

Forecasting: principles and practice

Rob J Hyndman

1.2 The forecaster's toolbox

Outline

1 The statistical forecasting perspective

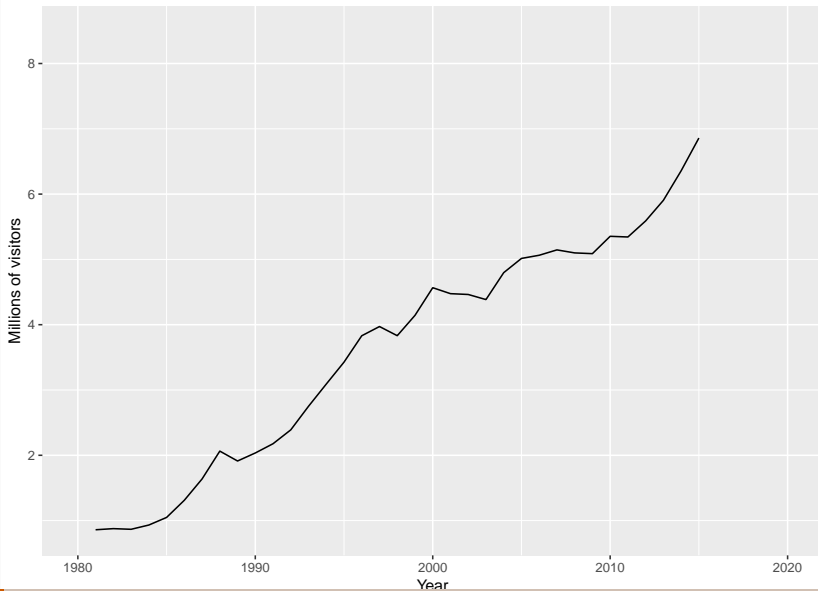
2 Some simple forecasting methods

3 Forecasting residuals

4 Evaluating forecast accuracy

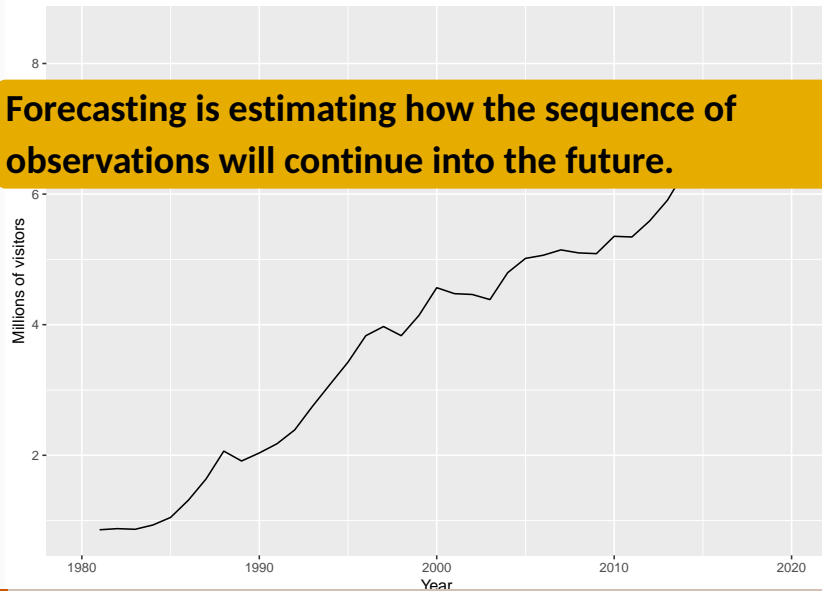
The statistical forecasting perspective

Total international visitors to Australia



The statistical forecasting perspective

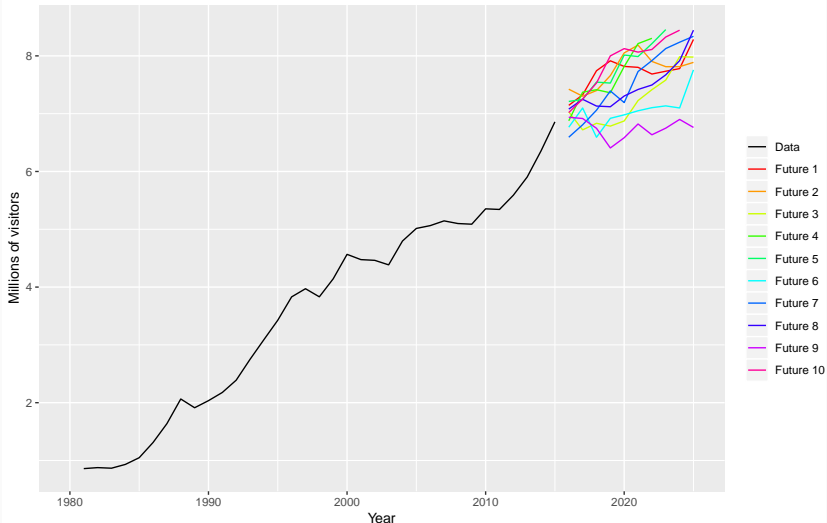
Total international visitors to Australia



Forecasting is estimating how the sequence of observations will continue into the future.

