Q.1.1:

We denote $\bar{V}^{\pi}$ the old value function. Then we have by definition:

$$\bar{V}^{\pi} = R^{\pi} + \gamma P^{\pi} \bar{V}^{\pi} \quad (1)$$

$$\bar{V}^{\pi} = (I - \gamma P^{\pi})^{-1} R^{\pi} \quad (2)$$

We now apply affine transformations to reward function:

$$R^{\pi}_{new} = \alpha R^{\pi} + \beta \mathbb{1} \quad (3)$$

We plug (3) into (2):

$$V^{\pi} = \alpha R^{\pi} + \beta \mathbb{1} + \gamma P^{\pi} V^{\pi}$$

$$V^{\pi} - \gamma P^{\pi} V^{\pi} = \alpha R^{\pi} + \beta \mathbb{1}$$

$$\Rightarrow V^{\pi} = (I - \gamma P^{\pi})^{-1} (\alpha R^{\pi} + \beta \mathbb{1})$$

$$\Rightarrow V'^{\pi} = (I - \gamma P^{\pi})^{-1} \alpha R^{\pi} + (1 - \gamma P^{\pi})^{-1} \beta \mathbb{1}$$

$$V^{\pi} = \alpha \bar{V}^{\pi} + (1 - \gamma P^{\pi})^{-1} \beta \mathbb{1}.$$

So essentially our new value function $V^{\pi} = \alpha \bar{V}^{\pi} + C$, where C is a constant. Optimal policy is not preserved. We give a contre-example with affine transformation where $\alpha = -1$, $\beta = 0$. π^* is not optimal for V^{π} obviously.