

# Forecasting: principles and practice

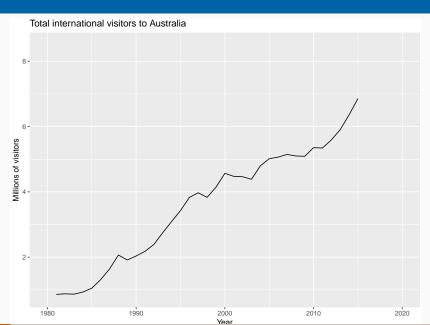
Rob J Hyndman

1.2 The forecaster's toolbox

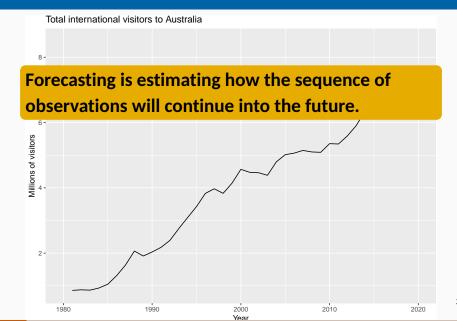
## **Outline**

- 1 The statistical forecasting perspective
- 2 Some simple forecasting methods
- **3** Forecasting residuals
- 4 Evaluating forecast accuracy

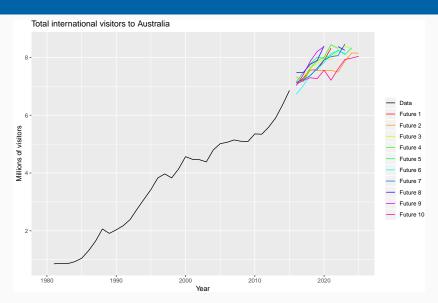
# The statistical forecasting perspective



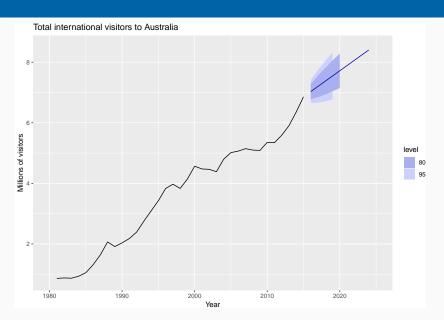
## The statistical forecasting perspective



# Sample futures

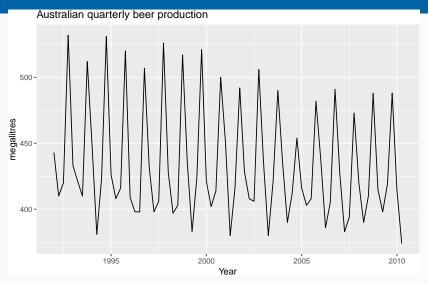


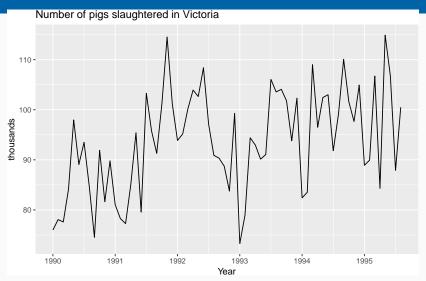
## **Forecast intervals**

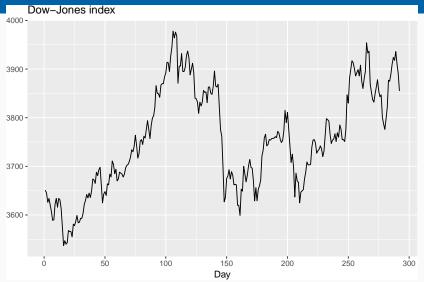


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#### Average method

- Forecast of all future values is equal to mean of historical data  $\{y_1, \dots, y_T\}$ .
- Forecasts:  $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$

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#### Naïve method

- Forecasts equal to last observed value.
- Forecasts:  $\hat{y}_{T+h|T} = y_T$ .
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#### Naïve method

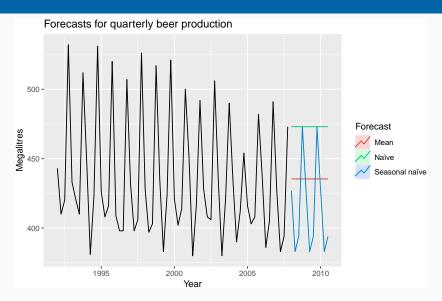
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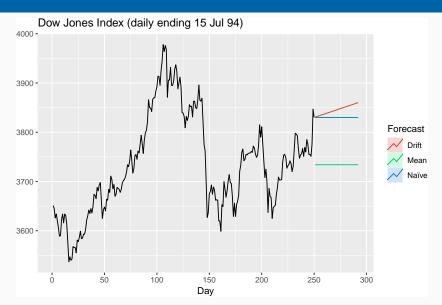
#### Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts:  $\hat{y}_{T+h|T} = y_{T+h-km}$  where m = seasonal period and k = |(h-1)/m|+1.

