MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

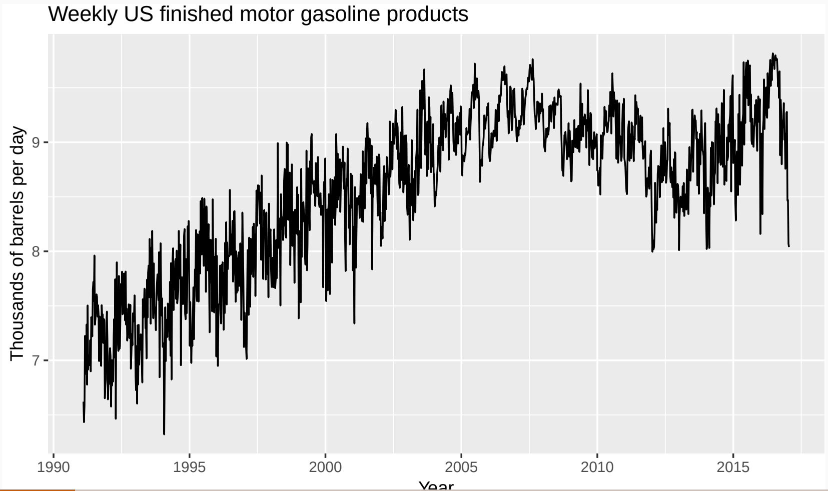
Rob J Hyndman

3.2 Forecasting with multiple seasonality

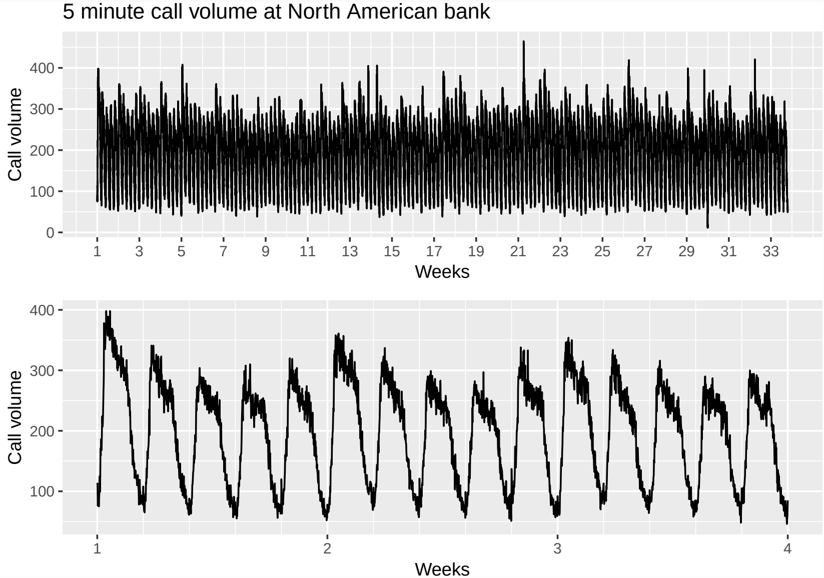
Outline

Examples

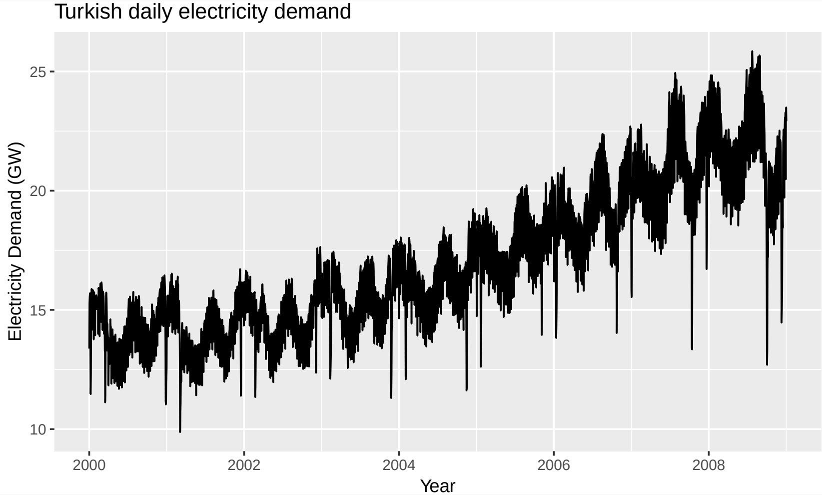
```r
autoplot(gasoline) +
  xlab("Year") + ylab("Thousands of barrels per day") +
  ggtitle("Weekly US finished motor gasoline products")
```



