Institut für Internationale Ökonomie

Advanced Macroeconomics (PhD level)

Problem set 3

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Exercise 1: Baxter and King (1993)

Ex1.1

A scientific experiment always consists of parameters, endogenous and exogenous variables. Parameters are coefficients of the experiments equations with given values, not depending on the experiments or other circumstances, for example physical constants like the force of gravity on earth. Exogenous variables are independent variables that can be freely chosen to observe different effects on the endogenous variables. Endogenous variables are the measurements that depend on exogenous variables and parameters. The theory will be supported by these measurement, if the endogenous variables accords to the forecasting outcome for different conditions of exogenous variables.

Parameters, endogenous and exogenous variables in the model of Baxter and King (1993):

endogenous variables		exogenous variables		parameters	
C	Private consumption	ε^{G^B}	shock in public consumption	α	productivity of private capital
dC	change in private consumption	ε^{I^B}	shock in public investment	β	discount factor
dG^B	change in public consumption	ε^{τ}	shock in rate of taxes	δ	depreciation rate
dI	change in public investments	ε^{z}	shock in productivity	η	productivity of public capital
dN	change in real labor supply			ρ_{G^B}	persistence factor of public consumption
$d\omega$	change in real wage			ρ_{I^B}	persistence factor of public investment
dr	change in real interest rate			ρ_{τ}	persistence factor of rate of taxes
dTR	change in real lump-sum transfers			ρ_{z}	persistence factor of productivity
dY	change in real outcome			θ_l	Frisch elasticity
G^B	public consumption				
I	private investments				
I^B	public investments				
K	private capital stock				
K^B	public capital stock				
λ	marginal utility of consumption				
N	real labor supply				
ω	real wage				
r	real interest rate				
τ	rate of taxes				
TR	real lump-sum transfers				
Y	real output				
z	productivity				

Ex1.2

Economic intuitions:

- Public-capital productivity $(\eta) <$ capital share in production (α)
- Mild persistence:
  - $\rho_{G^B} = 0.75$
  - $\rho_{I^B} = 0.75$
  - $\rho_\tau = 0.75$
  - $\rho_z = 0.75$
- Small stock standard errors of 1%
  - $\varepsilon^{G^B} = 0.01$
  - $\varepsilon^{I^B} = 0.01$
  - $\varepsilon^\tau = 0.01$
  - $\varepsilon^z = 0.01$

Targets:

- Steady-state output level: $\bar{Y} = 1$
- Steady-state public consumption: $\overline{G^B} = 0.2\bar{Y}$
- Steady-state public investment: $\overline{I^B} = 0.02\bar{Y}$
- Steady-state transfers:$\overline{TR} = 0$
- Steady-state real wage: $\bar{\omega} = 2$
- Steady-state labor supply: $\bar{N} = 1/3$

Reasonable values for β, δ and η

- Discount rate: $\beta = 0.988$ (see Robert G. King, Charles I. Plosser and Sergio T.Rebelo: Production, Growth and Buisness cycles, Journal of Monetary Economics 21 (1988), page 208)
- Depreciation rate: $\delta = 0.05$ (standard value)
- Productivity of public capital: $\eta = 0.05$ (see Marianne Baxter and Robert G. King: Fiscal Policy in General Equilibrium, The American Economic Review (1993), page 330)

→Steady-State formation:

1) Computation of α

$$(7)\ \bar{\omega}\bar{N} = (1-\alpha)\bar{Y}$$

$$\leftrightarrow \alpha = 1 - \frac{\overline{\omega}\overline{N}}{\overline{Y}}$$

$$\rightarrow \alpha = \frac{1}{3}$$

(Public-capital productivity $(\eta) <$ capital share in production (α))

