

DSGE-Models

Calibration and Introduction to Dynare

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Calibration and Introduction to Dynare

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- Econometrically, a DSGE-Model is a state-space model of which one has to determine the parameters.
- Three concepts:
 - ① **Calibration:** The parameters are set in such a way, that they closely correspond to some theoretical moment or stylized fact of data.
 - ② **Methods of limited information** or weak econometric interpretation: Minimize the distance between theoretical and empirical moments, i.e. *General-Method-of-Moments* or *Indirect Inference*.
 - ③ **Methods of full information** or strong econometric interpretation: The goal is an exact characterization of observed data, i.e. *Maximum-Likelihood* or *bayesian methods*.

- Goal: To answer a specific quantitative research question using a structural model.
- Construct and parameterize the model such, that it corresponds to certain properties of the true economy.
- Use steady-state-characteristics for choosing the parameters in accordance with observed data.
- Often: stable long-run averages (wages, working-hours, interest rates, inflation, consumption-shares, government-spending-ratios, etc.).
- You can use micro-studies as well, however, one has to be careful about the aggregation!

Calibration

Hints for calibrating a model

- Use long-term averages of interest rates, inflation, average growth of productivity, etc. for *steady-state* values.
- BUT: Weil (1989) shows, that in models with representative agents there is an overestimation of *steady-state* interest rates (*risk-free rate puzzle*). It is possible that you get absurd constellation of parameters, like a discount-factor of $\beta > 1$.
- Usual mark-up on prices is around 1.15 (Corsetti et al (2012)).
- Intertemporal elasticity of substitution $1/\sigma$ somewhere between $\sigma = 1$ and $\sigma = 3$ (King, Plosser and Rebelo (1988), Rotemberg and Woodford (1992), Lucas (2003)).

Calibration

Hints for calibrating a model

- Rigidity of prices: For an average price adjustment of 12-15 months see Keen and Wang (2007).
- Coefficients of monetary policy: Often Taylor-Rule, you can use the relative coefficients to put more emphasize/weight on the stability of prices or on smoothing the business cycle.
- Parameters of stochastic processes: Often persistent, small standard-deviations, otherwise you get high oscillations. You can also estimate the production function via OLS (Solow-residual).
- How to choose shocks: Look at similar studies: Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003), etc..
- Ultimately: Try & Error!

Exercise 2:

Calibration of a RBC-model with monopolistic competition

Households maximize expected utility over consumption c_t and leisure $f_t = 1 - l_t$, where l_t denotes labor:

$$E_t \sum_{t=0}^{\infty} \beta [\log(c_t) + \psi \log(1 - l_t)],$$

taking account of the following budget constraint:

$$c_t + k_{t+1} = \underbrace{w_t l_t + r_t k_t}_{=y_t} + (1 - \delta)k_t.$$

k_t is the capital stock of the economy, w_t the real wage, r_t the real interest rate and δ the rate of depreciation. Further, investment is given by:

$$i_t = y_t - c_t.$$

Exercise 2:

Calibration of a RBC-model with monopolistic competition

In the market for intermediate goods there is monopolistic competition, whereas perfect competition applies to the market for final goods. The production-function of a firm $i \in [0; 1]$, that sells intermediate goods, is given by:

$$y_{it} = A_t k_{it}^\alpha l_{it}^{1-\alpha}, \quad 0 < \alpha < 1,$$
$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),$$

with A_t denoting the level of technology. Firms cannot influence the real wage w_t or the real interest rate r_t . However, they have market power over their price p_{it} for their good y_{it} . The intermediate goods are combined into a final good by a Dixit/Stiglitz-type aggregator:

$$y_t = \left(\int_0^1 (y_{it})^{\frac{\varepsilon}{\varepsilon-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}},$$

with ε being the elasticity of substitution.

Exercise 2:

Calibration of a RBC-model with monopolistic competition

- (a) Show that, the structural form of the DSGE-model is given by the following equations and interpret these.

