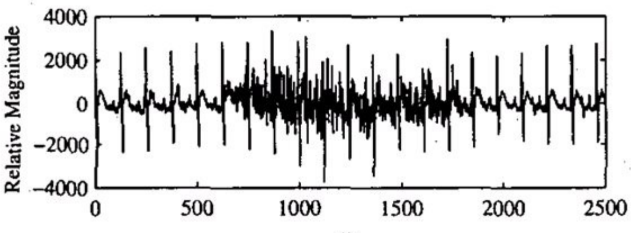
Signal segmentation and waveform characterization

Biosignal processing I, 521273S

Short-time analysis of signals

- Signal statistics may vary in time: nonstationary
  - how to compute signal characterizations?
- Signal must be partitioned into homogeneous segments
  - Operation is called segmentation
  - Statistics remain approximately the same in each segment: quasi-stationary segments
  - Characteristics is then computed for each segment separately
- Fixed and adaptive segmentation

Distortion caused by electromyogram:

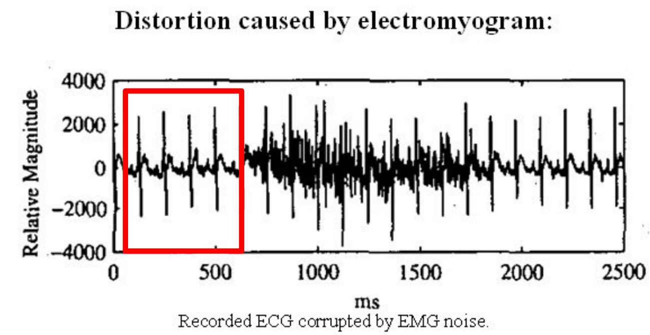


Recorded ECG corrupted by EMG noise.

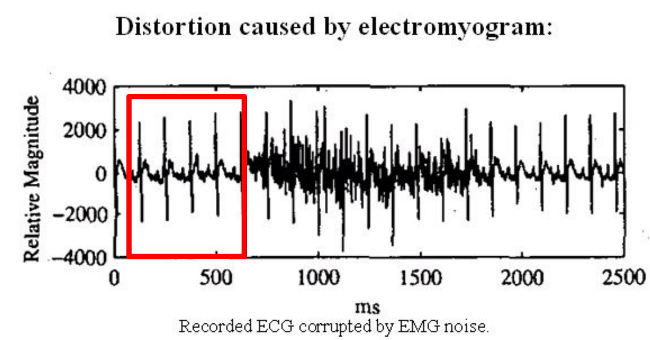
Fixed segmentation

- Partitioning of a signal into fixed-duration segments of length M samples
  - M must be selected manually (through trial and error, or via domain knowledge)
  - Each segment is then analyzed for some characteristics of interest
  - Segments can have partial overlap to smooth out the abrupt changes between them

- An example: Short-Time Fourier Transform (STFT)
  - STFT is computed for each segment (FFT, windowing, zero-padding)
  - Power spectrum is usually then computed for analyzing rhythmic components
  - Partial overlap of segments is often used for smoother output: e.g. 50%



Segment windows:
no overlap



Segment windows:
partial overlap

Adaptive segmentation

- Partitioning of a nonstationary signal into quasi-stationary segments of variable duration
- Typically, a small window (anchor window) is set in the beginning of a segment, and a second window (test window) slides forward for comparisons with the anchor
  - Difference in the contents of the two windows can be measured from their spectrums
  - If the difference exceeds a preset threshold, the ending boundary of the segment has been detected
  - A new anchor window is set, and the process starts over again

