MONASH University

MONASH BUSINESS SCHOOL

# Forecasting: principles and practice

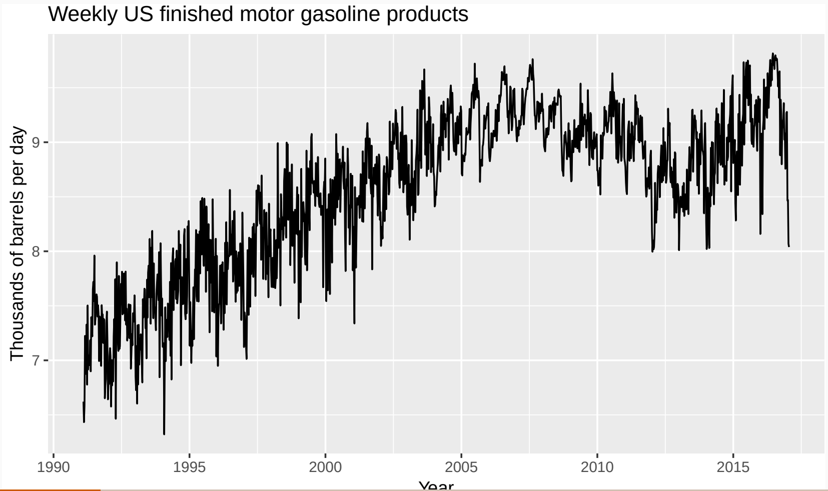
Rob J Hyndman

3.2 Forecasting with multiple seasonality

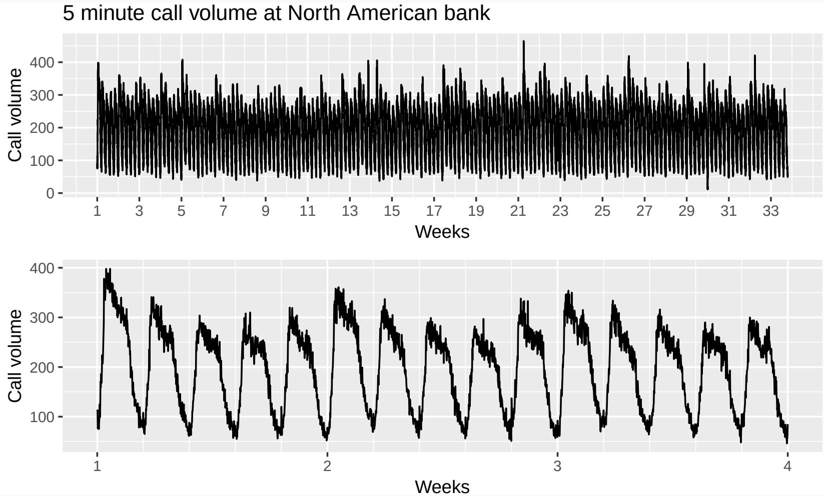
# Outline

# Examples

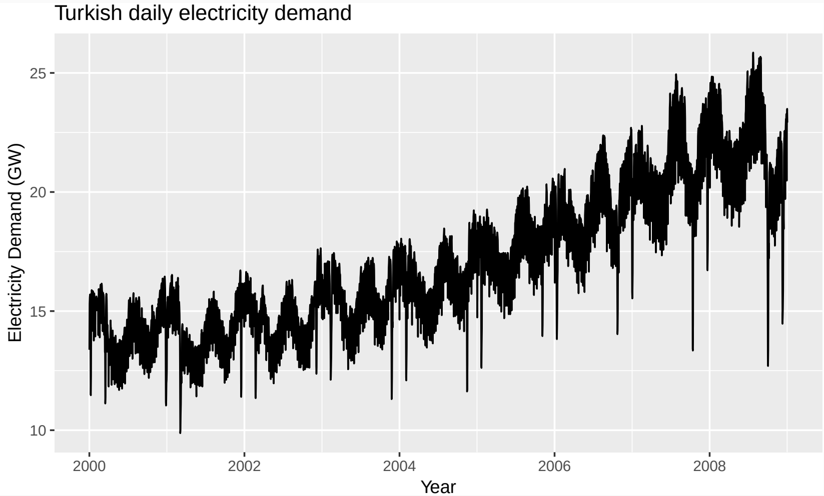
```
autoplot(gasoline) +
  xlab("Year") + ylab("Thousands of barrels per day") +
  ggtitle("Weekly US finished motor gasoline products")
```



# Examples


5 minute call volume at North American bank
Call volume
400
300
200
100
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
Weeks
Call volume
400
300
200
100
1
2
3
4
Weeks

# Examples


Turkish daily electricity demand
25
20
15
10
Electricity Demand (GW)
2000
2002
2004
2006
2008
Year

# TBATS model

## TBATS

**T**rigonometric terms for seasonality
**B**ox-Cox transformations for heterogeneity
**A**RMA errors for short-term dynamics
**T**rend (possibly damped)
**S**easonal (including multiple and non-integer periods)

# TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

$$s_t^{(i)} = \sum^{k_i} s_{j,t}^{(i)}$$

# TBATS model

$$y_t = \text{observation at time } t$$

Box-Cox transformation

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$M$ seasonal periods

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Box-Cox transformation

$M$ seasonal periods

global and local trend

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$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$M$ seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

global and local trend

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

ARMA error

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

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$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \; \mathrm{c} \ldots d_t$$

