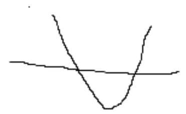
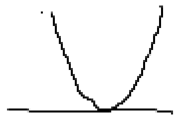
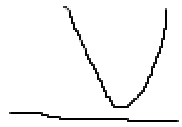
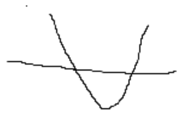
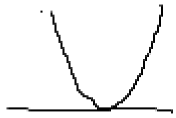
Discriminant

1. Introduction

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant:

$$b^2 - 4ac$$

Type of roots		Graph
1 Real roots and distinct roots Curve cuts **line at 2 points**	$b^2 - 4ac > 0$	
2 Real roots and equal roots Curve is at **tangent at line**	$b^2 - 4ac = 0$	
3 No Real roots Curve is **does not meet line** Equation **is always positive**	$b^2 - 4ac < 0$	
4 Real roots **Lines meets curve**	$b^2 - 4ac \geq 0$	 

2. Exercise – Discriminant and Nature of Roots

1	$Find\ the\ value\ of\ p\ for\ which\ px^2 = 2x - p\ has\ real\ and\ equal\ roots$ $px^2 - 2x + p$ $2^2 - 4pp = 0$ $4 = 4p^2$ $\pm 1 = p$ $p = -1, \quad p = 1$
2	$x^2 + 2kx + (k-1)(k-3) = 0\ has\ real\ roots. Find\ the\ range\ of\ values\ of\ k$ $x^2 + 2kx + k^2 - 4k + 3 = 0$ $4k^2 - 4(1)(k^2 - 4k + 3) \geq 0$ $4k^2 - 4k^2 + 16k - 12 \geq 0$ $16k - 12 \geq 0 \rightarrow k \geq \frac{3}{4}$
3	$Find\ range\ of\ m\ for\ 2x^2 - mx + 2 = 0\ has\ real\ and\ distinct\ roots$ $m^2 - 4(2)(2) > 0$ $m^2 > 16 \rightarrow m > \pm 4$ $m < -4, \quad m > 4$
4	$Find\ range\ of\ p\ for\ 3x^2 - px + 2p = 0\ has\ no\ real\ roots$ $(-p)^2 - 4(3)(2p) < 0$ $p^2 - 24p < 0$ $p(p - 24) < 0$ $0 < p < 24$

3. Exercise – Discriminant and Nature of Intersection between line and curves

1	$Find\ the\ value\ of\ k\ for\ which\ the\ line\ y+kx=12$ $is\ a\ tangent\ to\ the\ curve\ x^2+xy=12$ $y+kx=12 \rightarrow y=12-kx$ $x^2+x(12-kx)=12$ $x^2+12x-kx^2-12=0$ $(1-k)x^2+12x-12=0$ $12^2-4(1-k)(-12)=0$ $144+48-48k=0$ $k=4$
2	$Find\ range\ of\ m\ for\ line\ y=mx+1\ meets\ curve\ y^2=8x$ $(mx+1)^2-8x=0$ $m^2x^2+2mx+1-8x=0$ $m^2x^2+(2m-8)x+1=0$ $(2m-8)^2-4m^2\geq 0$ $4m^2-32m+64-4m^2\geq 0$ $2\geq m$
3	$Find\ range\ of c\ for\ line\ y=c-3x\ does\ not\ intersect\ curve\ xy=3$ $x(c-3x)=3$ $3x^2-cx+3=0$ $c^2-4(3)(3)<0$ $c<\pm 6$ $-6<c<6$
4	$Find\ range\ of\ m\ for\ line\ y=mx-5\ intersect\ curve\ y=x^2-1\ with\ 2\ distinct\ points$ $mx-5\ =x^2-1$ $x^2-mx+4=0$ $m^2-4(1)(4)>0$ $m>\pm 4$ $m<-4, \qquad m>4$
5	$x^2+2x+k=3k-1\ has\ no\ real\ roots, what\ can\ be\ deduced\ about\ the\ curve\ y=(x+1)^2$ $and\ the\ line\ y=3x-1$ $(x+1)^2=3x-1$ $x^2+2x+1=3x-1$ $x^2+2x+k=3k-1 \rightarrow x^2+2x+k=3kx-1$ $x^2+2x+k=3kx-1$ $x^2+(2-3k)x+k+1=0$ $b^2-4ac<0$ $(2-3k)^2-4(1)(k+1)<0$ $4-12k+9k^2-4k-4<0$ $9k^2-16k<0$ $k(9k-16)<0$ $0<k<\frac{16}{9}$

