Advanced Macroeconomics PhD

Solution to Problem-Set 3

DSGE modelling Keynesian Macroeconomics

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1) Willi, a fellow student, wants to assess how changes in fiscal policy (taxation & spending) affect the real economy.

1. From his class on introductory macroeconomics Willi remembers that as a first step it is always important to distinguish between endogenous and exogenous variables as well as model parameters. Can you help him with that? In the present model, all quantities, i. e. output Y_t, private consumption C_t, public consumption G_t^B, private investment I_t, public investment I_t^B, private capital K_t, public capital K_t^B, public transfers TR_t, the tax rate τ_t, labor supply N_t, the real wage w_t and the real interest rate r_t. These are also observables. The productivity z_t, the Frisch elasticity of labor θ_t and the marginal utility of consumption λ_t are also endogenously determined but not observable. The exogenous variables are the four shocks ϵ_t^z, $\epsilon_t^{G^B}$, $\epsilon_t^{I^B}$ and ϵ_t^τ to productivity, public consumption, public investment and the tax rate respectively, all begin normally distributed with mean 0 and standard deviation ω. The fixed model parameters are the discount rate β, the depreciation rate δ, the productivity of public capital η, the share of capital in production α and the smoothing parameters ρ_z, ρ_{G^B}, ρ_{I^B} and ρ_τ.

2. Willi is quite clever about calibrating the model parameters. In particular, he is interested in targeting steady-state values of the model. Furthermore, he thinks that public-capital productivity should be lower than the capital share in production. Regarding the exogenous processes he would like mild persistence (ρ's equal to 0.75) and small shock standard errors of 1%. Can you provide a calibration for all model parameters meeting his targets and economic intuition? At first, the discount factor is set to $\beta = 0.988$, the depreciation rate is set to $\delta = 0.05$ and the productivity of public capital is set to $\eta = 0.2$.
From eq. (7), the steady state for α follows immediately with the help of the known values:

$$\overline{w}\overline{N} = (1-\alpha)\overline{Y}$$
$$\Leftrightarrow 2\frac{1}{3} = (1-\alpha)$$
$$\Leftrightarrow \alpha = \frac{1}{3}$$

Plugging this into eq. (8), it follows:

$$\overline{r}\overline{K} = \alpha\overline{Y}$$
$$\Leftrightarrow \overline{r}\overline{K} = \frac{1}{3}$$

This can be used in eq. (10):

$$\overline{G}^B + \overline{I}^B + \overline{TR} = \overline{\tau}(\overline{w}\overline{N} + \overline{r}\overline{K})$$
$$\Leftrightarrow 0.2 + 0.02 + 0 = \overline{\tau}(\frac{2}{3} + \frac{1}{3})$$
$$\Leftrightarrow \overline{\tau} = 0.22$$

Plugging this into the household's budget restriction leads to:

$$\overline{C} + \overline{I} = (1-\overline{\tau})(\overline{w}\overline{N} + \overline{r}\overline{K}) + \overline{TR}$$
$$\Leftrightarrow \overline{C} + \overline{I} = (1-0.22)(\frac{2}{3} + \frac{1}{3}) + 0$$
$$\Leftrightarrow \overline{C} + \overline{I} = 0.78$$

The steady state of K^B follows from eq. (5):

$$\overline{K}^B = (1-\delta)\overline{K}^B + \overline{I}^B$$
$$\Leftrightarrow \overline{K}^B = (1-0.05)\overline{K}^B + 0.02$$
$$\Leftrightarrow \overline{K}^B = 0.4$$

Eq. (2) now gives a solution for the steady state of the interest rate:

$$\overline{\lambda} = \beta[\overline{\lambda}((1-\delta) + (1-\tau)\overline{r})]$$
$$\Leftrightarrow 1 = 0.988(0.95 + 0.78\overline{r})$$
$$\Leftrightarrow 1 = 0.9386 + 0.77064\overline{r}$$
$$\Leftrightarrow \overline{r} = 0.0797$$

