MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

Rob J Hyndman

3.1 Dynamic regression

Regression with ARIMA errors

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Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + e_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- In regression, we assume that e_t was WN.
- Now we want to allow e_t to be autocorrelated.

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Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + n_t,$$

$$(1 - \phi_1 B)(1 - B)n_t = (1 + \theta_1 B)e_t,$$

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Stochastic and deterministic trends

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Deterministic trend

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where n_t is ARMA process.

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Periodic seasonality

Fourier terms for seasonality

Periodic seasonality can be handled using pairs of Fourier terms:

$$s_k(t) = \sin\left(\frac{2\pi kt}{m}\right) \qquad c_k(t) = \cos\left(\frac{2\pi kt}{m}\right)$$

$$y_t = \sum_{k=1}^{K} [\alpha_k s_k(t) + \beta_k c_k(t)] + n_t$$

- n_t is non-seasonal ARIMA process.
- Every periodic function can be approximated by sums of sin and cos terms for large enough K.
- Choose K by minimizing AICc.

US Accidental Deaths

Lab Session 14

Dynamic regression models

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- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

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- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged explanatory variables

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