Outline

Stationarity

Definition

If $\{y_t\}$ is a stationary time series, then for all s, the distribution of $(y_t, \ldots, y_{t+s})$ does not depend on t.
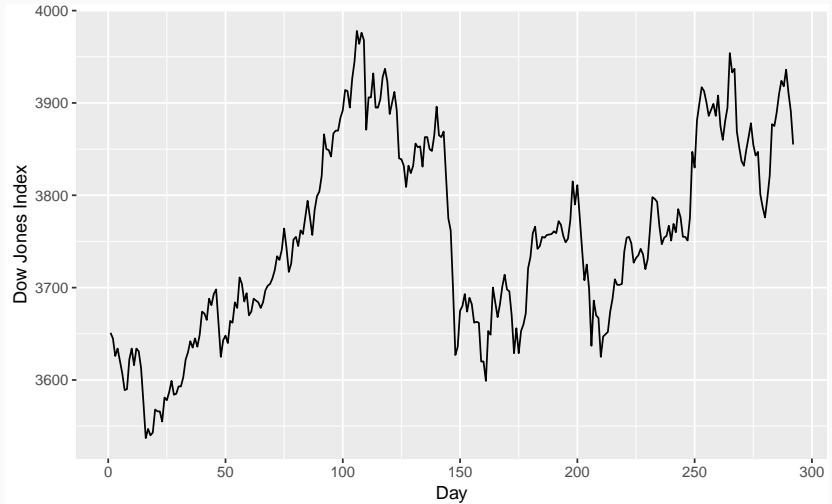
Stationarity

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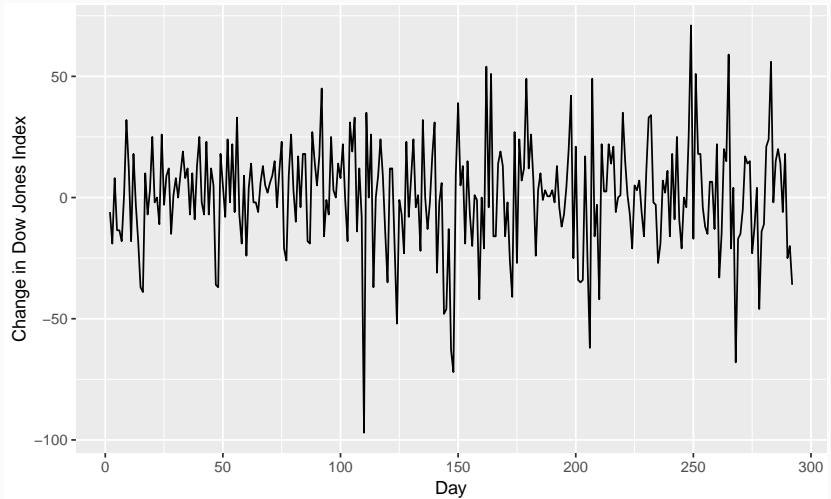
A **stationary series** is:

- roughly horizontal
- constant variance
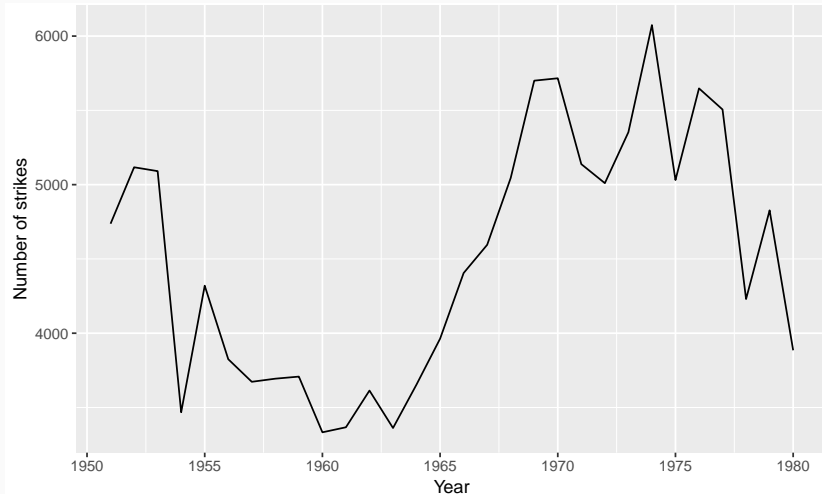- no patterns predictable in the long-term

Stationary?