It then follows for K:

$$\overline{r}\overline{K} = \alpha\overline{Y}$$
$$\Leftrightarrow 0.0797\overline{K} = \frac{1}{3}$$
$$\Leftrightarrow \overline{K} = 4.1824$$

This can be used to derive the steady state for I:

$$\overline{K} = (1-\delta)\overline{K} + \overline{I}$$
$$\Leftrightarrow 4.18240.95 \cdot 4.1824 + \overline{I}$$
$$\Leftrightarrow \overline{I} = 0.2091$$

From what we had derived already, we can conclude that

$$\overline{C} + \overline{I} = 0.78$$
$$\Leftrightarrow \overline{C} = 0.78 - 0.2091$$
$$\Leftrightarrow \overline{C} = 0.5709$$

Eq. (3) then gives:

$$\overline{\lambda} = \frac{1}{\overline{C}}$$
$$\Leftrightarrow \overline{\lambda} = \frac{1}{0.5709}$$
$$\Leftrightarrow \overline{\lambda} = 1.7516$$

and eq. (1) gives:

$$(1-\overline{\tau})\overline{w} = \overline{\theta}_l \frac{\overline{C}}{1-\overline{N}}$$
$$\Leftrightarrow 0.78 \cdot 2 = \overline{\theta}_l \frac{0.5709}{\frac{2}{3}}$$
$$\Leftrightarrow \overline{\theta}_l = 1.8217$$

For the technology shock z, we arrive at:

$$\overline{Y} = \overline{z}(\overline{K}^B)^\eta(\overline{K})^\alpha(\overline{N})^{(1-\alpha)}$$
$$\Leftrightarrow 1 = \overline{z}(0.4)^{0.2}(4.1824)^{\frac{1}{3}}(\frac{1}{3})^{\frac{2}{3}}$$
$$\Leftrightarrow 1 = \overline{z}0.64487$$
$$\Leftrightarrow \overline{z} = 1.5507$$

3. Willi is not sure if he wants to simulate the deterministic or stochastic model. Can you provide some guidance when to use which one? Since Willi is impatient, please be brief and try to explain it in a maximum of 10 sentences. In the present case, the stochastic model should be estimated. Simulating a deterministic model makes sense whenever the future shocks can perfectly be foreseen. Individuals are certain about the time and size of future shocks and furthermore know whether there will be more shocks in the future facing one shock today. Here, this is not the case since the exact occurence of future shocks is unknown, only their distribution is. Shocks can be observed today but the future expectation of further shocks is always zero as the shocks are normally distributed with mean 0. Therefore, a stochastic model should be simulated requiring a linearization of model equations before. If there are permanent changes in variables or expected shocks, these cannot be represented anymore when a Taylor approximation is used for linearization.

4. Willi has heard of the powerful toolbox DYNARE, so he asks you to help him set up this model in DYNARE. Write a DYNARE mod-file for this model, commenting each step such that Willi clearly understands each block. See mod-file!

5. How does (i) an unexpected temporary public consumption and (ii) an unexpected temporary public investment shock (of the same size) feed through the model? Simulate these two shocks and compare the reactions of the observables. Does it affect the real economy? Try to provide economic intuition behind the results. How do your results change if the productivity of public capital increases? (i) In this case, the public sector increases its consumption affecting the real economy in several ways: Output Y_t rises immediately while private consumption C_t immediately falls. This might be due to a crowding-out-effect. As capacities are fixed in the short run, the increase in public consumption crowds out households' consumption on the markets. The marginal utility of consumption λ_t increases as a logical consequences. As the state faces a fixed budget constraint, described by eq. (10), it is obvious that an increase in public consumption with an unchanged tax rate can only be financed by a reduction in public investment and/or transfer payments, therefore, I_t^B and TR_t both fall. As output Y_t increases following the positive demand shock, so does the amout of labor N_t. As eq. (7) must be fullfilled, rising Y_t and N_t lead to a fall wages w_t. Economically, this might be due to a . As public investment decreases, so does the public capital stock K_t^B. Since wages fall, households' incomes are reduces leading to less private investment I_t and reducing the private capital stock K_t. This reduces the interest rate r_t as well. After a time of about ten periods, the output falls again, even below its initial value. As total capital in the economy are fixed in the short run, production takes places overutilizing capacities. Because of the mentioned fall in investment, there is a lack of capital to expand capacities which would be necessary to keep the higher level of production. The government's consumption shock was just temporary, private consumption falls again due to lower wages and overall demand decreases. Labor demand falls again and the demand for capital instead rises. After a span of 50 time periods, all variables have reached their initial values again. The public consumption shock has only had temporary effects, leading to more output and labor demand, but less private consumption and lower wages. There is no long run effect.

