Indices Law

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1. What is Indices?

Indices are expressions in the form of a^n, $where\ a = base,\ \ n = index\ OR\ power$

2. Why learn Indices?

To express very big or very small number like 10^{100}, 10^{-9}, $and\ learn\ how\ to\ evalute\ the\ expression$

3. 9 Laws of Indices

Law		Example
1	$a^m * a^n = a^{m+n}$	$2^3 * 2^4 = 2^7$
2	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^8}{2^6} = 2^2$
3	$(a^m)^n = a^{mn} = (a^n)^m$	$(5^5)^7 = 5^{35} = (5^7)^5$
4	$a^m * b^m = (ab)^m$	$2^3 * 3^3 = (2 * 3)^3 = 6^3$
5	$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$	$8^3 * 3^3 = \left(\frac{8}{3}\right)^3 = 6^3$
6	$a^0 = 1$	
7	$a^{-n} = \frac{1}{a^n}, \qquad a^n = \frac{1}{a^{-n}}$	$x^{-3} = 8 \to (x^{-3})^{-\frac{1}{3}} = 8^{-\frac{1}{3}} \to x = \frac{1}{\sqrt[3]{8}} \to x = \frac{1}{2}$
8	$a^{\frac{1}{n}} = \sqrt[n]{a}\ , a > 0$ $index\ form = radical\ form$	Example 1: $49^{\frac{1}{2}} = \sqrt{49} = 7$ Example 2: $27^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$ Example 3: $\frac{1}{16^4} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$
9	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\ \right)^m, a > 0$	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

4. Exercise

1	$8^{\frac{2}{3}}$	$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
2	$9^{-\frac{3}{2}}$	$9^{-\frac{3}{2}} = \frac{1}{\left(\sqrt[2]{9}\right)^3} = \frac{1}{3^3} = \frac{1}{27}$
3	$100^{1.5}$	$100^{1.5} = 1000$
4	$\frac{1}{16^{\frac{3}{4}}}$	$\frac{1}{16^{\frac{3}{4}}} = \frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{8}$
5	$\frac{1}{32^{-\frac{4}{5}}}$	$\frac{1}{32^{-\frac{4}{5}}} = \left(\sqrt[5]{32}\right)^4 = 16$

	Find unknown in the power	Find unknown in the base
1	$9^x = 27$ $3^{2x} = 3^3$ $x = 1.5$	$x^{-3} = 8$ $(x^{-3})^{-\frac{1}{3}} = 8^{-\frac{1}{3}}$ $x = \frac{1}{\sqrt[3]{8}} \rightarrow x = \frac{1}{2}$
2	$4^{-x} = 32$ $2^{-2x} = 2^5 \rightarrow x = -\frac{5}{2}$	$x^{\frac{2}{3}} = 64 \rightarrow x = 64^{\frac{3}{2}}$ $x = (\sqrt{64})^3 \rightarrow x = 512$
3	$(a^x)(a^{2x-6}) = 1$ $a^{3x-6} = a^0$ $3x - 6 = 0$ $x = 2$	$(24a)^{\frac{1}{2}} = 8^{\frac{2}{3}} \rightarrow 24a = 8^{\frac{4}{3}}$ $a = \frac{2^4}{24} = \frac{16}{24} \rightarrow a = \frac{2}{3}$
4	$(a^{x+1})^3 = a^{12}$ $a^{3x+3} = a^{12}$ $3x + 3 = 12$ $x = 3$	$\frac{128a^{-3}}{2a^{-\frac{3}{2}}} = 27 \rightarrow \quad a^{-3--\frac{3}{2}} = \frac{27}{64} \rightarrow a^{-\frac{3}{2}} = \frac{27}{64}$ $a = \left(\frac{27}{64}\right)^{-\frac{2}{3}} = a = \left(\frac{3}{4}\right)^{-2} = a = \frac{16}{9}$

