<u>Surds Law</u>

Contents

1. Definition

a. Evaluate $\sqrt{4} = 2,\ \sqrt[3]{27} = 3,$ this expression can be evaluated exactly
b. Evaluate $\sqrt{3} = 1.732,\ \sqrt{2} = 1.414,$ this expression **<u>cannot</u>** be evaluated exactly

In b. this are called SURDS, which are irrational numbers that cannot be expressed as $\frac{m}{n}$ where m and n are integers

2. Why learn surds?

As indicated above, it is to simplify irrational numbers in radical form $(\sqrt{2})$ without the need to evaluate the expression first

3. Rules

	Rules	Example
1	$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$	$4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$
2	$a\sqrt{x} - b\sqrt{x} = (a-b)\sqrt{x}$	$2\sqrt{3} - 8\sqrt{3} = -6\sqrt{3}$
3	$\sqrt{x} * \sqrt{y} = \sqrt{xy}$	$\sqrt{5} * \sqrt{10} = \sqrt{50}$
4	$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$	$\frac{\sqrt{10}}{\sqrt{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$
5	$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$	$\frac{8^3}{3^3} = \left(\frac{8}{3}\right)^3 = 6^3$
6	$\sqrt{x} + \sqrt{y}$ *and* $\sqrt{x} - \sqrt{y}$ *are conjugates* $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ $= x - y$	Prove: $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$ $= \sqrt{x}\sqrt{x} - \sqrt{x}\sqrt{y} + \sqrt{y}\sqrt{x} - \sqrt{y}\sqrt{y}$ $= x - y$

4. Exercise - Example

E.g. 1 Solve	$\sqrt{48} + \sqrt{147} - \sqrt{75}$ $= \sqrt{16 * 3} + \sqrt{49 * 3} - \sqrt{25 * 3}$ $= \sqrt{16}\sqrt{3} + \sqrt{49}\sqrt{3} - \sqrt{25}\sqrt{3}$ $= 4\sqrt{3} + 7\sqrt{3} - 5\sqrt{3}$ $= 6\sqrt{3}$
E.g. 2 Solve	$\sqrt{50} + \sqrt{72} - \sqrt{160} \div \sqrt{5}$ $= \sqrt{25}\sqrt{2} + \sqrt{36}\sqrt{2} - \sqrt{\frac{160}{5}}$ $= 5\sqrt{2} + 6\sqrt{2} - \sqrt{32}$ $= 11\sqrt{2} - 4\sqrt{2}$ $= 7\sqrt{2}$
E.g. 3 Reduce, using conjugates	$\frac{5\sqrt{5} + 3\sqrt{7}}{4\sqrt{7} - 3\sqrt{5}} = \frac{5\sqrt{5} + 3\sqrt{7}}{4\sqrt{7} - 3\sqrt{5}} + \frac{4\sqrt{7} + 3\sqrt{5}}{4\sqrt{7} + 3\sqrt{5}}$ $= \frac{5\sqrt{5}\ 4\sqrt{7} + 3\sqrt{7}\ 4\sqrt{7} + 3\sqrt{5}\ 5\sqrt{5}\ + 3\sqrt{5}3\sqrt{7}}{16 * 7 - 9 * 5}$ $= \frac{20\sqrt{35}\ + 84 + 75\ + 9\sqrt{35}}{112 - 45}$ $= \frac{29\sqrt{35}\ + 159}{67}$

5. Exercise -

1 Simplify	$\sqrt{175}+\sqrt{112}-\sqrt{28}$ $=\sqrt{25}\sqrt{7}+\sqrt{16}\sqrt{7}-\sqrt{4}\sqrt{7}$ $=5\sqrt{7}+4\sqrt{7}-2\sqrt{7}$ $=7\sqrt{7}$
2 Simplify	$2\sqrt{21}\times\sqrt{27}\div\sqrt{343}$ $=2\sqrt{567}\div\sqrt{343}$ $=2\sqrt{81}\sqrt{7}\div\sqrt{49}\sqrt{7}$ $=18\sqrt{7}\div 7\sqrt{7}$ $=\frac{18}{7}$
3 Simplify	$(6\sqrt{5}-2\sqrt{2})^2=(6\sqrt{5}-2\sqrt{2})(6\sqrt{5}-2\sqrt{2})$ $=180-12\sqrt{10}-12\sqrt{10}+8$ $=188-24\sqrt{10}$
4 Find a and b	$(a+\sqrt{5})(3+b\sqrt{5})=26+11\sqrt{5}$ $3a+ab\sqrt{5}+3\sqrt{5}+5b=26+11\sqrt{5}$ $3a+5b+(ab+3)\sqrt{5}=26+11\sqrt{5}$ $3a+5b=26, \quad ab+3=11$ $3a+5b=26, \quad ab=8\rightarrow a=\frac{8}{b}$ $3\frac{8}{b}+5b=26\rightarrow 5b^2-26b+24=0$ $(5b-6)(b-4)=0, \quad b=\frac{6}{5}, \quad b=4, a=2$
5	$roots\ of\ x^2-\sqrt{20}x+2=0\ are\ c\ and\ d$ $show\ that\ \frac{1}{c}+\frac{1}{d}=\sqrt{5}$ $(x-c)(x-d)=0$ $x^2-(c+d)x+cd=0$ $sum\ of\ roots\rightarrow c+d=-\frac{-\sqrt{20}}{1}=\sqrt{20}$ $product\ of\ roots\rightarrow cd=2$ $\frac{1}{c}+\frac{1}{d}=\frac{c+d}{cd}=\frac{\sqrt{20}}{2}=\frac{\sqrt{4}\sqrt{5}}{2}=\sqrt{5}\ (shown)$