Binomial Theorem

Contents

1. **What is Binomial?**

<u>Expression that contain 2 terms</u>

2. **Pascal Triangle**

Power	Pascal Triangle pattern											
0	1											
1	1	1										
2	1	2	1									
3	1	3	3	1								
4	1	4	6	4	1							
5	1	5	10	10	5	1						
6	1	6	15	20	15	6	1					
7	1	7	21	35	35	21	7	1				
8	1	8	28	56	70	56	28	8	1			
9	1	9	36	84	126	126	84	36	9	1		
10	1	10	45	120	210	252	210	120	45	10	1	
11	1	11	55	165	330	462	462	330	165	55	11	1

3. **Example Observe Expansion of Power and its' coefficient**

Power	Expand	Number of terms
$(a+b)^0$	1	1
$(a+b)^1$	$a+b$	2
$(a+b)^2$	$a^2+2ab+b^2$	3
$(a+b)^3$	$a^3+3a^2b+3ab^2+b^3$	4
$(a+b)^4$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$	5
$(a+b)^5$	$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	6

The coefficient follows the patterns of Pascal Triangle!

4. Exercise – Expand the following (basic)

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

$$(a+b)^6 = a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$$

5. Exercise – Expand the following (patterns)

a. $(a + b)^n \rightarrow$ a and b can be any value

$$(2 + 3x)^4 = 2^4 + 4(2^3)(3x) + 6(2^2)(3x)^2 + 4(2)(3x)^3 + (3x)^4$$

$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

b. Any term = 1, can be ignored

$$\left(1 + \frac{x}{2}\right)^4 = 1^4 + 4(1^3)\left(\frac{x}{2}\right) + 6(1^2)\left(\frac{x}{2}\right)^2 + 4(1)\left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^4$$

$$= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$$

c. $(a - b)^n = \left(a + (-b)\right)^n, \textit{expand in the same way, but with alternating signs}$

$$\left(1 - \frac{x}{2}\right)^4 = 1^4 - 4(1^3)\left(\frac{x}{2}\right) + 6(1^2)\left(\frac{x}{2}\right)^2 - 4(1)\left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^4$$

$$= 1 - 2x + \frac{3}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{16}x^4$$

6. Limitations of Pascal Triangle

- Pascal triangle can only do this much, how about the power of 20? Or 30?
- It becomes tedious and not practical to memorize the patterns

a. $\mathbf{nC_r}$ *is the key*

n = the power of expansion

r = the term of the Pascal triangle of power n, starts from 0

Example:

$$5C_0, \quad 5C_1, \quad 5C_2, \quad 5C_3, \quad 5C_4, \quad 5C_5$$

$$1, \quad 5, \quad 10, \quad 10, \quad 5, \quad 1$$

Calculate nC_r by using factorial:

a. Factorial

Term	Formula	Result
0!	1	1
1!	1	1
2!	2 * 1	2
3!	3 * 2 * 1	6
4!	4 * 3 * 2 * 1	24
5!	5 * 4 * 3 * 2 * 1	120
n!	n * (n-1) * (n-2) * … * 1	

b. $\mathbf{nC_r}$ General Fomula:

$$nC_r = \frac{n!}{r!\,(n - r)!}$$

Use your calculator to calculate $\mathbf{nC_r}$

c. **Binomial Theorem Full Formula**

$$(\mathrm{a}+\mathrm{b})^{\mathrm{n}}=$$

$$a^{n}+\mathrm{nC}_1\ \mathrm{a}^{\mathrm{n}-1}\mathrm{b}^1+\mathrm{nC}_2\ \mathrm{a}^{\mathrm{n}-2}\mathrm{b}^2+\cdots+\mathrm{nC}_\mathrm{r}\ \mathrm{a}^{\mathrm{n}-\mathrm{r}}\mathrm{b}^\mathrm{r}+\cdots+\mathrm{nC}_\mathrm{n}\ \mathrm{b}^\mathrm{n}$$

$$\mathrm{nC}_0=\mathrm{nC}_\mathrm{n}=1, \text{it will always be 1 as seen from pascal triangle}$$

d. **Binomial Theorem General Formula**

$$\therefore\ \mathrm{a}^{\mathrm{n}}+\mathrm{nC}_1\ \mathrm{a}^{\mathrm{n}-1}\mathrm{b}^1+\mathrm{nC}_2\ \mathrm{a}^{\mathrm{n}-2}\mathrm{b}^2+\cdots+\mathrm{nC}_\mathrm{r}\ \mathrm{a}^{\mathrm{n}-\mathrm{r}}\mathrm{b}^\mathrm{r}+\cdots+\ \mathrm{b}^\mathrm{n}$$

