

WEAK AND STRUCTURAL INDUCTION – REVIEW SET 2
CSC 335

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Again, I strongly recommend David Liben-Nowell's excellent text, Discrete Mathematics for Computer Science, published by Wiley and available (under CSc 104) through our bookstore. These problems are from his Chapter 5.

- (1) In chess, a knight at position $\langle r, c \rangle$ can move in an L-shaped pattern to any of eight positions: moving over one row and up/down two columns, or two rows over and one column up/down. A *knight's walk* is a sequence of legal moves, starting from a square of your choice, that visits *every* square of the board. Prove by induction that there exists a knight's walk for any n -by- n chessboard, for any $n \geq 4$. Note that the knight (horse) may visit squares more than once.
- (2) A string of balanced parentheses $[,]$ is one of the following:
 - (a) the empty string;
 - (b) a string $[S]$ where S is a string of balanced parentheses; or
 - (c) a string S_1S_2 where S_1 and S_2 are both balanced strings of parentheses.

Prove by structural induction

- (a) Every string of balanced parentheses has exactly the same number of open parentheses as close parentheses;
- (b) Every prefix of a string of balanced parentheses has at least as many open parentheses as it does close parentheses