



# Forecasting: principles and practice

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2.2 Transformations

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Logarithms, in particular, are useful because they are

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