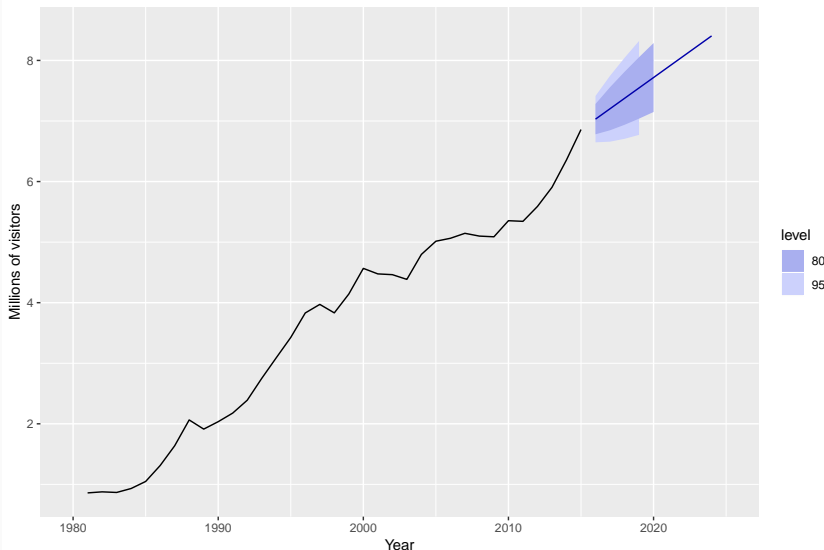
Sample futures

Total international visitors to Australia



Forecast intervals

Total international visitors to Australia



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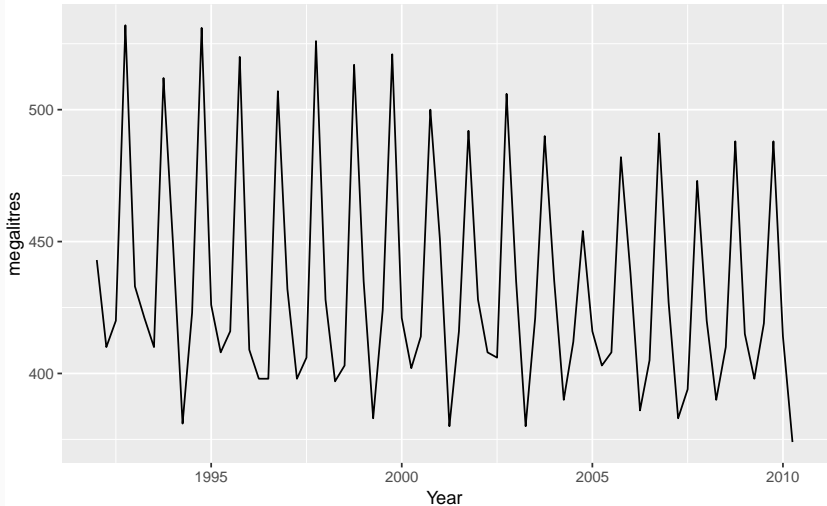
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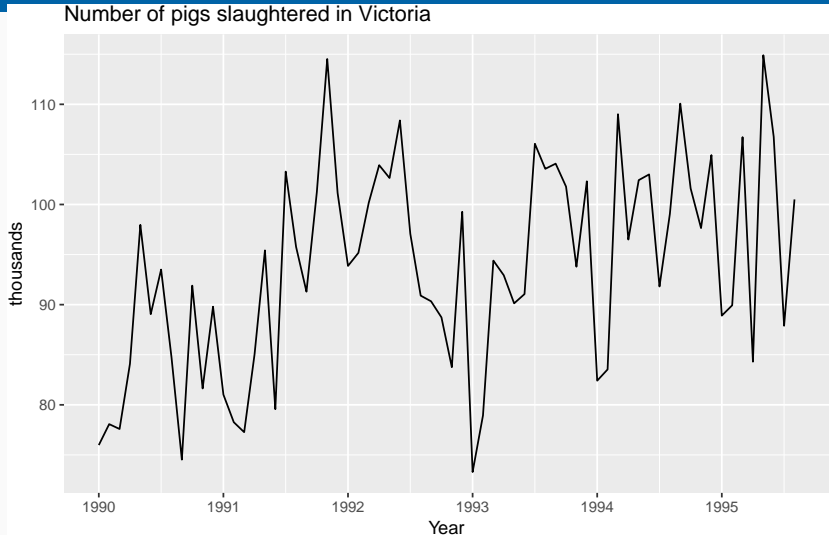
Some simple forecasting methods

Australian quarterly beer production



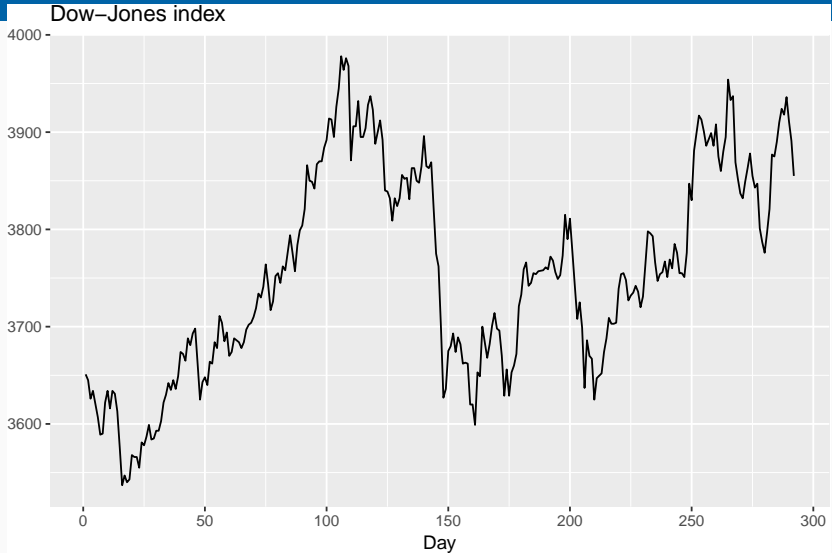
How would you forecast these data?

Some simple forecasting methods



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Some simple forecasting methods



How would you forecast these data?

Some simple forecasting methods

Average method

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$

Some simple forecasting methods

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Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Some simple forecasting methods

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Naïve method

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Consequence of efficient market hypothesis.

Seasonal naïve method

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-km}$ where m = seasonal period and $k = \lfloor (h-1)/m \rfloor + 1$.

