

Forecasting: principles and practice

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2.1 Transformations

Outline

- 1 Variance stabilization
- **2** Box-Cox transformations
- 3 Back-transformation
- 4 Lab session 11

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Mathematical transformations for stabilizing variation

Square root
$$w_t = \sqrt{y_t}$$

Cube root $w_t = \sqrt[3]{y_t}$ Increasing

Logarithm $w_t = \log(y_t)$ strength

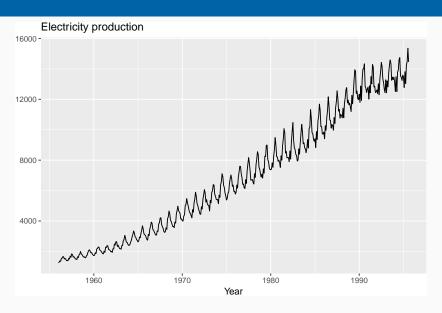
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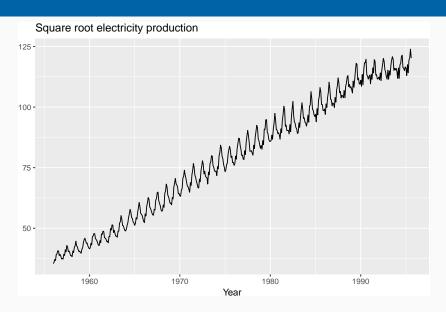
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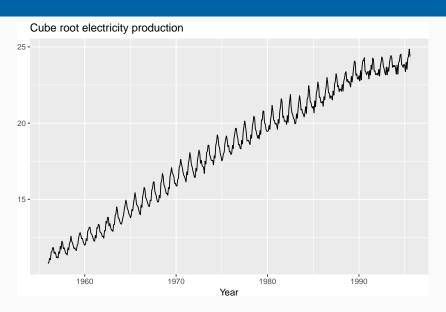
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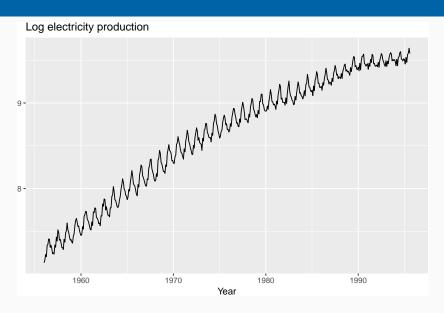
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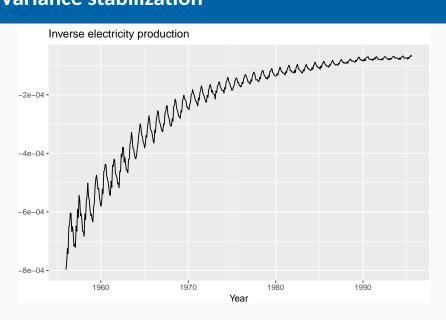
Logarithms, in particular, are useful because they are more interpretable: changes in a log value are **relative** (percent) changes on the original scale.











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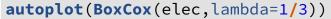
Each of these transformations is close to a member of the family of **Box-Cox transformations**:

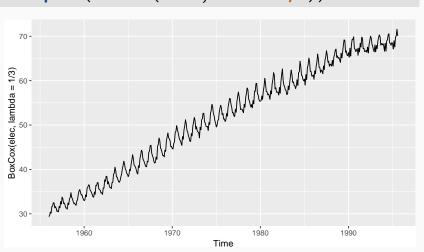
$$w_t = \begin{cases} \log(y_t), & \lambda = 0; \\ (y_t^{\lambda} - 1)/\lambda, & \lambda \neq 0. \end{cases}$$

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- λ = 1: (No substantive transformation)
- $\lambda = \frac{1}{2}$: (Square root plus linear transformation)
- λ = 0: (Natural logarithm)
- $\lambda = -1$: (Inverse plus 1)





