s[top-1];
83       top--;
84     }
85 }
86
87 int main()
88 {
89   //freopen("TD.in","r",stdin);
90   //freopen("TDM.out","w",stdout);
91   bit[0] = 1;
92   for (int i = 1; i < 12; i++)  bit[i] = bit[i-1]*5;
93   int t;
94   scanf("%d",&t);
95   dp[0].init();
96   dp[1].init();
97   for (int ft = 1; ft <= t; ft++)
98   {
99     scanf("%d%d",&n,&m);
100    res = 0;
101    memset(mp,0,sizeof(mp));
102    memset(pre,0,sizeof(pre));
103    memset(preuse,0,sizeof(preuse));
104    for (int i = 0; i < n; i++)
105    {
106      scanf("%s",buf[i]);
107      for (int j = 0; j < m; j++)
108        if (buf[i][j] == '.')
109          mp[i][j] = 1;
110        else if (buf[i][j] != 'B')
111          mp[i][j] = 2;
112    }
113    dp[0].clear();
114    dp[1].clear();
```

```
115     flag = 0;
116     dp[flag][0] = 0;
117     int res = 0;
118     now = 0;
119     for (int i = 0; i < n; i++)
120     {
121       for (int j = 0; j < m; j++)
122       {
123         dp[!flag].clear();
124         for (int k = 0; k < dp[flag].N; k++)
125         {
126           msk = dp[flag].ele[k].key;
127           pr = k;
128           val = dp[flag].ele[k].val;
129           decode(msk,key,cov);
130           l = getbit(key,j);
131           u = getbit(key,j+1);
132           if (mp[i][j] == 0)//是障碍
133           {
134             if (l == 0 && u == 0)
135             {
136               pru = 0;
137               update(!flag,key,setbit(setbit(cov,j,0),j+1,0),val);
138             }
139           }
140           else
141           {
142             if (mp[i][j] == 1 && l == 0 && u == 0)//不要插头
143             {
144               pru = 1;
145               update(!flag,key,setbit(setbit(cov,j,0),j+1,0),val);
146             }
147             if (getbit(cov,j) == 1 && l == 0)   continue;//不可以在
                  这里搞插头
148             if (getbit(cov,j+1) == 1 && u == 0) continue;
149             cov = setbit(setbit(cov,j,1),j+1,1);//更新覆盖情况
150             upd = setbit(setbit(key,j,0),j+1,0);
151             pru = 2;
152             if (mp[i][j] == 2)
153             {
154               if (l == 0 && u == 0)
155               {
156                 if (count3(key) < 2)//可以新建独立插头
157                 {
158                   if (mp[i][j+1] != 0)
159                     update(!flag,setbit(setbit(key,j,0),j+1,3),cov,
                          val+1);
160                   if (mp[i+1][j] != 0)
161                     update(!flag,setbit(setbit(key,j,3),j+1,0),cov,
                          val+1);
162                 }
```

```
163                 }
164                 else if (l == 0 || u == 0)
165                 {
166                   if (l+u < 3 && count3(key) < 2)//可以用一个独立插头来
                        结束这条路径
167                   {
168                     expand(key);
169                     if (l > 0)
170                       update(!flag,setbit(upd,w[j],3),cov,val+1);
171                     else
172                       update(!flag,setbit(upd,w[j+1],3),cov,val+1);
173                   }
174                   else if (l+u == 3 && upd == 0)//路径的一端
175                   {
176                     if (res < val+1)
177                     {
178                       res = val+1;
179                       resnow = now-1;
180                       resmsk = k;
181                     }
182                   }
183                 }
184               }
185               else if (l == 0 && u == 0)
186               {
187                 if (mp[i][j+1] != 0 && mp[i+1][j] != 0)//可以新建插头
188                   update(!flag,setbit(setbit(key,j,1),j+1,2),cov,val
                        +1);
189               }
190               else if (l == 0 || u == 0)
191               {
192                 if (mp[i][j+1] != 0)//可以延续插头
193                   update(!flag,setbit(upd,j+1,l+u),cov,val+1);
194                 if (mp[i+1][j] != 0)//可以延续插头
195                   update(!flag,setbit(upd,j,l+u),cov,val+1);
196               }
197               else if (l == u)
198               {
199                 if (l < 3) //合并两个相同的括号
200                 {
201                   expand(key);
202                   if (l == 1)
203                     update(!flag,setbit(upd,w[j+1],1),cov,val+1);
204                   else
205                     update(!flag,setbit(upd,w[j],2),cov,val+1);
206                 }
207                 else if (upd == 0)//合并两个独立插头
208                 {
209                   if (res < val+1)
210                   {
211                     res = val+1;
```