7. Exercises

$$\left(\mathrm{x}+\frac{1}{\mathrm{x}^2}\right)^6=\mathrm{x}^6+6x^5\frac{1}{\mathrm{x}^2}+15x^4\left(\frac{1}{\mathrm{x}^2}\right)^2+20x^3\left(\frac{1}{\mathrm{x}^2}\right)^3+15x^2\left(\frac{1}{\mathrm{x}^2}\right)^4+6x\left(\frac{1}{\mathrm{x}^2}\right)^5+\left(\frac{1}{\mathrm{x}^2}\right)^6$$

$$=\mathrm{x}^6+6x^3+15+\frac{20}{\mathrm{x}^3}+\frac{15}{\mathrm{x}^6}+\frac{6}{\mathrm{x}^9}+\frac{1}{\mathrm{x}^{12}}$$

$$6\mathrm{C}_2=\frac{6!}{2!\,4!}=\frac{6*5*4*3*2*1}{2*1*4*3*2*1}=\frac{6*5}{2*1}=\frac{30}{2}=15$$

$$10\mathrm{C}_6=\frac{10!}{6!\,4!}=\frac{10*9*8*7*6*5*4*3*2*1}{6*5*4*3*2*1*4*3*2*1}=\frac{10*9*8*7}{4*3*2*1}=\frac{5040}{24}=210$$

8. Application I

a. $Simplify\ the\ 1st\ 3\ term\ of\ (1-2x)^7, use\ the\ expandsion\ to\ esitmate\ (0.98)^7$

$$(1-2x)^7=1+7\mathrm{C}_1(-2\mathrm{x})+7\mathrm{C}_2(-2\mathrm{x})^2+\cdots$$

$$=1-14x+84\mathrm{x}^2+\cdots$$

$$1-2x=0.98\rightarrow x=0.01$$

$$1-14(0.01)+84(0.01)^2=0.8684$$

$$\text{check }(0.98)^7=0.86812553324672$$

b. $Simplify\ the\ 1st\ 4\ term\ of\ \left(2+\frac{x}{2}\right)^8,\ esitmate\ (2.01)^8\ to\ 3\ decimal\ places$

$$\left(2+\frac{x}{2}\right)^8=2^8+8\mathrm{C}_1 2^7\left(\frac{x}{2}\right)+8\mathrm{C}_2\ 2^6\left(\frac{x}{2}\right)^2+8\mathrm{C}_3\ 2^5\left(\frac{x}{2}\right)^3+\cdots$$

$$=256+8(128)\frac{x}{2}+28\ 64\frac{x}{4}+56\ 32\frac{x}{8}+\cdots$$

$$=256+512\mathrm{x}+448\mathrm{x}^2+224\mathrm{x}^3+\cdots$$

$$2+\frac{x}{2}=2.01\rightarrow x=0.005$$

$$256+512(0.005)+448(0.005)^2+224(0.005)^3=258.571228\approx 258.571$$

$$\text{check }(2.01)^8\ =266.4210032449121601$$

c. $Obtain\ 1st\ 4\ term\ of\ (1+p)^6, and\ then\ \ subtitude\ p=x-x^2,$
$esitmate\ (1.099)^6\ up\ to\ the\ term\ x^2$

