

### Solving Quadratic Equation

Intro	$ax^2 + bx + c = 0$ <p><i>a is the coefficient of <math>x^2</math> (Quadratic term)</i>  <i>b is the coefficient of <math>x^1</math> (Linear term)</i>  <i>c is the coefficient of <math>x^0</math> (Constant value)</i></p>
1	<p><u>Type 1: No linear term</u></p> <p>e.g. 1  Mth 1: <math>x^2 - 9 = 0</math></p> $x^2 = 9$ $x = \pm 3$ <p>Mth 2: <math>x^2 - 9 = 0</math></p> $(x + 3)(x - 3) = 0$ $x = \pm 3$ <p>e.g. 2</p> $\frac{v}{8} = \frac{18}{v}$ $v^2 = 144$ $v = \pm 12$
2	<p><u>Type 2: No constant term</u></p> <p>(Note: Do not divide by x, as it will become a linear equation instead)</p> <p>e.g. 1</p> $2x^2 = 7x$ $x(2x - 7) = 0$ $x = 0, x = \frac{7}{2}$
3	<p><u>Type 3: Solve by Factorization</u></p> <p>e.g. 1</p> $3x^2 - 5x - 8 = 0$ $(3x - 8)(x + 1) = 0$ $x = \frac{8}{3}, x = -1$
4	<p><u>Type 4: Completed Square form</u> <math>(x + a)^2 = b</math></p> <p>e.g. 1</p> $(x + 2)^2 = 16$ $x + 2 = \pm 4$ $x = 2, x = -6$

	<p>e.g. 2</p> $(3x + 2)^2 = 43$ $3x + 2 = \pm\sqrt{43}$ $x = \frac{\sqrt{43} - 2}{3}, x = \frac{-\sqrt{43} - 2}{3}$
5	<p><u>Type 5: Solve by completing Square</u></p> <p>e.g. 1</p> $x^2 + 8x + 9 = 0$ $x^2 + 8x + 4^2 = -9 + 4^2$ $(x + 4)^2 = 7$ $x + 4 = \pm\sqrt{7}$ $x = \sqrt{7} - 4, \quad x = -\sqrt{7} - 4$ <p>Condition:</p> <ol style="list-style-type: none"> <li>1. <i>Coefficient of <math>x^2</math> must be 1</i></li> <li>2. <i>Add coefficient of <math>x \rightarrow \left(\frac{\text{coefficient of } x^1}{2}\right)^2</math></i></li> <li>3. <i>Works for negative coefficient too.</i></li> </ol>
6	<p><u>Type 6: Solve by Formulae</u></p> $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>Prove (Solve by Type 5 Completing square):</p> $ax^2 + bx + c = 0$ $x^2 + \frac{b}{a}x = -\frac{c}{a}$ $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$ $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$ $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ (#proved)}$$