```
212                     resnow = now-1;
213                     resmsk = k;
214                   }
215                 }
216               }
217               else if (l == 3 || u == 3)//合并独立插头与括号
218               {
219                 expand(key);
220                 if (l == 3)
221                   update(!flag,setbit(upd,w[j+1],3),cov,val+1);
222                 else
223                   update(!flag,setbit(upd,w[j],3),cov,val+1);
224               }
225               else if (l == 2 || u == 1) //合并)(
226                 update(!flag,upd,cov,val+1);
227             }
228           }
229           flag = !flag;
230           now++;
231         }
232         if (i+1 == n)   break;
233 
234         dp[!flag].clear();
235         for (int k = 0; k < dp[flag].N; k++)
236         {
237           msk = dp[flag].ele[k].key;
238           pr = k;
239           val = dp[flag].ele[k].val;
240           pru = 0;
241           decode(msk,key,cov);
242           update(!flag,key*bit[1],cov*bit[1],val);
243         }
244         now++;
245         flag = !flag;
246       }
247 
248       printf("Case␣%d:␣%d\n",ft,res);
249       for (int i = resnow; i >= 0; i--)
250       {
251         if (preuse[i][resmsk] == 1)
252           buf[i/(m+1)][i%(m+1)] = 'W';
253         resmsk = pre[i][resmsk];
254       }
255       for (int i = 0; i < n; i++)
256         printf("%s\n",buf[i]);
257       printf("\n");
258     }
259     return 0;
260 }
```

9 杂物

9.1 高精度数

支持乘以整数和加法。

```
struct BigInt
{
  const static int mod = 100000000;
  int a[600],len;
  BigInt (){}
  BigInt (int v)
  {
    len = 0;
    do
    {
      a[len++] = v%mod;
      v /= mod;
    }while(v);
  }
  BigInt operator *(const int& b) const
  {
    BigInt res;
    res.len = len;
    for (int i = 0; i <= len; ++i)
      res.a[i] = 0;
    for (int i = 0; i < len; ++i)
    {
      res.a[i] += a[i]*b;
      res.a[i+1] += res.a[i]/mod;
      res.a[i] %= mod;
    }
    if (res.a[len] > 0)  res.len++;
    return res;
  }
  BigInt operator +(const BigInt& b) const
  {
    BigInt res;
    res.len = max(len,b.len);
    for (int i = 0; i <= res.len; ++i)
      res.a[i] = 0;
    for (int i = 0; i < res.len; ++i)
    {
      res.a[i] += ((i < len)?a[i]:0)+((i < b.len)?b.a[i]:0);
      res.a[i+1] += res.a[i]/mod;
      res.a[i] %= mod;
    }
    if (res.a[res.len] > 0)  res.len++;
    return res;
  }
  void output()
```

```
46    {
47      printf("%d",a[len-1]);
48      for (int i = len-2; i >= 0; --i)
49        printf("%08d",a[i]);
50      printf("\n");
51    }
52  };
```

9.2 整数外挂

```
 1  int wg;
 2  char ch;
 3  bool ng;
 4
 5  inline int readint()
 6  {
 7    ch = getchar();
 8    while (ch != '-' && (ch < '0' || ch > '9')) ch = getchar();
 9    if (ch == '-')
10    {
11      ng = true;
12      ch = getchar();
13    }
14    else
15      ng = false;
16    wg = ch-'0';
17    ch = getchar();
18    while (ch >= '0' && ch <= '9')
19    {
20      wg = wg*10+ch-'0';
21      ch = getchar();
22    }
23    if (ng == true) wg = -wg;
24    return wg;
25  }
```

9.3 Java

9.3.1 文件操作

