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Topic: Assignment on Linear Regression

Task-01: Compute the slope (M) and y -Intercept (C).

To compute the slope (M) and y -intercept (C) using ordinary least squares Linear Regression, we use the following formulas:

→ Slope (M):

$$M = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

→ y -Intercept (C):

$$C = \frac{(\sum y) - M(\sum x)}{n}$$

Where:

- n is the number of data points.
- $\sum xy$ is the sum of the products of each pair of x and y .
- $\sum x$ and $\sum y$ are the sums of x values and y values, respectively.
- $\sum x^2$ is the sum of each x value squared.

Now,

compute these values with the given dataset:

• Weight (x): 2, 4, 5, 3, 6, 5, 7

• Price (y): 35, 60, 20, 50, 50, 55, 60

1. Calculate the Sums:

$$* \sum x = 2 + 4 + 5 + 3 + 6 + 5 + 7 = 32$$

$$* \sum y = 35 + 60 + 20 + 50 + 50 + 55 + 60 = 330$$

$$\begin{aligned} * \sum xy &= (2 \times 35) + (4 \times 60) + (5 \times 20) + (3 \times 50) + (6 \times 50) \\ &\quad + (5 \times 55) + (7 \times 60) \\ &= 1555 \end{aligned}$$

$$* \sum x^2 = 2^2 + 4^2 + 5^2 + 3^2 + 6^2 + 5^2 + 7^2 = 164$$

$$* N = 7$$

2. Compute M and C:

$$\begin{aligned} * M &= \frac{(7 \times 1555) - (32 \times 330)}{(7 \times 164) - 32^2} = \frac{10885 - 10560}{1148 - 1024} \\ &= \frac{325}{124} \end{aligned}$$

$$= 2.621 \text{ (approx)}$$

$$\begin{aligned} * C &= \frac{330 - (2.621 \times 32)}{7} = \frac{330 - 83.872}{7} \\ &= \frac{246.128}{7} \\ &= 35.161 \end{aligned}$$

So, the equation of the line is approximately:

$$y = 2.621x + 35.161 \quad \text{--- (1)}$$

Now, Predict the Price for a vegetable weight of 6

Using the equation no (1);

$$\begin{aligned} y &= (2.621 \times 6) + 35.161 \\ &= 15.726 + 35.161 \\ &= 50.887 \end{aligned}$$

Task-02:

Compute the Residuals.

The residual for each data point is the difference between the observed value and the predicted value.

The formula for each residual is:

$$\text{Residual} = y_i - (Mx_i + c)$$

Let's compute the residuals for each data point:

1. Weight = 2:

- Predicted price $= (2.621 \times 2) + 35.161 = 40.403$
- Residual $= 35 - 40.403 = -5.403$

2. Weight = 4:

$$* \text{ Predicted price} = (2.621 \times 4) + 35.161 = 45.645$$

$$* \text{ Residual} = 60 - 45.645 = 14.355$$

3. Weight = 5:

$$* \text{ predicted price} = (2.621 \times 5) + 35.161 = 48.266$$

$$* \text{ Residual} = 20 - 48.266 = -28.266$$

4. Weight = 3:

$$* \text{ Predicted price} = (2.621 \times 3) + 35.161 = 43.024$$

$$* \text{ Residual} = 50 - 43.024 = 6.976$$

5. Weight = 6:

$$* \text{ Predicted price} = 50.887 \quad (\text{calculated before})$$

$$* \text{ Residual} = 50 - 50.887 = -0.887$$

6. Weight = 5:

$$* \text{ Predicted price} = 48.266 \quad (\text{same as above})$$

$$* \text{ Residual} = 55 - 48.266 = 6.734$$

7. Weight = 7:

* Predicted price = $(6.621 \times 7) + 35.161 = 53.508$

* Residual = $60 - 53.508 = 6.492$

So, the residual for each data point are approximately

* Weight Value

2 - 5.493

4 - 14.355

5 - 28.266

3 - 6.976

6 - -0.887

5 - 6.734

7 - 6.492

Task-03:

Compute the Mean Squared Error (MSE) and Mean Absolute Error (MAE)

1. MSE is the average of the squared residuals:

$$MSE = \frac{1}{N} \sum (Y_i - \hat{Y}_i)^2$$

$$\begin{aligned}
 MSE &= \frac{1}{7} [(-5.403)^2 + (14.355)^2 + (-28.266)^2 \\
 &\quad + (6.976)^2 + (-0.887)^2 + (6.734)^2 + (6.492)^2] \\
 &= \frac{1}{7} [29.217 + 206.090 + 798.976 + 48.663 + 0.787 \\
 &\quad + 45.343 + 42.156] \\
 &= \frac{1}{7} [1170.232] \\
 &= 167.176
 \end{aligned}$$

2. MAE

MAE is the average of the absolute residuals:

$$MAE = \frac{1}{N} \sum |Y_i - \hat{Y}_i|$$

$$\begin{aligned}
 MAE &= \frac{1}{7} [| -5.403 | + | 14.355 | + | -28.266 | + | 6.976 | \\
 &\quad + | -0.887 | + | 6.734 | + | 6.492 |] \\
 &= \frac{1}{7} [5.403 + 14.355 + 28.266 + 6.976 + 0.887 \\
 &\quad + 6.734 + 6.492] \\
 &= \frac{1}{7} [69.113] \\
 &= 9.873
 \end{aligned}$$