MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

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1.4 Exponential smoothing

#Simple exponential smoothing

##Simple methods

Time series $y_1, y_2, \dots, y_T$.

Random walk forecasts

$$\hat{y}_{T+h|T} = y_T$$

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Average forecasts

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- Want something in between that weights most recent data more highly.
- Simple exponential smoothing uses a weighted moving average with weights that decrease

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Trend methods

##Holt’s linear trend

Component form

$$
\begin{aligned}
\text{Forecast} \quad & \hat{y}_{t+h|t} = \ell_t + hb_t \\
\text{Level} \quad & \ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
\text{Trend} \quad & b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1},
\end{aligned}
$$

Trend methods

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Forecast	$\hat{y}_{t+h\|t} = \ell_t + hb_t$
Level	$\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1})$
Trend	$b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1},$

- Two smoothing parameters α and β^* ($0 \le \alpha, \beta^* \le 1$).
- ℓ_t level: weighted average between y_t one-step ahead forecast for time t, ($\ell_{t-1} + b_{t-1} = \hat{y}_{t|t-1}$)
- b_t slope: weighted average of ($\ell_t - \ell_{t-1}$) and b_{t-1}, current and previous estimate of slope

Trend methods

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Taxonomy of exponential smoothing methods

Trend	Seasonal N	Seasonal A	Seasonal M
N	$\hat{y}_{t+h\|t} = \ell_t$ $\ell_t = \alpha y_t + (1-\alpha)\ell_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)\ell_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = \ell_t s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)\ell_{t-1}$ $s_t = \gamma(y_t/\ell_{t-1}) + (1-\gamma)s_{t-m}$
A	$\hat{y}_{t+h\|t} = \ell_t + hb_t$ $\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + hb_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = (\ell_t + hb_t)s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)(\ell_{t-1} + b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1}$ $s_t = \gamma(y_t/(\ell_{t-1} - b_{t-1})) + (1-\gamma)s_{t-m}$
A_d	$\hat{y}_{t+h\|t} = \ell_t + \phi_h b_t$ $\ell_t = \alpha y_t + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$	$\hat{y}_{t+h\|t} = \ell_t + \phi_h b_t + s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - \phi b_{t-1}) + (1-\gamma)s_{t-m}$	$\hat{y}_{t+h\|t} = (\ell_t + \phi_h b_t)s_{t-m+h_m^+}$ $\ell_t = \alpha(y_t / s_{t-m}) + (1-\alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)\phi b_{t-1}$ $s_t = \gamma(y_t/(\ell_{t-1} - \phi b_{t-1})) + (1-\gamma)s_{t-m}$

##Exponential smoothing methods

Trend Component		Seasonal Component N (None)	A (Additive)	M (Multiplicative)
N	(None)	(N,N)	(N,A)	(N,M)
A	(Additive)	(A,N)	(A,A)	(A,M)
A_d	(Additive damped)	(A_d,N)	(A_d,A)	(A_d,M)

(N,N): Simple exponential smoothing

(A,N): Holt's linear method

(A_d,N): Additive damped trend method

(A,A): Additive Holt-Winters' method