Some simple forecasting methods

Drift method

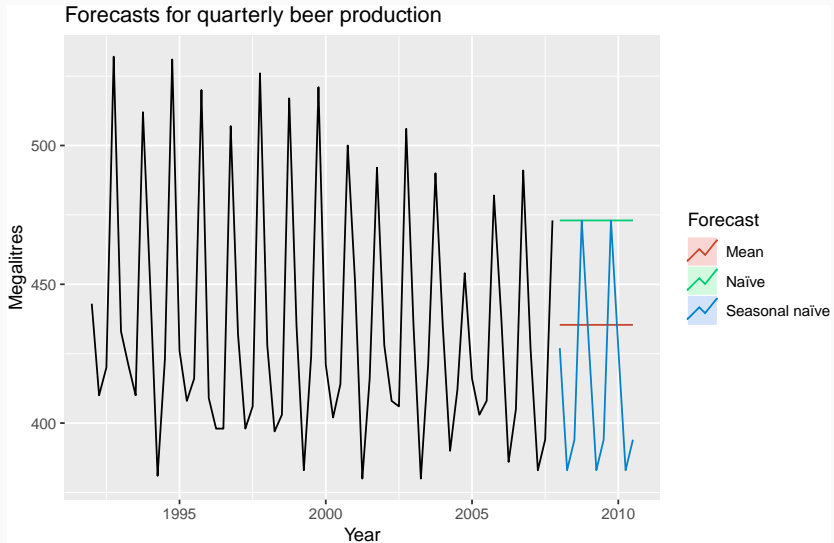
- Forecasts equal to last value plus average change.

- Forecasts:

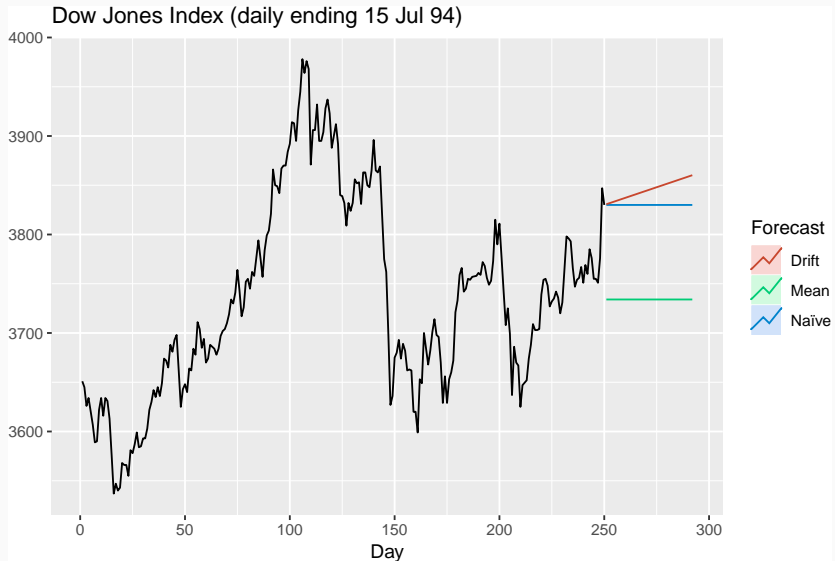
$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.

Some simple forecasting methods



Some simple forecasting methods



Some simple forecasting methods

- Mean: `meanf(y, h=20)`
- Naïve: `naive(y, h=20)`
- Seasonal naïve: `snaive(y, h=20)`
- Drift: `rwf(y, drift=TRUE, h=20)`

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Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations y_1, \dots, y_t .
- We call these “fitted values”.
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

###For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

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Assumptions

- 1 $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
- 2 $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