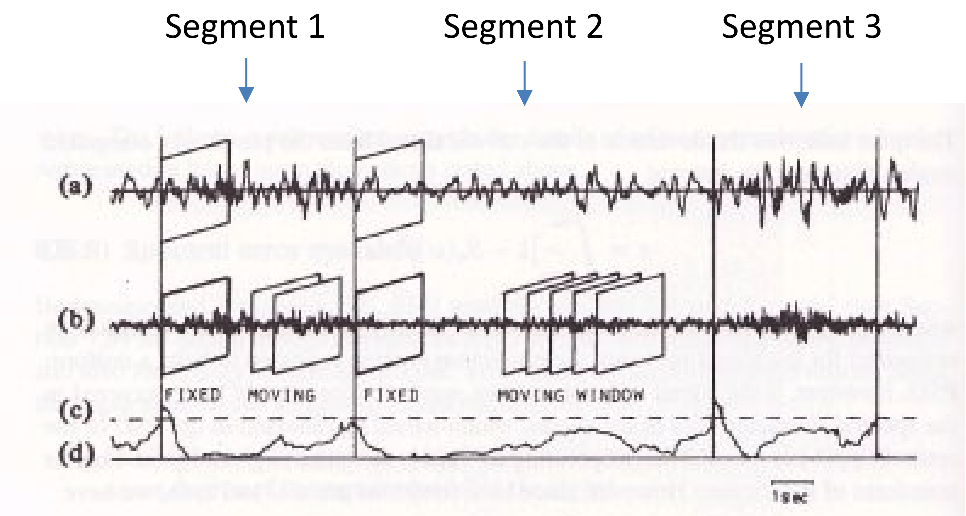
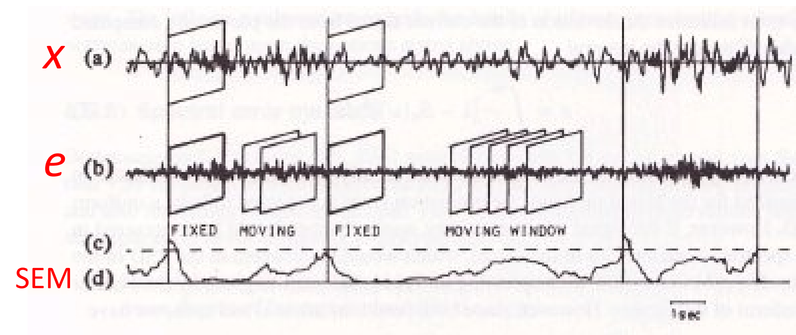


Figure 8.7 Adaptive segmentation of EEG signals via use of *SEM*. (a) Original EEG signal. The rectangular window at the beginning of each adaptive segment indicates the signal window to which the AR model has been optimized. (b) Prediction error. The initial ACF of the error is computed over the fixed window; the running ACF of the error is computed over the moving window. (c) Segmentation threshold. (d) *SEM*. The vertical lines represent the segmentation boundaries. Reproduced with permission from G. Bodenstein and H.M. Praetorius, Feature extraction from the electroencephalogram by adaptive segmentation, *Proceedings of the IEEE*, 65(5):642–652, 1977. ©IEEE.

Spectral Error Measure method (SEM)

- SEM applies autoregressive modeling (AR) of the signal spectrum
  - All-pole signal modeling in stochastic process theory: useful when signal contains oscillating components in one or more frequencies
  - Linear prediction of signal samples from p previous samples



- AR(p) model is computed from the signal contained in the anchor window (at time index n=0 in SEM equation)
  - Anchor window and sliding window are N samples long

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t$$

- The AR(p) model based linear prediction of signal x_t is applied to the test window (at time n) to generate prediction error signal e_t for the test window
- Autocorrelation function $\phi_e(n,m)$ of e_t is computed
- SEM value is computed from the autocorrelation function
  - First term compares error signal power in the windows 0 and n
  - Second term considers prediction error whiteness in window n
  - $1 < M \leq N$, to be specified by application design

$$e_t = x_t - \sum_{k=1}^{p} a_k(t) x_{t-k}$$

$$\varphi_e(n,m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} e_t(n) e_t(n+m)$$

- If the SEM value is larger than a preset threshold, the current segment ends at n
  - Reset the anchor window at n+1 to initialize a new segment
  - Establish a new AR model and start sliding the test window
- Otherwise, the current segment still continues
  - Increment time index n to slide the test window to the next sample
  - Compute error signal and autocorrelation function and SEM value for the new window

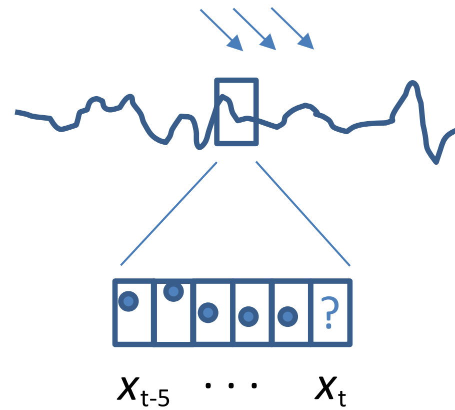
$$\mathrm{SEM}(n) = \left[\frac{\varphi_e(0,0)}{\varphi_e(n,0)} - 1\right]^2 + 2\sum_{m=1}^{M}\left[\frac{\varphi_e(n,m)}{\varphi_e(n,0)}\right]^2$$