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right] \quad (1)$$

$$w_t = \psi \frac{c_t}{1 - l_t} \quad (2)$$

$$y_t = c_t + i_t \quad (3)$$

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} \quad (4)$$

$$w_t = (1 - \alpha) \frac{y_t}{l_t} \frac{\varepsilon - 1}{\varepsilon} \quad (5)$$

$$r_t = \alpha \frac{y_t}{k_t} \frac{\varepsilon - 1}{\varepsilon} \quad (6)$$

$$i_t = k_{t+1} - (1 - \delta)k_t \quad (7)$$

$$\log(A_t) = \rho \log(A_{t-1}) + \epsilon_t \quad (8)$$

Exercise 2:

Calibration of a RBC-model with monopolistic competition

- (b) What are the parameters of the model?
- (c) Write a mod-file for the model and calibrate the vector of parameters μ . Simulate the model for 1000 periods with Dynare. Save the middle 100 observations of c_t, y_t, i_t, w_t and r_t into an Excel-file as well as into a mat-file. Plot the path of consumption.
- (d) Reformulate the structural equations such that variables are expressed as percentage deviations from *steady-state*:
$$x_t = e^{\log(x_t) - \log(x) + \log(x)} = x e^{\hat{x}_t}.$$
 Write a mod-file for this model.
What has changed?

Calibration

Pros

- Calibration is commonly used in the literature. It gives a first impression, a flavor of the strengths and weaknesses of a model.
- A good calibration can provide a valuable and precise image of data.
- Using different calibrations, one can assess interesting implications of different policies:
 - How does the economy react, if the central bank focuses more on smoothing the business cycle than on price stability?
 - What happens to consumption, if the households have a strong intertemporal elasticity of substitution? What if it is low?

Calibration

Cons

- This Ad-hoc-approach is at the center of criticism of DSGE-models.
- There is no statistical foundation, it is based upon subjective views, assessments and opinions.
- Many parameter, such as those of the exogenous processes, leave room for different values and interpretations (intertemporal elasticity of substitution, monetary and fiscal parameters, coefficients of rigidity, standard deviations, etc.).

Prescott (1986, S. 10) regarding RBC-models:

The models constructed within this theoretical framework are necessarily **highly abstract**. Consequently, they are necessarily false, and statistical hypothesis testing will reject them. This does not imply, however, that nothing can be learned from such a **quantitative theoretical exercise**.

Exercise 3: A simple RBC model

Consider the following model of an economy.

- Representative agent preferences

$$U = \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho} \right)^{t-1} E_t \left[\log(C_t) - \frac{L_t^{1+\gamma}}{1+\gamma} \right].$$

The household supplies labor and rents capital to the corporate sector.

- L_t is labor services.
- $\rho \in (0, \infty)$ is the rate of time preference.
- $\gamma \in (0, \infty)$ is a labor supply parameter.
- C_t is consumption.
- w_t is the real wage.
- r_t is the real rental rate.

Exercise 3: A simple RBC model

- The household faces the sequence of budget constraints

$$K_t = K_{t-1} (1 - \delta) + w_t L_t + r_t K_{t-1} - C_t,$$

where

- K_t is capital at the end of period.
- $\delta \in (0, 1)$ is the rate of depreciation.
- The production function is given by the expression

$$Y_t = A_t K_{t-1}^\alpha \left((1 + g)^t L_t \right)^{1-\alpha},$$

where $g \in (0, \infty)$ is the growth rate and α and β are parameters.

- A_t is a technology shock that follows the process

$$A_t = A_{t-1}^\lambda \exp(e_t),$$

where e_t is an i.i.d. zero mean normally distributed error with standard deviation σ and $\lambda \in (0, 1)$ is a parameter.

