Prooving the non-solvability of the friends' problem

(from a certain number of friends)

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Quick recap of the problem

- N friends, after eating in a pizzeria, decide to move to a pub
- Half of them study Computer Engineering (C), the other half Data Science (D)
- They have only a tandem (two seats) to move
- NEVER have less D than C in either side, otherwise the data scientists will get annoyed by the geeks (C) and will leave

Thesis

The problem, this formulated, cannot be solved from a certain number of friends: in our case, we will consider $N \geq 14$ (let's call M the number of both C and D: we will have $M \geq 7$)

It doesn't mean that for $N < 14$ the problem is always solvable: we focused to proof that the problem is unsolvable for $N \geq 14$

Demonstration

Since the tandem can bring only two people, we cannot see an increase higher than 2 (i.e. it's impossible to go from 2 to 5 data scientists at the pub)

At a certain point, we will have 3 or 4 D at the pub, so all the possible states are:

*C[m-α]D[m-3] – C[α]D[3]

C[m-α]D[m-3] – *C[α]D[3]

*C[m-α]D[m-4] – C[α]D[4]

C[m-α]D[m-4] – *C[α]D[4]

What should be the value of α? Ignoring the bike:

$$C[m-\alpha]D[m-3] - C[\alpha]D[3]$$

$$C[m-\alpha]D[m-4] - C[\alpha]D[4]$$

It should be 3 or 4 otherwise the constraints would be violated, so:

$$*C[m-3]D[m-3] - C[3]D[3]$$

$$C[m-3]D[m-3] - *C[3]D[3]$$

$$*C[m-4]D[m-4] - C[4]D[4]$$

$$C[m-4]D[m-4] - *C[4]D[4]$$

Let’s consider the first state:

*C[m-3]D[m-3] – C[3]D[3]

Possible choices:

- Moving one or two C: constraint violated at the pub
- Moving one or two D: constraint violated at the pizzeria
- Moving one C and one D: the only valid option, ending in

C[m-4]D[m-4] – *C[4]D[4]

C[m-4]D[m-4] – *C[4]D[4]

And now? Again, possible choices:

- Moving one or two C: constraint violated at the pizzeria
- Moving one or two D: constraint violated at the pub
- Moving one C and one D: again, the only valid option, ending in

*C[m-3]D[m-3] – C[3]D[3]

We are stuck! We can’t reach the goal

Similar reasoning can be conducted for the other states

To sum up

- Due to the problem formulation, we are forced to go through one of the possible states we showed
- Since none of them allow us to reach the goal, the problem is not solvable