



Forecasting: principles and practice

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1.5 Exponential smoothing

#Simple exponential smoothing

##Simple methods

Time series y_1, y_2, \dots, y_T .

Random walk forecasts

$$\hat{y}_{T+h|T} = y_T$$

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- Simple exponential smoothing uses a weighted moving average with weights that decrease

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Trend methods

##Holt's linear trend

Component form

Forecast $\hat{y}_{t+h|t} = \ell_t + hb_t$

Level $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1})$

Trend $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1},$

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- Two smoothing parameters α and β^*
($0 \leq \alpha, \beta^* \leq 1$).
- ℓ_t level: weighted average between y_t one-step ahead forecast for time t , ($\ell_{t-1} + b_{t-1} = \hat{y}_{t|t-1}$)
- b_t slope: weighted average of $(\ell_t - \ell_{t-1})$ and b_{t-1} current and previous estimate of slope

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Taxonomy of exponential smoothing methods

Trend		Seasonal			
##Exponential smoothing methods		A		M	
N	Trend	$\hat{y}_{t+h t} = \ell_t$ $\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$	$\hat{y}_{t+h t} = \ell_t + s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)\ell_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1}) + (1 - \gamma)s_{t-m}$	$\hat{y}_{t+h t} = \ell_t s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t / s_{t-m}) + (1 - \alpha)\ell_{t-1}$ $s_t = \gamma(y_t / \ell_{t-1}) + (1 - \gamma)s_{t-m}$	
		Seasonal Component			
A	Trend	$\hat{y}_{t+h t} = \ell_t + hb_t$ $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + hb_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$	$\hat{y}_{t+h t} = \ell_t + hb_t + s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + hb_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$	$\hat{y}_{t+h t} = (\ell_t + hb_t)s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t / s_{t-m}) + (1 - \alpha)(\ell_{t-1} + hb_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t / (\ell_{t-1} - b_{t-1})) + (1 - \gamma)s_{t-m}$	
		Seasonal Component			
A _d	Trend	$\hat{y}_{t+h t} = \ell_t + \phi_h b_t$ $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$	$\hat{y}_{t+h t} = \ell_t + \phi_h b_t + s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t - \ell_{t-1} - \phi b_{t-1}) + (1 - \gamma)s_{t-m}$	$\hat{y}_{t+h t} = (\ell_t + \phi_h b_t)s_{t-m+h_m}^+$ $\ell_t = \alpha(y_t / s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$ $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$ $s_t = \gamma(y_t / (\ell_{t-1} - \phi b_{t-1})) + (1 - \gamma)s_{t-m}$	
		Seasonal Component			
A _d		(Additive damped)	(A _d ,N)	(A _d ,A)	(A _d ,M)

- (N,N): Simple exponential smoothing
- (A,N): Holt's linear method
- (A_d,N): Additive damped trend method
- (A,A): Additive Holt-Winters' method