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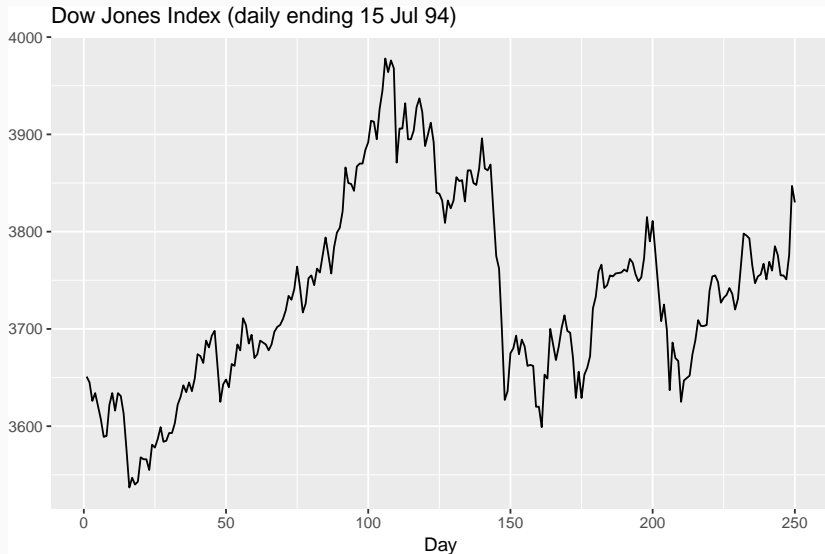
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Useful properties (for prediction intervals)

- 3 $\{e_t\}$ have constant variance.
- 4 $\{e_t\}$ are normally distributed

Example: Dow-Jones index



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Naïve forecast:

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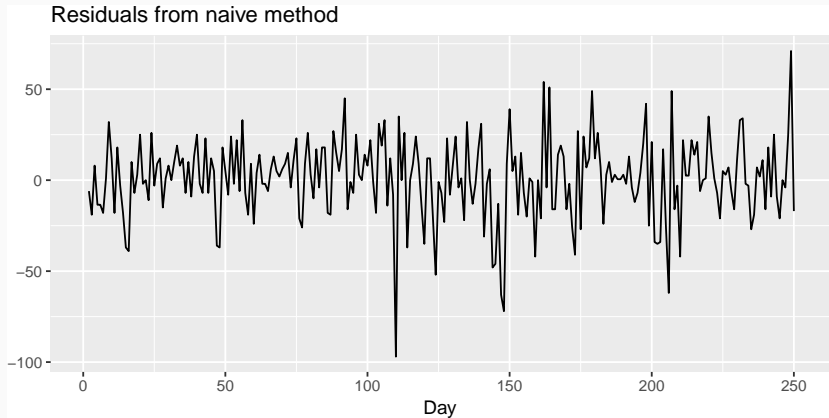
Naïve forecast:

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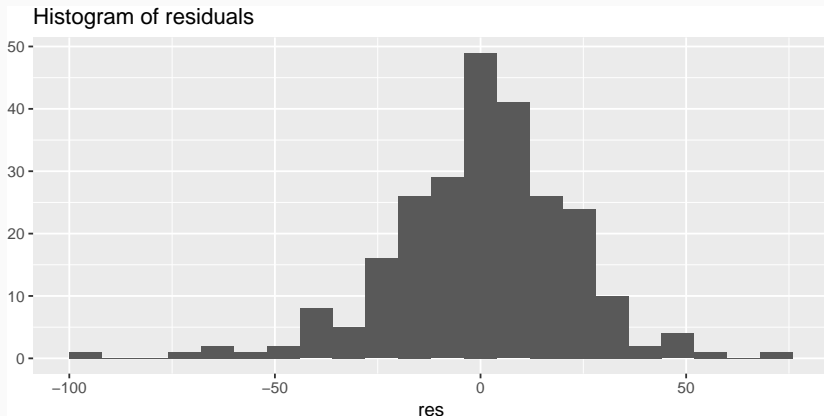
$$e_t = y_t - y_{t-1}$$