Stationary?

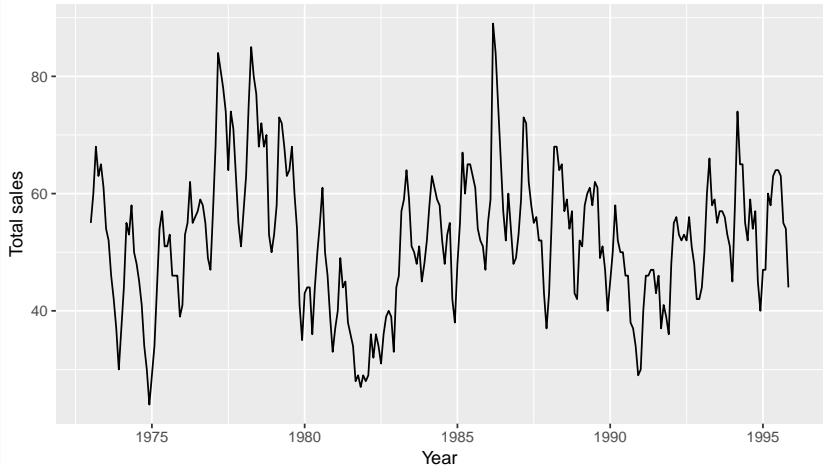# Stationary?

Stationary?

Sales of new one–family houses, USA

Stationary?

Price of a dozen eggs in 1993 dollars

Stationary?

Number of pigs slaughtered in Victoria

Stationary?

Annual Canadian Lynx Trappings
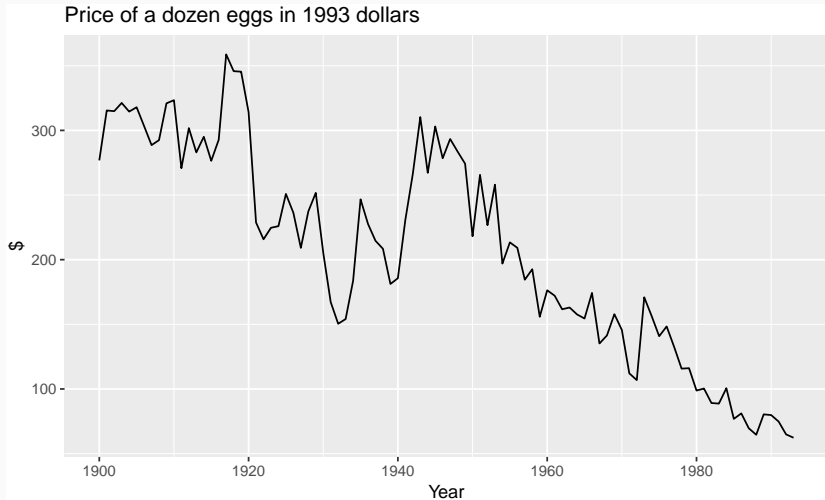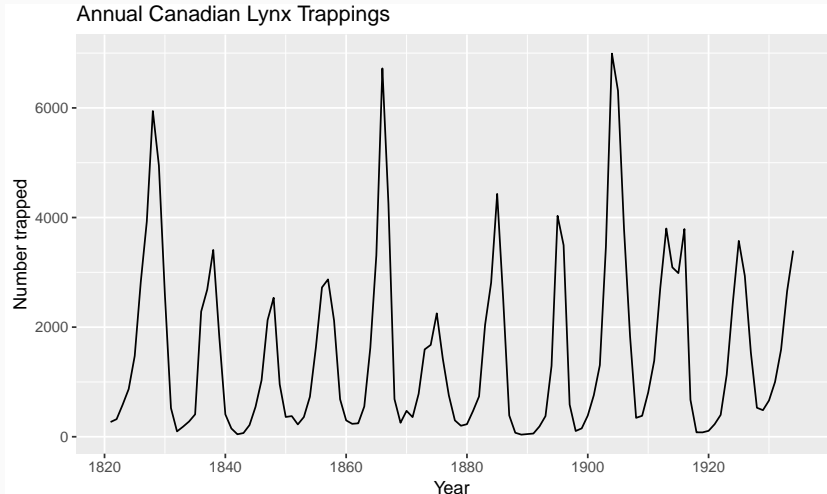
Stationary?

