② е) Плотность распределения медианы выборки

$$f(t) = F'(t)$$

$$\xi_{(k)} \sim \sum_{i=k}^{n} C_n^i \, F^i(t)\,(1-F(t))^{n-i} = \Phi(t)$$

$$p(t) = \frac{d}{dt}\Phi(t) = \sum_{i=k}^{n} C_n^i \Big[i\cdot F^{i-1}(t)\cdot f(t)(1-F(t))^{n-i} - (n-i)\cdot F^i(t)\cdot(1-F(t))^{n-i-1}\cdot f(t)\Big] = f(t)\sum_{i=k}^{n} C_n^i \cdot$$

$$\cdot\Big[i\cdot F^{i-1}\cdot(1-F)^{n-i} - (n-i)\,F^i(1-F)^{n-i-1}\Big] =$$

$$= f(t)\sum_{i=k}^{n}\Big(\frac{n!}{(i-1)!\,(n-i)!}F^{i-1}(1-F)^{n-i} - \frac{n!}{i!\,(n-i-1)!}F^i(1-F)^{n-i-1}\Big) =$$

$$= f(t)\Big(\frac{n!}{(k-1)!\,(n-k)!}F^{k-1}(1-F)^{n-k} - \cancel{\frac{n!}{k!\,(n-k-1)!}F^k(1-F)^{n-k-1}} +$$

$$+ \cancel{\frac{n!}{k!\,(n-k-1)!}F^k(1-F)^{n-k-1}} - \dots - \overset{0}{\cancel{\frac{n!}{n!\,(n-n-1)!}F^n(1-F)^{n-n-1}}}\Big) =$$

$$= f(t) \cdot \frac{n!}{(k-1)!\,(n-k)!} \cdot F^{k-1} \cdot (1-F)^{n-k}$$

В нашем случае $k = 13$, $f(t) = e^{-x}\ \{x \geq 0\}$

$n = 25$ $\quad F(t) = \int_0^{t} e^{-t}\,dt = 1 - e^{-t}$

$$p(t) = \frac{25!}{12!\,12!} \cdot e^{-t} \cdot (1-e^{-t})^{12} \cdot e^{-12t}$$