



# Forecasting: principles and practice

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3.4 Extras

# Outline

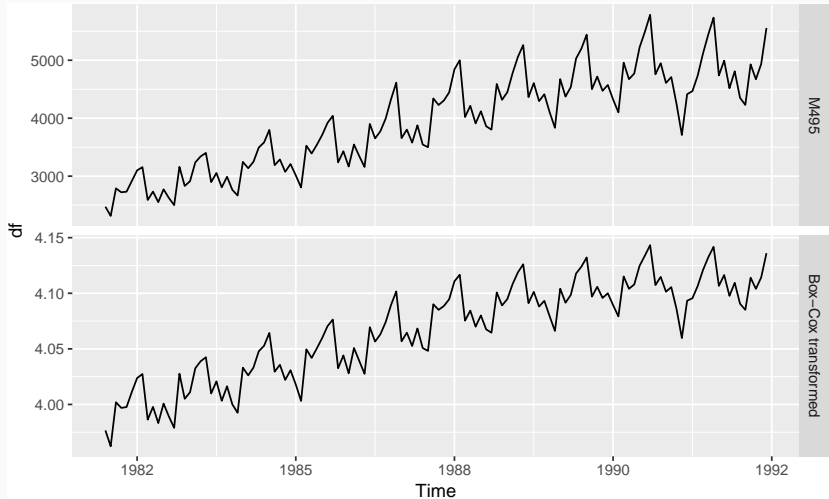
- 1 Bagged ETS
- 2 Models for different frequencies
- 3 Ensuring forecasts stay within limits
- 4 Forecast combinations
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# Bagged ETS

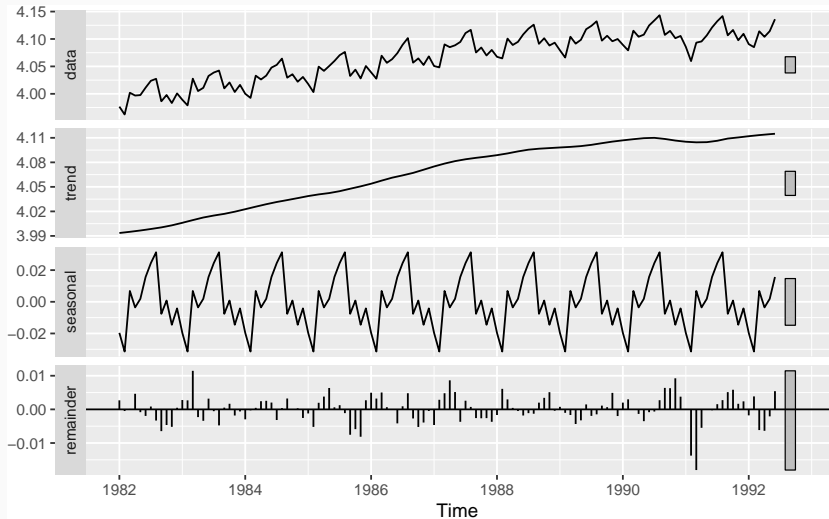
## Algorithm: Generating bootstrapped series

```
bootstrap ← function(ts, num.boot) {  
  lambda ← BoxCox.lambda(ts, min=0, max=1)  
  ts.bc ← BoxCox(ts, lambda)  
  if(ts is seasonal) {  
    [trend, seasonal, remainder] ← stl(ts.bc)  
  }  
  else {  
    seasonal ← 0  
    [trend, remainder] ← loess(ts.bc)  
  }  
  recon.series[1] ← ts  
  for(i in 2:num.boot) {  
    boot.sample[i] ← MBB(remainder)  
    recon.series.bc[i] ← trend + seasonal + boot.sample[i]  
    recon.series[i] ← InvBoxCox(recon.series.bc[i], lambda)  
  }  
  return(recon.series)
```