```
 1  import java.io.*;
 2  import java.util.*;
 3  import java.math.*;
 4  import java.text.*;
 5
 6  public class Main
 7  {
 8
 9    public static void main(String args[]) throws
         FileNotFoundException, IOException
10    {
11      Scanner sc = new Scanner(new FileReader("a.in"));
```

```
12      PrintWriter pw = new PrintWriter(new FileWriter("a.out"));
13      int n,m;
14      n=sc.nextInt();//读入下一个INT
15      m=sc.nextInt();
16
17      for(ci=1; ci<=c; ++ci)
18      {
19        pw.println("Case␣#"+ci+":␣easy␣for␣output");
20      }
21
22      pw.close();//关闭流并释放，这个很重要，否则是没有输出的
23      sc.close();//关闭流并释放
24    }
25  }
```

9.3.2 优先队列

```
1  PriorityQueue queue = new PriorityQueue( 1, new Comparator()
2  {
3    public int compare( Point a, Point b )
4    {
5    if( a.x < b.x || a.x == b.x && a.y < b.y )
6      return -1;
7    else if( a.x == b.x && a.y == b.y )
8      return 0;
9    else
10     return 1;
11   }
12 });
```

9.3.3 Map

```
1  Map map = new HashMap();
2  map.put("sa","dd");
3  String str = map.get("sa").toString;
4
5  for(Object obj : map.keySet()){
6    Object value = map.get(obj );
7  }
```

9.3.4 sort

```
1  static class cmp implements Comparator
2  {
3    public int compare(Object o1,Object o2)
4    {
5    BigInteger b1=(BigInteger)o1;
6    BigInteger b2=(BigInteger)o2;
7    return b1.compareTo(b2);
8    }
9  }
10 public static void main(String[] args) throws IOException
11 {
```

```
12    Scanner cin = new Scanner(System.in);
13    int n;
14    n=cin.nextInt();
15    BigInteger[] seg = new BigInteger[n];
16    for (int i=0;i<n;i++)
17    seg[i]=cin.nextBigInteger();
18    Arrays.sort(seg,new cmp());
19  }
```

9.4 hashmap

```
 1  struct hash_map
 2  {
 3    const static int mod=10007;
 4    int head[mod];
 5    struct hash_tables
 6    {
 7      int key;
 8      int val;
 9      int next;
10    } ele[10007];
11    int N;
12    int getHash(int x)
13    {
14      return x%mod; //小心负数
15    }
16    void init()
17    {
18      memset(head,255,sizeof(head));
19      N=0;
20    }
21    void clear()
22    {
23      for (int i = 0; i < N; i++)
24        head[getHash(ele[i].key)] = -1;
25      N = 0;
26    }
27    int fint(int x)
28    {
29      for (int i=head[getHash(x)]; i!=-1; i=ele[i].next)
30        if (ele[i].key==x) return i;
31      return -1;
32    }
33    void insert(int x)
34    {
35      int tmp=getHash(x);
36      ele[N].key=x;
37      ele[N].val=0;
38      ele[N].next=head[tmp];
39      head[tmp]=N++;
40    }
41    int& operator [](int x)
```

```
42    {
43      int tmp=fint(x);
44      if (tmp==-1)
45      {
46        insert(x);
47        return ele[N-1].val;
48      }
49      else
50        return ele[tmp].val;
51    }
52  };
```

9.5 C++&STL 常用函数

9.5.1 lower_bound/upper_bound

不解释