Australian quarterly beer production
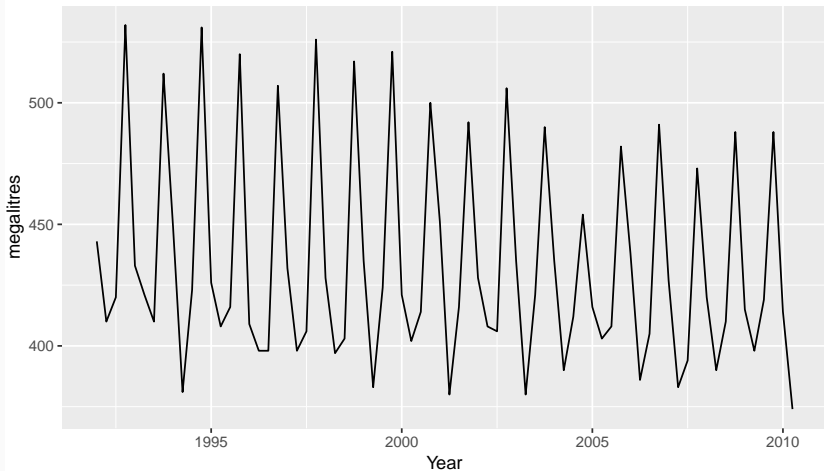
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Transformations help to **stabilize the variance**.

For ARIMA modelling, we also need to **stabilize the mean**.

Non-stationarity in the mean

Identifying non-stationary series

- time plot.

- The ACF of stationary data drops to zero relatively quickly

- The ACF of non-stationary data decreases slowly.

- For non-stationary data, the value of r_1 is often large and positive.

Example: Dow-Jones index

```
autoplot(dj) + ylab("Dow Jones Index") + xlab("Day")
```
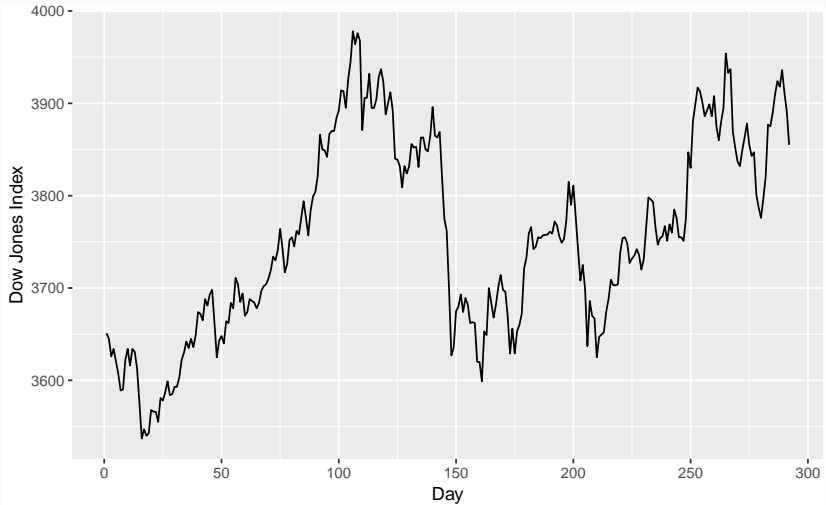
Example: Dow-Jones index

```
ggAcf(dj)
```