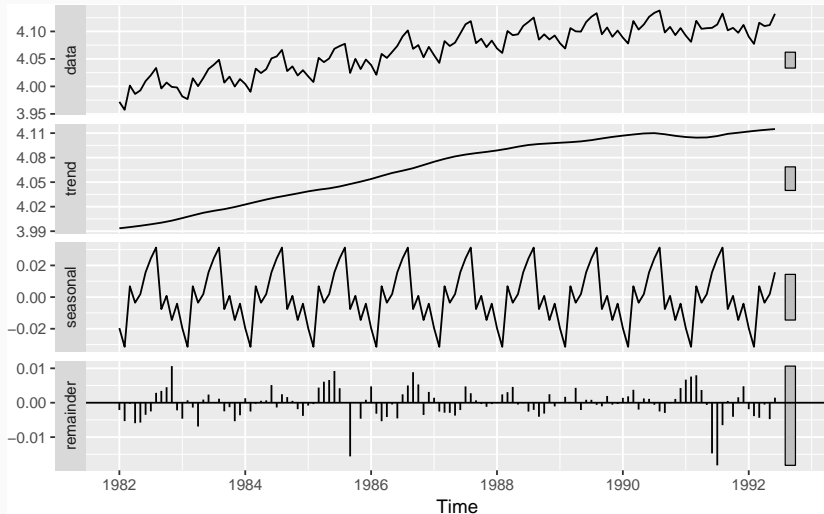
# Bagged ETS



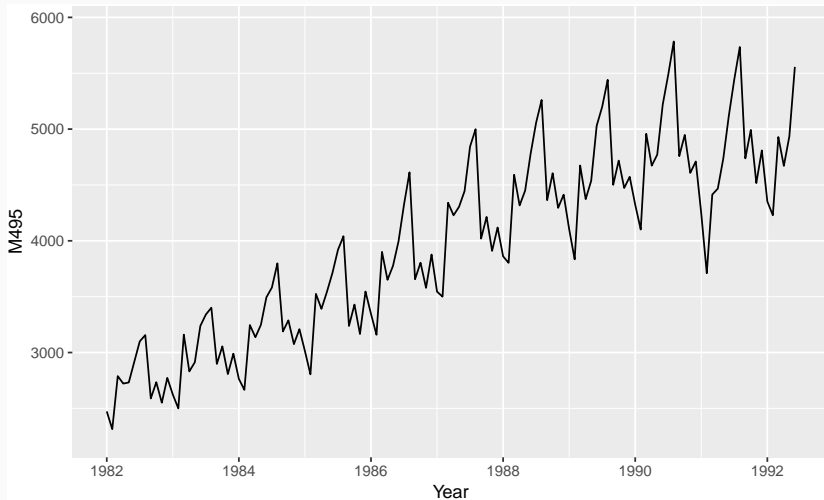
# Bagged ETS



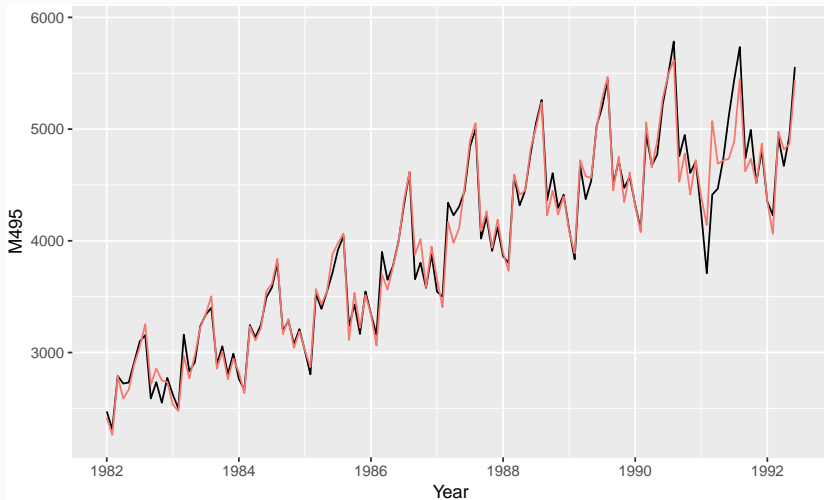
# Bagged ETS



# Bagged ETS

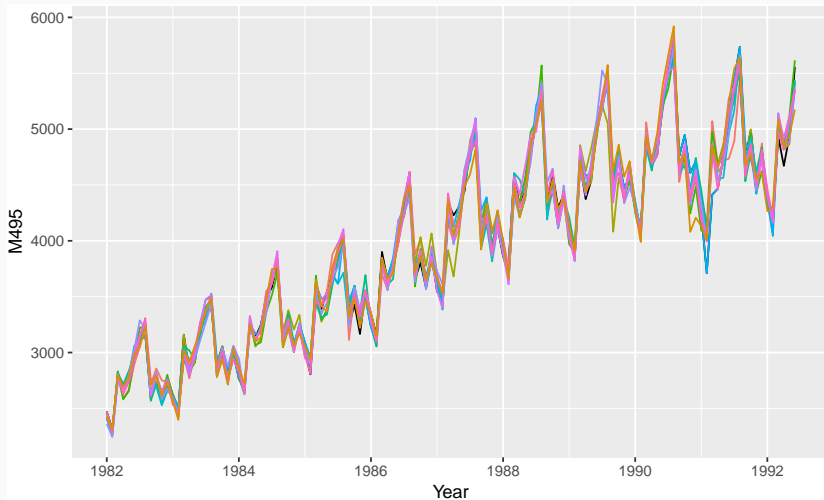


# Bagged ETS



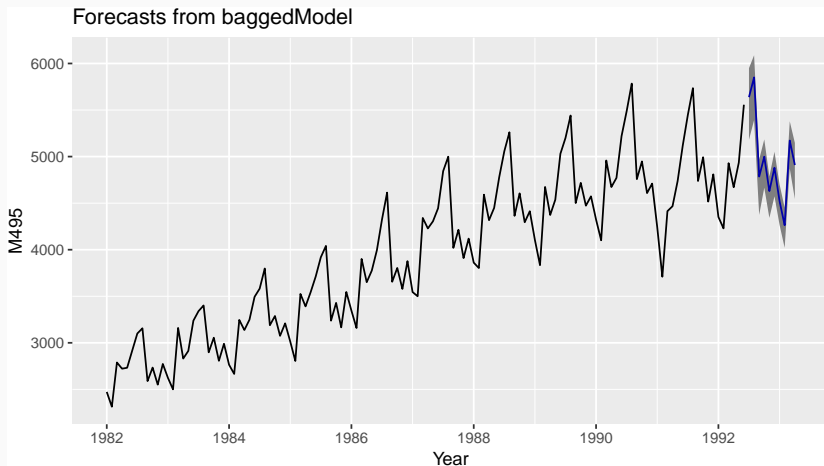


# Bagged ETS

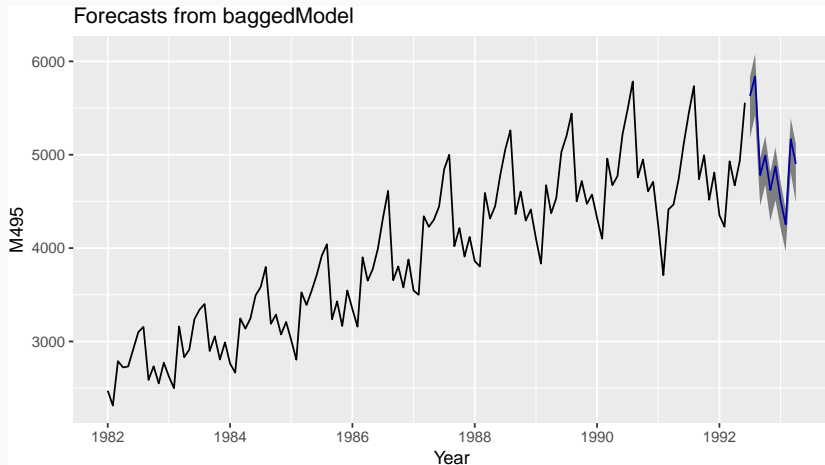


# Bagged ETS

```
baggedETS(Mcomp::M3[[1896]]$x) %>%  
  forecast %>% autoplot +  
  xlab("Year") + ylab("M495")
```



# Bagged ETS



- Intervals show range of point forecasts
- They are not prediction intervals

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# Models for different frequencies