$$(1+p)^6 = 1^6 + 6p + 15p^2 + 20p^3$$

$$1 + 6(x - x^2) + 15(x - x^2)^2 + 20(x - x^2)^3$$

$$= 1 + 6x - 6x^2 + 15x^2 - 30x^3 + 15x^4 + \cdots$$

$$\cancel{1 + x - x^2 = 1.099 \rightarrow x(1 - x) = 0.099}$$

$$\cancel{x = 0.099, \qquad 0.901 = x}$$

$$\cancel{1 + 6(0.099) - 9(0.099)^2 = 1.0609}$$

9. Application II – Solve for Unknown

a. 1^{st} 4 term of $(4+\mathrm{bx})^6$, the coefficient of $\mathrm{x}^2 = x^3, find\ b$

$$(4+\mathrm{bx})^6 = 4^6 + 6(4^5)bx + 15(4^4)b^2x^2 + 20(4^3)b^3x^3 + \cdots$$

$$= 4096 + 6144bx + 3840b^2x^2 + 1280b^3x^3 + \cdots$$

$$3840b^2 = 1280b^3 \rightarrow \frac{3840}{1280} = b \rightarrow b = 3$$

b. in $\left(1+\frac{1}{4}\right)^{\mathrm{n}}$, the 3rd term is 2 times the 4th term, find n, and then evaluate the middle term.

$$\left(1+\frac{1}{4}\right)^{\mathrm{n}} = 1 + nC_1\frac{1}{4} + nC_2\frac{1^2}{4} + nC_3\frac{1^3}{4} + \cdots = 1 + nC_1\frac{1}{4} + nC_2\frac{1}{16} + nC_3\frac{1}{64} + \cdots$$

$$nC_2\frac{1}{16} = 2nC_3\frac{1}{64} \rightarrow \frac{n(n-1)}{2*1}\frac{1}{16} = \frac{n(n-1)(n-2)}{3*2*1}\frac{1}{32}$$

$$\frac{\cancel{n(n-1)}}{\cancel{2*1}}\frac{\cancel{1}}{\cancel{16}} = \frac{\cancel{n(n-1)}(n-2)}{3*\cancel{2*1}}\frac{\cancel{1}\ 1}{\cancel{32}\ 2} \rightarrow 1 = \frac{\mathrm{n}-2}{6}$$

$$6 = \mathrm{n} - 2 \rightarrow \mathrm{n} = 8$$

$$8C_4\frac{1^4}{4} = \frac{70}{256}$$

c. Write the 1st 3 term of $(1+\mathrm{ax})^5$, Given that the 1st 3 term of $(\mathrm{b}+2\mathrm{x})(1+\mathrm{ax})^5\ is\ 3 - 28x + cx^2$, $find\ the\ value\ of\ a, b, c$

$$(1+\mathrm{ax})^5 = 1 + 5ax + 10a^2x^2 + \cdots$$

$$(\mathrm{b}+2\mathrm{x})(1+\mathrm{ax})^5 = (\mathrm{b}+2\mathrm{x})(1 + 5ax + 10a^2x^2 + \cdots)$$

$$= b + 5abx + 10ba^2x^2 + 2x + 10ax^2 + 20a^2x^3$$

$$\rightarrow b + (5ab + 2)x + (10ba^2 + 10a)x^2 + 20a^2x^3 = 3 - 28x + cx^2$$

$$coeff^n\ of\ x^0 \rightarrow b = 3$$

$$coeff^n\ of\ x \rightarrow 15a + 2 = -28 \rightarrow \mathrm{a} = -2$$

$$coeff^n\ of\ x^2 \rightarrow (10(3)(-2)^2 + 10(-2)) = c \rightarrow \mathrm{c} = 100$$

10. Application III A – Finding the specific term or coefficient

a. Write the 1st 3 term of $(1-2\mathrm{x})^7$, Obtain the coefficient of $\mathrm{x}^2 in\ the\ expandsion\ of (2+x)(1-2\mathrm{x})^7$

