Forecasting: principles and practice

Rob J Hyndman

3.2 Forecasting with multiple seasonality
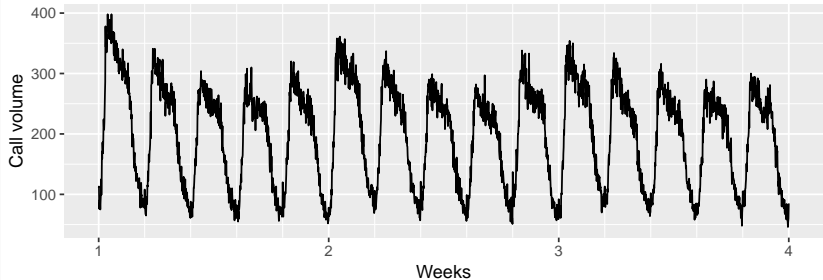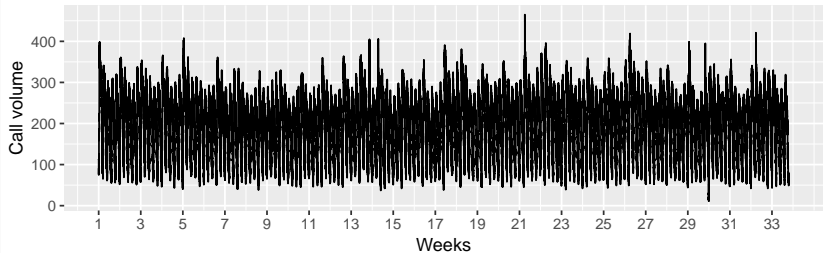
Outline

Examples

```
autoplot(gasoline) +
  xlab("Year") + ylab("Thousands of barrels per day") +
  ggtitle("Weekly US finished motor gasoline products")
```


Weekly US finished motor gasoline products
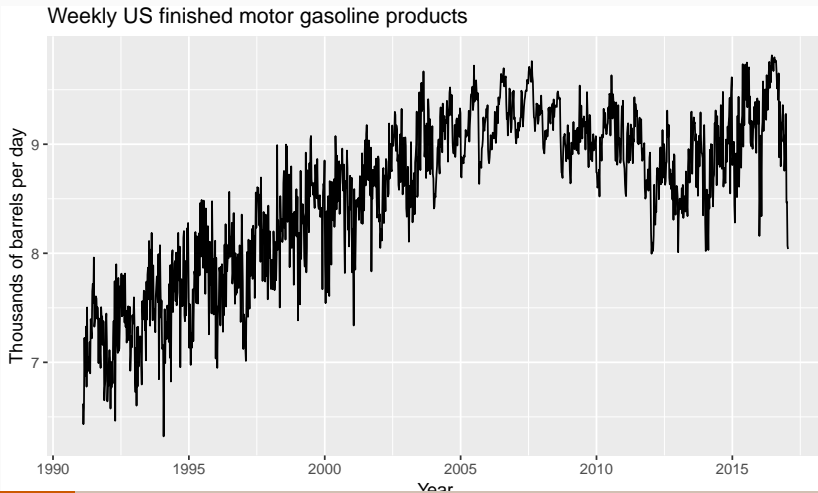
Examples

5 minute call volume at North American bank

Examples

Turkish daily electricity demand

Outline

STL with multiple seasonal periods

```
calls %>% mstl() %>% autoplot() + xlab("Week")
```

STL with multiple seasonal periods

```
calls %>% stlf() %>% autoplot() + xlab("Week")
```


Forecasts from STL + ETS(M,N,N)

STL with multiple seasonal periods

```
calls %>% stlf() %>% autoplot(include=5*169) + xlab("Week")
```
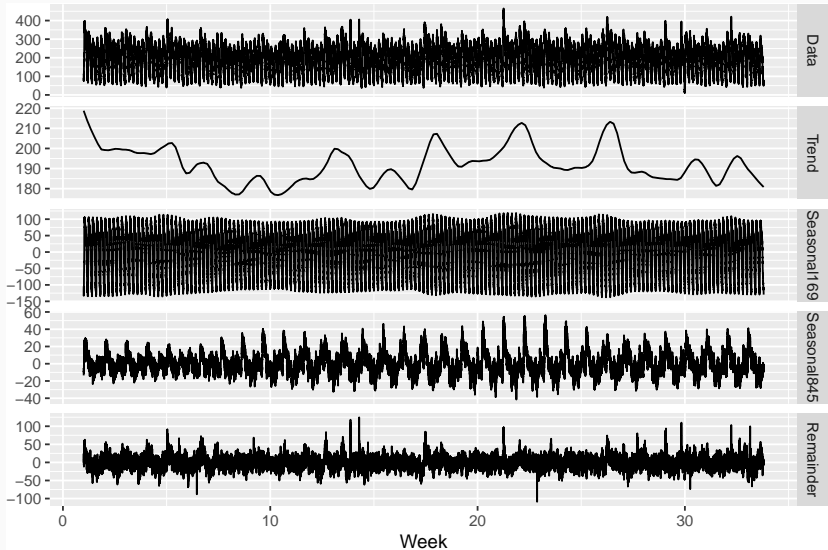

