Linear Algebra

Why Linear Algebra?

Visualization techniques of LA

Ways of solving equations.

Examples

Singular Case

Inverse

Eigenvalues and their subsequent use - Next class

Why Linear Algebra

1. Easy to solve
2. Integral to approaching solutions for many problems, especially of polynomial nature
3. Plethora of applications (first approximations to non-linear systems)

Applications - examples

Let’s make some tea?
Or bake a cake?
Magic trick!
Biological context from the audience

New Age Applications

Facebook - Social Network

Google Search

Fluid Dynamics

Quantum mechanics

Statistics

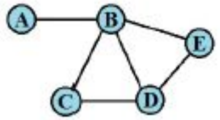
Special relativity

Machine Learning (SVD)

[Iron Wolf Man]

Encode Network in a Friendship Matrix

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

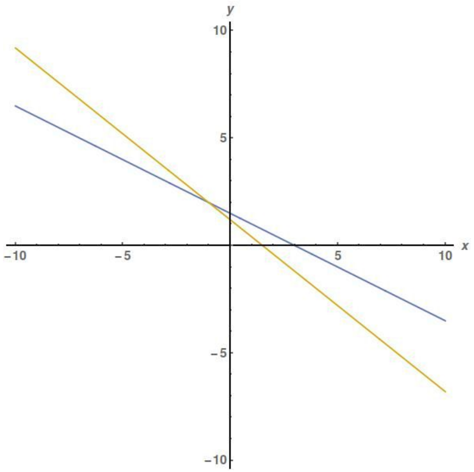


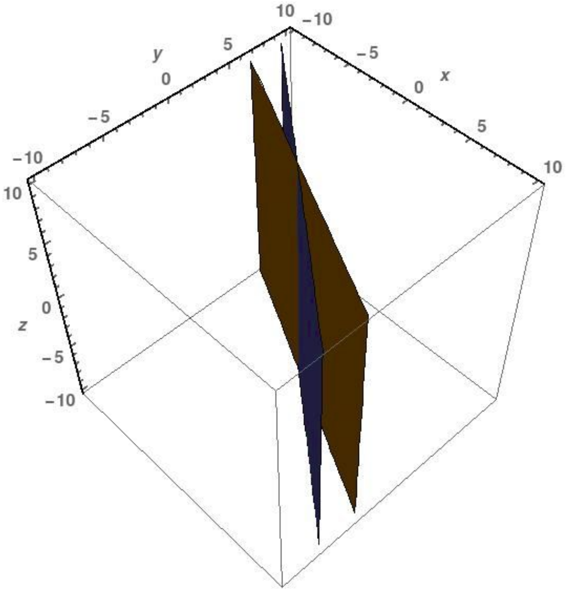
Yes $= 1$, N/A or No $= 0$.

Friendship?	Alice	Brandon	Cynthia	Dion	Enrique
Alice	N/A	Yes			
Brandon	Yes	N/A	Yes	Yes	Yes
Cynthia		Yes	N/A	Yes	
Dion		Yes	Yes	N/A	Yes
Enrique		Yes		Yes	N/A

