

Problem Set 3

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Exercise 1:

- i.) Exogenous variables are variables that are not influenced or determined by the economic model itself. This means that changes of the variable's values can't be explained by the other variables used in a model, they have to be seen as given. In economic models the exogenous variables cause the other variables to move out of its steady state (and maybe return to it later). In contrast to that, endogenous variables are such variables that can be explained by the model itself. Their development is dependent on other variables and their own past realizations. The interaction of the endogenous variables, after a shock hits the system, can be seen as the adjustment path to the new or back to the old steady state.
- ii.) Determining values for the parameters and the steady state levels.

Given are: $\bar{Y}=1$; $\bar{G}^B=0,2*\bar{Y}=0,2$; $\bar{I}^B=0,02*\bar{Y}=0,02$; $\bar{TR}=0$; $\bar{\omega}=2$; $\bar{N}=1/3$

Setting reasonable parameter values:

$\beta = 0,97$, same value as in the model of Clarida-Gali-Gertler for the discount rate

$\rho_z = \rho_\tau = \rho_{I^B} = \rho_{G^B} = 0,75$, persistence of exogenous processes

$\delta = 0,02$, as in the real business cycle model for depreciation of the capital stock

$\alpha = 1/3$, because of (7) when plugging in the given values for N , ω and Y

$\eta = 1/4$, because the productivity of the public capital stock should be lower than the share of capital in production

Writing the equations (1), (2), (3), (4), (5), (6), (8), (10), (14) in steady state level

$$(1) (1 - \tau) * 2 = \theta * C * 3/2$$

$$(2) \lambda = 0,97 * [\lambda((1 - 0,02) + (1 - \tau) * r)]$$

$$(3) \lambda = 1/C$$

$$(4) K = (1 - 0,02) * K + I$$

$$(5) K^B = (1 - 0,02) * K^B + 0,02$$

$$(6) 1 = z * K^{B^{1/4}} * K^{1/3} * 1/3^{2/3}$$

$$(8) r * K = 1/3 * 1$$

$$(10) 0,2 + 0,02 + 0 = \tau * (2 * 1/3 + r * K)$$

$$(14) 1 = C + I + 0,2 + 0,02$$

Combining (8) and (10) $\rightarrow 0,22 = \tau * (2/3 + 1/3) \rightarrow \tau = 0,22$

From (5) it becomes clear that $K^B = 1$

Plugging in $\tau=0,22$ into (2) $\rightarrow \lambda = 0,97 * \lambda[(0,98 + 0,78 * r)] \rightarrow r \approx 0,065$ (6,5%)

Plugging r into (8) $\rightarrow K \approx 5,10$

Plugging K and K^B into (6) $\rightarrow z \approx 1,2$

Plugging K into (4) $\rightarrow I \approx 0,10$

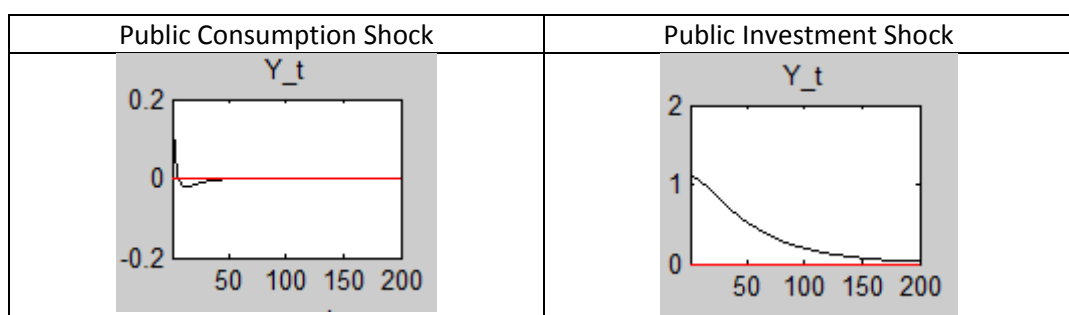
Plugging I into (14) $\rightarrow C \approx 0,68$