Forecasts from STL + ETS(M,N,N)
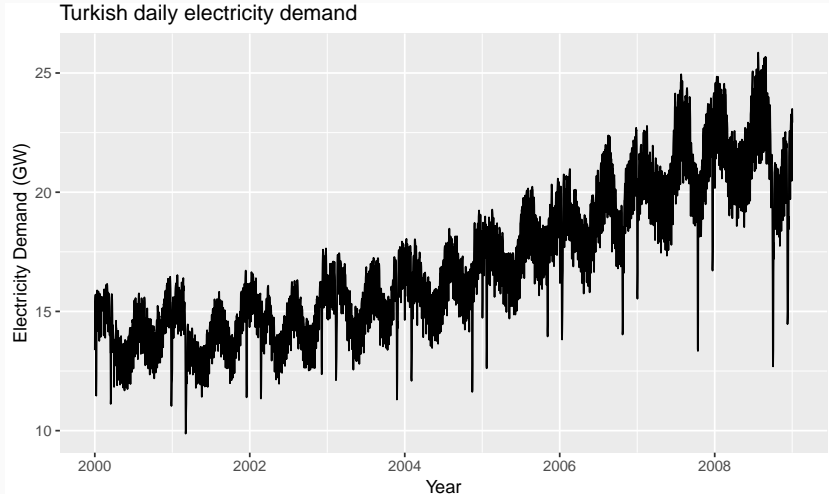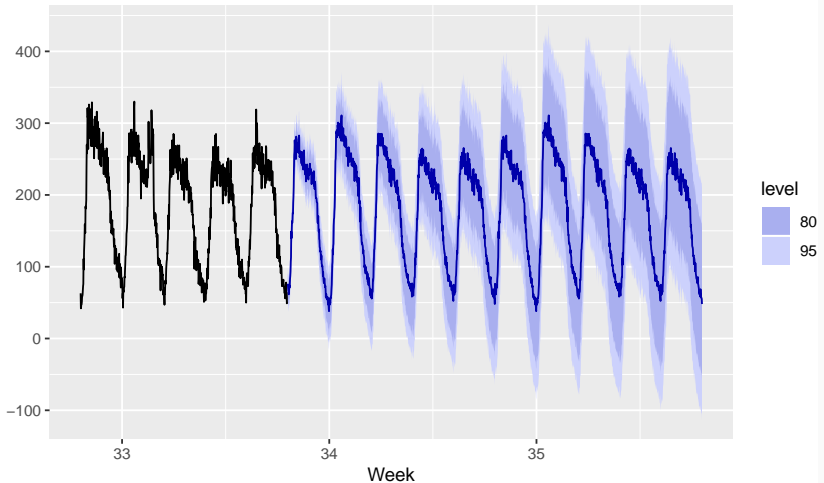
Outline

1. Time series with complex seasonality

2. STL with multiple seasonal periods

3. **Dynamic harmonic regression**

4. TBATS model

5. Lab session 20

Dynamic harmonic regression

```
fit <- auto.arima(calls, seasonal=FALSE, lambda=0,
        xreg=fourier(calls, K=c(10,10)))
fit %>%
  forecast(xreg=fourier(calls, K=c(10,10), h=2*169)) %>%
  autoplot(include=5*169) +
    ylab("Call volume") + xlab("Weeks")
```


Forecasts from Regression with ARIMA(3,1,4) errors

Outline

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TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

y_t = observation at time t

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad \begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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Box-Cox transformation

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M seasonal periods

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$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

global and local trend

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ARMA error

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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ARMA error