SEM cont’d: explanations

- **Linear prediction** with AR(p) model
  - Establish an AR(p) model for the signal *x*
    - Set the desired model order p
    - Optimize the parameters a_k, k=1,…,p
  - By using the model, predict each sample x_t by using the p previous samples x_{t-i}
  - The model is not perfect which results in a prediction error e_t
    - For example, white additive noise cannot be predicted, or
    - p may be too small to enable modeling of all signal components

-> after going through the entire signal in the window, we get a prediction error signal *e*

predict each sample



(p=5)

$$x_t = \sum_{k=1}^{5} \underbrace{a_k(t) x_{t-k}}_{\text{Predicted value}} + e_t$$

Real value

Predicted value

Prediction error

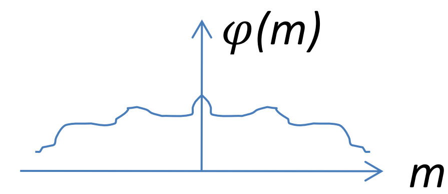
SEM cont'd: explanations

- **Autocorrelation function of prediction error signal**: the correlation of the error signal with itself at different time lags
  - self-similarity of the error signal as a function of time lag *m*
- It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal obscured by noise
- Signal power spectrum P(ω) can be computed by Fourier transforming the autocorrelation function

Signal segment and it's copy:

Time lag *m*

$$\varphi(m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} e(n)e(n+m)$$



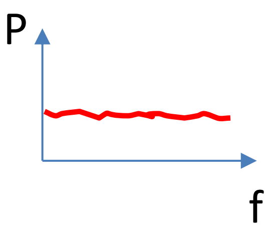
$$P(\omega) = \sum_{m=-(N-1)}^{N-1} \varphi(m)e^{-j\omega m}$$

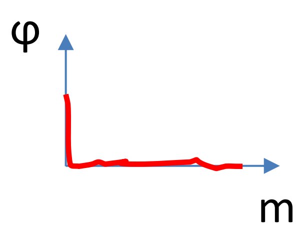
SEM cont'd: explanations

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t$$

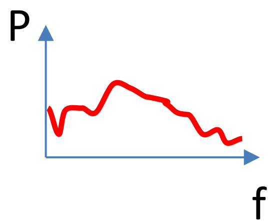
- If the signal x follows the set AR model, the prediction error e_t is white noise
  - The model then explains all structure in the signal (still in the same segment!)
  - Flat power spectrum P of e_t
  - Autocorrelation function of e_t: close to Dirac delta function

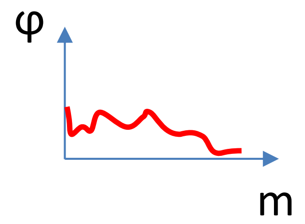
$$\varphi_e(n,m) > 0, \text{only for } m = 0$$





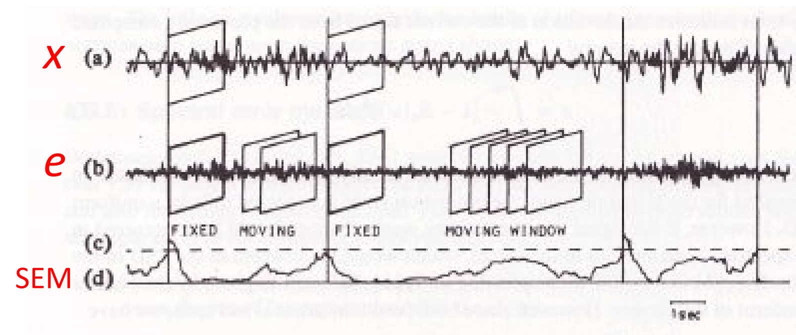
- If the signal does not follow the set AR model, the prediction error e_t is not white noise
  - There is some structure in the prediction error signal (possibly the next segment!)
  - Power spectrum P is not flat for e_t
  - Autocorrelation function of e_t is more complex containing many non-zero values





Spectral Error Measure method (SEM)