Series: dj
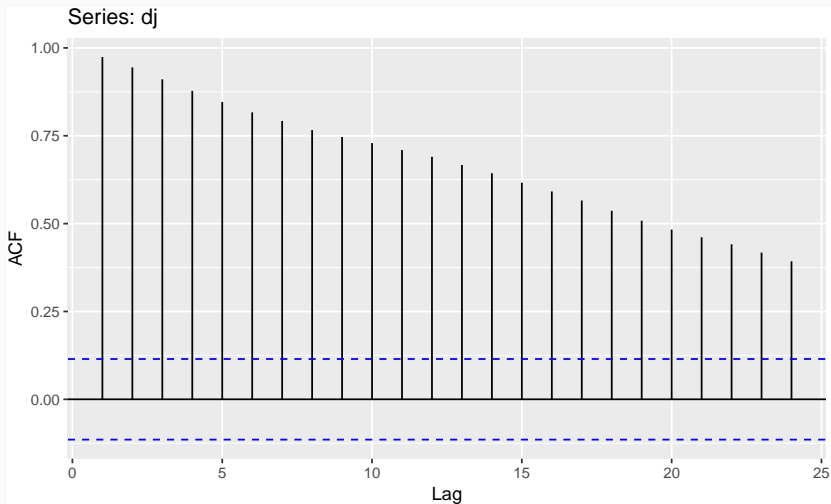
Example: Dow-Jones index

```
autoplot(diff(dj)) +
  ylab("Change in Dow Jones Index") + xlab("Day")
```
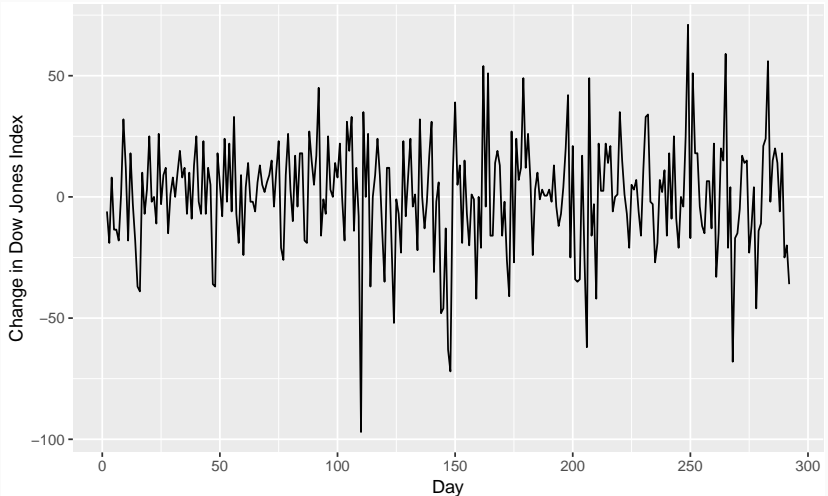
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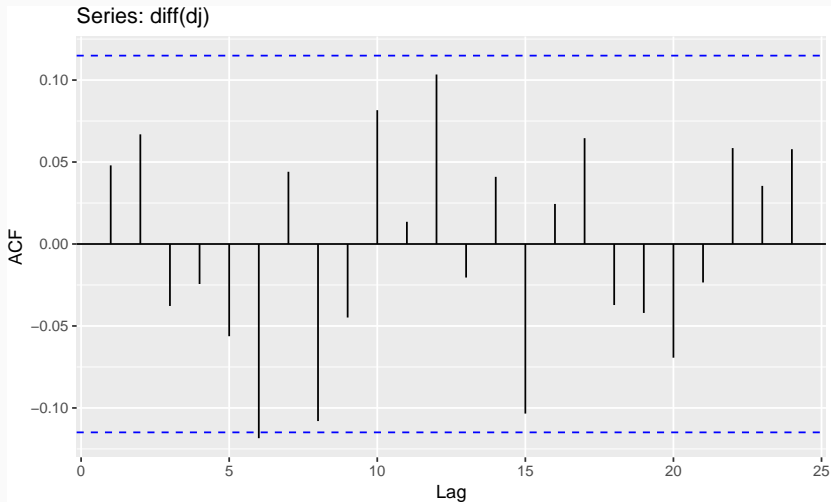
Outline

Differencing

- Differencing helps to **stabilize the mean**.
- The differenced series is the *change* between each observation in the original series:

$$y'_t = y_t - y_{t-1}.$$

- The differenced series will have only $T - 1$ values since it is not possible to calculate a difference y'_1 for the first observation.

Second-order differencing

Occasionally the differenced data will not appear stationary and it may be necessary to difference the data a second time:

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$$y_t'' = y_t' - y_{t-1}'$$
$$= (y_t - y_{t-1}) - (y_{t-1} - y_{t-2})$$
$$= y_t - 2y_{t-1} + y_{t-2}.$$

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- y_t'' will have $T - 2$ values.
- In practice, it is almost never necessary to go beyond second-order differences.

Seasonal differencing

A seasonal difference is the difference between an observation and the corresponding observation from the previous year.

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where m = number of seasons.

- For monthly data m = 12.
- For quarterly data m = 4.

