



Forecasting: principles and practice

Rob J Hyndman

2.1 State space models

Innovations state space models

Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} + (\alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + (\alpha \varepsilon_t)$	$y_t \equiv (\ell_{t-1} + s_{t-m})(\alpha_t + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + \alpha(\ell_{t-1} + s_{t-m})\varepsilon_t$ $s_t \equiv s_{t-m} + \gamma(\ell_{t-1} + s_{t-m})\varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m}(\alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + (\alpha \varepsilon_t)_{t-m}$ $s_t \equiv s_{t-m}(\alpha \varepsilon_t)_{t-1}$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1})(\alpha_t + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1})(\alpha \varepsilon_t)$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m})(\alpha_t + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$ $s_t \equiv s_{t-m} + \gamma(\ell_{t-1} + b_{t-1} + s_{t-m})\varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1})s_{t-m}(\alpha_t \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1})(\alpha \varepsilon_t)_{t-m}$ $b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$ $s_t \equiv s_{t-m}(\alpha \varepsilon_t)_{t-1}$
Algorithms that return point forecasts:			
A _d	$y_t \equiv (\ell_{t-1} + \phi b_{t-1})(\alpha_t + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1})(\alpha \varepsilon_t)$ $b_t \equiv \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m})(\alpha_t + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$ $s_t \equiv s_{t-m} + \gamma(\ell_{t-1} + \phi b_{t-1} + s_{t-m})\varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1})s_{t-m}(\alpha_t \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1}(\alpha \varepsilon_t)_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta(\ell_{t-1} + \phi b_{t-1})\varepsilon_t$ $s_t \equiv s_{t-m}(\alpha \varepsilon_t)_{t-1}$

Algorithms that return point forecasts.

Innovations state space models

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##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} (1 + \alpha \varepsilon_t)$	$y_t \equiv (\ell_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} (1 + \alpha \varepsilon_t) s_{t-m}$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) \ell_{t-1}$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t)$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + b_{t-1})$
Algorithms that return point forecasts:			
Ad	$y_t \equiv \ell_{t-1} + \phi b_{t-1} (1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv \ell_{t-1} + \phi b_{t-1} + s_{t-m} (1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + \phi b_{t-1})$
###Innovations state space models			

- Generate same point forecasts but can also generate forecast intervals.
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Methods V Models		
N	A	M
$y_t \equiv \ell_{t-1} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t$	$y_t \equiv (\ell_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t s_{t-m}$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t \ell_{t-1}$
Exponential smoothing methods		
A		
$y_t = \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} - \ell_{t-1})$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1} - s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) + \alpha \varepsilon_t s_{t-m}$ $b_t = b_{t-1} + \beta (\ell_{t-1} - b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t (\ell_{t-1} + b_{t-1})$
Innovations state space models		
Ad		
$y_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha \varepsilon_t s_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} - \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t (\ell_{t-1} + \phi b_{t-1})$

- Generate same point forecasts but can also generate forecast intervals.
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Innovations state space models

Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t$	$y_t \equiv (\ell_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} - \ell_{t-1})$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$
Algorithms that return point forecasts:			
	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$
###Innovations state space models			
Ad			

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Innovations state space models

Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t$	$y_t \equiv (\ell_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \alpha \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} - \ell_{t-1})$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$
Algorithms that return point forecasts:			
Ad	$y_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} + \alpha \varepsilon_t$ $\ell_t \equiv \ell_{t-1} + \phi b_{t-1} + \alpha \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma \varepsilon_t$

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Innovations state space models

Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} (1 + \alpha \varepsilon_t)$	$y_t \equiv (\ell_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} (1 + \alpha \varepsilon_t) s_{t-m}$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) \ell_{t-1}$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t)$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + b_{t-1})$
Algorithms that return point forecasts:			
Ad	$y_t \equiv \ell_{t-1} + \phi b_{t-1} (1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv \ell_{t-1} + \phi b_{t-1} + s_{t-m} (1 + \varepsilon_t)$ $\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + \phi b_{t-1})$
###Innovations state space models			

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Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} (1 + \alpha \varepsilon_t)$	$y_t \equiv (\ell_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} (1 + \alpha \varepsilon_t) s_{t-m}$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) \ell_{t-1}$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t)$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + b_{t-1})$
Algorithms that return point forecasts:			
Ad	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + \phi b_{t-1})$
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Trend

Seasonal

M

##Methods V Models		A	M
N	$y_t \equiv \ell_{t-1} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t=1} (1 + \alpha \varepsilon_t)$	$y_t \equiv (\ell_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + \alpha (\ell_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv \ell_{t-1} s_{t-m} (1 + \alpha_t \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} (1 + \alpha \varepsilon_t) s_{t-m}$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) \ell_{t-1}$
###Exponential smoothing methods			
A	$y_t = (\ell_{t-1} + b_{t-1}) (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t)$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t \equiv \ell_{t-1} + b_{t-1} + \alpha (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t \equiv (\ell_{t-1} + b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t = b_{t-1} + \beta (\ell_{t-1} + b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + b_{t-1})$
Algorithms that return point forecasts:			
Ad	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) (1 + \alpha \varepsilon_t) + s_{t-m} \varepsilon_t$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$ $s_t \equiv s_{t-m} + \gamma (\ell_{t-1} + \phi b_{t-1} + s_{t-m}) \varepsilon_t$	$y_t \equiv (\ell_{t-1} + \phi b_{t-1}) s_{t-m} (1 + \varepsilon_t)$ $\ell_t = (\ell_{t-1} + \phi b_{t-1}) (1 + \alpha \varepsilon_t) s_{t-m}$ $b_t \equiv \phi b_{t-1} + \beta (\ell_{t-1} + \phi b_{t-1}) \varepsilon_t$ $s_t \equiv s_{t-m} (1 + \gamma \varepsilon_t) (\ell_{t-1} + \phi b_{t-1})$
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##Example: drug sales

```
ets(h02)
```

```
## ETS(M,Ad,M)
##
## Call:
## ets(y = h02)
##
## Smoothing parameters:
##   alpha = 0.1953
##   beta  = 1e-04
##   gamma = 1e-04
##   phi   = 0.9798
##
## Initial states:
##   l = 0.3945
##   b = 0.0085
##   s = 0.874 0.8197 0.7644 0.7693 0.6941 1.2838
##       1.326 1.1765 1.1621 1.0955 1.0422 0.9924
##
## sigma: 0.0676
##
```