Note: e_t are one-step-forecast residuals

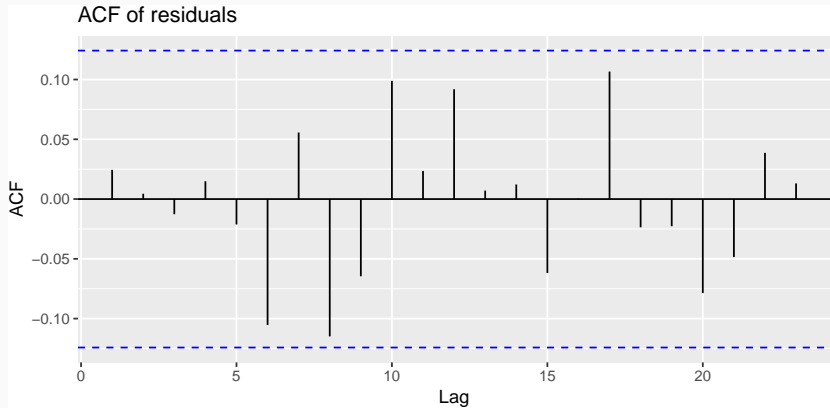
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ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

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Box-Pierce test

$$Q = T \sum_{k=1}^h r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- If each r_k close to zero, Q will be **small**.

Portmanteau tests

Consider a *whole* set of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2) \sum_{k=1}^h (T-k)^{-1} r_k^2$$

where h is max lag being considered and T is number of observations.

- My preferences: $h = 10$ for non-seasonal data, $h = 2m$ for seasonal data.
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(h - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.
- For the Dow-Jones example,

```
# lag=h and fitdf=K  
Box.test(res, lag=10, fitdf=0)
```

```
##  
## Box-Pierce test  
##  
## data: res  
## X-squared = 10.655, df = 10, p-value = 0.385
```

Portmanteau tests

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- For the Dow-Jones example,

```
# lag=h and fitdf=K
```

```
Box.test(res, lag=10, fitdf=0, type="Lj")
```

```
##
```

```
## Box-Ljung test
```

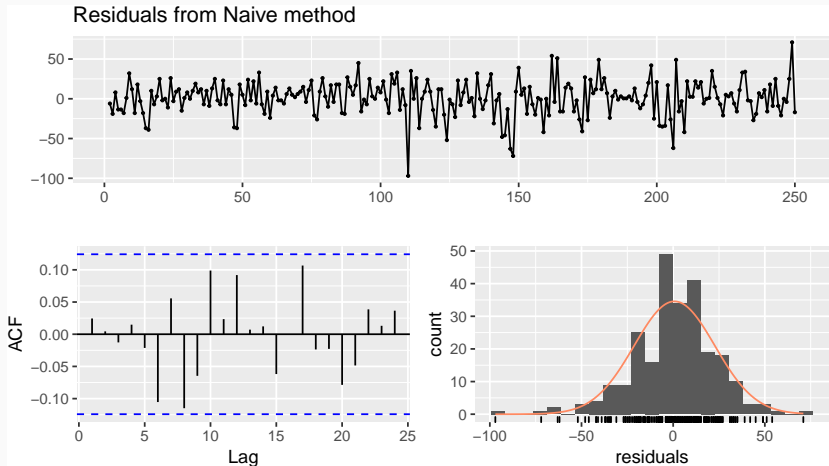
```
##
```

```
## data: res
```

```
## X-squared = 11.088, df = 10, p-value = 0.3507
```

checkresiduals function

```
checkresiduals(naive(dj2))
```



checkresiduals function

```
##  
##  Ljung-Box test  
##  
## data:  Residuals from Naive method  
## Q* = 11.088, df = 10, p-value = 0.3507  
## Model df: 0.    Total lags used: 10  
#Lab session 3 ##
```

Lab Session 3

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Measures of forecast accuracy

Let y_t denote the t th observation and $\hat{y}_{t|t-1}$ denote its forecast based on all previous data, where $t = 1, \dots, T$. Then the following measures are useful.