5. Exercise – Simplify the expressions

1	$\frac{(x^3y)^3(2xy)^{-2}}{4x^{-4}y^{-5}}$	2	$\frac{2^5 * 9^{-2}}{27^{-3} * 8^{-4}}$	3	$\frac{\frac{3^{-2}}{4}}{\frac{4^3}{9} * \frac{27^{-1}}{16}}$
4	$\frac{\frac{3^{-2}}{4}}{\frac{4^3}{9} * \frac{27^{-1}}{16}}$	5	$(2^{-3}a^4b)^{-1} * (4^{-2}b^{-5})$	6	$\frac{(pqr^2)^{-2}}{(p^2r^2q)^{-5}}$
7	$\frac{(pqr)^{-2}(p^2q^3)^2}{(p^4r^3s)^{-7}}$				

1	$\frac{(x^3y)^3(2xy)^{-2}}{4x^{-4}y^{-5}} = \frac{2^{-2}x^{3*3+(-2)-(-4)}y^{3+(-2)-(-5)}}{4} = \frac{x^{11}y^6}{16}$
2	$\frac{2^5 * 9^{-2}}{27^{-3} * 8^{-4}} = \frac{2^5 * 27^3 * 8^4}{9^2} = \frac{2^5 * (3^3)^3 * (2^3)^4}{(3^2)^2} = \frac{2^5 * 2^{12} * 3^9}{3^4} = 2^{17} * 3^5$
3	$\frac{\frac{3^{-2}}{4}}{\frac{4^3}{9} * \frac{27^{-1}}{16}} = \frac{4^2}{3} * \frac{9^3}{4} * \frac{16}{27} = \frac{2^4}{3^2} * \frac{3^6}{2^6} * \frac{2^4}{3^3} = 2^2 3^1$
4	$\frac{\frac{3^{-2}}{4}}{\frac{4^3}{9} * \frac{27^{-1}}{16}} = \frac{4^2}{3} * \frac{9^3}{4} * \frac{16}{27} = \frac{2^4}{3^2} * \frac{3^6}{2^6} * \frac{2^4}{3^3} = 2^2 3^1$
5	$(2^{-3}a^4b)^{-1} * (4^{-2}b^{-5}) = 2^3a^{-4}b^{-1}2^{-4}b^{-5} = 2^{-1}a^{-4}b^{-6} = \frac{1}{2a^4b^6}$
6	$\frac{(pqr^2)^{-2}}{(p^2r^2q)^{-5}} = \frac{p^{-2}q^{-2}r^{-4}}{p^{-10}r^{-10}q^{-5}} = p^{-2-(-10)}q^{-2-(-5)}r^{-4-(-10)} = p^8q^3r^6$
7	$\frac{(pqr)^{-2}(p^2q^3)^2}{(p^4r^3s)^{-7}} = \frac{p^{-2}q^{-2}r^{-2}p^4q^6}{p^{-28}r^{-21}s^{-7}} = p^{-2+4-(-28)}q^{-2+6}r^{-2-(-21)}s^{-(-7)} = p^{30}q^4r^{19}s^7$