```
1  iterator lower_bound(const key_type &key )
2  \\返回一个迭代器，指向键值 >= key 的第一个元素。
3  iterator upper_bound(const key_type &key )
4  \\返回一个迭代器，指向键值 > key 的第一个元素。
5
6  #include <iostream>
7  #include <algorithm>
8  #include <vector>
9  using namespace std;
10
11 int main () {
12   int myints[] = {10,20,30,30,20,10,10,20};
13   vector<int> v(myints,myints+8);
14   // 10 20 30 30 20 10 10 20
15   vector<int>::iterator low,up;
16
17   sort (v.begin(), v.end());
18   // 10 10 10 20 20 20 30 30
19
20   low=lower_bound (v.begin(), v.end(), 20);
21   // 10 10 10 20 20 20 30 30
22   //          ^
23   up= upper_bound (v.begin(), v.end(), 20);
24   // 10 10 10 20 20 20 30 30
25   //                   ^
26
27   cout << "lower_bound␣at␣position␣" << int(low- v.begin()) << endl
        ;
28   cout << "upper_bound␣at␣position␣" << int(up - v.begin()) << endl
        ;
29
30   return 0;
31 }
```

Output:

```
lower_bound at position 3
upper_bound at position 6
```

9.5.2 rotate

把数组后一半搬到前面

```
template <class ForwardIterator>
  void rotate ( ForwardIterator first, ForwardIterator middle,
                ForwardIterator last );
```

9.5.3 nth_element

```
template <class RandomAccessIterator>
  void nth_element ( RandomAccessIterator first,
     RandomAccessIterator nth,
                     RandomAccessIterator last );

template <class RandomAccessIterator, class Comapre>
  void nth_element ( RandomAccessIterator first,
     RandomAccessIterator nth,
                     RandomAccessIterator last, Compare comp );
```

9.5.4 bitset

取用

```
bitset<4> mybits;

mybits[1]=1;              // 0010
mybits[2]=mybits[1];      // 0110
```

翻转

```
bitset<4> mybits (string("0001"));

cout << mybits.flip(2) << endl;     // 0101
cout << mybits.flip() << endl;      // 1010
```

运算

```
bitset<4> first (string("1001"));
bitset<4> second (string("0011"));

cout << (first^=second) << endl;          // 1010 (XOR,assign)
cout << (first&=second) << endl;          // 0010 (AND,assign)
cout << (first|=second) << endl;          // 0011 (OR,assign)

cout << (first<<=2) << endl;              // 1100 (SHL,assign)
```

```
 9 cout << (first>>=1) << endl;              // 0110 (SHR,assign)
10
11 cout << (~second) << endl;                // 1100 (NOT)
12 cout << (second<<1) << endl;              // 0110 (SHL)
13 cout << (second>>1) << endl;              // 0001 (SHR)
14
15 cout << (first==second) << endl;          // false (0110==0011)
16 cout << (first!=second) << endl;          // true  (0110!=0011)
17
18 cout << (first&second) << endl;           // 0010
19 cout << (first|second) << endl;           // 0111
20 cout << (first^second) << endl;           // 0101
```

9.5.5 multimap

遍历

```
 1 multimap<char,int> mymm;
 2 multimap<char,int>::iterator it;
 3 char c;
 4
 5 mymm.insert(pair<char,int>('x',50));
 6 mymm.insert(pair<char,int>('y',100));
 7 mymm.insert(pair<char,int>('y',150));
 8 mymm.insert(pair<char,int>('y',200));
 9 mymm.insert(pair<char,int>('z',250));
10 mymm.insert(pair<char,int>('z',300));
11
12 for (c='x'; c<='z'; c++)
13 {
14   cout << "There␣are␣" << (int)mymm.count(c);
15   cout << "␣elements␣with␣key␣" << c << ":";
16   for (it=mymm.equal_range(c).first; it!=mymm.equal_range(c).second
        ; ++it)
17     cout << "␣" << (*it).second;
18   cout << endl;
19 }
20 /*
21 Output:
22
23 There are 1 elements with key x: 50
24 There are 3 elements with key y: 100 150 200
25 There are 2 elements with key z: 250 300
26 */
```

二分查找

```
1 multimap<char,int> mymultimap;
2 multimap<char,int>::iterator it,itlow,itup;
3
4 mymultimap.insert(pair<char,int>('a',10));
```