Examples


5 minute call volume at North American bank
Call volume
400
300
200
100
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
Weeks
Call volume
400
300
200
100
1
2
3
4
Weeks

Examples

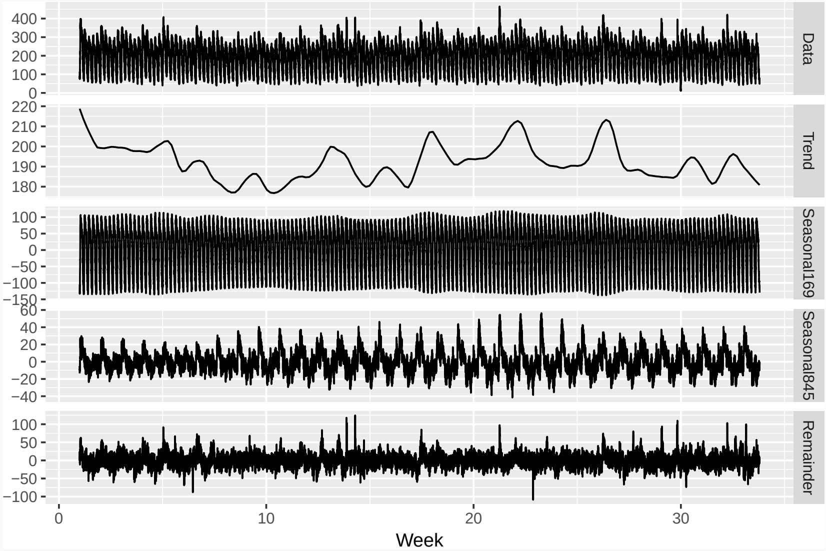


Turkish daily electricity demand
25
20
15
10
Electricity Demand (GW)
2000
2002
2004
2006
2008
Year

Outline

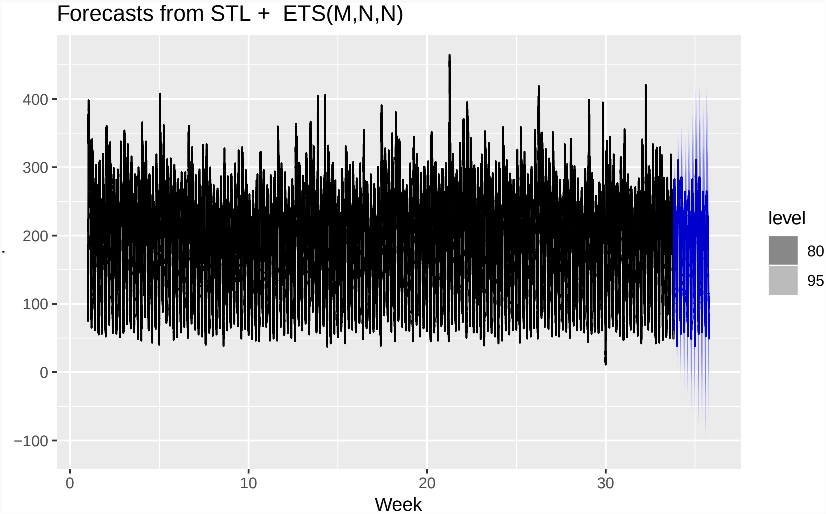
STL with multiple seasonal periods

```r
calls %>% mstl() %>% autoplot() + xlab("Week")
```