## Models for annual data

- ETS, ARIMA, Dynamic regression

# Models for different frequencies

## Models for annual data

- ETS, ARIMA, Dynamic regression

## Models for quarterly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

# Models for different frequencies

## Models for annual data

- ETS, ARIMA, Dynamic regression

## Models for quarterly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

## Models for monthly data

- ETS, ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA

# Models for different frequencies

## Models for weekly data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS



# Models for different frequencies

## Models for weekly data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS

## Models for daily, hourly and other sub-daily data

- ARIMA/SARIMA, Dynamic regression, Dynamic harmonic regression, STL+ETS, STL+ARIMA, TBATS

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# Positive forecasts

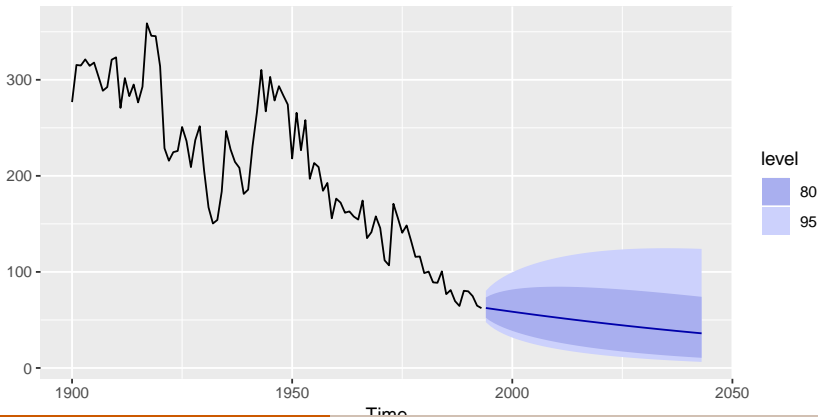
eggs %>%

```
ets(model="AAN", damped=FALSE, lambda=0) %>%
```

```
forecast(h=50, biasadj=TRUE) %>%
```

```
autoplot()
```

Forecasts from ETS(A,A,N)



# Forecasts constrained to an interval

Suppose egg prices constrained to lie within  $a = 50$  and  $b = 400$ .

Transform data using scaled logit transform:

$$y = \log \left( \frac{x - a}{b - x} \right),$$

where  $x$  is on the original scale and  $y$  is the transformed data. To reverse the transformation, we will use

$$x = \frac{(b - a)e^y}{1 + e^y} + a.$$

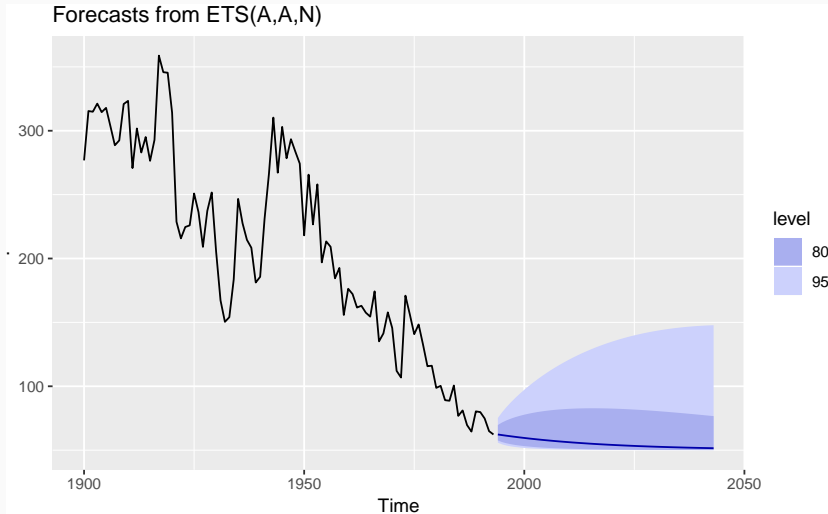
# Forecasts constrained to an interval

```
# Bounds
a <- 50
b <- 400

# Transform data and fit model
fit <- log((eggs-a)/(b-eggs)) %>%
  ets(model="AAN", damped=FALSE)
fc <- forecast(fit, h=50)

# Back-transform forecasts
fc[["mean"]] <- (b-a)*exp(fc[["mean"]]) /
  (1+exp(fc[["mean"]])) + a
fc[["lower"]] <- (b-a)*exp(fc[["lower"]]) /
  (1+exp(fc[["lower"]])) + a
fc[["upper"]] <- (b-a)*exp(fc[["upper"]]) /
  (1+exp(fc[["upper"]])) + a
fc[["x"]] <- eggs
autoplot(fc)
```

