**Partial Fractions**

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## Why learn Partial Fraction?

* 1. To be able to get the derivative or get the integral more easily in Calculus

## What is Partial Fraction?

* 1. It’s a **method to simplify algebraic fraction**
  2. Initially we learned how to add or subtract 2 **proper fraction**:
  3. **Partial Fraction** is to **reverse the process** by decomposing:

## Definitions (introduction)

* 1. A ratio of 2 polynomials f(x) and g(x), that is is called an **algebraic fraction**.
     1. Example:
  2. If the **degree of the numerator** f(x) is **less than the degree of the denominator** g(x), then the fraction is said to be **proper fraction**.
     1. Example:
  3. If the **degree of the numerator** f(x) is **greater than or equal to the degree** of the denominator g(x), the fraction is said to be **improper fraction**.
     1. Example:
  4. A **simple fraction** is a **proper fraction** of which the **denominator cannot be factorized**
     1. Example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **🗴** | **🗸** | **🗴** | **🗴** | **🗸** |
|  |  |  |  |  |

## Objective

* 1. To express **algebraic fraction** as **partial fraction**
     1. It must be a **proper Fraction**
     2. The **denominator must be factorized completely**

## Patterns and Forms

* 1. The **denominator** of the single fraction determine the forms of the partial fractions

|  |  |  |
| --- | --- | --- |
| **Name** | **Single fractions** | **Possible Forms of Partial Fractions** |
| Distinct Linear |  |  |
| Repeated Linear |  |  |
| Distinct Quadratic |  |  |

## Practice, example using different method to solve

|  |  |
| --- | --- |
| 1a. Linear |  |

Solve:

Let: a = 1, b = -1, c = 1, d = 2

Find A, Input x = 1:

Find B, Input x = -2

Answer:

Check:

|  |  |
| --- | --- |
| 1b. Linear, Factorize base |  |

Factorize base:

Solve:

Find A, Input x = -1:

Find B, Input x = 4

Answer:

Check:

|  |  |
| --- | --- |
| 2a. Repeated Linear |  |

Solve:

Find B, Input x = 1:

Find A, Input x = 0:

Answer:

Check:

|  |  |
| --- | --- |
| 2b. Repeated Linear |  |

Solve:

Find B, Input x = -3:

Find A, Input x = 0:

Answer:

Check:

|  |  |
| --- | --- |
| 3a. Distinct Quadratic |  |

Solve:

Find A, Input x =:

Expand:

Find B,

Find C,

Answer:

Check:

|  |  |
| --- | --- |
| 3b. Distinct Quadratic |  |

Solve:

Find A, Input x = -2

Find C, Input x = 0

Find B, Input x = 1

Answer:

Check:

|  |  |
| --- | --- |
| 4. Tricky Denominator (repeated) |  |

Solve:

Find C, Input x = 1

Find A, Input x = -1

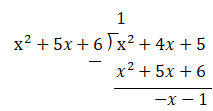
Find B, Input x = 0

Answer:

Check:

|  |  |
| --- | --- |
| 5. Long Division |  |

Long Division:



Solve:

Find A, Input x = -3

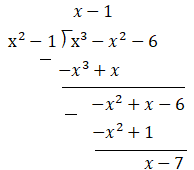
Find B, Input x = -2

Answer:

Check:

|  |  |
| --- | --- |
| 6. Long Division (Integral) |  |

Long Division:



Solve:

Find A, Input x = 1

Find B, Input x = -1

Answer:

Check:

## Tips/Summary

* 1. **Use long division** for **improper fraction,** which is the **degree of numerator is greater than or equal than denominator. It is an extra needed step.**
     1. You may have notice the term **improper fraction, it is the exact same term** used for normal, non-polynomial fraction example like,
     2. Proper Fraction will be example like
  2. See, notice and use the pattern for **linear, repeated and distinct quadratic respectively**
  3. **Mainly 2 method to find A, B, C, D …** 
     1. Substitute x = value
     2. Expand the equation and use coefficient of x