$$\text{MAE} = T^{-1} \sum_{t=1}^T |y_t - \hat{y}_{t|t-1}|$$

$$\text{MSE} = T^{-1} \sum_{t=1}^T (y_t - \hat{y}_{t|t-1})^2 \quad \text{RMSE} = \sqrt{T^{-1} \sum_{t=1}^T (y_t - \hat{y}_{t|t-1})^2}$$

$$\text{MAPE} = 100 T^{-1} \sum_{t=1}^T |y_t - \hat{y}_{t|t-1}| / |y_t|$$

Measures of forecast accuracy

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$$\text{MAPE} = 100 T^{-1} \sum_{t=1}^T |y_t - \hat{y}_{t|t-1}| / |y_t|$$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if

Measures of forecast accuracy

Mean Absolute Scaled Error

$$\text{MASE} = T^{-1} \sum_{t=1}^T |y_t - \hat{y}_{t|t-1}| / Q$$

where Q is a stable measure of the scale of the time series $\{y_t\}$.

Proposed by Hyndman and Koehler (IJF, 2006).

For non-seasonal time series,

$$Q = (T - 1)^{-1} \sum_{t=2}^T |y_t - y_{t-1}|$$

works well. Then MASE is equivalent to MAE relative to a naïve method.

Measures of forecast accuracy

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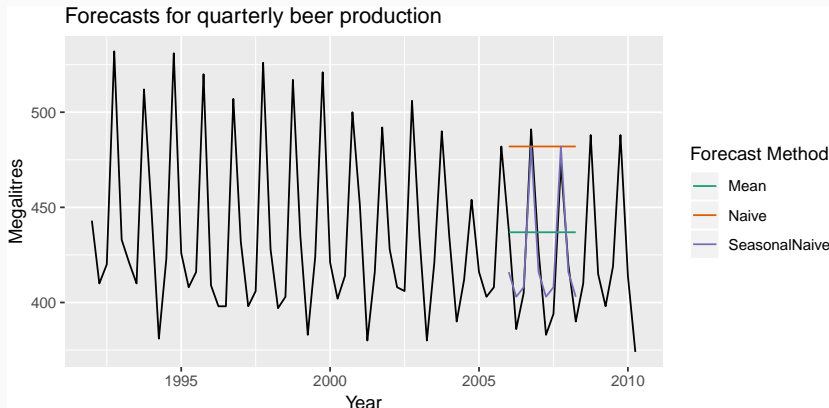
Proposed by Hyndman and Koehler (IJF, 2006).

For seasonal time series,

$$Q = (T - m)^{-1} \sum_{t=m+1}^T |y_t - y_{t-m}|$$

works well. Then MASE is equivalent to MAE relative to a seasonal naïve method.

Measures of forecast accuracy



Measures of forecast accuracy

	RMSE	MAE	MAPE	MASE
Mean method	38.95	34.46	8.33	2.35
Naïve method	70.80	63.10	15.71	4.29
Seasonal naïve method	13.59	12.20	2.95	0.83

Training and test sets

Available data

Training set (e.g., 80%)	Test set (e.g., 20%)
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- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Training and test sets

```
beer2 <- window(ausbeer, start=1992, end=c(2005,4))  
fc <- snaive(beer2, h=10)  
accuracy(fc, ausbeer)
```

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accuracy(fc, ausbeer)
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In-sample accuracy (one-step forecasts)

```
accuracy(fc)
```

Beware of over-fitting

- A model which fits the data well does not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters. (Compare R^2)
- Over-fitting a model to data is as bad as failing to identify the systematic pattern in the data.
- Problems can be overcome by measuring true *out-of-sample* forecast accuracy. That is, total data divided into “training” set and “test” set. Training set used to estimate parameters.

Poll: true or false?

- 1 Good forecast methods should have normally distributed residuals.
- 2 A model with small residuals will give good forecasts.
- 3 The best measure of forecast accuracy is MAPE.
- 4 If your model doesn't forecast well, you should make it more complicated.
- 5 Always choose the model with the best forecast accuracy as measured on the test set.

#Lab session 4 ##