# Forecasts constrained to an interval



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# Forecast combinations

## Clemen (1989)

“The results have been virtually unanimous: combining multiple forecasts leads to increased forecast accuracy. ... In many cases one can make dramatic performance improvements by simply averaging the forecasts.”



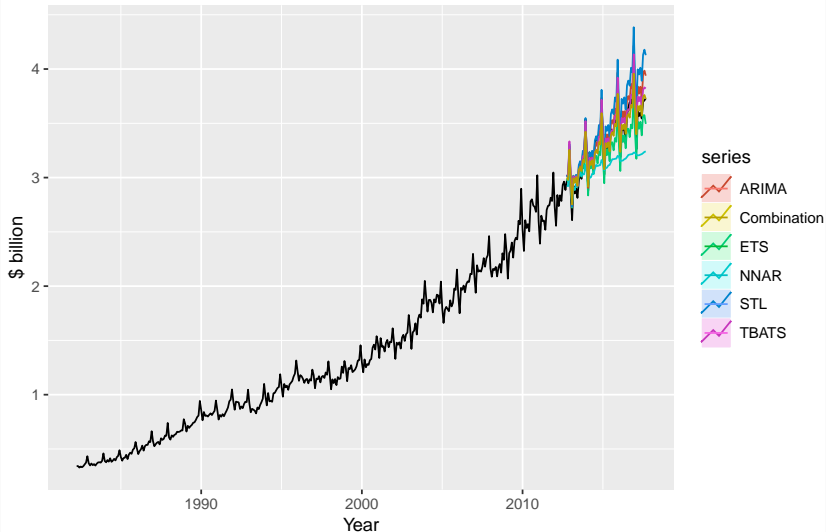
# Forecast combinations

```
train <- window(auscafe, end=c(2012,9))
h <- length(auscafe) - length(train)
ETS <- forecast(ets(train), h=h)
ARIMA <- forecast(auto.arima(train, lambda=0, biasadj=TRUE),
  h=h)
STL <- stlf(train, lambda=0, h=h, biasadj=TRUE)
NNAR <- forecast(nnetar(train), h=h)
TBATS <- forecast(tbats(train, biasadj=TRUE), h=h)
Combination <- (ETS[["mean"]] + ARIMA[["mean"]] +
  STL[["mean"]] + NNAR[["mean"]] + TBATS[["mean"]])/5

autoplot(auscafe) +
  autolayer(ETS, series="ETS", PI=FALSE) +
  autolayer(ARIMA, series="ARIMA", PI=FALSE) +
  autolayer(STL, series="STL", PI=FALSE) +
  autolayer(NNAR, series="NNAR", PI=FALSE) +
  autolayer(TBATS, series="TBATS", PI=FALSE) +
  autolayer(Combination, series="Combination") +
  xlab("Year") + ylab("$ billion") +
  ggtitle("Australian monthly expenditure on eating out")
```

# Forecast combinations

Australian monthly expenditure on eating out



# Forecast combinations

```
c(ETS = accuracy(ETS, auscafe)["Test set", "RMSE"],  
  ARIMA = accuracy(ARIMA, auscafe)["Test set", "RMSE"],  
  STL-ETS = accuracy(STL, auscafe)["Test set", "RMSE"],  
  NNAR = accuracy(NNAR, auscafe)["Test set", "RMSE"],  
  TBATS = accuracy(TBATS, auscafe)["Test set", "RMSE"],  
  Combination =  
    accuracy(Combination, auscafe)["Test set", "RMSE"])
```