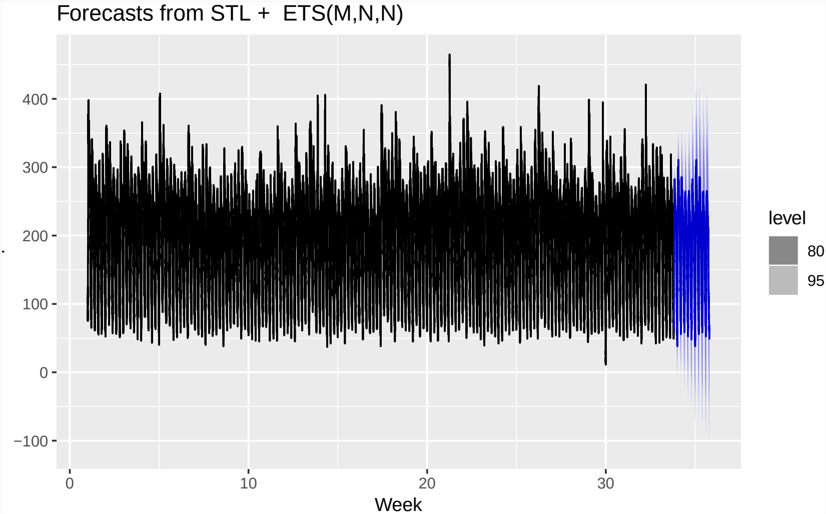
STL with multiple seasonal periods

```
calls %>%  stlf() %>% autoplot() + xlab("Week")
```



STL with multiple seasonal periods

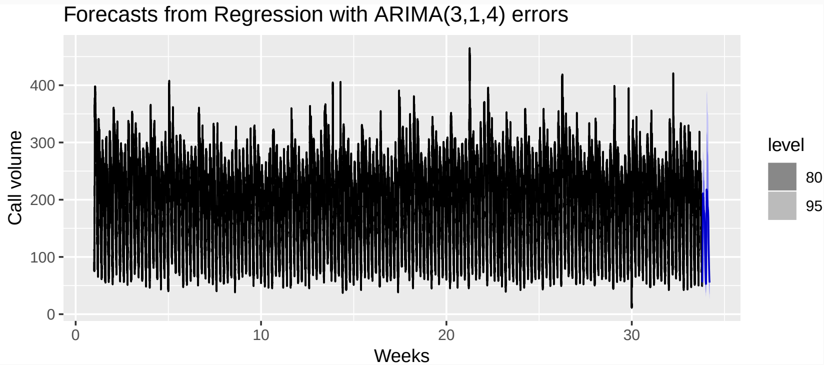
```
calls %>%  stlf() %>% autoplot(include=5*169) + xlab("Week")
```



Outline

Dynamic harmonic regression

```r
fit <- auto.arima(calls, seasonal=FALSE, lambda=0,
         xreg=fourier(calls, K=c(10,10)))
fit %>%
  forecast(xreg=fourier(calls, K=c(10,10), h=2*169)) %>%
  autoplot(include=5*169) +
    ylab("Call volume") + xlab("Weeks")
```



Outline

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

$$\begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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M seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

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M seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

global and local trend

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

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$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

ARMA error

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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ARMA error

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Fourier-like seasonal terms

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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TBATS model

y_t = observation at time t

$y_t^{(\omega)} = \begin{cases}(y_t^\omega - 1)/\omega & \text{if } \omega \neq 0;\end{cases}$