- SEM applies autoregressive modeling (AR) of the signal spectrum
  - All-pole signal modeling in stochastic process theory: useful when signal contains oscillating components in one or more frequencies
  - Linear prediction of signal samples from p previous samples
- AR(p) model is computed from the signal contained in the anchor window (at time index n=0 in SEM equation)
  - Anchor window and sliding window are N samples long
- The AR(p) model based linear prediction of signal x_t is applied to the test window (at time n) to generate prediction error signal e_t for the test window
- Autocorrelation function $\phi_e(n,m)$ of e_t is computed
- SEM value is computed from the autocorrelation function
  - First term compares error signal power in the windows 0 and n
  - Second term considers prediction error whiteness in window n
  - $1 < M \leq N$, to be specified by application design
- If the SEM value is larger than a preset threshold, the current segment ends at n
  - Reset the anchor window at n+1 to initialize a new segment
  - Establish a new AR model and start sliding the test window
- Otherwise, the current segment still continues
  - Increment time index n to slide the test window to the next sample
  - Compute error signal and autocorrelation function and SEM value for the new window



$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t$$

$$e_t = x_t - \sum_{k=1}^{p} a_k(t) x_{t-k}$$

$$\varphi_e(n,m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} e_t(n) e_t(n+m)$$

$$\mathrm{SEM}(n) = \left[\frac{\varphi_e(0,0)}{\varphi_e(n,0)} - 1\right]^2 + 2 \sum_{m=1}^{M} \left[\frac{\varphi_e(n,m)}{\varphi_e(n,0)}\right]^2$$

Three methods of plotting STFT-spectrums of consecutive segments

- Separate sub-plots of spectrums
- Landscape plot of spectrums
- Spectrograms of spectrums

STFT example: phonocardiogram

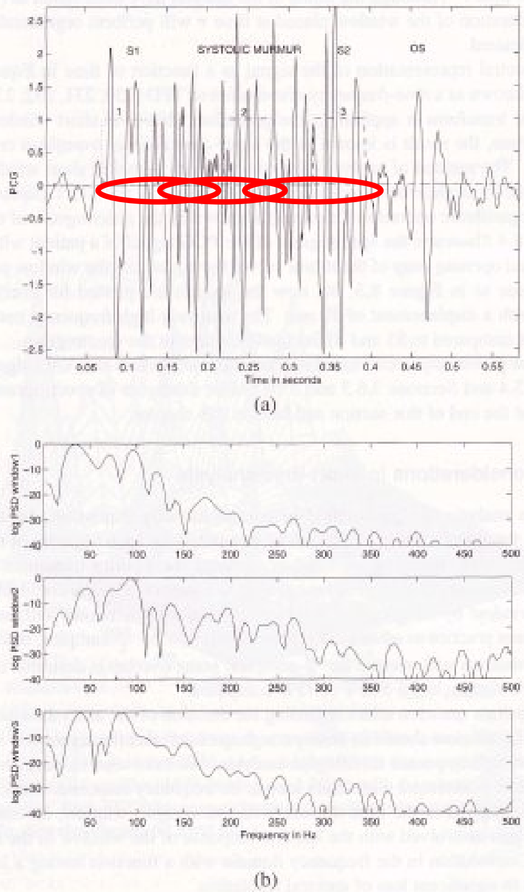


Figure 8.3 (a) PCG signal of a patient (female, 14 months) with systolic murmur and opening snap (OS) of the mitral valve. Three short-time analysis windows are superimposed, each one being a rectangular window of duration 64 ms. (b) Log PSDs of the three windowed signal segments. Each FFT was computed with zero-padding to a total length 256 samples. $f_s = 1$ kHz. See also Figure 6.12.

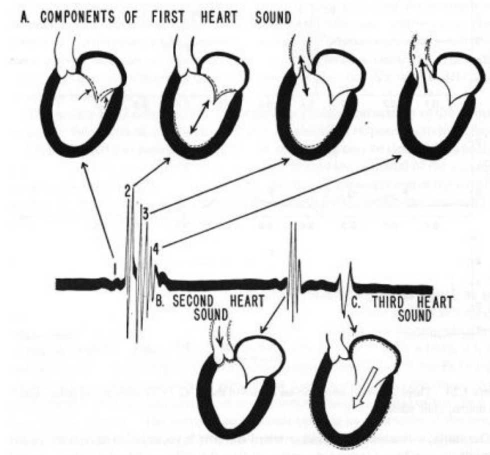


Figure 1.25 Schematic representation of the genesis of heart sounds. Only the left portion of the heart is illustrated as it is the major source of the heart sounds. The corresponding events in the right portion also contribute to the sounds. The atria do not contribute much to the heart sounds. Reproduced with permission from R.F. Rushmer, *Cardiovascular Dynamics*, 4th edition, ©W.B. Saunders, Philadelphia, PA, 1976.

