

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

Rob J Hyndman

2.1 State space models

| ##Methods V Models | Seasonal A | М |
|---|---|---|
| ###Exponential smoothing | methods | $y_t \equiv \ell_{t-1} s_{t-m}(\mathfrak{t} \oplus_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1}(\mathfrak{t} \oplus_t \varepsilon_t) \ell_{t-m}$ |
| $s_t = s_{t=m} + \gamma \epsilon t$ | $(\epsilon_{t-1} + s_{t-m})\varepsilon_t$ | $s_t = s_{t=m}(4 $ |
| | +1 $+1$ $+1$ $+1$ $+1$ $+1$ $+1$ $+1$ | $\begin{aligned} 5 & \underbrace{7_{t} = (\ell_{t-1} + b_{t-1}) s_{t-m} (\mathbb{1} \exists_{t} \varepsilon_{t})}_{\ell_{t} = (\ell_{t-1} + i \ell_{t+1}) \uparrow (\mathfrak{a} \exists_{t} \ell \varepsilon_{t+m})} \\ b_{t} &= b_{t-1} + \beta (\ell_{t} \exists_{t-m} b_{t-1}) \varepsilon_{t} \\ s_{t} &= s_{t-m} (\mathbb{1} \forall \varepsilon_{t} \ell \ell_{t-1} + b_{t-1}) \end{aligned}$ |
| $\mathbf{A_d} = \begin{pmatrix} \ell_t = (\ell_{t-1}) + (\phi b_{t-1}) & \ell_t = \ell_{t-1} + \phi b_{t-1} \\ b_t = \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) & b_t = \phi b_{t-1} + \beta \end{pmatrix}$ | $\begin{aligned} & \rho_{-1,1} + s_{t-mn} \right) (\mathbf{d}_{t} + \varepsilon_{t}) \\ & = 1 + \alpha (\mathbf{d}_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_{t} \\ & \in \mathbf{d}_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_{t} \\ & \in \mathbf{d}_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_{t} \end{aligned}$ | $\begin{aligned} y_t &\equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (\mathbb{I} \stackrel{d_t}{\leftarrow} \epsilon_t) \\ \ell_t &= (\ell_{t-1} + \phi b_{t-1}) (\Delta \stackrel{d_t}{\leftarrow} \epsilon_t \not= \epsilon_t) \\ b_t &\equiv \phi b_{t-1} + \beta (\ell_t \not= \epsilon_t \not= \epsilon_t) \\ s_t &\equiv s_{t-m} (\mathbb{I} \not= \epsilon_t \not= \epsilon_t \not= \epsilon_t) \end{aligned}$ |

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| ^d ■ LGEnerate same boil | ht forecasts byt | |
| forecast intervals. | $\mathbf{n} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ | $s_t \equiv s_{t=m}(t \not\approx s_t \not\in s_{t-1} + \phi b_{t-1})$ |

- A stochastic (or random) data generating process that can generate an entire forecast distribution.
- Allow for "proper" model selection.

| ##Methods, V Models | Seasonal A | M |
|---|--|--|
| ###Exponential smoothi | | $y_t \equiv \ell_{t-1} s_{t-m} (\mathfrak{t} s_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} (\mathfrak{t} \mathfrak{c} \varepsilon_t \widetilde{s}_t)_{-m}$ |
| | $\gamma + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$ | $s_t = s_{t=m}(\mathbb{1} \mathcal{HSyEl})_{-1}$ |
| $\mathbf{A} = \underset{\ell_t = (\ell_{t-1} + \ell_{t-1})(\alpha \in \mathcal{A}, \alpha \in \mathcal{E}_t)}{\text{Plane of the problem}} \text{ that } \underset{\ell_t = \ell_{t-1}}{\text{Plane of the problem}}$ | $ \mathbf{U} \mathbf{P}^{1} \mathbf{P} \mathbf{D} \mathbf{P}^{1} \mathbf{T}^{1} \mathbf{T} \mathbf{O} \mathbf{C} $ $ \mathbf{P} \mathbf{P}^{1} \mathbf{P} \mathbf{D} \mathbf{P}^{1} \mathbf{T}^{1} \mathbf{T} \mathbf{O} \mathbf{C} $ $ \mathbf{P} \mathbf{P}^{1} \mathbf{P} \mathbf{D} \mathbf{P}^{1} \mathbf{T}^{1} \mathbf{T} \mathbf{O} \mathbf{C} $ | asts $v_t = (\ell_{t-1} + b_{t-1})s_{t-m}(1 \ \mathbf{s}_t \varepsilon_t)$ $\ell_t = (\ell_{t+1} + b_{t+1}) \cdot (1 \ \mathbf{s}_t \ell \varepsilon_{t+1})$ |
| ###Innovations state spa | $\mathbf{p} \leftarrow \mathbf{p} \in \mathcal{C}_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ | $\begin{aligned} b_t &= b_{t-1} + \beta \notin \{ \ell_{s_t - m} b_{t-1} \} \varepsilon_t \\ s_t &\equiv s_{t-m} (\# \not \approx \ell_{s_{t-1}} + b_{t-1}) \end{aligned}$ |
| $y_t \equiv (\ell_{t+1} + \phi h_{t+1}) \cdot (\ell_{t+1} + \epsilon_t) \qquad y_t \equiv (\ell_{t+1} + \epsilon_t)$ | $_{1}+\phi b_{t+1}+s_{t+mn})(4_{t}+\varepsilon _{t})$ | $y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (\mathbb{1} e_t \varepsilon_t)$ |
| | <u></u> ht* f ōrecasts:_bt*1 | t"can, also generate |
| forecast intervals. **** | $q \neq \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ | $s_t \equiv s_{t-m}(\# \mathcal{L}(\beta_{t-1} + \phi b_{t-1}))$ |

- A stochastic (or random) data generating process that can generate an entire forecast distribution.
- Allow for "proper" model selection.

ETS in R

##Example: drug sales

```
ets(h02)
## ETS(M.Ad.M)
##
## Call:
##
    ets(v = h02)
##
##
     Smoothing parameters:
##
       alpha = 0.1953
##
       beta = 1e-04
##
    gamma = 1e-04
##
       phi = 0.9798
##
##
     Initial states:
##
    1 = 0.3945
##
    b = 0.0085
##
       s = 0.874 \ 0.8197 \ 0.7644 \ 0.7693 \ 0.6941 \ 1.2838
##
              1.326 1.1765 1.1621 1.0955 1.0422 0.9924
##
##
     sigma: 0.0676
##
          ATC
                    ATCc
                              BTC
##
## -122.90601 -119.20871 -63.17985
```