Electricity production

```
usmelec %>% autoplot()
```
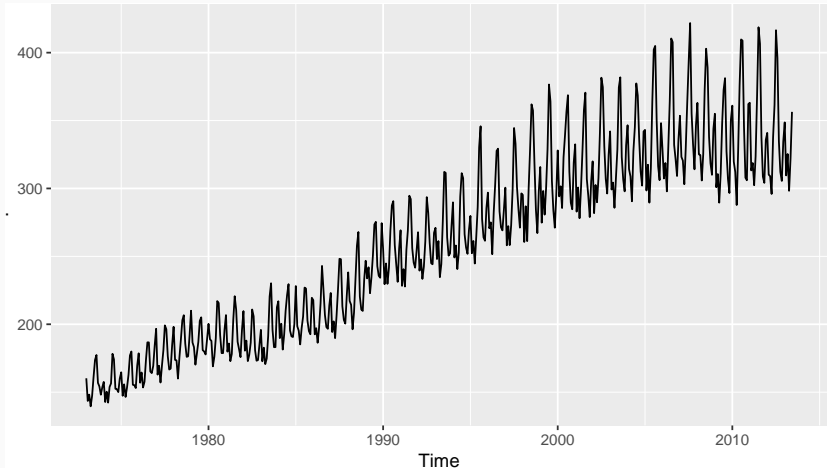
Electricity production

```
usmelec %>% log() %>% autoplot()
```
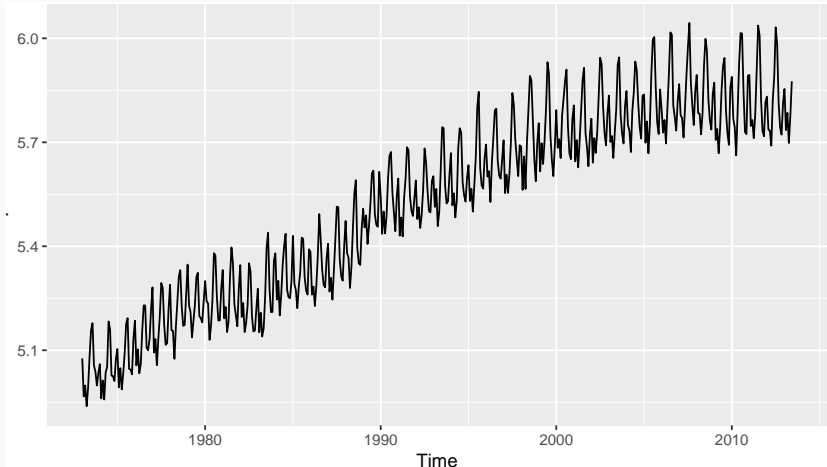
Electricity production

```
usmelec %>% log() %>% diff(lag=12) %>%
  autoplot()
```
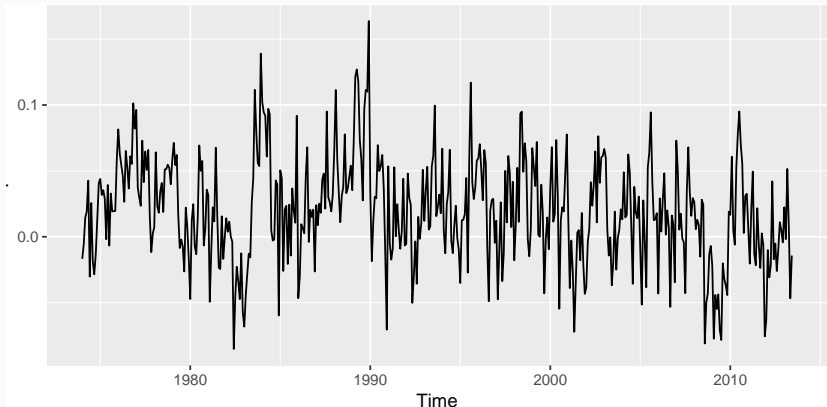
Electricity production

```
usmelec %>% log() %>% diff(lag=12) %>%
  diff(lag=1) %>% autoplot()
```
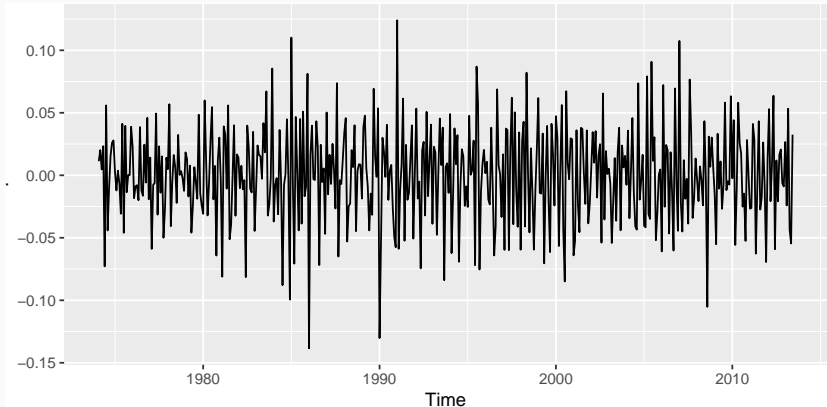
Electricity production

- Seasonally differenced series is closer to being stationary.
- Remaining non-stationarity can be removed with further first difference.