2) Computation of $\bar{r}\overline{K}$

$$(8)\ \bar{r}\overline{K} = \alpha\overline{Y}$$

$$\rightarrow \bar{r}\overline{K} = \frac{1}{3}$$

3) Computation of $\bar{\tau}$

$$(10)\ \overline{G^B} + \overline{I^B} + \overline{TR} = \bar{\tau}(\overline{\omega}\overline{N} + \bar{r}\overline{K})$$

$$\leftrightarrow\ \bar{\tau} = \frac{\left(\overline{G^B} + \overline{I^B} + \overline{TR}\right)}{(\overline{\omega}\overline{N} + \bar{r}\overline{K})}$$

$$\rightarrow \bar{\tau} = 0.22$$

4) Computation of $\bar{r}$

$$(2)\ \bar{\lambda} = \beta\bar{\lambda}[(1-\delta) + (1-\bar{\tau})\,\bar{r}]$$

$$\leftrightarrow\ \bar{r} = \frac{1-\beta(1-\delta)}{\beta(1-\bar{\tau})}$$

$$\rightarrow \bar{r} = 0.0797$$

5) Computation of $\overline{K}$

$$(8)\ \bar{r}\overline{K} = \alpha\overline{Y}$$

$$\leftrightarrow \overline{K} = \frac{\alpha\overline{Y}}{\bar{r}}$$

$$\rightarrow \overline{K} = 4.1837$$

6) Computation of $\bar{I}$

$$(4)\ \overline{K} = (1-\delta)\overline{K} + \bar{I}$$

$$\leftrightarrow \bar{I} = \delta\overline{K}$$

$$\rightarrow \bar{I} = 0.2092$$

7) Computation of $\bar{C}$

$$(14)\ \overline{Y} = \bar{C} + \bar{I} + \overline{G^B} + \overline{I^B}$$

$$\leftrightarrow \bar{C} = \ \overline{Y} - \left(\bar{I} + \overline{G^B} + \overline{I^B}\right)$$

$$\rightarrow \bar{C} = 0.5708$$

8) Computation of $\bar{\lambda}$

$$(3)\bar{\lambda} = \frac{1}{\bar{C}}$$

$$\rightarrow \bar{\lambda} = 1.7519$$

9) Computation of θ_l

$$(1)\ (1-\bar{\tau})\bar{\omega} = \theta_l \frac{\bar{C}}{1-\bar{N}}$$

$$\leftrightarrow \theta_l = \frac{(1-\bar{\tau})\bar{\omega}(1-\bar{N})}{\bar{C}}$$

$$\rightarrow \theta_l = 1.8220$$

10) Computation of $\overline{K^B}$

$$(5)\ \overline{K^B} = (1-\delta)\overline{K^B} + \overline{I^B}$$

$$\leftrightarrow \overline{K^B} = \frac{\overline{I^B}}{\delta}$$

$$\rightarrow \overline{K^B} = 0.4000$$

11) Computation of $\bar{z}$

$$\bar{Y} = \bar{z}\left(\overline{K^B}\right)^{\eta}(\bar{K})^{\alpha}(\bar{N})^{1-\alpha}$$

$$\leftrightarrow\ \bar{z} = \frac{\bar{Y}}{\left(\overline{K^B}\right)^{\eta}(\bar{K})^{\alpha}(\bar{N})^{1-\alpha}}$$

$$\rightarrow \bar{z} = 1.3514$$

Ex1.3

Willi should simulate the stochastic model, because the occurrence of all future shocks is unknown in opposite to the distribution of these shocks, which indicates the stochastic model. Deterministic models need full information and no uncertainty around shocks, so that shocks are expected with perfect foresight. Here, the distribution is the only thing that is known of the shocks. They are observed today and the expectation of a shock in the future is zero, so the shocks are normally distributed with the mean of zero. However, if there are expected future shocks or permanent changes in exogenous variables, the stochastic models cannot be used, because of the application of the Taylor approximations around a steady state for required linearization of the model equations.

Ex1.4

See mod-file Ex1_1.mod, scenario 0.

Ex1.5

i) Unexpected temporary public consumption shock

See mod-file Ex1_1.mod, scenario 1.