$$(1-2\mathrm{x})^7 = 1 - 14x + 84x^2 + \cdots$$

$$(2+x)(1-2\mathrm{x})^7 = (2+x)(1-14x+84x^2)$$

$$= 2 - 28x + 168x^2 + x - 14x^2 + 84x^3$$

$$coeff^n\ of\ x^2 = 154$$

b. $Find\ in\ ascending\ powers\ of\ x, the\ 1st\ 4\ terms\ in\ the\ expansion,$

$$(1+3x)^6, \qquad (1-4x)^5$$

$$hence\ find\ the\ coeff^n\ of\ x^2 in\ the\ expansion\ of\ (1+3x)^6(1-4x)^5$$

$$(1+3x)^6 = 1 + 6C_1 3x + 6C_2(3x)^2 + 6C_3(3x)^3 + \cdots$$

$$= 1 + 18x + 135x^2 + 540x^3$$

$$(1-4x)^5 = 1 + 6C_1 - 4x + 6C_2(-4x)^2 + 6C_3(-4x)^3 + \cdots$$

$$= 1 - 20x + 160x^2 - 640x^3$$

$$(1 + 18x + 135x^2 + 540x^3)(\ 1 - 20x + 160x^2 - 640x^3)$$

$$coeff^n\ of\ x^2 \to 160 - 360 + 135 = -65$$

11. Application III B – Finding the specific term or coefficient using formulae (r+1)

$$(r+1)^{th}\ term\ fomulae$$

Recall:

$$(\mathrm{a}+\mathrm{b})^{\mathrm{n}} = a^n + \mathrm{nC_1}\ \mathrm{a^{n-1}b^1} + \mathrm{nC_2}\ \mathrm{a^{n-2}b^2} + \cdots + \mathrm{nC_r}\ \mathrm{a^{n-r}b^r} + \cdots + \mathrm{nC_n}\ \mathrm{b^n}$$

a. $r = 0,\ 1st\ term \rightarrow\ a^n$
b. $r = 1,\ 2nd\ term \rightarrow\ \mathrm{nC_1}\ \mathrm{a^{n-1}b^1}$
c. $r = 2,\ 3rd\ term \rightarrow\ \mathrm{nC_2}\ \mathrm{a^{n-2}b^2}$
d. $r = 3,\ 4th\ term \rightarrow\ \mathrm{nC_3}\ \mathrm{a^{n-3}b^3}$

$$(r+1)^{th}\ term = \mathrm{nC_r}\ \mathrm{a^{n-r}b^r}$$

a. Find the term in x^4 *in the expansion of* $(3+2x)^7$
   a. Method 1 by expansion:

$$(3+2x)^7 = 3^7 + 7C_1 3^6 2x + 7C_2 3^5 (2x)^2 + 7C_3 3^4 (2x)^3 + 7C_4 3^3 (2x)^4 + \cdots$$

$$coeff^n x^4 = 7C_4 3^3 (2x)^4 = 15120x^4$$

   b. Method 2 by r+1:

$$(r+1)^{th}\ term = \mathrm{nC_r}\ \mathrm{a^{n-r}b^r}$$

$$(3+2x)^7\ term = 7\mathrm{C_r}\ 3^{7-\mathrm{r}}(2\mathrm{x})^{\mathrm{r}}$$

$$for\ term\ x^4 \rightarrow x^r = x^4 \rightarrow r = 4$$

$$5th\ term \rightarrow 7\mathrm{C_4}\ 3^{7-4}(2\mathrm{x})^4 = 15120x^4$$

b. Find the $\mathrm{coeff^n}$ of x^5 *in the expansion of* $(1-3x)^8$

$$(1-3x)^8 = 8\mathrm{C_r}\ 1^{8-\mathrm{r}}(-3x)^{\mathrm{r}}$$

$$for\ term\ x^5 \rightarrow x^r = x^5 \rightarrow r = 5$$

$$r = 5, \qquad 6th\ term \rightarrow 8\mathrm{C_5}\ 1^{7-\mathrm{r}}(-3x)^5 = -13608x^5$$

c. *in the expansion of* $\left(2x^2 + \frac{1}{x}\right)^9$
   a. The term independent of x

$$\left(2x^2 + \frac{1}{x}\right)^9 = 9\mathrm{C_r}\ (2\mathrm{x}^2)^{9-\mathrm{r}}(\frac{1}{x})^{\mathrm{r}} = 9\mathrm{C_r}\ 2^{9-\mathrm{r}}\mathrm{x}^{18-2\mathrm{r}}x^{-r}$$
$$= 9\mathrm{C_r}\ 2^{9-\mathrm{r}}\mathrm{x}^{18-3\mathrm{r}}$$

