MONASH University

MONASH BUSINESS SCHOOL

Forecasting: principles and practice

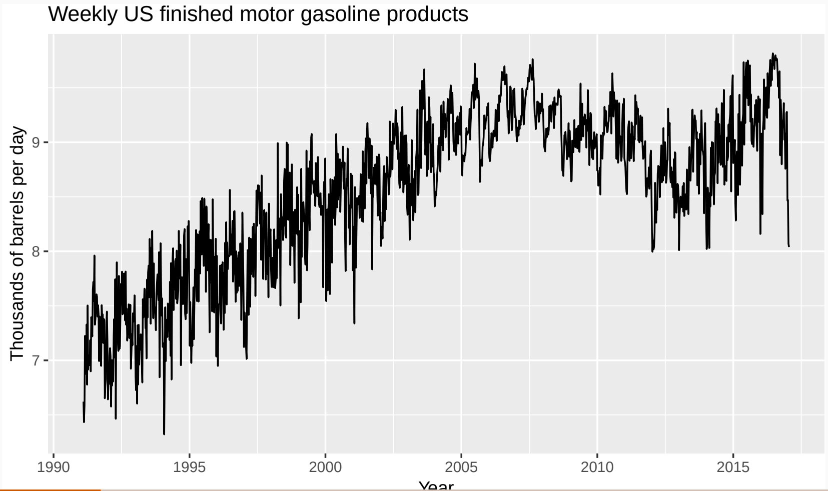
Rob J Hyndman

3.2 Forecasting with multiple seasonality

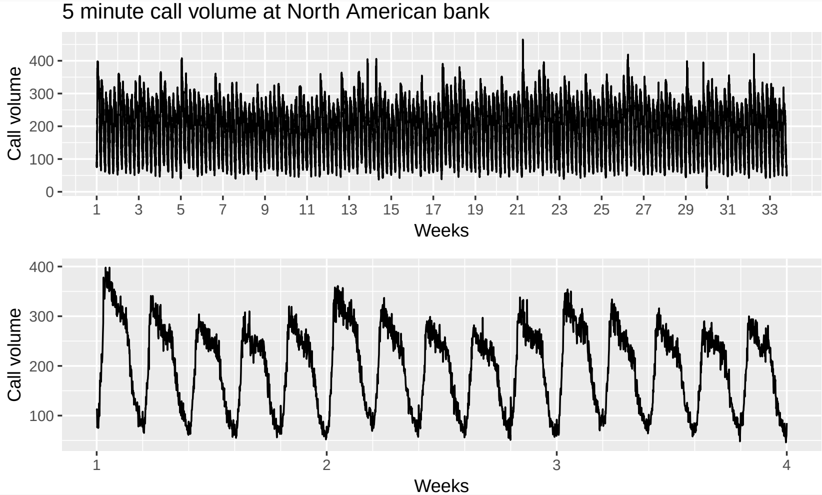
Outline

Examples

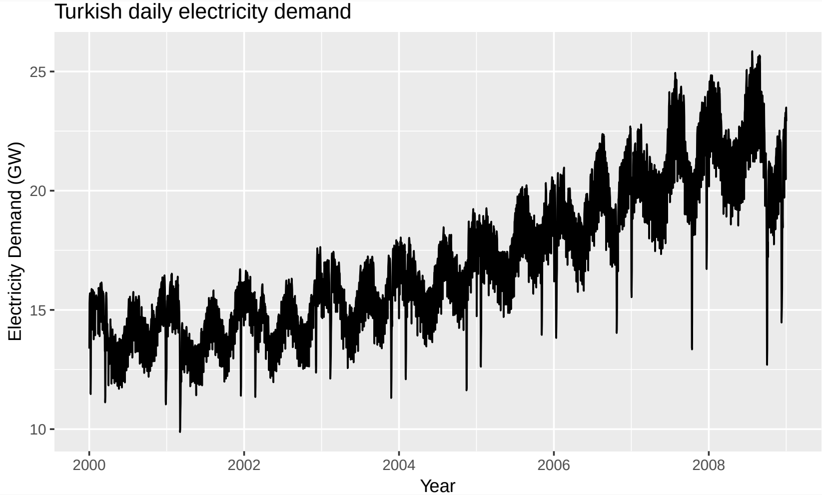
```r
autoplot(gasoline) +
  xlab("Year") + ylab("Thousands of barrels per day") +
  ggtitle("Weekly US finished motor gasoline products")
```



Examples


5 minute call volume at North American bank
Call volume
400
300
200
100
0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
Weeks
Call volume
400
300
200
100
1
2
3
4
Weeks

Examples



Turkish daily electricity demand
Electricity Demand (GW)
25
20
15
10
2000
2002
2004
2006
2008
Year

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$
\begin{aligned}
y_t &= \text{observation at time } t \\
y_t^{(\omega)} &= \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases} \\
y_t^{(\omega)} &= \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t \\
\ell_t &= \ell_{t-1} + \phi b_{t-1} + \alpha d_t \\
b_t &= (1-\phi)b + \phi b_{t-1} + \beta d_t \\
d_t &= \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t
\end{aligned}
$$

$$
s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad
\begin{aligned}
s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t \\
s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t
\end{aligned}
$$

TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

Box-Cox transformation

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

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$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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M seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

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M seasonal periods

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

global and local trend

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

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TBATS model

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global and local trend

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

ARMA error

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

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global and local trend

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

ARMA error

Fourier-like seasonal terms

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

$$\begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

TBATS model

y_t = observation at time t

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \end{cases}$$

Box-Cox transformation

TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

$y_t^{(\omega)} = \ell_t \; \ldots \; d_t$

M seasonal periods

$\ell_t = \ell_t \ldots$

$b_t = (1 \ldots$

global and local trend

$d_t = \sum_{i=\ldots}^{p} \ldots$

ARMA error

Fourier-like seasonal terms

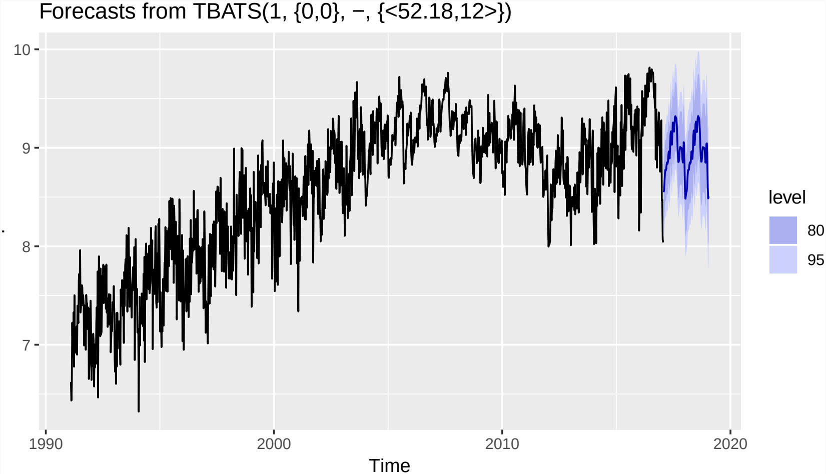
$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

$$s_{j,t}^{(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

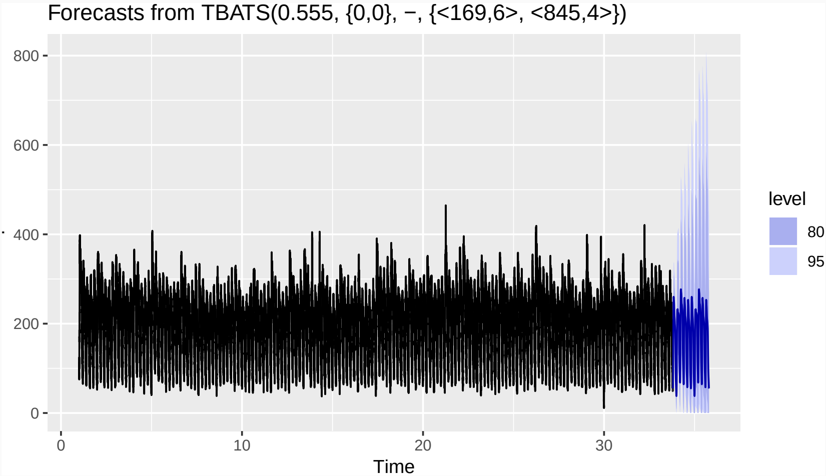
Complex seasonality

```
gasoline %>% tbats() %>% forecast() %>% autoplot()
```



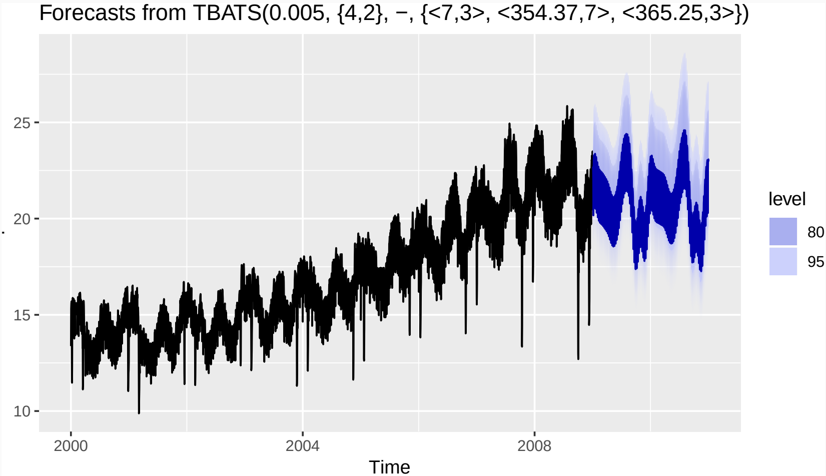
Complex seasonality

```r
calls %>% tbats() %>% forecast() %>% autoplot()
```



Complex seasonality

```r
telec %>% tbats() %>% forecast() %>% autoplot()
```



TBATS model

TBATS

Trigonometric terms for seasonality

Box-Cox transformations for heterogeneity

ARMA errors for short-term dynamics

Trend (possibly damped)

Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

Lab Session 17

Outline

Lab Session 18

Outline

Lab Session 19