

We firstly prove bound on $\|V^* - \hat{V}\|$:

$$\begin{aligned}\|V^* - \hat{V}\| &= \|\tilde{T}V^* - \hat{T}\hat{V}\| \leq \|\tilde{T}V^* - \hat{T}V^*\| + \|\hat{T}V^* - \hat{T}\hat{V}^*\| \\ &\leq \gamma \|P V^* - \hat{P} V^*\| + \gamma \|V^* - \hat{V}^*\|\end{aligned}$$

By solving this inequality with respect to $\|V^* - \hat{V}\|$ we deduce

$$\|V^* - \hat{V}\| \leq \gamma b \|(P - \hat{P})V^*\|$$

By Hoeffding's inequality to bound $|(P - \hat{P})V^*(x)|$ for all $x \in \mathcal{X}$ in high probability:

$$\mathbb{P}(|(P - \hat{P})V^*(x)| \geq \varepsilon) \leq 2e^{\left(-\frac{N\varepsilon^2}{2b^2}\right)}$$

We deduce

$$\mathbb{P}(\|(P - \hat{P})V^*\| \geq \varepsilon) \leq 2|\mathcal{X}|e^{\left(-\frac{N\varepsilon^2}{2b^2}\right)}$$

We then define δ as

$$\delta \triangleq 2|\mathcal{X}|e^{\left(-\frac{N\varepsilon^2}{2b^2}\right)}$$

By plugging last two (in)equalities we deduce

$$\mathbb{P}\left[\|(P - \hat{P})V^*\| < b\sqrt{\frac{\ln(2|\mathcal{X}|)}{\delta}}\right] \geq 1 - \delta$$

(in our situation $b=1$, as V^* is in $[0,1]$.)