$$for\ term\ x^0 \rightarrow \mathrm{x}^{18-3\mathrm{r}} = x^0 \rightarrow 18 - 3\mathrm{r} = 0, \qquad \mathrm{r} = 6$$

$$r = 6, \qquad \rightarrow 9\mathrm{C_6}\ 2^{9-6}\mathrm{x}^{18-3(6)} = 672$$

   b. *the* $coeff^n of\ x^3$

$$for\ term\ x^3 \rightarrow \mathrm{x}^{18-3\mathrm{r}} = x^3 \rightarrow 18 - 3\mathrm{r} = 3, \qquad \mathrm{r} = 5$$

$$r = 5, \qquad \rightarrow 9\mathrm{C_5}\ 2^{9-5}\mathrm{x}^{18-3(5)} = 9\mathrm{C_5}\ 2^4 x^3 = 2016x^3$$

12. Exercise

a. Expand the following

   a. $(2+x)^5$

$$= 2^5 + 5C_1\, 2^4x + 5C_2 2^3x^2 + 5C_3 2^2x^3 + 5C_4 2x^4 + x^5$$

$$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$$

   b. $(3-x)^4$

$$3^4 + 4C_1\,(3)^3(-x) + 4C_2(3)^2(-x)^2 + 4C_3(3)(-x)^3 + (-x)^4$$
$$= 81 - 108x + 54x^2 - 12x^3 + x^4$$

b. Expand the 1st 3 term

   a. $(1-2x^2)^7$

$$1 + 7C_1\,(-2x^2) + 7C_2(-2x^2)^2 + \cdots$$
$$= 1 - 14x^2 + 84x^4$$

   b. $\left(1+\frac{1}{x}\right)^8$

$$1 + 8C_1\left(\frac{1}{x}\right) + 8C_2\left(\frac{1}{x}\right)^2 + \cdots$$
$$= 1 + \frac{8}{x} + \frac{28}{x^2}$$

   c. $\left(x+\frac{1}{2x}\right)^7$

$$x^7 + 7C_1\,(x)^6\left(\frac{1}{2x}\right) + 7C_2(x)^5\left(\frac{1}{2x}\right)^2 + \cdots$$
$$= x^7 + \frac{7x^6}{2x} + \frac{21x^5}{4x^2}$$
$$= x^7 + \frac{7x^5}{2} + \frac{21x^3}{4}$$

c. $Expand\ \left(2+\frac{x}{4}\right)^8\ to\ 1st\ 4\ term\ .Find\ Appoximate\ value\ for\ (2.0025)^8\ to\ 3\ d.p.$

$$\left(2+\frac{x}{4}\right)^8 = 2^8 + 8C_1\,(2)^7\left(\frac{x}{4}\right) + 8C_2(2)^6\left(\frac{x}{4}\right)^2 + 8C_3(2)^5\left(\frac{x}{4}\right)^3 + \cdots$$

$$= 256 + 256x + 112x^2 + 28x^3$$

$$2 + \frac{x}{4} = 2.0025 \rightarrow x = 0.01$$

$$256 + 256(0.01) + 112(0.01)^2 + 28(0.01)^3 = 258.571$$

d. $Expand\ (2+x)^5\ ,\ find\ \left(2+\sqrt{3}\right)^5\ in\ surd\ form\ a+b\sqrt{3}$

$$(2+x)^5 = 2^5 + 5C_1\,(2)^4(x) + 5C_2(2)^3(x)^2 + 5C_3(2)^2(x)^3 + 5C_4 2(x)^4 + x^5$$

$$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$$

$$\rightarrow 32 + 80\sqrt{3} + 80(\sqrt{3})^2 + 40(\sqrt{3})^3 + 10(\sqrt{3})^4 + (\sqrt{3})^5$$

$$= 32 + 80\sqrt{3} + 240 + 120\sqrt{3} + 90 + 9\sqrt{3} = 362 + 200\sqrt{3}$$

e. $Expand\ (2+a)^5\ ,\ find\ coeff^n of\ x^2\ where\ a = 2x - 5x^2$

$$(2+a)^5 = 32 + 80(2x-5x^2) + 80(2x-5x^2)^2 + 40(2x-5x^2)^3 + 10(2x-5x^2)^4 + (2x-5x^2)^5$$