(ii) The effects just mentioned can be compared to a temporary shock in public investment. First, it can be stated that this shock has no long run effects neither whereas the reaction of the observables differs. A larger amount of public investment I_t^B of course leads to a higher public capital stock K_t^B according to its law of motion. As the fiscal budget constraint must be respected again, government's consumption sharply decreases. Taking a look at the production function makes clear that output Y_t also increases due to the positive public capital development. A slight fall in private capital is observable, probably due to a reduced firm demand for private capital. A higher output leads to a higher demand for labor and to more private consumption which results in a lower marginal utility of consumption. The higher demand for labor induces higher wages. Whereas in (i) individuals might have increased labor supply due to the crowding out and their reduced consumption level, there is now an excess demand and an upward pressure on wages. Both private investment an capital rise. After about 10 periods, a peak can be seen in most figures. The public investment shock is only temporary. It has led to a higher price of capital, say the interest rate r_t. As the share of private capital in production is larger than the public capital productivity, there is no perfect substitution. The price for private capital is high. Output and labor demand fall again, thereby private consumption and wages. Private investments are also reduced, leading to less private capital and an even lower price of capital r_t than prior to the shock. All variables come back to their initial values but only after 100 time periods which is much more than in (i). So although, there are no long run effects, the positive impact of a public investment shock on wages and private consumption is larger.

With a higher productivity of public capital, here η increases from 0.2 to 0.4, the direction of impacts is not changed at all. In (i) nothing changes because of the fact that an increase in public consumption is not connected to η. In (ii) however, the size of impact varies as all effects on variables are larger, i. e. output, wages and labor demand rise by more than with a smaller productivity of capital which seems to be logical: If public capital is more productive, its increase leads to a larger output raise, resulting in also more private consumption and demand for labor. So, the direction of effects is the same whereas the size differs depending on the parameter specification.

6. How does a permanent increase in the tax rate of 1 percentage point affect the long-run equilibrium of the economy? Compute the new steady-state and compare it with the old one. Also, show the transition path from the old to the new steady state. Try to provide economic intuition. A permanent increase has, in contrast to what could be seen when considering temporary shocks, a permanent impact on the real economy. Of course, since the tax rate τ_t is set to a new higher value, tax revenues increase as well which lead, for some periods, to more a higher fiscal budget and more transfer payments. Later, the transfers fall again which is due to the fact that the households know about the permanent tax raise and reoptimize their behaviour: Income falls according to the higher tax burden, therefore consumption and aggregate output are reduced. The amount of labor used in production, N_t falls sharply at first as a consequence of the smaller goods demand. After about 10 periods, labor rises again a little. A reason could be that households want to work more hours now to earn some more money and compensate the higher tax rate partly. As a reaction to the lower income, households also lower their investments, at first strongly, after 10 periods, the amount of investment rises again a little. This probably goes hand in hand with more hours worked and therefore an income rise. All in all, it it obvious that the real economy is permanently affected here. Output Y_t, consumption C_t, labor N_t and investment I_t are all reduced in the long-run. Therefore, the tax raise can be considered to have a negative effect on the economy. As labor and investment first undershoot their new long-term level, this can be a sign of household reoptimization because of the permanently higher tax rate.