Logarithmic PSD (power spectral density) in three consecutive windows (64 ms), normalized power axis

$$P_{dB}(f) = 10 \log_{10}(P(f)/P_{max})$$

- 64 sample windows with 32 sample overlap
- 256 sample FFT, zero-padding

STFT example: phonocardiogram

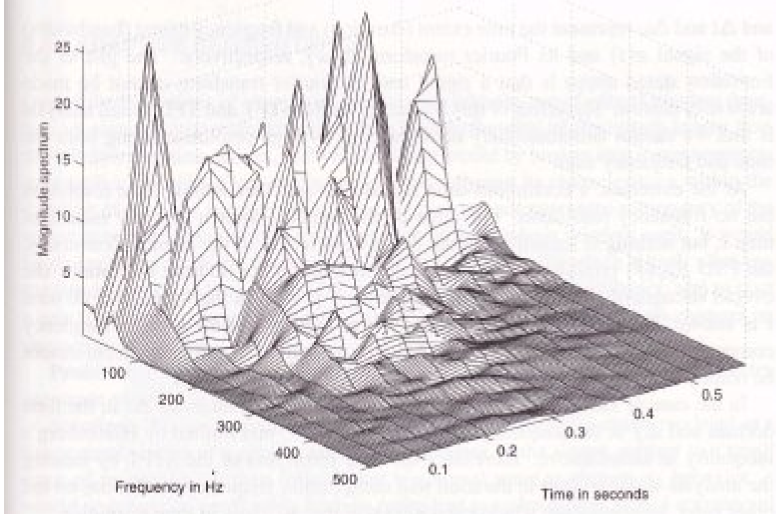


Figure 8.4 Spectrogram of the PCG signal of a patient (female, 14 months) with systolic murmur and opening snap of the mitral valve, computed with a moving short-time analysis window of duration 64 samples (64 ms with $f_s = 1\ kHz$), with the window advance interval being 32 samples. Each FFT was computed with zero-padding to a total length 256 samples. $f_s = 1\ kHz$. See also Figures 6.12 and 8.3.

An alternative representation of the spectrogram: interpolated magnitude spectrum as a **landscape plot**

For example:

- 64 sample windows with 32 sample overlap
- 256 sample FFT, zero-padding

More spectrums can be visualized than if plotted separately

STFT example: speech signal

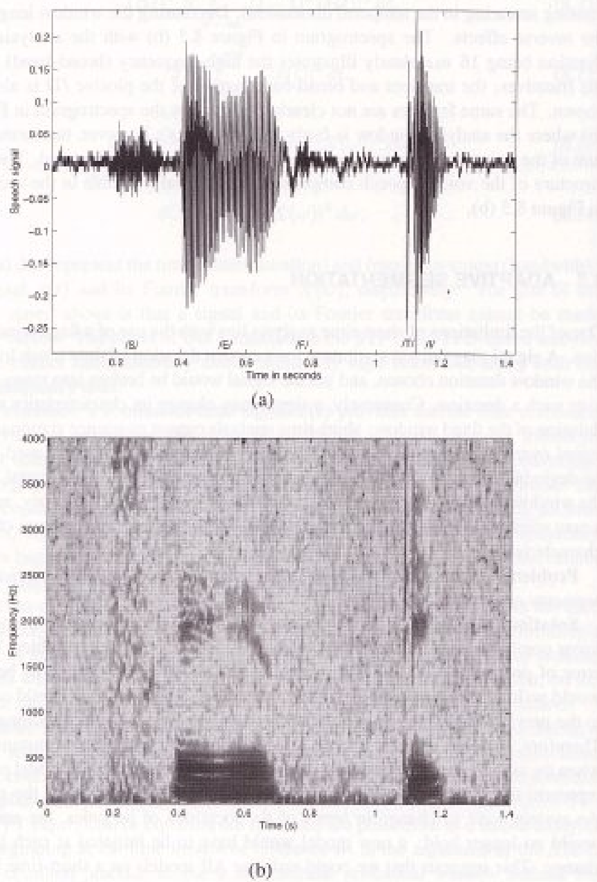


Figure 8.5 (a) Time-domain speech signal of the word "safety" uttered by a male speaker. (The signal is also illustrated in Figures 1.29, 3.1, and 3.3.) (b) Spectrogram (log PSD) of the signal computed with a moving short-time analysis window of duration 16 ms (128 samples with $f_s = 8\ kHz$), with the window advance interval being 8 ms.

