Solving Quadratic Equation

Contents

1. Why learn?

- As the title implies, learning to solve quadratic equation

2. Introduction

$$ax^2 + bx + c = 0$$

$$a\ is\ the\ coefficient\ of\ x^2\ (Quadratic\ term)$$

$$b\ is\ the\ coefficient\ of\ x^1\ (Linear\ term)$$

$$c\ is\ the\ coefficient\ of\ x^0\ (Constant\ value)$$

3. Type of quadratic equation

1 No linear Term	e.g. 1 $Mth\ 1:\ x^2 - 9 = 0$ $x^2 = 9$ $x = \pm 3$ $Mth\ 2:\ x^2 - 9 = 0$ $(x+3)(x-3) = 0$ $x = \pm 3$ e.g. 2 Solve: $\frac{v}{8} = \frac{18}{v}$ $v^2 = 144$ $v = \pm 12$
2 No constant Term (Do not divide by x, as it will become a linear equation, meaning, loss of information)	e.g. 1 $2x^2 = 7x$ $x(2x-7) = 0$ $x_1 = 0, \quad x_2 = \frac{7}{2}$
3 Solve by Factorization $min\ or\ max = \frac{x_1 + x_2}{2}$	e.g. 1 $3x^2 - 5x - 8 = 0$ $(3x-8)(x+1) = 0$ $x_1 = \frac{8}{3}, \quad x_2 = -1$
4 Completed Square form $(x+a)^2 = b$	e.g. 1 $(x+2)^2 = 16$ $x + 2 = \pm 4$ $x_1 = 2, \quad x_2 = -6$ e.g. 2 $(3x+2)^2 = 43$ $3x + 2 = \pm\sqrt{43}$ $x = \frac{\sqrt{43}-2}{3}, x = \frac{-\sqrt{43}-2}{3}$

5 Solve by Completing Square	e.g. 1 $$x^2 + 8x + 9 = 0$$ $$x^2 + 8x + 4^2 = -9 + 4^2$$ $$(x+4)^2 = 7$$ $$x + 4 = \pm\sqrt{7}$$ $$x = \sqrt{7} - 4\,, \qquad x = -\sqrt{7} - 4$$ Condition: 1. $coeff^n\ of\ x^2\ must\ be\ 1$ 2. $Add\ oeff^n\ of\ x \rightarrow \left(\frac{coeff^n\ of\ x}{2}\right)^2$ 3. $Works\ for\ negative\ oeff^n\ too.$
6 Solve by Formulae (always works, but use it for last resort, extensive calculation)	$$ax^2 + bx + c = 0$$ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Prove (Solve by using **Type 5 Completing square**): $$ax^2 + bx + c = 0$$ $$x^2 + \frac{b}{a}x = -\frac{c}{a}$$ $$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$ $$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$ $$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$ $$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{4ac}{4a^2}}$$ $$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$ $$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$ $$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\ (\#proved)$$

4. Exercise – Solve by completing Square (type 5)

1	$x^2 - 10x + 5 = 0$ $x^2 - 10x + (-5)^2 = -5 + (-5)^2$ $(x-5)^2 = 20$ $x - 5 = \pm\sqrt{20}$ $x_1 = \sqrt{20} + 5\,, \qquad x_2 = -\sqrt{20} + 5$
2	$3x^2 - 4x - 6 = 0 \rightarrow x^2 - \frac{4}{3}x - 2 = 0$ $x^2 - \frac{4}{3}x - \left(-\frac{4}{6}\right)^2 = 2 + \left(-\frac{4}{6}\right)^2$ $\left(x - \frac{2}{3}\right)^2 = 2 + \frac{4}{9}$ $x - \frac{2}{3} = \pm\sqrt{\frac{22}{9}}$ $x_1 = \sqrt{\frac{22}{9}} + \frac{2}{3}\,, \qquad x_2 = -\sqrt{\frac{22}{9}} + \frac{2}{3}$