6 UNIT

TIME AND WORK, WORK AND WAGES

INTRODUCTION

In our daily life, we come across situations where we need to complete a particular job in resolvable time. We have to complete the project earlier or later depending upon the needs. Accordingly, the men on duty have to be increased or decreased, i.e., the time allowed and the men engaged for a project are inversely proportional to each other, i.e., the more the number of men involved, the lesser is the time required to finish a job. We also come across situations where time and work or men and work are in direct proportion to each other.

KEY FACTS

- If 'A' can do a piece of work in n days, then at a uniform rate of working 'A' will finish
- $\frac{1}{n}$th work in one day.
- If $\frac{1}{n}$ of a work is done by 'A' in one day, then 'A' will take n days to complete the full work.
- If 'A' does $\frac{1}{n}$th of a work in one hour then to complete the full work, 'A' will take n hours.
- If 'A' does three times faster work than 'B' then ratio of work done by A and B is 3:1 and ratio of time taken by A and B is 1:3.
- A, B and C can do a piece of work in T_1, T_2 and T_3 days, respectively. If they have worked for D_1, D_2 and D_3 days respectively, then

  Amount of work done by $A = \frac{D_1}{T_1}$

  Amount of work done by $B = \frac{D_2}{T_2}$.

  And, Amount of work done by $C = \frac{D_3}{T_3}$.

  Also, the amount of work done by A, B and C together

  $$= \frac{D_1}{T_1} + \frac{D_2}{T_2} + \frac{D_3}{T_3}$$

  Which will be equal to 1, if the work is completed

KEY FACT:

If A can do a piece of work in X days and B can do the same work in Y days, then both of them working together will do the same work in $\frac{XY}{X+Y}$ days.

Example 1: **A can finish a piece of work by working alone in 6 days and B while working alone, can finish the same work in 12 days. If both of them work together, then in how many days, the work will be finished?**

Solution:

Here, $X = 6$ and $Y = 12$

$\therefore$ Working together, A and B will complete the work in $= \frac{XY}{X+Y}$ days $= \frac{6 \times 12}{6+12}$ days

$$= \frac{72}{18} \text{ days} = 4 \text{ days.}$$

KEY FACT

If A, B and C, while working alone can complete a work in X, Y and Z days respectively, then they will together complete the work in $\frac{XYZ}{XY+YZ+ZX}$ days.

Example 2: **A, B and C can complete a piece of work in 10, 15 and 18 days, respectively. In how many days would all of them complete the same work working together?**

Solution:

Here, $X = 10, Y = 15$ and $Z = 18$.

Therefore, the work will be completed in $= \frac{XYZ}{XY+YZ+ZX}$ days

$$= \frac{10 \times 15 \times 18}{10 \times 15 + 15 \times 18 + 18 \times 10}$$

$$= \frac{2700}{600} = 4\frac{1}{2} \text{ days.}$$

KEY FACTS:

Two persons A and B, working together, can complete a piece of work in X days. If A, working alone can complete the work in Y days, then B, working alone will complete the work in $\frac{XY}{Y-X}$ days.

Example 3: **A and B working together take 15 days to complete a piece of work. If A alone can do this work in 20 days, how long would B take to complete the same work?**

Solution:

Here, $X = 15$ and $Y = 20$

Therefore, B alone can complete the work in $\frac{XY}{Y-X} = \frac{15 \times 20}{20-15} = 60\, days.$

KEY FACTS:

If A and B, working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days, then

(a) A, B and C working together, will finish the job in $\left(\frac{2XYZ}{XY+YZ+ZX}\right)$ days.

(b) A alone will finish the job in $\left(\frac{2XYZ}{XY+YZ-ZX}\right)$ days.

(c) B alone will finish the job in $\left(\frac{2XYZ}{ZX+YZ-XY}\right)$ days.

(d) C alone can do the work in $\left(\frac{2XYZ}{ZX+XY-YZ}\right)$ days.

Example 4: **A and B can do a piece of work in 12 days, B and C in 15 days, C and A in 20 days. How long would each take separately to do the same work?**

Solution: Here, $X = 12, Y = 15$ and $Z = 20$.