An unexpected temporary public consumption shock ($G^B \uparrow$) leads to a decrease of the real lump-sum transfers ($TR \downarrow$) and an increase of the real interest rate($r \uparrow$) in the short-run, because this consumption shock is not linked with a tax increase ($\tau = constant$). The upward shift of the real interest rate is grounded by the increased demand in the money and capital market through the increased public consumption. The private demand decreased ($C \downarrow, I \downarrow$) through the upward shifting real interest rate, so public demand partly replaces private demand, this is the so called crowding out effect. However, the upward shift of public demand overshoots the downward shift of private demand, thus, the real output ($Y \uparrow$) raises due to the market clearing equation. Due to the decreased demand level of the households, the marginal utility of work and so the supplied labor increases ($N \uparrow$). The resulting downward shift of the wage ($\omega \downarrow$) offers the opposite movement of the real interest rate in terms of relative factor price implications. The private capital has got a lagged decreased in the short-run, due to the private investment ($K \downarrow$) behavior and the evolving of the private capital stock.

The decreasing of the public consumption to the steady state level leads to the steady state level in private consumption ($C \uparrow$), real lump-sum transfers ($TR \uparrow$), labor supply ($N \downarrow$), real wage ($\omega \uparrow$) and private capital ($K \uparrow$) in the long run. The private investments even overshoot the steady state ($I \uparrow$) for a few periods due to the rebuild of the private capital, but in the long run, the private investments reach the steady state, too($I \downarrow$). The private consumption needs a longer time to converge to the steady state level, than the public consumption or the private investment, due to the lagged marginal utility of consumption. This causes that the real output falls below the steady state level, too ($Y \downarrow$), for a few periods, before it reaches the steady state ($Y \uparrow$). Thus, in the long run, the whole real economy get to the steady state again.

ii) Unexpected temporary public investment shock

See mod-file Ex1_1.mod, scenario 2.

An unexpected temporary public investment shock ($I^B \uparrow$) causes three main effects. First, there is a crowding out effect which is similar to the crowding out effect in the public consumption shock ($TR \downarrow, I \downarrow, C \downarrow,$). Secound, short-run increase of the public investments shifts up the public capital and this reinforces the increase of the real output besides the increase due to the crowding out effect. It follows, that after initial decreasing, the private consumption, private investments and lump-sum transfers shifts over the steady state level in the middle-term($TR \uparrow, I \uparrow, C \uparrow,$). Third, the high public capital stock influences the real interest rate and the real wage, too. In fact, the initial real interest rate increases and the initial real wage decreases due to the crowding out effect ($N \uparrow, \omega \downarrow, r \uparrow,$), but with the raising demand at the money and capital market shifts the real interest rate under the steady state level , in the middle term ($r \downarrow$). So does the labor supply ($N \downarrow$), because of the decreasing marginal utility of work due to the increase in private investment and private consumption, correspondingly, the real wage shifts above the steady state level in the middle term ($\omega \uparrow$). The private capital follows the private investments in lagged movement ($K \downarrow, K \uparrow$). In the long run, all variables move like the public capital to the steady state due to steady state level of the public investments. Thus, like in the unexpected temporary public consumption shock, the whole real economy get to the steady state again.

iii) Change in the results due to an increasing productivity of public capital ($\eta \uparrow$)

See mod-file Ex1_2.mod, scenario 3 and 4.

The productivity of public capital only refers to the public capital. However, the public capital in the unexpected temporary public consumption shock does not change at all, so with the affected productivity ($z \downarrow$) of an increased productivity of public capital, the observables act in the same way like in i).

In terms of the unexpected temporary public investment shock, almost all movements happen in the same way, but get reinforced by the increasing productivity of public capital. This is grounded by the increasing public capital and its key role in the functional chain of the unexpected temporary public investment shock.