Finally, the solution for $\lambda \approx 1,47$ and $\theta \approx 1,53$ are taken from (1) and (3)

$Y = 1$	$K = 5,10$	$\delta = 0,02$
$G^B = 0,2$	$K^B = 1$	$\alpha = 1/3$
$I^B = 0,02$	$z = 1,2$	$\eta = 1/4$
$TR = 0$	$r = 0,065$	$\rho_{G^B} = 0,75$
$\omega = 2$	$\lambda = 1,47$	$\rho_{I^B} = 0,75$
$N = 1/3$	$\tau = 0,22$	$\rho_z = 0,75$
$C = 0,68$	$\Theta = 1,53$	$\rho_\tau = 0,75$
$I = 0,10$	$\beta = 0,97$	

- iii.) The difference between running a deterministic model or a stochastic model lies in the assumption about how good the agents in the model are informed about the shocks. In the deterministic case agents are fully informed about future shocks. They know when they take place and how strong they are. So everyone is able to include the shock into its optimizing behavior. In contrast, when we are in stochastic model, agents are only informed about the distribution of shocks and, therefore, always optimize their behavior taking the average realization of shocks into consideration. This means that when a shock materializes the agents are hit by the shock surprisingly. Whether a deterministic or a stochastic model is optimal depends on the assumption how well households are able to anticipate these shocks.
- v.) The reaction of the economy concerning a public consumption shock and a public investment shock differ in some respect. While output increases in both scenarios in the short-run, the impact is much stronger for a public investment shock. Furthermore, the impact on the output level lasts much longer than for a public consumption shock.

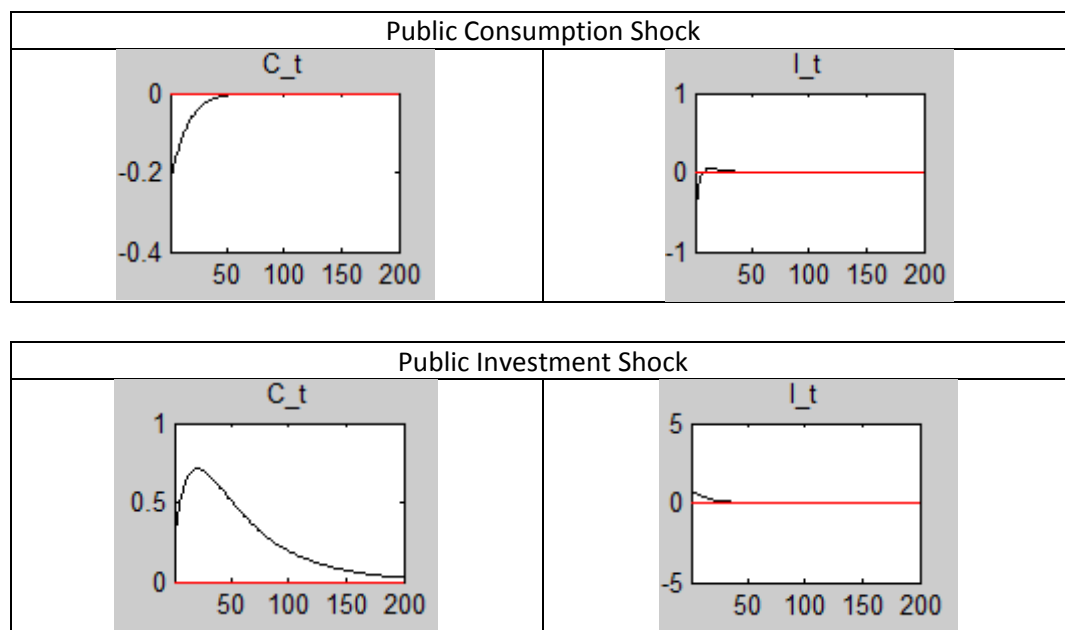
Output Response:



