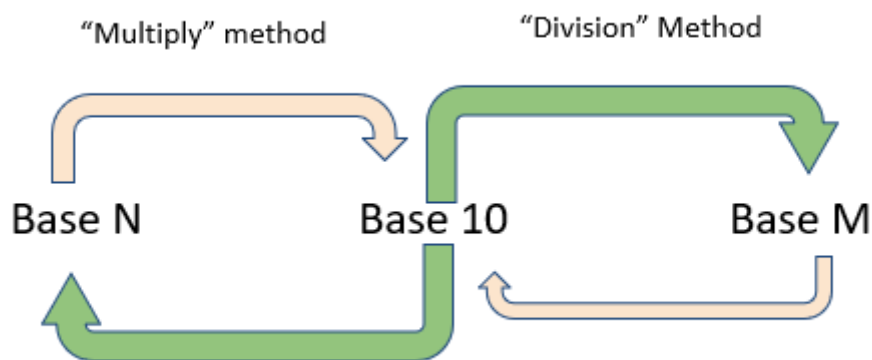# Contents

1. Objective
- Learn how numbers can be represented in different ways.
- Also known as Change of Bases.

2. Purpose
- Computers only read binaries (Base 2).
- Understand how number can be represented in different bases
- Learn to convert between bases
- Requires this topics knowledge for future lessons

3. Summary

"Multiply" method   "Division" Method

Base N   Base 10   Base M

4. Introduction

4.1. Base 10
- We have 10 digits to represent numbers starting from 0 to 9
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

4.2. Base 2 (Binary)
- We have 10 digits to represent numbers starting from 0 to 1
- 0, 1

4.3. Base 8 (Octal)
- We have 10 digits to represent numbers starting from 0 to 7
- 0, 1, 2, 3, 4, 5, 6, 7

4.4. Base 16 (Hexadecimal)
- We have 10 digits to represent numbers starting from 0 to 9, A to F
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

5. Convert Base N to Base 10 (Multiply method)

5.1. Example 1 - convert base 2 to 10

$$01110_2 \ to \ base \ 10$$

$$0 = 0 * 2^0$$

$$1 = 1 * 2^1$$

$$1 = 1 * 2^2$$

$$1 = 1 * 2^3$$

$$0 = 0 * 2^4$$

$$01110_2 = 0 * 2^4 + 1 * 2^3 + 1 * 2^2 + 1 * 2^1 + 0 * 2^0$$

$$= 0 + 8 + 4 + 2 + 0$$

$$= 14$$

5.2. Example 2 - convert base 3 to 10

$$10122_3 \ to \ base \ 10$$

$$10122_3 = 1 * 3^4 + 0 * 3^3 + 1 * 3^2 + 2 * 3^1 + 2 * 3^0$$

$$= 81 + 0 + 9 + 6 + 2$$

$$= 98$$

5.3. Example 3 - convert base 16 to 10

$$AC953_{16} \ to \ base \ 10$$

$$AC953_{16} = A * 16^4 + C * 16^3 + 9 * 16^2 + 5 * 16^1 + 3 * 16^0$$

$$= (10 * 65536) + (12 * 4096) + (9 * 256) + (5 * 16) + 3$$

$$= 655360 + 49152 + 2304 + 80 + 3$$

$$= 706899$$

6. Convert Base 10 to Base N (Division Method)

6.1. Example 1 – convert base 10 to base 2

$$convert\ 14_{10}\ to\ base\ 2$$

```
2   | 14        | Remainder
    | 7         | 0      (7>=2, continue)
    | 3         | 1      (3>=2, continue)
    | 1         | 1      (1<2, stop)
    | 1
```

$$answer: 1110, \quad (read\ from\ bottom\ up)$$

6.2. Example 2 – convert base 10 to base 16

```
16  | 666       | Remainder
    | 41        | 10     (41>=16, continue)
    | 2         | 9      (2<16, stop)
    | 2
```

$$answer: 29A, \quad (read\ from\ bottom\ up)$$

7. Tips

7.1. HEX to BIN ("4-bit partition method")

    7.1.1. Convert 0xAD3EF23750F

        1010_1101_0011_1110_1111_0010_0011_0111_0101_0000_1111

7.2. BIN to HEX ("4-bit partition method")

    7.2.1. Convert 1010_1101_0011_1110_1111_0010_0011_0111_0101_0000_1111

        0xAD3EF23750F

Base 10 to Base 2 to Base 16 (longer route)

7.3. OCT to BIN ("3-bit partition method")

    7.3.1. Convert 075643742153

        111_101_110_100_011_111_100_010_001_101_011

    7.3.2. Convert 111_101_110_100_011_111_100_010_001_101_011

        075643742153

8. Bit Conversion - Questions

Convert the following from its respective bases

Question	Bin (Base 2)	Oct (Base 8)	Dec (Base 10)	Hex (Base 16)
110_2				
11011_2				
101001_2				
101101_2				
176_8				
22_8				
441_8				
244_8				
50_{10}				
666_{10}				
333_{10}				
434_{10}				
441_8				
$0xAB_{16}$				
$0x123_{16}$				
$0x2F_{16}$				
$0x13_{16}$				
$0xA2C_{16}$				
$0xFFF_{16}$				

Operations

$101101_2 + 111011_2$	
$110000_2 - 00111_2$	
$111111_2 * 101_2$	
$111111_2 / 111_2$	
$0x123_{16} + 0xABC_{16}$	
$0xABC_{16} - 0x891_{16}$	
$0xB_{16} * 0x2_{16}$	
$0x14_{16} / 0xA_{16}$	

Convert to Base 10 OR evaluate the expressions

35.2_6	Base 10 =
11.3_8	Base 10 =
11.52_9	Base 10 =
$33.3_8 - 11.1_8$	
111.111_2	Base 10 =

9. Bit Conversion - Answers

Convert the following from its respective bases

Question	Bin	Oct	Dec	Hex
110_2	110	6	6	6
11011_2	11011	33	27	1B
101001_2	101001	51	41	29
101101_2	101101	55	25	2D
176_8	1111110	176	126	7E
22_8	10010	22	18	12
441_8	100100001	441	289	121
244_8	10100100	244	164	A4
50_{10}	110010	62	50	32
666_{10}	1010011010	1232	666	29A
333_{10}	101001101	515	333	14D
434_{10}	110110010	662	434	1B2
441_8	100100001	441	289	121
$0xAB_{16}$	10101011	253	171	0xAB
$0x123_{16}$	100100011	443	291	0x123
$0x2F_{16}$	101111	57	47	0x2D
$0x13_{16}$	10011	23	19	0x13
$0xA2C_{16}$	101000101100	5054	2604	0xA2C
$0xFFF_{16}$	111111111111	7777	4095	0xFFF

Operations

101101_2+ 111011_2	1001000_2
110000_2- 00111_2	101001_2
111111_2* 101_2	100111011_2
111111_2/111_2	1001_2
$0x123_{16}$+$0xABC_{16}$	$0xBDF_{16}$
$0xABC_{16}$-$0x891_{16}$	$0x22B_{16}$
$0xB_{16}$*$0x2_{16}$	$0x16_{16}$
$0x14_{16}$/$0xA_{16}$	$0x2_{16}$

Convert to Base 10 OR evaluate the expressions

35.2_6	Base 10 = 24/3
11.3_8	Base 10 = 2/3
11.52_9	Base 10 = 57/81
33.3_8-11.1_8	22.2_8
111.111_2	Base 10 = 14/8