A Alone can do the work in $= \frac{2XYZ}{XY+YZ-ZX}$ days $= \frac{2\times12\times15\times20}{12\times15+15\times20-20\times12}$ days

$$= \frac{7200}{240} = 30 \text{ days.}$$

B Alone can do the work in $= \frac{2XYZ}{YZ+ZX-XY}$ days $= \frac{2\times12\times15\times20}{15\times20+20\times12-12\times15}$ days

$$= \frac{7200}{360} = 20 \text{ days.}$$

C Alone can do the work in $= \frac{2XYZ}{ZX+XY-YZ}$ day $= \frac{2\times12\times15\times20}{20\times12+12\times15-15\times20}$ days

$$= \frac{7200}{120} = 60 \text{ days}$$

KEY FACTS:

- If A can finish a work in X days and B is k times efficient than A, then the time taken by both A and B working together to complete the work is $\frac{X}{1+k}$
- If A and B working together can finish a work in X days and B is k times efficient thanA, then the time taken by

(i) A, working alone, to complete the work is $(k+1)X$.

(ii) B, working alone, to complete the work is $\left(\frac{k+1}{k}\right)X$.

Example 5: **Asad can do a piece of work in 24 days. If Zafar works twice as fast as Asad, how long would they take to finish the work working together?**

Solution:

Here, $X = 24$ and $k = 2$.

$\therefore$ Time taken by Asad and Zafar, working together, to complete the work

$$= \left(\frac{X}{1+k}\right) \text{days} = \left(\frac{24}{1+2}\right) \text{days} = \left(\frac{24}{1+2}\right) \text{days} = 8 \text{ days.}$$

Example 6: **A and B together can do a piece of work in 3 days. If A does thrice as much work as B in a given time. Find how long A alone would take to do the work?**

Solution:

Here, $X = 3$ and $k = 3$.

Time taken by A working alone, to complete the work $= \left(\frac{k+1}{k}\right) X$ days

$$= \left(\frac{3+1}{3}\right) 3 = 4 \text{ days}$$

KEY FACTS:

If A can complete $\frac{a}{b}$ part of work in X days, then $\frac{c}{d}$ part of the work will be done in $\frac{b \times c \times X}{a \times d}$ days.

Example 7:

A can do $\frac{3}{4}$ of a work in 12 days. In how many days can he finish $\frac{1}{8}$ of the work?

Solution:

Here, $a = 3$, $b = 4$, $X = 12$, $c = 1$ and $d = 8$.

$\therefore$ Therefore, number of days required to finish $\frac{1}{8}$ of the work $= \left(\frac{b \times c \times X}{a \times d}\right)$ days.

$$= \left(\frac{4 \times 1 \times 12}{3 \times 8}\right) = 2 \text{ days.}$$

KEY FACTS:

- There are two groups of people with same efficiency. In one group M_1 person can do W_1 works in D_1 time and in the other M_2 persons can do W_2 works in D_2 time. The relationship between the two groups is given by

$$M_1D_1W_2 = M_2D_2W_1$$

- There are two groups of people with same efficiency. In one group M_1 persons can do W_1 works in D_1 times working t_1 hours a day and in other M_2 persons can do W_2 works in D_2 time working t_2 hours a day. The relationship between the two groups is given by $M_1D_1t_1W_2 = M_2D_2t_2W_1$

Example 8: **If 10 persons can complete 2/5th of a work in 8 days, then find the number of persons required to complete the remaining work in 12 days.**

Solution:

We Here, $M_1 = 10$, $W_1 = \frac{2}{5}$, $D_1 = 8$.

$M_2 = ?$, $W_2 = \frac{3}{5}$, $D_2 = 12$

$\therefore$ $M_1D_1W_2 = M_2D_2W_1$

$\Rightarrow 10\times 8\times\frac{3}{5} = M_2\times 12\times\frac{2}{5}$

$\Rightarrow M_2 = 10$

Example 9: If 10 persons can cut 20 trees in 3 days working 12 hours a day. Then, in how many days can 24 persons cut 32 trees working 4 hours a days.

Solution:

We Here, $M_1 = 10$, $W_1 = 20$, $D_1 = 3$, $t_1 = 12$.

$M_2 = 24$, $W_2 = 32$, $D_2 = ?$, $t_2 = 4$

$\therefore$ $M_1D_1t_1W_2 = M_2D_2t_2W_1$

$\Rightarrow 10\times 3\times 12\times 32 = 24\times D_2\times 4\times 20$

$\Rightarrow D_2 = 6$ days

KEY FACTS:

If a men and b women can do a piece of work in n days, then c men and d women can do the work in $\left(\frac{nab}{bc+ad}\right)$ days

Example 10: **12 men and 15 women can do a work in 14 days. In how many days, 7 men and 5 women would complete the work.**

Solution:

Here, $a = 12$, $b = 15$, $n = 14$, $c = 7$ and $d = 5$.

Required number of days $= \frac{nab}{bc+ad} = \left(\frac{14\times 12\times 15}{15\times 7+12\times 5}\right)$ days $= \frac{168}{11}$ days $= 15\frac{3}{11}$ days