Box-Cox transformation

$y_t^{(\omega)} = \ell_t \ldots d_t$

M seasonal periods

$\ell_t = \ell_t \ldots$

$b_t = (1 \ldots$

global and local trend

$d_t = \sum_{i=\ldots}^{p} \ldots$

ARMA error

TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

Fourier-like seasonal terms

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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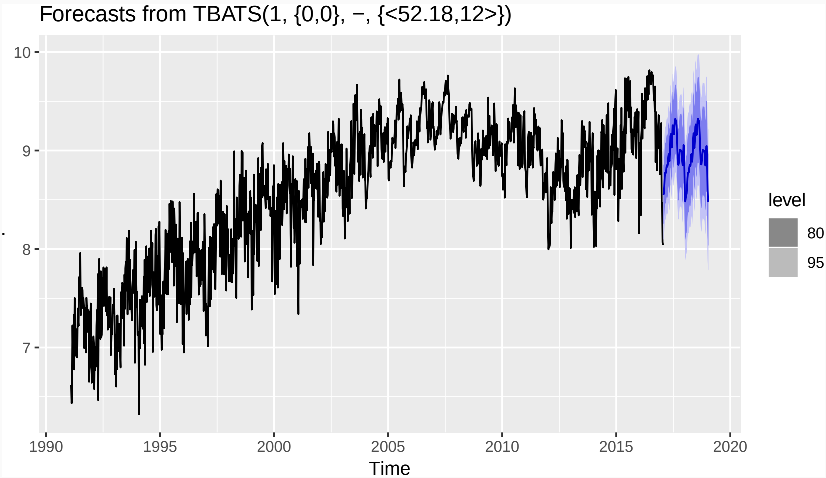
Complex seasonality

```r
gasoline %>% tbats() %>% forecast() %>% autoplot()
```



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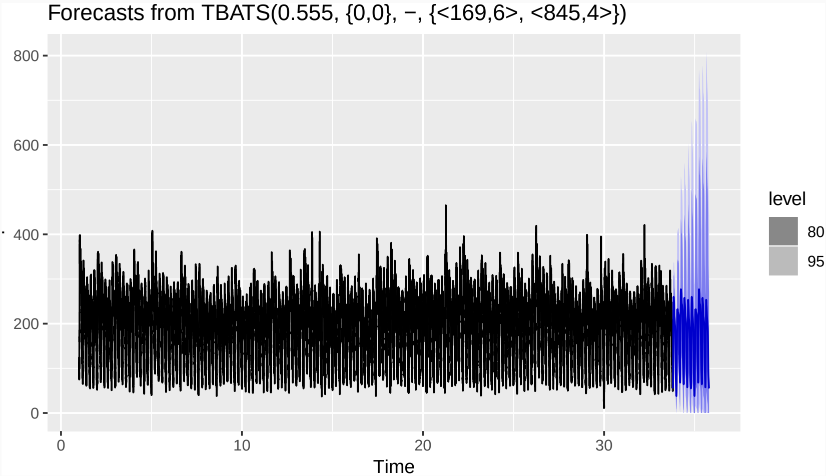
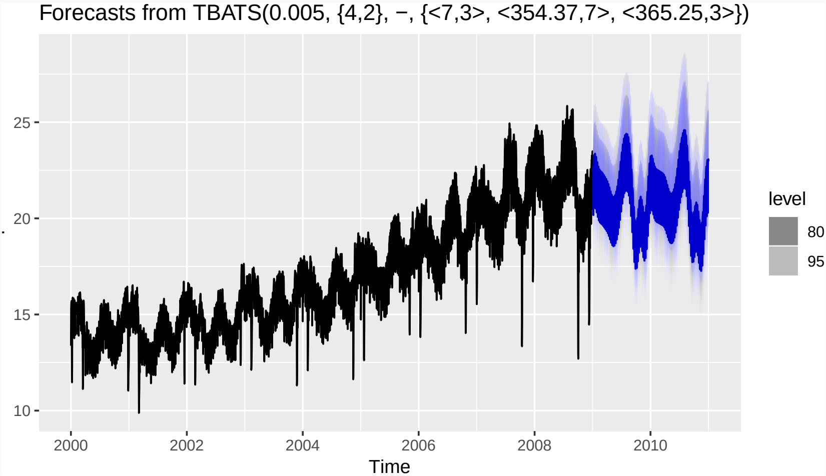


Complex seasonality

```r
telec %>% tbats() %>% forecast() %>% autoplot()
```



TBATS model

TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

1. Time series with complex seasonality
2. STL with multiple seasonal periods
3. Dynamic harmonic regression
4. TBATS model
5. **Lab session 20**

Lab Session 20