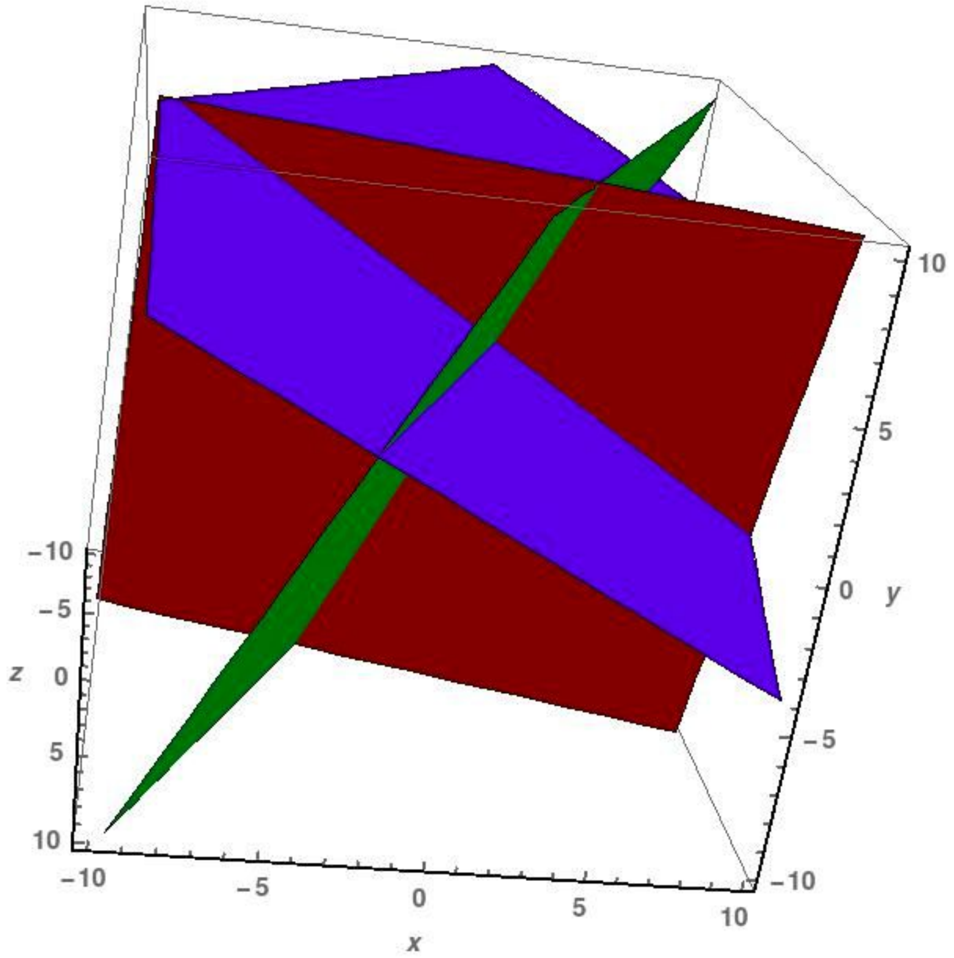
Visualizations

$$1x + 2y = 3$$
$$4x + 5y = 6$$

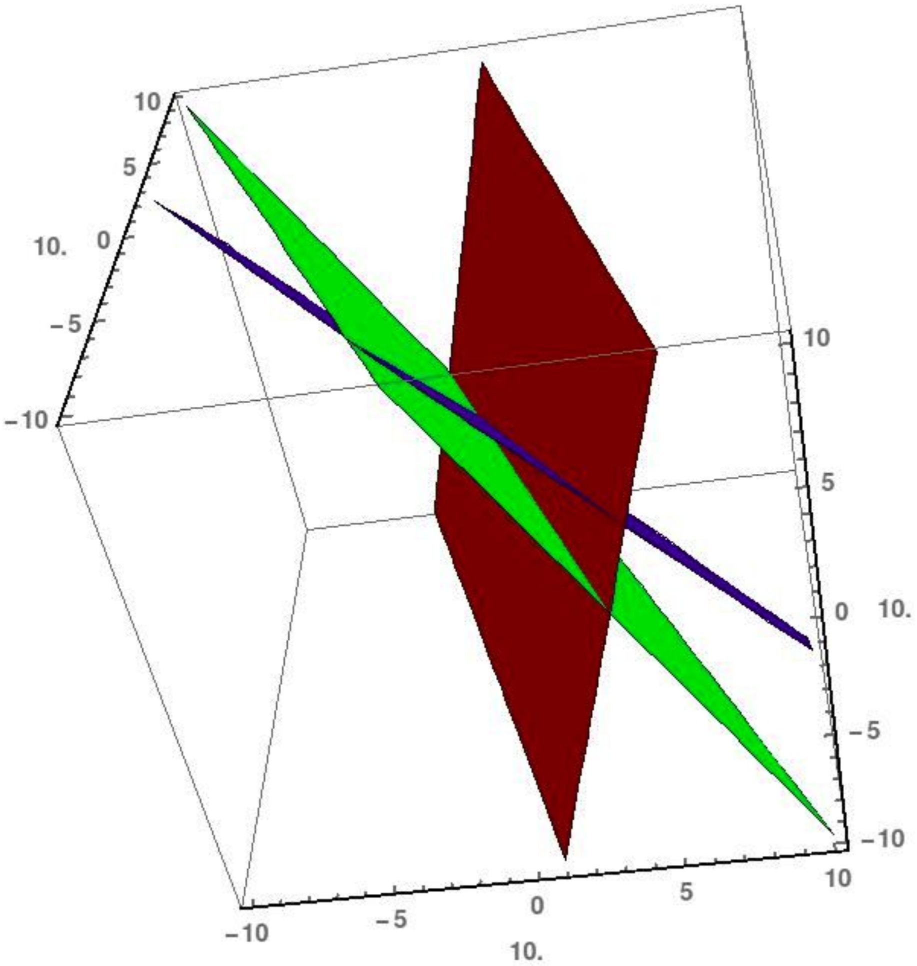




x + 2y + z = 12
2x - y +z = 1
X + y - 3z= -4



$6u + 7v + 8w = 8$
$4u + 5v + 9w = 0$
$2u - 2v + 7w = 7$

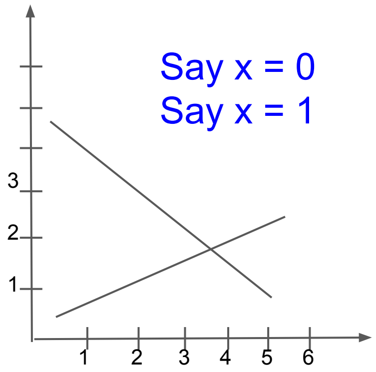


Solving Linear Equations

1. Row picture

$$1x + 2y = 3$$
$$4x + 5y = 6$$

Graph based method



Say x = 0
Say x = 1

Geometrical representation

Catch the thief--
Thief runs at 0.2 km/mt
You chase him on a cycle at 0.5 km/mt. But it took you 6 mts to get get going.
How far will the thief run before you catch him?

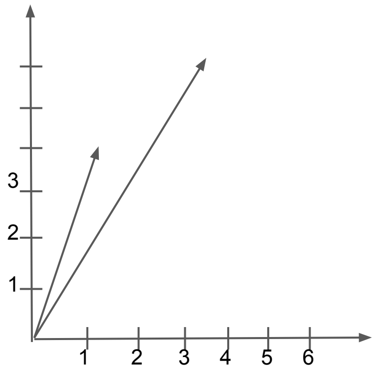
Solving Linear Equations

2. Column picture:

Linear Combination of Vectors

$$1x + 2y = 3$$
$$4x + 5y = 6$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 5 \end{bmatrix} y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$



Geometrical representation

Vector space, V, over a field of scalars, F

Some More Examples

$x - 2y = 0$
$x + y = 6$

$x + 2y = 2$
$x - y = 2$
$y = 1$

$x + y = 4$
$2x - 2y = 4$

$x + y + z = 2$
$x + 2y + z = 3$
$2x + 3y + 2z = 5$

Solving multiple variables in multiple equations is not easy with graphs

Solving Linear Equations

3. Matrix format

$1x + 2y = 3$
$4x + 5y = 6$

$$\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$AX = B$

So,
$X = A^{-1}B$

Other algebraic methods: Elimination & Substitution

Matrix Transformations

1. Inverse

$$A = \begin{pmatrix} 2 & 4 & 1 & 0 \\ 7 & 2 & 9 & -5 \\ 6 & 11 & 8 & 1 \\ 1 & 9 & 3 & 1 \end{pmatrix}$$