##	ETS	ARIMA	STL-ETS	NNAR
##	0.13700	0.12146	0.21446	0.30407
##	TBATS	Combination		
##	0.09406	0.07115		

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# Prediction intervals for aggregates

```
# First fit a model to the data
```

```
fit <- ets(gas/1000)
```

```
# Forecast six months ahead
```

```
fc <- forecast(fit, h=6)
```

```
sum(fc[["mean"]][1:6])
```

```
## [1] 281.8
```

```
# Simulate 10000 future sample paths
```

```
nsim <- 10000
```

```
h <- 6
```

```
sim <- numeric(nsim)
```

```
for(i in seq_len(nsim))
```

```
  sim[i] <- sum(simulate(fit, future=TRUE, nsim=h))
```

```
mean(sim)
```

```
## [1] 281.9
```

# Prediction intervals for aggregates

*#80% interval:*

```
quantile(sim, prob=c(0.1, 0.9))
```

```
##      10%      90%
```

```
## 263.4 301.0
```

*#95% interval:*

```
quantile(sim, prob=c(0.025, 0.975))
```

```
##      2.5%     97.5%
```

```
## 254.4 311.4
```

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# Backcasting

```
# Function to reverse time
reverse_ts <- function(y)
{
  ts(rev(y), start=tsp(y)[1L], frequency=frequency(y))
}

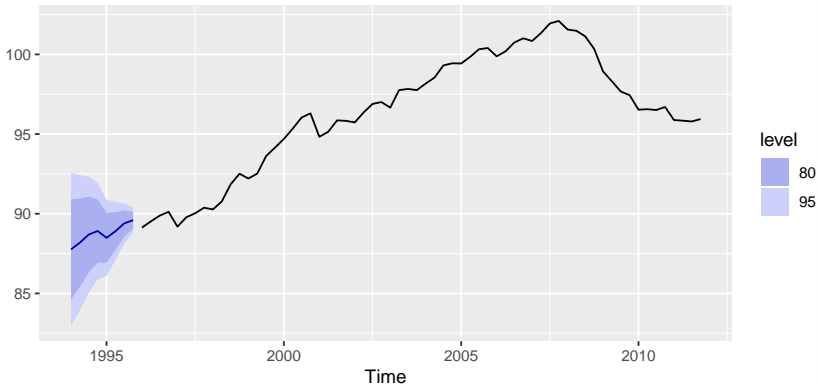
# Function to reverse a forecast
reverse_forecast <- function(object)
{
  h <- length(object[["mean"]])
  f <- frequency(object[["mean"]])
  object[["x"]] <- reverse_ts(object[["x"]])
  object[["mean"]] <- ts(rev(object[["mean"]]),
    end=tsp(object[["x"]])[1L]-1/f, frequency=f)
  object[["lower"]] <- object[["lower"]][h:1L,]
  object[["upper"]] <- object[["upper"]][h:1L,]
  return(object)
}
```



# Backcasting

```
euretail %>% reverse_ts() %>%  
  auto.arima() %>%  
  forecast() %>% reverse_forecast() -> bc  
autoplot(bc) +  
  ggtitle(paste("Backcasts from", bc[["method"]]))
```

Backcasts from ARIMA(1,1,2)(0,1,1)[4]