Fourier-like seasonal terms

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TBATS model

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Box-Cox transformation

TBATS
Trigonometric
Box-Cox
ARMA
Trend
Seasonal

$$y_t^{(\omega)} = \ell_t \qquad \qquad d_t$$

M seasonal periods

$$\ell_t = \ell_t$$

$$b_t = (1$$

global and local trend

$$d_t = \sum_{i=}^{p}$$

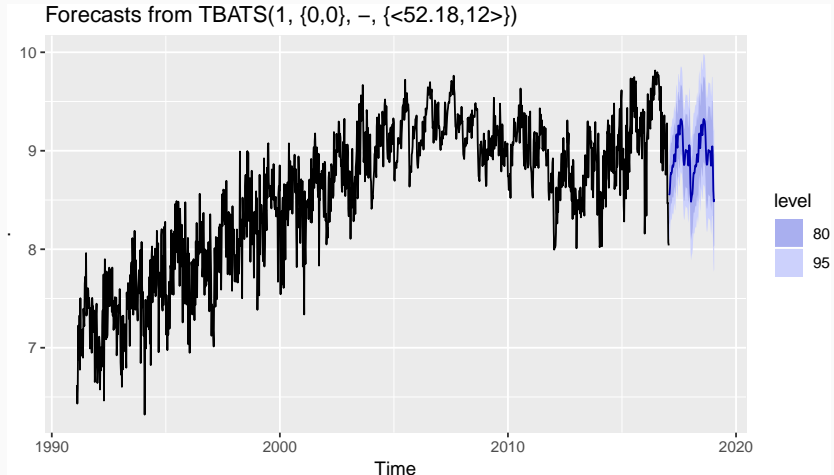ARMA error

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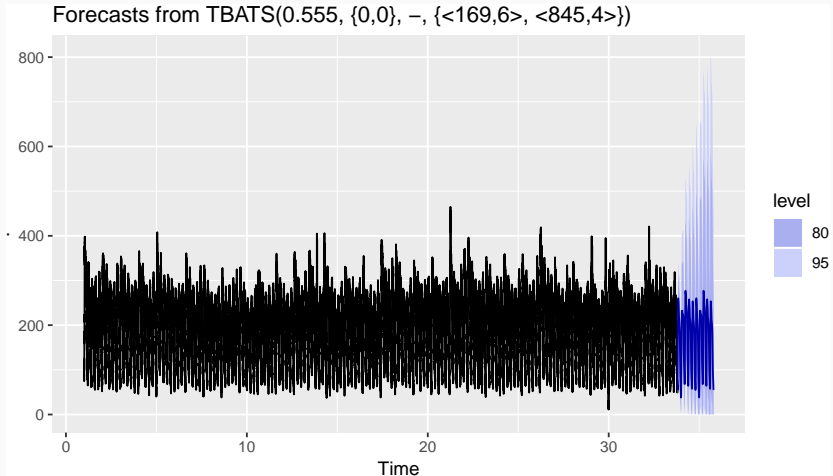
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Complex seasonality

```
gasoline %>% tbats() %>% forecast() %>% autoplot()
```
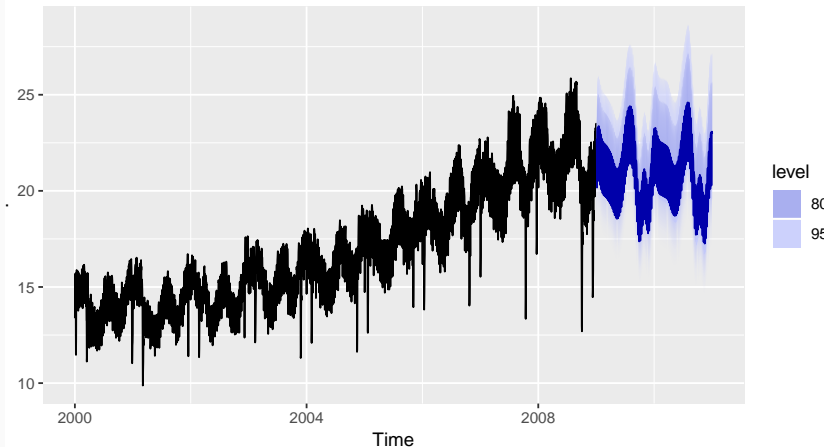

Forecasts from TBATS(1, {0,0}, −, {<52.18,12>})

Complex seasonality

```
calls %>% tbats() %>% forecast() %>% autoplot()
```


Forecasts from TBATS(0.555, {0,0}, –, {<169,6>, <845,4>})

Complex seasonality

```
telec %>% tbats() %>% forecast() %>% autoplot()
```


Forecasts from TBATS(0.005, {4,2}, −, {<7,3>, <354.37,7>, <365.25,3>})

TBATS model

TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

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Lab Session 20