4 x 4 square matrix (n equations with n variables)

$$A.A^{-1} = A.A^{-1} = I$$

where,
I = Identity Matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a. Finding inverse by Adjoint and Determinants

$$\begin{pmatrix} -3 & -2 \\ 7 & 4 \end{pmatrix} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$|A|$ Determinant of A

$$\begin{matrix} 2 & 4 & 1 & 0 \\ 7 & 2 & 9 & -5 \\ 6 & 11 & 8 & 1 \\ 1 & 9 & 3 & 1 \end{matrix}$$

b. Finding inverse by Elementary Row Operations

- More fun way
- Play around with rows, swap, multiply till you get an identity matrix
- Do the same operations on an identity matrix, till you get the inverse.

Pivot/ Upper Triangular

$2u + v + w = 5$
$4u - 6v = -2$
$-2u + 7v + 2w = 9$

$$\begin{pmatrix} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{pmatrix}$$

Subtract 2 times the first equation from the second
Subtract -1 times thefirst equation from the third
Subtract -1 times the second equation from the third

$\mathbf{2}u + v + w = 5$ → ?
$\mathbf{-8}v - 2w = -12$
$1w = 2$

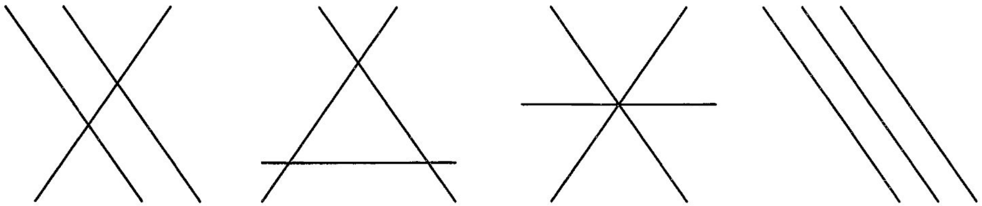
↓

Back substitution to get u and v

When can equations be solved?

a. When for a matrix A, its inverse exists
b. A unique solution exists
c. You can perform elimination

Singular case - No solution exists; Columns are not independent or elimination doesn't work



x + y + z = 2
x +2y + z= 3
2x + 3y + 2z= 5

Row 1 & 2 add to 3rd
Column 1 & 3 are not independent

u + v + w = 2
2u + 3w = 5
3u + v + 4w = 6

If 3u + v + 4w =7, then a solution was possible