$$= 32 + 160x - 400x^2 + 80(4x^2 - 20x^3 + 25x^4$$

$$= 32 + 160x - 400x^2 + 320x^2$$

$$coeff^n of\ x^2 = -80$$

f. $Expand\ (1-p)^5,\ \ p = x + x^2\ ,\ to\ the\ coeff^n of\ x^3,$
$Find\ Appoximate\ value\ for\ (0.9899)^5\ to\ 3\ d.p.$

$$(1-p)^5 = 1^5 + 5C_1\,(-p) + 5C_2(-p)^2 + 5C_3(-p)^3 + 5C_4(-p)^4 + -p^5$$

$$= 1 - 5p + 10p^2 - 10p^3 + 5p^4 - p^5$$

$$= 1 - 5(x+x^2) + 10(x+x^2)^2 - 10(x+x^2)^3 + 5(x+x^2)^4 - (x+x^2)^5$$

$$= 1 - 5x - 5x^2 + 10x^2 + 10x^3 + \cdots$$

$$= 1 - 5x + 5x^2 + 10x^3$$

$$1 - x + x^2 = (0.9899) \rightarrow x^2 + x - 0.0101 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-0.0101)}}{2(1)} \rightarrow x = 0.01, -1.01(NA)$$

$$(0.9899)^5 = 1 - 5(0.01) + 5(0.01)^2 + 10(0.01)^3 = 0.951$$

g. $Find\ in\ terms\ of\ a\ the\ coeff^n of\ x^2 in\ the\ expandsion\ of\ (1-3x)(1+ax)^6,$
$Given\ that\ the\ coeff^n of x^2 = 24, a > 0$

   a. Find a

$$(1+ax)^6 = 1^6 + 6C_1\,(ax) + 6C_2(ax)^2 + 6C_3(ax)^3 + \cdots$$

$$= 1 + 6ax + 15a^2x^2 + \cdots$$

$$(1-3x)(1+ax)^6 = (1-3x)(1 + 6ax + 15a^2x^2)$$

$$= 1 + 6ax + 15a^2x^2 - 3x - 18ax^2 - 45a^2x^3$$

$$\rightarrow 15a^2 - 18a = 24$$

$$a = \frac{-(-18) \pm \sqrt{(-18)^2 - 4(15)(-24)}}{2(15)} \rightarrow a = 2, -\frac{4}{5}$$

   b. $Find\ coeff^n of\ x$

$$\rightarrow x = 6a - 3 = 9$$

h. *Write down the middle term of* $\left(2x+\frac{1}{3}\right)^{8}$ *and simplify it*

$$\left(2x+\frac{1}{3}\right)^{8} = 8\mathrm{C}_{\mathrm{r}}\ (2x)^{8-\mathrm{r}}(\frac{1}{3})^{\mathrm{r}}, \qquad 5th\ term \rightarrow r = 4$$

$$8\mathrm{C}_4\ (2x)^4(\frac{1}{3})^4 = \frac{1120x^4}{81} = 13\frac{67}{81}x^4$$

i. *3rd term of* $\left(3+\frac{1}{10}\right)^{n}$ *is 10 times the 4th term, calculate n*

$$\mathrm{nC}_2\ (3)^{\mathrm{n}-2}(\frac{1}{10})^2 = 10\ \mathrm{nC}_4\ (3)^{\mathrm{n}-4}(\frac{1}{10})^4$$

$$\frac{\mathrm{n}(\mathrm{n}-1)}{1*2}\frac{(3)^2(3)^{\mathrm{n}}}{100} = \frac{\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2)}{1*2*3}\frac{(3)^{-3}(3)^{\mathrm{n}}}{100}$$

$$\frac{(3)^{-2}}{100} = \frac{(\mathrm{n}-2)}{3}\frac{(3)^{-3}}{100}$$

$$\frac{1}{900} = \frac{\mathrm{n}-2}{8100}$$

$$8100 = 900\mathrm{n} - 1800$$

$$n = 11$$