Exercise 3: A simple RBC model

The household problem

Lagrangian

$$L = \max_{C_t, L_t, K_t} \sum_{t=1}^{\infty} \left(\frac{1}{1+\rho} \right)^{t-1} E_t \left[\log(C_t) - \frac{L_t^{1+\gamma}}{1+\gamma} \right. \\ \left. - \mu_t (K_t - K_{t-1}(1-\delta) - w_t L_t - r_t K_{t-1} + C_t) \right].$$

First order conditions

$$\frac{\partial L}{\partial C_t} = \left(\frac{1}{1+\rho} \right)^{t-1} \left(\frac{1}{C_t} - \mu_t \right) = 0,$$

$$\frac{\partial L}{\partial L_t} = \left(\frac{1}{1+\rho} \right)^{t-1} (L_t^\gamma - \mu_t w_t) = 0,$$

$$\frac{\partial L}{\partial K_t} = - \left(\frac{1}{1+\rho} \right)^{t-1} \mu_t + \left(\frac{1}{1+\rho} \right)^t E_t (\mu_{t+1}(1-\delta + r_{t+1})) = 0.$$

Exercise 3: A simple RBC model

First order conditions

Eliminating the Lagrange multiplier, one obtains

$$L_t^\gamma = \frac{w_t}{C_t},$$
$$\frac{1}{C_t} = \frac{1}{1+\rho} E_t \left(\frac{1}{C_{t+1}} (r_{t+1} + 1 - \delta) \right).$$

Exercise 3: A simple RBC model

The firm problem

$$\max_{L_t, K_{t-1}} A_t K_{t-1}^\alpha \left((1+g)^t L_t \right)^{1-\alpha} - r_t K_{t-1} - w_t L_t.$$

First order conditions:

$$\begin{aligned} r_t &= \alpha A_t K_{t-1}^{\alpha-1} \left((1+g)^t L_t \right)^{1-\alpha}, \\ w_t &= (1-\alpha) A_t K_{t-1}^\alpha \left((1+g)^t \right)^{1-\alpha} L_t^{-\alpha}. \end{aligned}$$

Exercise 3: A simple RBC model

Goods market equilibrium

$$K_t + C_t = K_{t-1}(1 - \delta) + \underbrace{A_t K_{t-1}^\alpha ((1 + g)^t L_t)^{1-\alpha}}_{w_t L_t + r_t K_t}.$$

Exercise 3: A simple RBC model

Dynamic Equilibrium

$$\frac{1}{C_t} = \frac{1}{1+\rho} E_t \left(\frac{1}{C_{t+1}} (r_{t+1} + 1 - \delta) \right),$$

$$L_t^\gamma = \frac{w_t}{C_t},$$

$$r_t = \alpha A_t K_{t-1}^{\alpha-1} ((1+g)^t L_t)^{1-\alpha},$$

$$w_t = (1-\alpha) A_t K_{t-1}^\alpha ((1+g)^t L_t)^{1-\alpha} L_t^{-\alpha},$$

$$K_t + C_t = K_{t-1}(1-\delta) + A_t K_{t-1}^\alpha ((1+g)^t L_t)^{1-\alpha},$$

$$\log(A_t) = \lambda \log(A_{t-1}) + e_t.$$

Exercise 3: A simple RBC model

Existence of a balanced growth path

Good markets equilibrium for each period t :

$$K_t + C_t = K_{t-1}(1 - \delta) + A_t K_{t-1}^\alpha ((1 + g)^t L_t)^{1-\alpha}.$$

So, there must exist growth rates g_c and g_k such that

$$(1 + g_k)^t K_1 + (1 + g_c)^t C_1 = \frac{(1 + g_k)^t}{1 + g_k} K_1 (1 - \delta) + A_t \left(\frac{(1 + g_k)^t}{1 + g_k} K_1 \right)^\alpha ((1 + g)^t L_t)^{1-\alpha}$$

$$\Leftrightarrow K_1 + \left(\frac{1 + g_c}{1 + g_k} \right)^t C_1 = \underbrace{\frac{K_1}{1 + g_k}}_{K_0} (1 - \delta) + A_t \left(\frac{K_1}{1 + g_k} \right)^\alpha \left(\left(\frac{1 + g}{1 + g_k} \right)^t L_t \right)^{1-\alpha}.$$

This is only valid, if $g_c = g_k = g$.