$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j \ldots s_{j,t-1} \cos \lambda_j \ldots {}_2 d_t$$

$$s_t^{(i)} = \sum^{k_i} s_{j,t}^{(i)}$$

Box-Cox transformation

$M$ seasonal periods

global and local trend

ARMA error

Fourier-like seasonal terms

# TBATS model

$y_t$ = observation at time $t$

$y_t^{(\omega)} =$

$y_t^{(\omega)} =$ … $+ d_t$

$\ell_t =$

$b_t =$

$d_t = \sum_{i=1} \phi_i d_{t-i} + \sum_{j=1} \theta_j \varepsilon_{t-j} + \varepsilon_t$

$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$

$s_t^{(i)} = \sum^{k_i} s_{j,t}^{(i)}$

**TBATS**

**Trigonometric**

**Box-Cox**

**ARMA**

**Trend**

**Seasonal**

Box-Cox transformation

$M$ seasonal periods

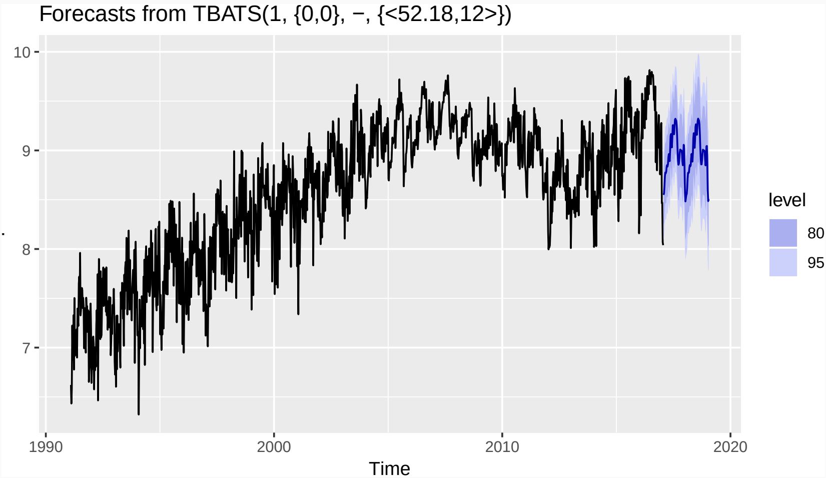
global and local trend

ARMA error

Fourier-like seasonal terms

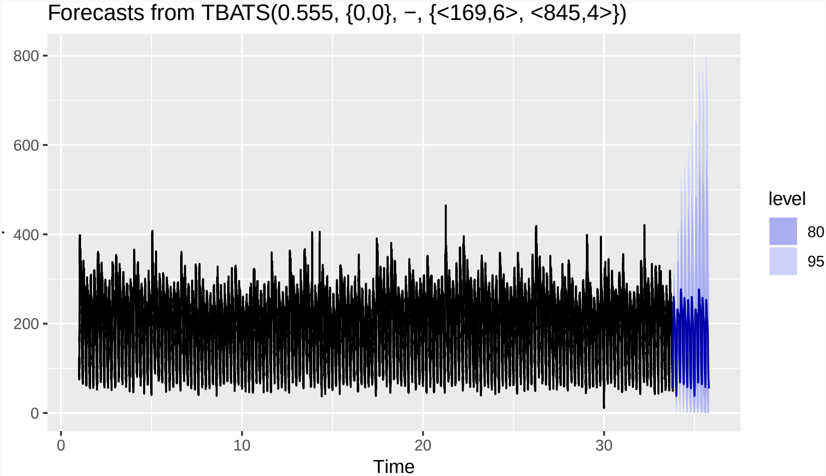
# Complex seasonality

```
gasoline %>% tbats() %>% forecast() %>% autoplot()
```



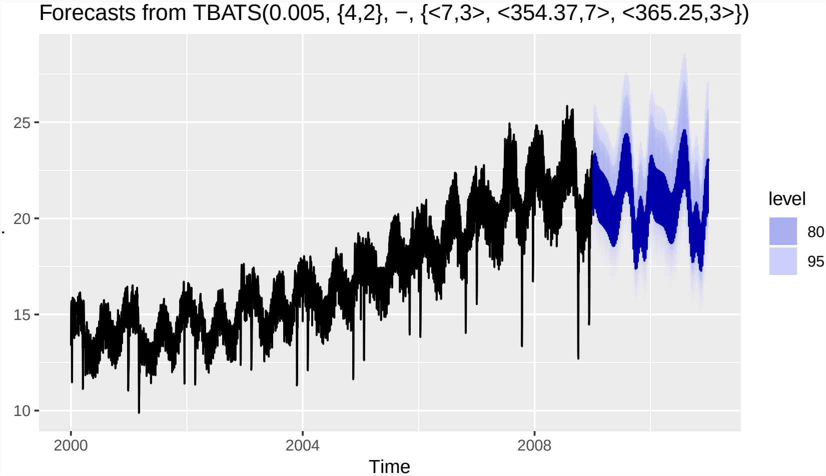
# Complex seasonality

```r
calls %>% tbats() %>% forecast() %>% autoplot()
```



# Complex seasonality

```
telec %>% tbats() %>% forecast() %>% autoplot()
```



# TBATS model

**TBATS**

**T**rigonometric terms for seasonality

**B**ox-Cox transformations for heterogeneity

**A**RMA errors for short-term dynamics

**T**rend (possibly damped)

**S**easonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

# Outline

# Lab Session 17

# Outline

# Lab Session 18

# Outline

# Lab Session 19