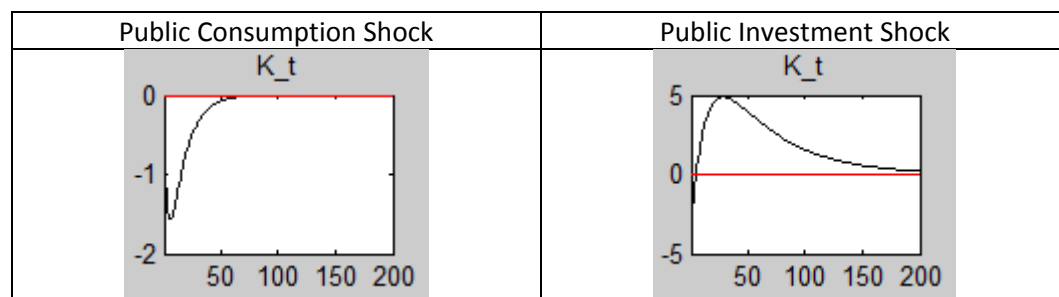
The explanation can be seen in the way the shocks influence the private investment of the economy and the private consumption. While an increase in public consumption crowds out private consumption (because of a necessary transfer from the private agents to the government) and private investment (because of higher real interest rates), this effect is overcompensated by an additional capital accumulation in the hands of the government when instead of public consumption the public investment is increased. The additional accumulation of capital pushes the output level up and generates extra income for the households which reduces the crowding out effect additionally. The capital stock of the economy, therefore, goes up after the investment shock occurs. When public consumption

is increased instead, the total stock of capital is reduced because of the crowding out effect of higher interest rates and the reduced income of the households through transfers to the government affect the household's budget.

Consumption and Investment Response:



Capital Formation Process:



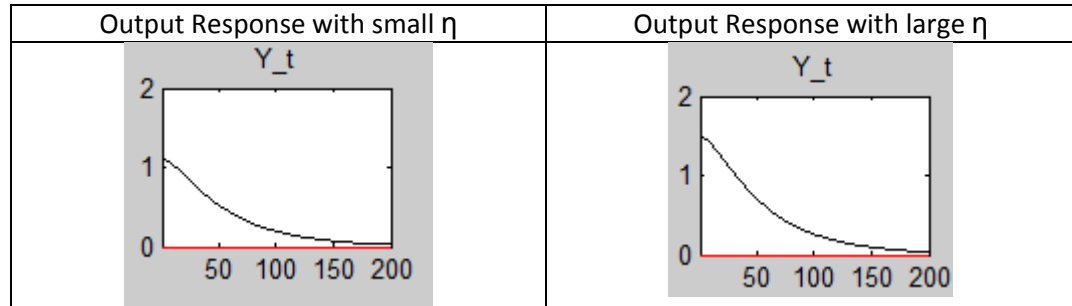
Since labor supply from the households increases only in the short-run and becomes smaller than in the steady state in the medium-term the utility increase for a representative household by an investment shock is larger (given the consumption increase mentioned above) than for a public consumption shock.

The reaction functions finally disclose that output levels for the public consumption shock fall under the old steady state level because of the misallocation between capital formation and consumption. However, in both scenarios it is obvious that fiscal policy has an impact of the real economy.

In the case the public productivity η increases the impact becomes obvious only in the scenario of a public investment shock. In the case that only public consumption increases, the effect is equal to zero because capital owned by the government is not affected in the aftermath of a consumption shock. When instead public investment is increased the productivity of the public capital becomes important. The higher the productivity is, the

stronger is the response of output, private consumption and private investment. This is intuitive because the more productive the public capital is the stronger is the output level effected by an increase in public capital formation causing additional income for the households (which can be spent on consumption and investment).