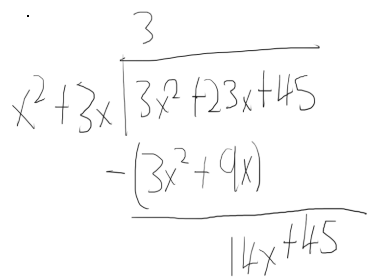
## Exercises

|  |  |
| --- | --- |
| [1](#_1) |  |
| [2](#_2) |  |
| [3](#_3) |  |
| [4](#_4) |  |
| [5](#_5) |  |
| [6](#_6) |  |
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| [9](#_9) |  |
| [10](#_10) |  |
| [11](#_11) |  |
| [12](#_12) |  |
| [13](#_13) |  |
| [14](#_14) |  |
| [15](#_15) |  |
| [16](#_15) | Find a, b, c |

## Answers

|  |  |
| --- | --- |
| 1 |  |

Long Division:



Solve:

Find A, Input x = 0

Find B, Input x = -3

Answer:

|  |  |
| --- | --- |
| 2 |  |

Long Division:…

Solve:

Find A, Input x = 3

Find B, Input x = -3

Answer:

|  |  |
| --- | --- |
| 3 |  |

Solve:

Find C, Input x = 1

Find A, Input x = -2

Find B, Input x = 0

Answer:

|  |  |
| --- | --- |
| 4 |  |

Solve:

Find A, Input x = 1

Find C, Input x = -2

Find B, Input x = 0

Answer:

|  |  |
| --- | --- |
| 5 |  |

Solve:

Find A, Input x = 2

Find C, Input x = 0

Find B, Input x = 1

Answer:

|  |  |
| --- | --- |
| 6 |  |

Solve:

Find A, Input x = -2:

Find B, Input x = 5:

Answer:

|  |  |
| --- | --- |
| 7 |  |

Solve:

Find A, Input x = 7:

Find B, Input x = -7:

Answer:

|  |  |
| --- | --- |
| 8 |  |

Solve:

Find A, Input x = 0:

Find C, Input x = 1:

Find B, Input x =2:

Answer:

|  |  |
| --- | --- |
| 9 |  |

Solve:

Find B, Input x = 1/2:

Find A, Input x = -3/2:

Answer:

|  |  |
| --- | --- |
| 10 |  |

Solve:

Find A, Input x = 2:

Find C, Input x = 0:

Find B, Input x =1:

Answer:

|  |  |
| --- | --- |
| 11 |  |

Solve:

Find B, Input x = 0:

Expand

Find A, Coeff^n of x:

Find D, Coeff^n of x^2:

Find C, Coeff^n of x^3:

Answer:

|  |  |
| --- | --- |
| 12 |  |

Long Division:

Answer:

|  |  |
| --- | --- |
| 13 |  |

Long Division:

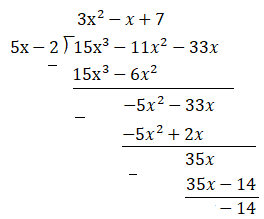
Find A, Input x = -1:

Find B, Input x = -2:

Answer:

|  |  |
| --- | --- |
| 14 |  |

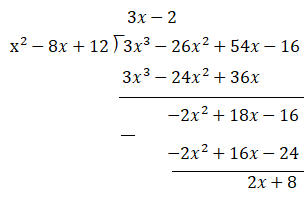
Long Division:



Answer:

|  |  |
| --- | --- |
| 15 |  |

Long Division:



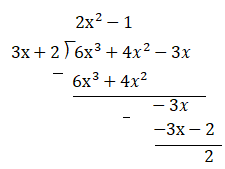
Find A, Input x = 6:

Find B, Input x = 2:

Answer:

|  |  |
| --- | --- |
| 16 | Find a, b, c |

Long Division:



Answer: