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VQS - DEFINITIONS

Let $T = [v, q, s] = [\text{position}, \text{quaternion}, \text{scale}]$

$\text{data types} = [\text{vector 3}, \text{quaternion}, \text{vector3}]$

OPERATORS

Let $r = \text{vector3}$, Let $T_1 = [v, q, s]$, $T_2 = [u, p, t]$, $T_n[\dots] \dots$

ADDITION AND SUBTRATION

$$T_2 \pm T_1 = [u, p, t] \pm [v, q, s] = [u \pm v, p \pm q, t \pm s]$$

IDENTITY

$$I = [\text{vector3}(0,0,0), \quad \text{quaternion}(0,0,0,1), \quad \text{vector3}(1,1,1)]$$

INVERSE

$$T_1^{-1} = [q^{-1}(s^{-1} * -v)q, \quad q^{-1}, \quad s^{-1}] = [q^{-1}(s^{-1} * -v)q, \quad \text{quaternion}(-x, -y, -z, w), \quad \text{vector3}\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)]$$

$$T_1 T_1^{-1} = IT = T_1^{-1} T$$

MULTIPLICATION WITH VECTOR/POINT

$$T_1 * r = [v, q, s] * r = q(s * r)q^{-1} + v$$

$$T_1^{-1} * r = [q^{-1}(s^{-1} * -v)q, \quad \text{quaternion}(-x, -y, -z, w), \quad \text{vector3}\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)] = q^{-1}(s^{-1} * r)q + q^{-1}(s^{-1} * -v)q$$

CONCATENATION

$$\begin{aligned} T_2(T_1 * r) &= (T_2 T_1) * r = [u, p, t][v, q, s]r \\ &= [[u, p, t]v, pq, ts] * r \\ &= [p(tv)p^{-1} + u, \quad pq, \quad ts] * r \end{aligned}$$

Prove (hard to observe? Observe again):

$$\begin{aligned} T_2(T_1 * r) &= (T_2 T_1) * r = [u, p, t][v, q, s]r \\ &= [u, p, t] (q(sr)q^{-1} + v) = p(t(q(sr)q^{-1} + v))p^{-1} + u \\ &= p(q(tsr)q^{-1} + tv))p^{-1} + u, \quad (\text{multiply by } t) \\ &= pq(tsr)q^{-1}p^{-1} + (p(tv)p^{-1} + u), \\ &= [p(tv)p^{-1} + u, \quad pq, \quad ts] * r, \\ &= [[u, p, t]v, pq, ts] * r \end{aligned}$$

