**Algorithmic Game Theory**

*Project Assignment A.Y. 2024/2025*

Consider a setting with a set of agents and with a set of *skills*. Each agent  **N** is associated with a set of skills denoted by Note that there is no a-priori specified number of skills that each agent owns; for instance, an agent might have just one skill, while another might have all the available skills. In fact, there is a task to be completed and all the skills in are required to this end. Hence, agents are required to collaborate with each other and a number of strategic issues come into play.

For each of the following questions, implement in Python a method that can provide results for any possible pair . Report then the results obtained over the instance, with 4 agents and 3 skills, graphically depicted below according to an intuitive notation (an edge means that the agent owns the corresponding skill).

Immagine che contiene schermata, cerchio, linea, diagramma

Descrizione generata automaticamente

Provide arguments and explanations on the various design choices.

1. Assume that completing the task leads to a reward of **100$**. Then, compute the *Shapley value* associated with the agents as a fair way to distribute that reward among them. And, finally, check whether the *Shapley value* is in the *core* of the *coalitional game* induced by the setting.
2. Assume that each agent  **N** might freely decide whether to join the group in order to complete the task. In particular, each agent incurs a fixed cost of to join the group and s/he is selfish interested, so that s/he would like that the revenue s/he gets by collaboration covers this expense (in the example, let . Assume that the revenue is divided according to the *Shapley value* of the *coalitional game* induced by the agents that join the group (as in item 1). Then, check whether the resulting strategic setting, where each agent has two actions (joint, not join), admits a *pure Nash equilibrium* and computes one, if any.
3. Assume that all agents participate to the setting, but they might cheat on the cost , and consider a setting where a mechanism has to identify a group of agents that is capable of completing the task with the minimum overall cost. Then, compute a payment scheme that provides incentives to *truthfully* report such costs.