6. Exercise – Evaluate the expressions

1	$\frac{2^{x-3}}{8^{-x}} = \frac{32}{4^{\frac{1}{2}x}} \to 2^{x-3-(-3x)} = 2^{5-x} \to 4x-3=5-x \to x=\frac{8}{5}$
2	$\frac{a^x}{b^{3-x}} * \frac{b^y}{(a^{y+1})^2} = ab^6$ $a^{x-2y-2} * b^{y-3+x} = ab^6$ $x-2y-2=1, \quad y-3+x=6$ $Let\ 1 \to x-2y=3, \quad Let\ 2 \to y+x=9$ Sub 1 into 2: $y+3+2y=9$ $3y=6$ $y=2$ Therefore Sub y=2 into 2: $x=7$
3	$8^x \div 2^y = 64, \quad 3^{4x} * \left(\frac{1}{9}\right)^{y-1} = 81$ $2^{3x} \div 2^y = 2^6, \quad 3^{4x} * \left(\frac{1}{3^2}\right)^{y-1} = 3^4$ $2^{3x-y} = 2^6, \quad 3^{4x-2y+2} = 3^4$ $3x-y=6, \quad 4x-2y=2$ $Let\ 1 \to 3x-y=6, \quad Let2 \to 2x-y=1 \to 2x-1=y$ Sub 2 into 1: $3x-2x+1=6$ $x=5$ Therefore Sub x=5 into 1: $y=10-1$ $y=9$
4	$\frac{16^{x+1}+20(4^{2x})}{2^{x-3}8^{x+2}} = \frac{2^{4x+4}+20(2^{4x})}{2^{x-3}2^{3x+6}} = \frac{16(2^{4x})+20(2^{4x})}{2^{4x+3}} = \frac{16(2^{4x})+20(2^{4x})}{8(2^{4x})}$ $= \frac{36}{8} = 4.5$
5	$(a^{\frac{1}{3}}+b^{\frac{2}{3}})(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}})$ $= a^{\frac{1}{3}}a^{\frac{2}{3}} - a^{\frac{1}{3}}a^{\frac{1}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} + b^{\frac{2}{3}}a^{\frac{2}{3}} - b^{\frac{2}{3}}a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{2}{3}}b^{\frac{4}{3}}$ $= a - a^{\frac{2}{3}}b^{\frac{2}{3}} + a^{\frac{1}{3}}b^{\frac{4}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{4}{3}} + b^2$ $a+b^2$
6	$2^{2x+2}5^{x-1} = 8^x5^{2x}, \quad Evalute\ 10^x,$ $2^{2x+2}5^{x-1} = 2^{3x}5^{2x} \to 2^{2x}2^25^x5^{-1} = 2^{3x}5^{2x}$ $\frac{2^2}{5} = \frac{2^{3x}5^{2x}}{2^{2x}5^x} \to \frac{4}{5} = 2^x5^x = 10^x$
7	$(144p^4)^{\frac{3}{2}} \div (216p^{-3})^{-\frac{2}{3}} = 2^x3^yp^z, \quad 144 = 2*2*2*2*3*3, \quad 216 = 2*2*2*3*3*3$ $(2^43^2p^4)^{\frac{3}{2}} \div (2^33^3p^{-3})^{-\frac{2}{3}} = 2^x3^yp^z$ $2^63^3p^6 \div 2^{-2}3^{-2}p^2 = 2^x3^yp^z$ $2^83^5p^4 = 2^x3^yp^z$ $x=8, \quad y=5, \quad z=4$

7. Exercise – ALL 9 rules

1 Evaluate the expression	$\frac{1}{8}^{-\frac{3}{5}} = 8^{\frac{5}{3}} = \left(\sqrt[3]{8}\right)^{5} = 32$ $25^{-2.5} = \frac{1}{25^{\frac{5}{2}}} = \frac{1}{5^{5}} = \frac{1}{3125}$
2 Express the following in index form	$\sqrt[4]{16a^{2}b^{6}} = 2a^{\frac{2}{4}}b^{\frac{6}{4}} = 2a^{\frac{1}{2}}b^{\frac{3}{4}}$ $\sqrt[3]{\frac{a^{5}b^{6}}{c^{4}}} = \frac{a^{\frac{5}{3}}b^{2}}{c^{\frac{4}{3}}}$
3 Simplify, result in radical form	$\sqrt[4]{125}\,\sqrt[6]{5} = 5^{\frac{3}{4}}\,5^{\frac{1}{6}} = 5^{\frac{3}{4}+\frac{1}{6}} = 5^{\frac{11}{12}} = (\sqrt[12]{5})^{11}$ $\frac{\sqrt{27}}{\sqrt[12]{81}} = \frac{3^{\frac{3}{2}}}{3^{\frac{4}{12}}} = 3^{\frac{3}{2}-\frac{4}{12}} = 3^{\frac{14}{12}} = (\sqrt[6]{3})^{7}$ $a^{\frac{2}{3}}b^{\frac{5}{6}} \times a^{\frac{1}{2}}b \div (ab)^{\frac{1}{3}} = a^{\frac{2}{3}+\frac{1}{2}-\frac{1}{3}}b^{\frac{5}{6}+1-\frac{1}{3}} = a^{\frac{5}{6}}b^{\frac{3}{2}} = (\sqrt[6]{a})^{5}(\sqrt{b})^{3}$ $\left(x^{\frac{1}{4}}y^{3}\right)^{-\frac{1}{2}} \div \left(x^{\frac{1}{3}}y^{-\frac{1}{4}}\right)^{-5} = x^{-\frac{1}{8}+\frac{5}{3}}y^{-\frac{3}{2}-\frac{5}{4}} = x^{-\frac{3}{24}+\frac{40}{24}}y^{-\frac{6}{4}-\frac{5}{4}} = x^{\frac{37}{24}}y^{-\frac{11}{4}} = \frac{x^{\frac{37}{24}}}{y^{\frac{11}{4}}}$

8. Exercise – Solve the following

1	$(25^3)^2 \times 125 = 5^x$ $5^{12}5^3 = 5^x$ $x = 15$
2	$16 \times (4^x)^3 = 2^{2x}$ $2^4 2^{6x} = 2^{2x}$ $4 + 6x = 2x$ $x = -1$
3	$a^{2x} \div a^{5x-1} \times a^5 = \frac{1}{a^2}$ $a^{2x-5x+1+5} = a^{-2}$ $2x - 5x + 1 + 5 = -2 \rightarrow -3x = -8 \rightarrow x = \frac{8}{3}$
4	$3x^5 = 96$ $x^5 = 32$ $x = 2$
5	$x^{-\frac{1}{2}} = 5 \rightarrow x = 5^{-2}$ $x = \frac{1}{25}$
6	$x^{\frac{4}{3}} = 256 \rightarrow x = 256^{\frac{3}{4}}$ $x = 4^3$ $x = 64$

9. Exercise – Exponential Functions

1	$5^{2x} - 6(5^x) + 5 = 0$ $(5^x)^2 - 6(5^x) + 5 = 0$ $let\ y = 5^x \rightarrow y^2 - 6y + 5 = 0$ $(y-5)(y-1) = 0$ $y = 5, \quad y = 1$ $5^x = 5, \quad 5^x = 1$ $x = 1, \quad x = 0$
2	$2^{2x} - 10(2^x) + 16 = 0$ $(2^x)^2 - 10(2^x) + 16 = 0$ $let\ y = 2^x \rightarrow y^2 - 10y + 16 = 0$ $(y-8)(y-2) = 0$ $y = 8, \quad y = 2$ $2^x = 8, \quad 2^x = 2$ $x = 3, \quad x = 1$
3	$2(16^x) - 5(4^x) + 2 = 0$ $2(4^x)^2 - 5(4^x) + 2 = 0$ $let\ y = 4^x \rightarrow 2y^2 - 5y + 2 = 0$ $(2y-1)(y-2) = 0$ $y = \frac{1}{2}, \quad y = 2$ $4^x = \frac{1}{2}, \quad 4^x = 2$ $2^{2x} = 2^{-1}, \quad 2^{2x} = 2$ $2x = -1\ , \quad 2x = 1$ $x = -\frac{1}{2}, \quad x = \frac{1}{2}$
4	$9^{x+1} + 1 = 10(3^x)$ $9(9^x) - 10(3^x) + 1 = 0$ $9(3^x)^2 - 10(3^x) + 1 = 0$ $let\ y = 3^x \rightarrow 9y^2 - 10y + 1 = 0$ $(9y-1)(y-1) = 0$ $y = \frac{1}{9}, \quad y = 1$ $3^x = \frac{1}{9}, \quad 3^x = 1$ $x = -2, \quad x = 0$