Exercise 3: A simple RBC model

Stationarized model

Let's define

$$\hat{C}_t = C_t / (1 + g)^t,$$

$$\hat{K}_t = K_t / (1 + g)^t,$$

$$\hat{w}_t = w_t / (1 + g)^t.$$

Exercise 3: A simple RBC model

Stationarized model (continued)

$$\frac{1}{\widehat{C}_t(1+g)^t} = \frac{1}{1+\rho} E_t \left(\frac{1}{\widehat{C}_{t+1}(1+g)(1+g)^t} (r_{t+1} + 1 - \delta) \right),$$

$$L_t^\gamma = \frac{\widehat{w}_t(1+g)^t}{\widehat{C}_t(1+g)^t},$$

$$r_t = \alpha A_t \left(\widehat{K}_{t-1} \frac{(1+g)^t}{1+g} \right)^{\alpha-1} ((1+g)^t L_t)^{1-\alpha},$$

$$\widehat{w}_t(1+g)^t = (1-\alpha) A_t \left(\widehat{K}_{t-1} \frac{(1+g)^t}{1+g} \right)^\alpha ((1+g)^t)^{1-\alpha} L_t^{-\alpha},$$

$$\begin{aligned} (\widehat{K}_t + \widehat{C}_t) (1+g)^t &= \widehat{K}_{t-1} \frac{(1+g)^t}{1+g} (1-\delta) \\ &+ A_t \left(\widehat{K}_{t-1} \frac{(1+g)^t}{1+g} \right)^\alpha ((1+g)^t L_t)^{1-\alpha}. \end{aligned}$$

Exercise 3: A simple RBC model

Stationarized model (continued)

$$\frac{1}{\widehat{C}_t} = \frac{1}{1+\rho} E_t \left(\frac{1}{\widehat{C}_{t+1}(1+g)} (r_{t+1} + 1 - \delta) \right),$$

$$L_t^\gamma = \frac{\widehat{w}_t}{\widehat{C}_t},$$

$$r_t = \alpha A_t \left(\frac{\widehat{K}_{t-1}}{1+g} \right)^{\alpha-1} L_t^{1-\alpha},$$

$$\widehat{w}_t = (1-\alpha) A_t \left(\frac{\widehat{K}_{t-1}}{1+g} \right)^\alpha L_t^{-\alpha},$$

$$\widehat{K}_t + \widehat{C}_t = \frac{\widehat{K}_{t-1}}{1+g} (1-\delta) + A_t \left(\frac{\widehat{K}_{t-1}}{1+g} \right)^\alpha L_t^{1-\alpha},$$

$$\log(A_t) = \lambda \log(A_{t-1}) + e_t.$$

Exercise 3: A simple RBC model

Practicing Dynare

- (a) Write a mod-File for this simple RBC-model and use for calibration:

$$\alpha = 0.33, \delta = 0.1, \rho = 0.03, \lambda = 0.97, \gamma = 0, g = 0.015.$$

Use initval with these values:

$$C = 1, K = 3, L = 0.9, w = 1, r = 0.15, A = 1.$$

- (b) Show that the steady-state implies:

$$\begin{aligned} A &= 1, & r &= (1 + g)(1 + \delta) + \delta - 1 \\ L &= \left(\frac{1 - \alpha}{\frac{r}{\alpha} - \delta - g} \right) \left(\frac{r}{\alpha} \right), & K &= (1 + g) \left(\frac{r}{\alpha} \right)^{\frac{1}{\alpha-1}} L \\ C &= (1 - \delta) \frac{K}{1 + g} + \left(\frac{K}{1 + g} \right)^{\alpha} L^{1-\alpha} - K, & w &= C \end{aligned}$$

- (c) Use this analytical solution for the mod-file, i.e. use `steady_state_model` instead of `initval`. Dynare creates a steady-state m-file. Have a look at it.