- y_t^{λ} for λ close to zero behaves like logs.
- If some $y_t = 0$, then must have $\lambda > 0$
- if some $y_t < 0$, no power transformation is possible unless all y_t adjusted by adding a constant to all values.
- Simple values of λ are easier to explain.
- Results are relatively insensitive to λ .
- Often no transformation (λ = 1) needed.
- Transformation can have very large effect on PI.
- Choosing λ = 0 is a simple way to force forecasts to be positive

Automated Box-Cox transformations

```
(BoxCox.lambda(elec))
```

```
## [1] 0.2654
```

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- This attempts to balance the seasonal fluctuations and random variation across the series.
- Always check the results.
- A low value of λ can give extremely large prediction intervals.

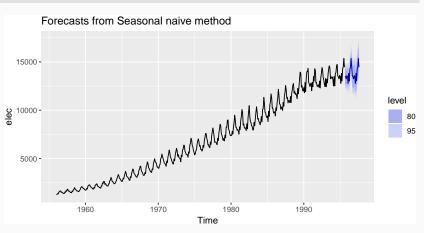
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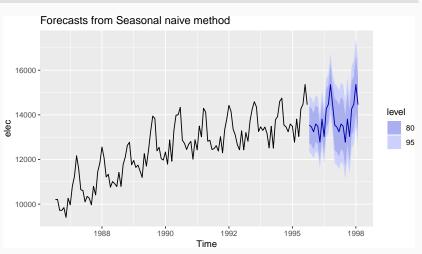
We must reverse the transformation (or back-transform) to obtain forecasts on the original scale. The reverse Box-Cox transformations are given by

$$y_t = \begin{cases} \exp(w_t), & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda}, & \lambda \neq 0. \end{cases}$$

```
fit <- snaive(elec, lambda=1/3)
autoplot(fit)</pre>
```







- Back-transformed point forecasts are medians.
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Back-transformed means

Let X be have mean μ and variance σ^2 .

Let f(x) be back-transformation function, and Y = f(X).

$$E[Y] = E[f(X)] = f(\mu) + \frac{1}{2}\sigma^2[f''(\mu)]^2.$$

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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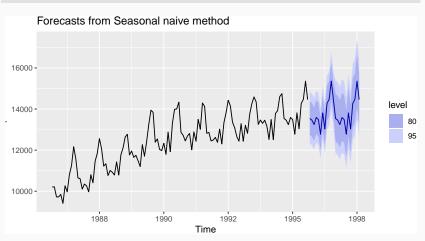
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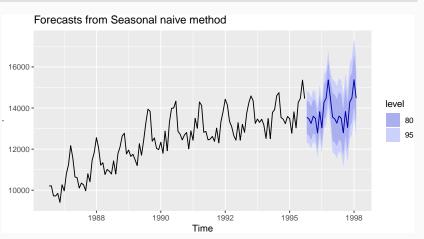
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$$\mathsf{E}[\mathsf{Y}] = \begin{cases} e^{\mu} \left[1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[1 + \frac{\sigma^2 (1 - \lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

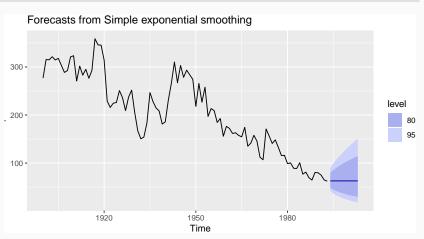
```
elec %>% snaive(lambda=1/3, biasadj=FALSE) %>%
  autoplot(include=120)
```



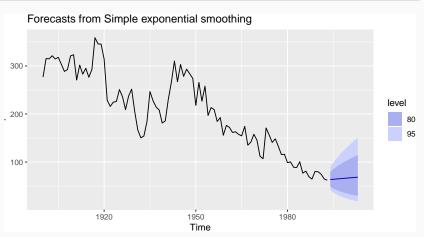
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```
eggs %>% ses(lambda=1/3, biasadj=FALSE) %>%
autoplot
```



```
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