Ex1.6

See mod-file Ex1_1.mod, scenario 5.

a permanent increase in the tax rate of 1 percentage point => $\tau_t = \tau_{t-1} = 0.23$ *and* $\bar{\tau} = 0.22$

$$(13)\ \log\left(\frac{\tau_t}{\bar{\tau}}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right) + \varepsilon_t^\tau$$

$$\leftrightarrow\ \varepsilon_t^\tau = \log\left(\frac{\tau_t}{\bar{\tau}}\right) - \rho_\tau \log\left(\frac{\tau_{t-1}}{\bar{\tau}}\right)$$

The permanent increase in the tax rate leads to upward shifting of the real lump-sum transfers ($TR \uparrow$) grounded by the budget constraint of the fiscal authority, because the public consumption and public investment do not change. The marginal utility of capital and labor decrease through the tax that is why private capital and labor supply shifts downwards($N \downarrow, K \downarrow$), too. In the following way, the real wage increased ($\omega \uparrow$) and the real interest rate decreased in the short run ($r \downarrow$). Although there is a positive movements in transfers, the resulting decrease of private consumption and private investments dominate and yield a decreasing real output($Y \downarrow$). This decreasing output and the decreasing labor supply are in such a relation, that the real wage has to shift downwards after the initial increase for the satisfaction of the factor demand ($\omega \downarrow$). This yields an upwards movement of the labor supply ($N \uparrow$)to the new steady state that lies under the old steady state. And the new steady state of real wage is under the initial steady state, too. The relation of the private capital demand causes an upward shift to the new steady state of the initial down sloping real interest rate ($r \uparrow$), grounded by the downward trend of private capital due to private investment. Thus, the real interest rate in the new steady state overshoots the old steady state. The decrease in private capital and labor supply and real wage leads to a smaller value for taxing, so the real lump-sum transfers reduce a bit ($TR \downarrow$), but the new steady state lies over the old steady state. Furthermore, it leads to an increase of the private investment ($I \uparrow$)after the initial decrease which yields a new steady state under the old one. This development of the private investment stops the decrease of private capital to a new steady state below the old steady state. In the end, the new steady state of real output and private consumption are below the old steady state, too.

Ex1.7

In consideration of Ex1.5, it turns out that there can be exist a boost in economy due to fiscal policy. In fact, when there is a positive shock of public investment, a multiplier effect due to the increased government capital shifts the real output upward for a few periods, so there exist a boost in the short-run. However, if the fiscal policy only includes a positive shock in the public consumption, then the multiplier effect get absorbed by the crowding-out of private consumption and private investment. But in the long-run the both ways of fiscal policy leads to the initial steady state.

Private and government capital distinguish in the formation, owner and concrete composition. Private capital evolve through private investment and it belongs to the HH. The private capital is subject of the taxes τ, is part of the households budget constraint and exist for example of private real estates, stocks and other objects of value. In contrast, government capital evolve through public investments and owns the state. Examples are highways, airports, public hospitals, police and fire protection and so on. Both forms of capital are included of the formation of output.

Exercise 2: An and Schorfheide (2007)

Ex2.1

The number of shocks have to be equivalent to the number of observable variables in a Bayesian estimation of a DSGE-model, so in this model, there are three observables required.

See mod-file Ex2.mod, scenario 0.

Ex2.2

See mod-file Ex2.mod, scenario 0.

Ex2.3

In Bayesian methods are prior-beliefs about the structure of fixed parameters θ. These beliefs based on economic modelling experience, certain assumptions and the structure of the database d which is supposed to be fix. In addition to a Maximum-Likelihood approach, an estimated parameter vector $\hat{\theta}_B$ can be developed that is the maximum of the sum of the likelihood function of parameters. Thus, this vector based on the prior beliefs and on the data and with certain methods the parameters of the vector can be estimated. With posterior distribution of parameters, the prior distribution can be checked and assessed in the end.

Ex2.4

See mod-file Ex2.mod, scenario 1.

Ex2.5

grey: prior estimation

black: posterior estimation

green line: posterior parameter estimation

The quality of figures can be graduate in three stages of accordance of prior and posterior estimation. For τ, ψ_1 and ψ_2, the estimations almost coincide each other, what presents an high quality of estimation. For $\kappa, \rho_R, p_A, std_z$ and std_g gives the prior estimation always a larger range of possible estimation values than the posterior estimation, but the value with the highest probability approximately coincides with the posterior parameter estimation. And for $\gamma_Q, std_R, \rho_g, \rho_z$ and r_A the range of the estimation and the posterior parameter estimation do not fit with the prior estimation, which makes the estimation for these parameters quite bad.