#### **Drift** method

- Forecasts equal to last value plus average change.
- Forecasts:  $\hat{y}_{T+h|T} = y_T + \frac{h}{T-1} \sum_{t=2}^{T} (y_t y_{t-1})$ =  $y_T + \frac{h}{T-1} (y_T - y_1)$ .
- Equivalent to extrapolating a line drawn between first and last observations.





- Mean: meanf(y, h=20)
- Naïve: naive(y, h=20)
- Seasonal naïve: snaive(y, h=20)
- Drift: rwf(y, drift=TRUE, h=20)

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## **Fitted values**

- $\hat{y}_{t|t-1}$  is the forecast of  $y_t$  based on observations  $y_1, \dots, y_t$ .
- We call these "fitted values".
- Sometimes drop the subscript:  $\hat{y}_t \equiv \hat{y}_{t|t-1}$ .
- Often not true forecasts since parameters are estimated on all data.

#### ###For example:

- $\hat{y}_t = \bar{y}$  for average method.
- $\hat{y}_t = y_{t-1} + (y_T y_1)/(T 1)$  for drift method.

## **Forecasting residuals**

**Residuals in forecasting:** difference between observed value and its fitted value:  $e_t = y_t - \hat{y}_{t|t-1}$ .

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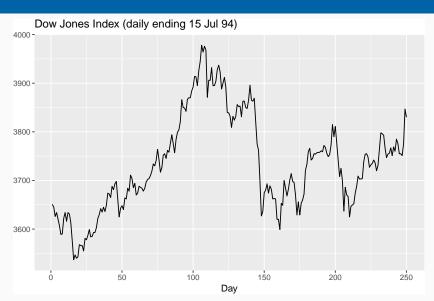
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(for prediction intervals)

 $\{e_t\}$  have constant variance.

 $\{e_{t}\}$  are normally distributed

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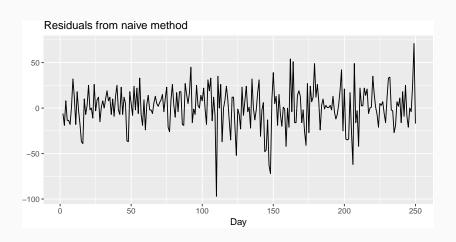
$$e_t = y_t - y_{t-1}$$

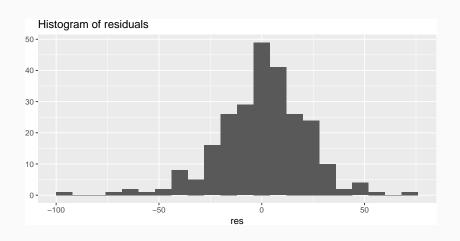
Naïve forecast:

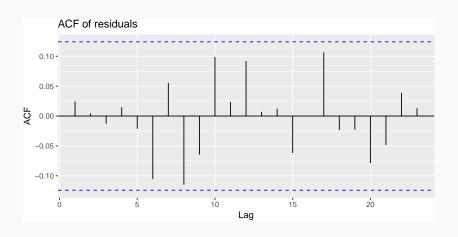
$$\hat{\mathbf{y}}_{t|t-1} = \mathbf{y}_{t-1}$$

$$e_t = y_t - y_{t-1}$$

Note:  $e_t$  are one-step-forecast residuals







## **ACF of residuals**

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We expect these to look like white noise.

Consider a whole set of  $r_k$  values, and develop a test to see whether the set is significantly different from a zero set.

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#### **Box-Pierce test**

$$Q = T \sum_{k=1}^{h} r_k^2$$

where *h* is max lag being considered and *T* is number of observations.

- My preferences: h = 10 for non-seasonal data,
   h = 2m for seasonal data.
- If each  $r_k$  close to zero, Q will be **small**.

Consider a whole set of  $r_k$  values, and develop a test to see whether the set is significantly different from a zero set.