If $y'_t = y_t - y_{t-12}$ denotes seasonally differenced series, then twice-differenced series is

$$
\begin{aligned}
y^*_t &= y'_t - y'_{t-1} \\
&= (y_t - y_{t-12}) - (y_{t-1} - y_{t-13}) \\
&= y_t - y_{t-1} - y_{t-12} + y_{t-13} \, .
\end{aligned}
$$

Seasonal differencing

When both seasonal and first differences are applied...

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- it makes no difference which is done first—the result will be the same.
- If seasonality is strong, we recommend that seasonal differencing be done first because sometimes the resulting series will be stationary and there will be no need for further first difference.

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When both seasonal and first differences are applied…

- it makes no difference which is done first—the result will be the same.
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It is important that if differencing is used, the differences are interpretable.

Interpretation of differencing

- first differences are the change between **one observation and the next**;
- seasonal differences are the change between **one year to the next**.

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But taking lag 3 differences for yearly data, for example, results in a model which cannot be sensibly interpreted.

Outline

Unit root tests

Statistical tests to determine the required order of differencing.

1. Augmented Dickey Fuller test: null hypothesis is that the data are non-stationary and non-seasonal.
2. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test: null hypothesis is that the data are stationary and non-seasonal.
3. Other tests available for seasonal data.

KPSS test

```r
library(urca)
summary(ur.kpss(goog))
```

```
##
## #######################
## # KPSS Unit Root Test #
## #######################
##
## Test is of type: mu with 7 lags.
##
## Value of test-statistic is: 10.72
##
## Critical value for a significance level of:
##                 10pct  5pct 2.5pct  1pct
## critical values 0.347 0.463  0.574 0.739
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```

```
ndiffs(goog)
```

```
## [1] 1
```

Automatically selecting differences

STL decomposition: $y_t = T_t + S_t + R_t$

Seasonal strength $F_s = \max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right)$

If $F_s > 0.64$, do one seasonal difference.

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If $F_s > 0.64$, do one seasonal difference.

```
usmelec %>% log() %>% nsdiffs()
```

```
## [1] 1
```

```
usmelec %>% log() %>% diff(lag=12) %>% ndiffs()
```

```
## [1] 1
```

Outline

Lab Session 14

Outline

35

Backshift notation

A very useful notational device is the backward shift operator, B, which is used as follows:

$$By_t = y_{t-1} \, .$$

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$$B(By_t) = B^2 y_t = y_{t-2} .$$

For monthly data, if we wish to shift attention to "the same month last year," then B^{12} is used, and the notation is $B^{12} y_t = y_{t-12}$.

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Note that a first difference is represented by $(1 - B)$.

Similarly, if second-order differences (i.e., first differences of first differences) have to be computed, then:

$$y''_t = y_t - 2y_{t-1} + y_{t-2} = (1 - B)^2 y_t \,.$$

Backshift notation

- Second-order difference is denoted $(1 - B)^2$.

- *Second-order difference* is not the same as a *second difference*, which would be denoted $1 - B^2$;

- In general, a *d*th-order difference can be written as

$$(1 - B)^d y_t.$$

- A seasonal difference followed by a first difference can be written as

$$(1 - B)(1 - B^m) y_t.$$

Backshift notation

The "backshift" notation is convenient because the terms can be multiplied together to see the combined effect.

$$(1 - B)(1 - B^m)y_t = (1 - B - B^m + B^{m+1})y_t$$
$$= y_t - y_{t-1} - y_{t-m} + y_{t-m-1}.$$

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The "backshift" notation is convenient because the terms can be multiplied together to see the combined effect.

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$$= y_t - y_{t-1} - y_{t-m} + y_{t-m-1}.$$

For monthly data, $m = 12$ and we obtain the same result as earlier.