4. Exercise – Show the Nature of roots or y > 0

1	$Show\ that\ x^2 + (1-p)x - p = 0\ are\ real\ for\ all\ real\ valus\ of\ p$ $(1-p)^2 - 4(1)(-p) =$ $1 - 2p + p^2 + 4p =$ $p^2 + 2p + 1 \geq 0 =$ $(p+1)^2 \geq 0\ (shown)$
2	$Show\ that\ x^2 - 2x - p + 2 = 0\ are\ real\ and\ distinct\ if\ p > 1$ $4 - 4(1)(-p+2) > 0$ $4 + 4p - 8 > 0$ $p > 4\ (shown)$
3	$Find\ the\ range\ of\ c\ for\ which\ 3x^2 + 5x + c\ is\ always\ positive$ $3x^2 + 5x + c > 0 \rightarrow \therefore b^2 - 4ac < 0$ $5^2 - 4(3)(c) < 0$ $25 - 12c < 0$ $c > \frac{25}{12}\ (shown)$
4	$Given\ that\ y = tx^2 + 8x + 10 - t$ $find\ the\ range\ of\ values\ of\ t\ for\ which\ y\ is\ always\ positive$ $y = tx^2 + 8x + 10 - t > 0 \rightarrow \therefore b^2 - 4ac < 0$ $8^2 - 4(t)(10-t) < 0$ $64 - 40t + 4t^2 < 0$ $4t^2 - 40t + 64 < 0$ $t^2 - 10t + 16 < 0$ $(t-2)(t-8) < 0$ $2 < t < 8$

5. Exercise – Show the Nature of roots or y > 0

1	$Show\ that\ x^2+(1-p)x-p=0\ are\ real\ for\ all\ real\ valus\ of\ p$ $(1-p)^2-4(1)(-p)=$ $1-2p+p^2+4p=$ $p^2+2p+1\geq 0=$ $(p+1)^2\geq 0\ (shown)$
2	$Show\ that\ x^2-2x-p+2=0\ are\ real\ and\ distinct\ if\ p>1$ $4-4(1)(-p+2)>0$ $4+4p-8>0$ $p>4\ (shown)$
3	$Find\ the\ range\ of\ c\ for\ which\ 3x^2+5x+c\ is\ always\ positive$ $3x^2+5x+c>0\rightarrow\therefore b^2-4ac<0$ $5^2-4(3)(c)<0$ $25-12c<0$ $c>\frac{25}{12}\ (shown)$
4	$Given\ that\ y=tx^2+8x+10-t$ $find\ the\ range\ of\ values\ of\ t\ for\ which\ y\ is\ always\ positive$ $y=tx^2+8x+10-t>0\rightarrow\therefore b^2-4ac<0$ $8^2-4(t)(10-t)<0$ $64-40t+4t^2<0$ $4t^2-40t+64<0$ $t^2-10t+16<0$ $(t-2)(t-8)<0$ $2<t<8$
5	$Find\ the\ range\ of\ values\ of\ k\ for\ which\ the\ graph\ of\ y=x^2+(k-4)x+1$ $Lies\ entirely\ above\ the\ x-axis$ $b^2-4ac<0$ $(k-4)^2-4(1)(1)<0$ $k^2-8k+16-4<0$ $k^2-8k+12<0$ $(k-2)(k-6)<0$ $\therefore 2<k<6$