Mean Mode Median

1. Mean

$$\text{The mean of a set of } n \text{ numbers}, x_1, x_2, x_3, \dots, x_n \text{ is denoted by } x \text{ where}$$

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x}{n} \; OR \; = \frac{\sum f(x)}{\sum f}$$

2. Median

$$\text{The median is the value of the middle term of a set of numbers,}$$
$$\text{arranged in either ascending or descending order}$$

$$\text{For } n \text{ numbers,}$$

$$\text{if } n \text{ is odd: median} = \frac{(n+1)}{2} \text{th term}$$

$$\text{if } n \text{ is even: median} = \frac{\frac{n}{2}\text{th term} + \frac{(n+1)}{2}\text{th term}}{2}$$

3. Mode

$$\text{The mode is the numberwith the highest frequency}$$

4. Example - MMM

1	$\text{The marks of } 12 \text{ pupils}: 14,13,16,20,11,16,19,14,16,8,17,19$ $\text{mean} \rightarrow \frac{14+13+16+20+11+16+19+14+16+8+17+19}{2} = 15.25$ $8,11,13,14,14,16,16,16,17,19,19,20$ $\text{median} \rightarrow \frac{16+16}{2} = 16$ $\text{mode} \rightarrow 16 \text{ (there are three } 16\text{)}$
2	$5x^2 + 3x - 2 = x^2 + \frac{3}{5}x + \left(\frac{3}{5}\right)^2 = \frac{2}{5} + \left(\frac{9}{25}\right)$ $= \left(x + \frac{3}{5}\right)^2 + \frac{19}{25}$
3	$(2x-1)^2 - 2 = 2^2\left(x - \frac{1}{2}\right)^2 - 2 = 4\left(x - \frac{1}{2}\right)^2 - 2$

5. Standard Deviation

$$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad OR \quad \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Set A

x	$x_i - \bar{x}$	$(x-\bar{x})^2$	x^2
3	-4	16	9
3	-4	16	9
5	-2	4	25
7	0	0	49
8	1	1	64
9	2	4	81
9	2	4	81
12	5	25	144
$\bar{x} = 7$		$\sum(x-\bar{x})^2 = 70$	$\sum x^2 = 462$

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{70}{8}} \approx 2.96, \qquad \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{462}{8} - 49} = \sqrt{\frac{70}{8}} \approx 2.96$$

Set B

x	x^2
0	0
1	1
1	1
1	1
4	16
11	121
15	225
23	529
$\bar{x} = 7$	$\sum x^2 = 894$

$$\sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{894}{8} - 49} \approx 7.52$$

$$Both\ SD\ have\ the\ same\ mean, it\ is\ meaningful\ to\ compare\ the\ SD$$

$$Set\ A\ has\ smaller\ SD, meaning\ narrow\ spread\ of\ marks, most\ number\ are\ around\ the\ mean$$

$$Set\ B\ has\ greater\ SD, meaning\ further\ spread\ of\ marks, wide\ spread\ of\ number, extreme\ performance$$

6. Standard Deviation for a Frequency Distribution

$$S = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{n}} \; OR \; \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$S = \sqrt{\frac{\sum_{i=1}^{n} f_i x_i^2}{n} - \bar{x}^2} \; OR \; \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f}\right)^2}$$

No. of People	1	2	3	4	5	6
No. of Units	20	35	40	55	91	59

x	f	fx	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
1	20	20	-3.13	9.796	195.938
2	35	70	-2.13	4.5368	159.7915
3	40	120	-1.13	1.276	51.076
4	55	220	-0.13	0.0169	0.9295
5	91	455	0.87	0.7569	68.8779
6	59	354	1.87	3.4969	206.3171
	$\sum f = 300$	$\sum f x = 1239$	$\bar{x} = \frac{1239}{300} \approx 4.13$		$\sum f(x - \bar{x})^2 = 681.93$

$$\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{681.93}{300}} \approx 1.51$$

x	f	fx	fx^2
1	20	20	40
2	35	70	140
3	40	120	360
4	55	220	880
5	91	455	2275
6	59	354	2124
	$\sum f = 300$	$\sum f x = 1239, \quad \bar{x} = \frac{1239}{300} \approx 4.13$	$\sum f x^2 = 5799$

$$\sqrt{\frac{\sum_{i=1}^{n} f_i x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{5799}{300} - (4.13)^2} = 1.51$$