

**WEAK AND STRONG INDUCTION – REVIEW SET 1**  
**CSC 335**

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Again, I strongly recommend David Liben-Nowell's excellent text, Discrete Mathematics for Computer Science, published by Wiley and available (under CSC 104) through our bookstore. These problems are from his Chapter 5.

- (1) Prove by (weak) induction on  $n$ :  $\sum_{i=0}^{i=n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- (2) Prove by (weak) induction on  $n$ :  $\sum_{i=0}^{i=n} i^3 = \frac{n^4+2n^3+n^2}{4}$
- (3) Prove by (strong) induction on  $n$  that, for every integer  $n \geq 4$ , it is possible to make  $n$  dollars using only two- and five-dollar bills.
- (4) Prove by (strong) induction on  $n$  that, for every integer  $n \geq 1$ , there exist  $k \geq 0$  prime numbers  $p_1, p_2, \dots, p_k$  such that  $n = \prod_{i=1}^{i=k} p_i$
- (5) What is the sum of the first  $n$  odd positive integers? Prove your claim by induction.