## Ljung-Box test

$$Q^* = T(T+2) \sum_{k=1}^{h} (T-k)^{-1} r_k^2$$

where *h* is max lag being considered and *T* is number of observations.

- My preferences: h = 10 for non-seasonal data,
   h = 2m for seasonal data.
- Better performance, especially in small samples.

##

- If data are WN,  $Q^*$  has  $\chi^2$  distribution with (h K) degrees of freedom where K = no. parameters in model.
- When applied to raw data, set K = 0.
- For the Dow-Jones example,

```
# lag=h and fitdf=K
Box.test(res, lag=10, fitdf=0)
```

```
## Box-Pierce test
##
## data: res
## X-squared = 10.655, df = 10, p-value = 0.385
```

##

##

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Box-Ljung test

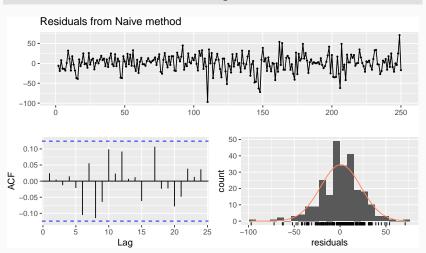
```
# lag=h and fitdf=K
Box.test(res, lag=10, fitdf=0, type="Lj")
```

```
##
## data: res
## X-squared = 11.088, df = 10, p-value = 0.3507
```

#### checkresiduals function

##

#### checkresiduals(naive(dj2))



#### checkresiduals function

```
##
## Ljung-Box test
##
## data: Residuals from Naive method
## Q* = 11.088, df = 10, p-value = 0.3507
## Model df: 0. Total lags used: 10
#Lab session 3 ##
```

# Lab Session 3

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Let  $y_t$  denote the tth observation and  $\hat{y}_{t|t-1}$  denote its forecast based on all previous data, where t = 1, ..., T. Then the following measures are useful.

$$\begin{aligned} \text{MAE} &= T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}| \\ \text{MSE} &= T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2 \quad \text{RMSE} \quad = \sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2} \\ \text{MAPE} &= 100 T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}| / |y_t| \end{aligned}$$

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MSE =  $T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2$  RMSE =  $\sqrt{T^{-1} \sum_{t=1}^{T} (y_t - \hat{y}_{t|t-1})^2}$ 

MAPE = 
$$100T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/|y_t|$$

MAE, MSE, RMSE are all scale dependent. MAPF is scale independent but is only sensible if

#### **Mean Absolute Scaled Error**

MASE = 
$$T^{-1} \sum_{t=1}^{T} |y_t - \hat{y}_{t|t-1}|/Q$$

where Q is a stable measure of the scale of the time series  $\{y_t\}$ .

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^{T} |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

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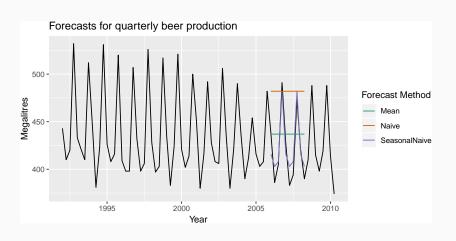
where Q is a stable measure of the scale of the time series  $\{y_t\}$ .

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For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=1}^{T} |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.



	RMSE	MAE	MAPE	MASE
Mean method	38.95	34.46	8.33	2.35
Naïve method	70.80	63.10	15.71	4.29
Seasonal naïve method	13.59	12.20	2.95	0.83

## **Training and test sets**

#### Available data

Training set (e.g., 80%)

Test set (e.g., 20%)

- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

#### **Training and test sets**

```
beer2 <- window(ausbeer, start=1992, end=c(2005,4))
fc <- snaive(beer2, h=10)
accuracy(fc, ausbeer)</pre>
```

## **Training and test sets**

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beer2 <- window(ausbeer, start=1992, end=c(2005,4))
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```

(one-step forecasts)

accuracy(fc)

# Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare  $R^2$ )
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true out-of-sample forecast accuracy. That is, total data divided into "training" set and "test" set.
   Training set used to estimate parameters.

#### Poll: true or false?

- Good forecast methods should have normally distributed residuals.
- A model with small residuals will give good forecasts.
- The best measure of forecast accuracy is MAPE.
- If your model doesn't forecast well, you should make it more complicated.
- Always choose the model with the best forecast accuracy as measured on the test set.

#Lab session 4 ##