An alternative representation of PSD in a sliding window: **spectrogram** - gray-level image of power spectrums

- Horizontal axis: start time of window
- Vertical axis: frequency
- Pixel value: signal power at time t and at frequency f

For example:

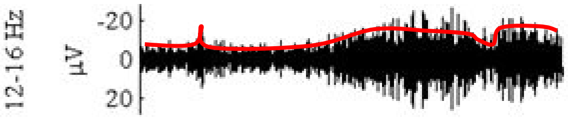
- 128 sample windows with 64 sample overlap
- 128 sample FFT

The most dense representation of spectral information

Activity level of signal

Envelope extraction

- Estimation of trends in signal activity or energy
  - Amplitude envelope



- A simple method:
  - Full-wave rectify the signal (taking absolute value)
  - Smooth the result (moving average, Butterworth low-pass filter,...)

$$y(t) = smooth(|x(t)|)$$

Envelogram

- Envelogram estimate: magnitude of the analytic signal y(t) formed using x(t) and its Hilbert transform x_H(t)

$$y(t) = x(t) + jx_H(t)$$

$$x_H(t) = \int_{-\infty}^{\infty} \frac{x(t)}{\pi(t-\tau)} d\tau$$

$$|y(t)| = \sqrt{x(t)^2 + x_H(t)^2}$$

- Practical algorithm:
  1. Compute DFT of x(t)
  2. Set the negative-frequency terms to zero
  3. Multiply positive-frequency terms by 2
  4. Compute inverse DFT of the result
  5. The magnitude of the result gives the envelogram estimate

(DFT = Discrete Fourier Transform)

Envelope examples (phonocardiogram)

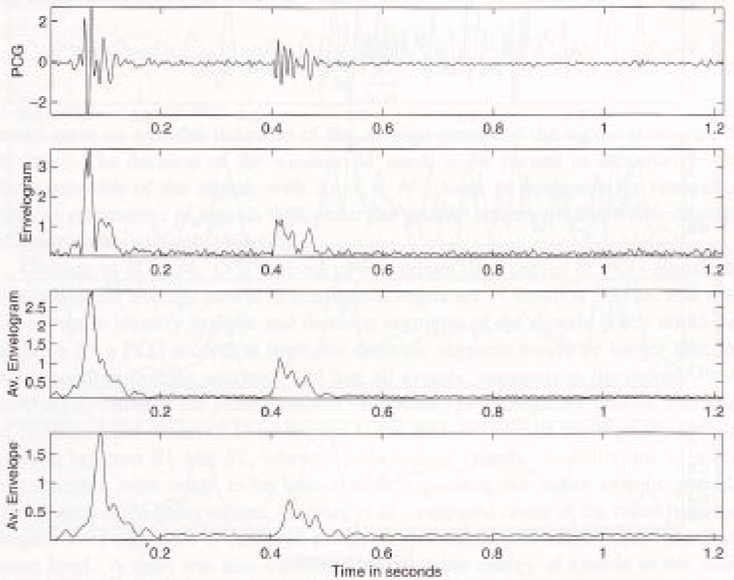


Figure 5.6 Top to bottom: PCG signal of a normal subject (male, 23 years); envelogram estimate of the signal shown; averaged envelogram over 16 cardiac cycles; averaged envelope over 16 cardiac cycles. The PCG signal starts with S1. See Figure 4.27 for an illustration of segmentation of the same signal.