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# Missing values

## Functions which can handle missing values

- `auto.arima()`, `Arima()`
- `tslm()`
- `nnetar()`

## Models which cannot handle missing values

- `ets()`
- `stl()`
- `stlf()`
- `tbats()`

# Missing values

## Functions which can handle missing values

- `auto.arima()`, `Arima()`
- `tslm()`
- `nnetar()`

## Models which cannot handle missing values

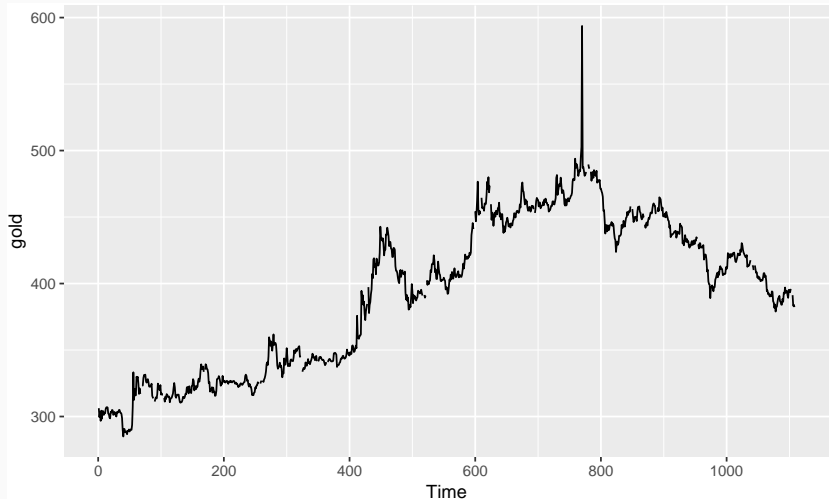
- `ets()`
- `stl()`
- `stlf()`
- `tbats()`

## What to do?

- 1 Model section of data after last missing value.
- 2 Estimate missing values with `na.interp()`.

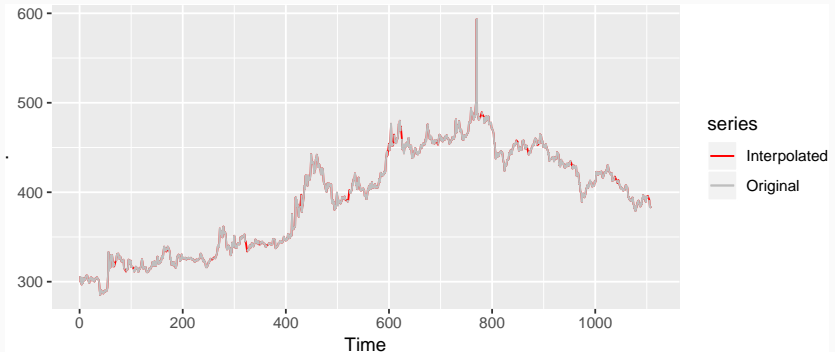
# Missing values

```
autoplot(gold)
```



# Missing values

```
gold %>% na.interp() %>%  
  autoplot(series="Interpolated") +  
  autolayer(gold, series="Original") +  
  scale_color_manual(  
    values=c(Interpolated="red",Original="gray"))
```

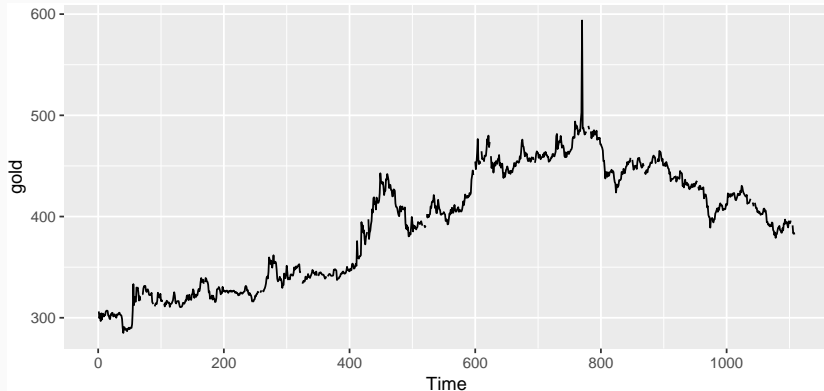


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# Outliers

```
autoplot(gold)
```





# Outliers

```
tsoutliers(gold)
```

```
## $index
```

```
## [1] 770
```

```
##
```

```
## $replacements
```

```
## [1] 494.9
```

# Outliers

```
gold %>% tsclean() %>% autoplot()
```