```
mymultimap.insert(pair<char,int>('b',121));
mymultimap.insert(pair<char,int>('c',1001));
mymultimap.insert(pair<char,int>('c',2002));
mymultimap.insert(pair<char,int>('d',11011));
mymultimap.insert(pair<char,int>('e',44));

itlow=mymultimap.lower_bound ('b');  // itlow points to b
itup=mymultimap.upper_bound ('d');   // itup points to e (not d)

// print range [itlow,itup):
for ( it=itlow ; it != itup; it++ )
  cout << (*it).first << "␣=>␣" << (*it).second << endl;

/*
Output:

b => 121
c => 1001
c => 2002
d => 11011
*/
```

删除

```
multimap<char,int> mymultimap;
multimap<char,int>::iterator it;

// insert some values:
mymultimap.insert(pair<char,int>('a',10));
mymultimap.insert(pair<char,int>('b',20));
mymultimap.insert(pair<char,int>('b',30));
mymultimap.insert(pair<char,int>('c',40));
mymultimap.insert(pair<char,int>('d',50));
mymultimap.insert(pair<char,int>('d',60));
mymultimap.insert(pair<char,int>('e',70));
mymultimap.insert(pair<char,int>('f',80));

it=mymultimap.find('b');
mymultimap.erase (it);
// erasing by iterator (1 element)

mymultimap.erase ('d');
// erasing by key (2 elements)

it=mymultimap.find ('e');
mymultimap.erase ( it, mymultimap.end() );
// erasing by range

// show content:
for ( it=mymultimap.begin() ; it != mymultimap.end(); it++ )
  cout << (*it).first << "␣=>␣" << (*it).second << endl;

```

```
/*
Output:

a => 10
b => 30
c => 40
*/
```

9.6 位运算

9.6.1 基本操作

注意括号

功能	示例	位运算
去掉最后一位	$(101101 \to 10110)$	x shr 1
在最后加一个 0	$(101101 \to 1011010)$	x shl 1
在最后加一个 1	$(101101 \to 1011011)$	x shl 1+1
把最后一位变成 1	$(101100 \to 101101)$	x or 1
把最后一位变成 0	$(101101 \to 101100)$	x or 1-1
最后一位取反	$(101101 \to 101100)$	x xor 1
把右数第 k 位变成 1	$(101001 \to 101101, k = 3)$	x or (1 shl (k-1))
把右数第 k 位变成 0	$(101101 \to 101001, k = 3)$	x and not (1 shl (k-1))
右数第 k 位取反	$(101001 \to 101101, k = 3)$	x xor (1 shl (k-1))
取末三位	$(1101101 \to 101)$	x and 7
取末 k 位	$(1101101 \to 1101, k = 5)$	x and (1 shl k-1)
取右数第 k 位	$(1101101 \to 1, k = 4)$	x shr (k-1) and 1
把末 k 位变成 1	$(101001 \to 101111, k = 4)$	x or (1 shl k-1)
末 k 位取反	$(101001 \to 100110, k = 4)$	x xor (1 shl k-1)
把右边连续的 1 变成 0	$(100101111 \to 100100000)$	x and (x+1)
把右起第一个 0 变成 1	$(100101111 \to 100111111)$	x or (x+1)
把右边连续的 0 变成 1	$(11011000 \to 11011111)$	x or (x-1)
取右边连续的 1	$(100101111 \to 1111)$	(x xor (x+1)) shr 1
去掉右起第一个 1 的左边	$(100101000 \to 1000)$	x and (x xor (x-1))

9.6.2 枚举长为 n 含 k 个 1 的 01 串

```
int n = 5,k = 3;
for (int s = (1 << k)-1,u = 1 << n; s < u;)
{
  for (int i = 0;i < n;i++)
    printf("%d",(((s>>(n-1-i))&1) == 1));
  printf("\n");

  int b = s & -s;
  s = (s+b)|(((s^(s+b))>>2)/b);
}
```

9.7 其它

9.7.1 对跑脚本

```
while true; do
  ./gen > input
  ./sol < input > output.sol
  ./bf < input > output.bf

  diff output.sol output.bf
  if [ $? -ne 0 ] ; then break; fi
done
```