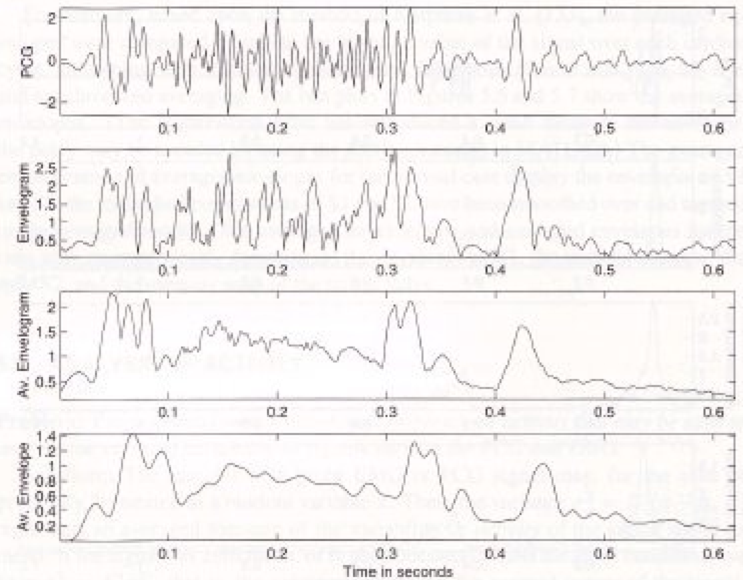


Figure 5.7 Top to bottom: PCG signal of a patient (female, 14 months) with systolic murmur (approximately $0.1 - 0.3\ s$), split S2 ($0.3 - 0.4\ s$), and opening snap of the mitral valve ($0.4 - 0.43\ s$); envelogram estimate of the signal shown; averaged envelogram over 26 cardiac cycles; averaged envelope over 26 cardiac cycles. The PCG signal starts with S1. See Figure 4.28 for an illustration of segmentation of the same signal.

- Signal
- Envelogram of the signal
- Synchronously averaged envelogram (N=16)
- Synchronously averaged envelope (N=16)

- Signal
- Envelogram of the signal
- Synchronously averaged envelogram (N=26)
- Synchronously averaged envelope (N=26)

Other descriptors of activity level

- Root-mean-square (RMS) value of the signal window
  - Computed over a sliding window of lenth N

$$RMS = \left[\frac{1}{N} \sum_{n=0}^{N-1} x^2(n) \right]^{1/2}$$

- Zero-crossing rate (ZCR): number of times the signal crosses zero-value per time unit
- Turns count: number of times that the signal amplitude changes direction per time unit

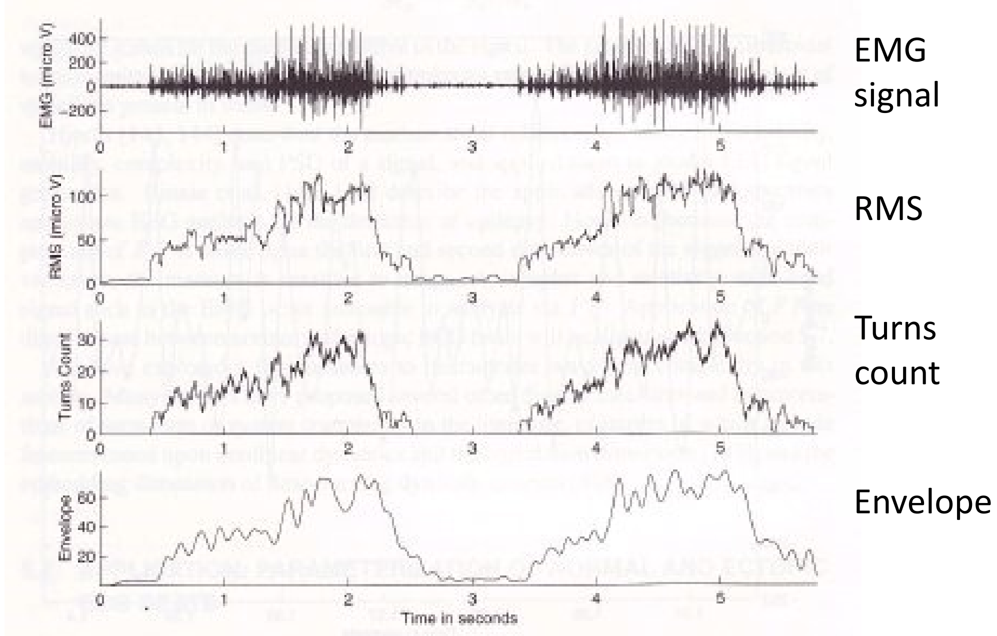


Figure 5.8 Top to bottom: EMG signal over two breath cycles from the crural diaphragm of a dog recorded via implanted fine-wire electrodes; short-time RMS values, turns count using Willison's procedure; and smoothed envelope of the signal. The RMS and turns count values were computed using a causal moving window of *70 ms* duration. EMG signal courtesy of R.S. Platt and P.A. Easton, Department of Clinical Neurosciences, University of Calgary.