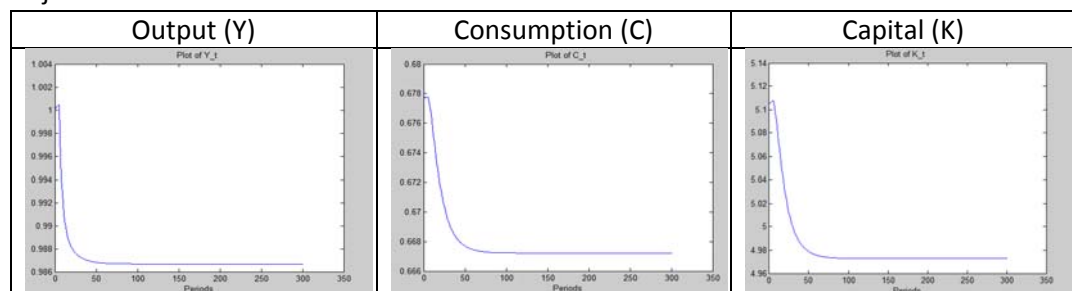
Output Response with different η :



- vi.) After a permanent increase in the tax rate by 1 %, the economy needs some time to adjust to the new equilibrium. In the long-run, the tax increase decreases the output level of the economy, because the taxes paid by the households can't be spend by them for consumption or capital formation. The extra money in the hands of the government finally causes a permanent reduction in private capital in the steady state. Since the capital in the government's hand is less productive than private capital, the redistribution of capital in the new steady state causes output level to be below the old one.

	Output (Y)	Consumption (C)	Investment (I)	Capital (K)
Old Steady-State	1	0,678	0,102	5,105
New Steady-State	0,987	0,667	0,0995	4,972

Adjustment Process:



- vii.) From the analysis above it becomes obvious that fiscal policy has an impact on the real economic activity. However, in the long-run fiscal policy hasn't to be an efficient tool to boost economic growth. The analysis shows that an oversized state budget causes a permanent decrease in output levels of the economy. The reason behind this development can be seen in the crowding out of the private agents who accumulate less capital than in a lean government regime. This finally causes (i) a larger consumption share of the entire economy, since consumption/investment ratio is higher for the government than for the private agents and (ii) more capital used in a less productive way because public capital is less productive than private capital. But the analysis also shows that fiscal policy can

especially in the short-run be a good tool to promote economic growth as long as the extra budget is almost entirely spend on investments.

Exercise 2:

- i.) To get proper results for the Bayesian estimations, it is necessary to have the same amount of observable variables as for the amount of shocks included in the model. Since in the An/Schorfheide (2007) model, there are three kinds of shocks, one needs three observable variables (here: INFL, YGR, INT).
- iii.) In contrast to use calibration techniques for determining parameter values, the Bayesian method is used to calculate the “real values” for the parameters from the underlying data. However, to prevent that solutions are found which seem to be completely unrealistic but may be suggested by the data, additional external beliefs (priors) are included in the estimation process. There is no rule how this beliefs have to be formed (it is just assumed that they are existent). For this reason the data is matched with some prior knowledge about the parameters. In estimating the values for the parameters the prior beliefs are used to place more focus on values which are considered to be more probable/realistic for the “real” parameter values. In this way the priors work as a kind of selection mechanism to make estimates more precise and estimation more efficient.
- v.) When looking at the prior/posterior some differences become obvious. First, for all parameters the distribution becomes steeper. This means that the knowledge about the “real” parameter becomes more precise. Second, in some cases the median of the posterior distribution differs from the median of the prior distribution. In these cases the estimation improves the quality of the model significantly, because the parameters do not only become more precise they also become more accurate. The difference in the means are not surprising. The parameters for which we see such a difference are those which were specified in such a way that they differ significantly from the values set in the calibration case. Since the simulated data were generated from the calibrated parameters, it should be assumed that the Bayesian Estimation of the posterior distribution converges by mean to the calibration values.