7. Willi argues that fiscal policy cannot boost the economy due to the implied crowding-out of private consumption and private investment. Do you agree given your results above? Also explain, in economic terms, the difference between private and government capital. Given the results in exercises 5 and 6, I do slightly agree with Willi. On the one hand, he is right that a temporary fiscal policy has no long run effects. All variables of the real economy have returned to their initial values after a certain time. However, the effect with a higher government investment only vanishes after 100 time periods which could be considered as relatively long lasting effect. Furthermore, exercise 6 has shown that a permanent rise of the tax rate in fact leads to a new long-term equilibrium. Here, consumption and output are lower. A simulation could show whether a tax reduction has the same but positive effect on income and consumption. An expansive fiscal policy is likely to have positive long-term effects on the real economy. In addition, as household income is affected, one cannot speak of a direct crowding-out effect here. Nevertheless, a difference is observable depending whether the public impact is temporary or permanent. If people know that a policy measure has a long-term character, they reoptimize their behaviour according to the new situation. The opposite seems to be the case if there is only one shock in the short run. So to sum up, I do not think that it is completely right to say that fiscal policy cannot boost the economy. It is the case that public consumption or investment can lead to crowding out and therefore to lower private consumption and investment. However, a permanent change that households can rely on results in permanent effects. It must be born in mind that an expansive fiscal policy has to be financed in some way. So, if there are permanent increases in public expenditure for example, there must also be contractory measures, such as higher taxes or lower transfers, to balance the fiscal budget. It might be the case that effects set off each other then.

2) Consider a version of the An and Schorfheide (2007) model. The mod-file AnScho.mod contains (incomplete) code to estimate the An and Schorfheide model with Bayesian methods.

1. How many observable variables do you need for a Bayesian estimation? Include varobs into the mod-file. See mod-file!

2. Simulate data for your observable variables and save these into a matfile called simdat.mat. See mod-file!

3. Briefly explain the intuition behind prior information in a Bayesian estimation (maximum 10 sentences). Bayesian approaches have the idea that data d are fixed and the model parameters θ are fixed. Furthermore, there are prior-beliefs about the structure of parameters: Based on what is known about economic modelling, certain assumptions about the parameters can be made. These are combined with a Maximum-Likelihood approach to determine the parameters from the given data. Thereby, the estimated parameter vector $\hat{\theta}_B$ is the maximum of the sum of the likelihood function of parameters given the data and the prior beliefs about the parameter distribution. With the help of several methods, e. g. the Monte Carlo-method, the parameters can be estimated. Afterwards, the posterior distribution of parameters can be compared with the prior distribution to determine the accordance.

4. Estimate the model with your simulated data and Bayesian methods. See mod-file!

5. Consider the figure Priors and Posteriors (ignore the other output). How would you assess the quality of this estimation exercise? The figures show the prior estimation in light grey, the posterior estimation in black and the posterior parameter estimation as a dashed green line. The quality of prior estimation differs substantially. For τ, κ, ψ_1 and ψ_2, one can see that the posterior distribution using the simulated data is quite close to what was assumed about the parameters before. For all four parameters, the prior mean is in the $95-\%-$confidence interval of posterior estimation (which is not shown in the figures, but in the DYNARE output). Especially for ψ_1 and ψ_2, prior beliefs seem to precisely match the estimated parameter values with simulated data. The same is true for the values of $\pi^{(A)}$ and $r^{(A)}$. The density functions for prior and posterior values are almost congruent. Furthermore, the prior assumption made for $\gamma^{(R)}$ also seems to be good as the posterior parameter is very close to the mean of the prior density function. However, the posterior estimation does not confirm all prior beliefs about the parameters. For ρ^R, ρ^z and ρ^g, the posterior densities are very different from the prior. Although the posterior parameter is close to the mean of the prior density, the posterior densities are tighter displaying less variance of possible parameter values. A similar result is found for the parameters σ^R, σ^g, σ^z. Considering again the non-graphical estimation output as well, one can see that the prior means of σ^R, σ^g and σ^z are all outside the $95-\%-$confidence interval of posterior estimation, confirming the graphical result that prior and posterior parameter values differ a lot. One can conclude that the estimation exercise was helpful as the beliefs about half the parameters could be confirmed as being quite reasonable whereas the assumptions about the other half of parameters do not seem to match the simulated model data well. Especially, the prior beliefs about these parameters' variances are larger than for posterior parameters. A further consideration might be helpful.