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Advanced Macroeconomics (PhD level)

Problem set 3

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Exercise 1: Baxter and King (1993)

1) Parameters, endogenous and exogenous Variables

Parameters: Time-invariant coefficients on habit, behavior or technology

Endogenous: Determined within the model

Exogenous: Not determined within the model

Parameters	Endogenous variables	Exogenous variables
$\boldsymbol{\theta_l}$: Frisch-elasticity of labor	$\boldsymbol{C_t}$: Private Consumption	$\boldsymbol{\varepsilon_t^z}$: Technology shock
$\boldsymbol{\delta}$: Time-invariant overall depreciation rate	$\boldsymbol{I_t}$: Private investment	$\boldsymbol{\varepsilon_t^{G^B}}$: Government consumption shock
$\boldsymbol{\beta}$: Time preference rate / discount factor	$\boldsymbol{\tau_t}$: Lump-sum tax rate	$\boldsymbol{\varepsilon_t^{I^B}}$: Government investment shock
$\boldsymbol{\eta}$: Public capital share	$\boldsymbol{w_t}$: Wage rate	$\boldsymbol{\varepsilon_t^{\tau}}$: Tax rate shock
$\boldsymbol{\alpha}$: Capital share	$\boldsymbol{N_t}$: Labor input	
$\boldsymbol{\rho_z}$: Smoothing parameter of z	$\boldsymbol{r_t}$: Real interest rate	
$\boldsymbol{\rho_{G^B}}$: Smoothing parameter of G^B	$\boldsymbol{K_t}$: Private capital stock	
$\boldsymbol{\rho_{I^B}}$: Smoothing parameter of I^B	$\boldsymbol{TR_t}$: Real lump-sum Transfers to households (Public debt also enters the model by this)	
$\boldsymbol{\rho_\tau}$: Smoothing parameter of τ	$\boldsymbol{\lambda_t}$: Marginal utility of consumption	
	$\boldsymbol{G_t^B}$: Government consumption	
	$\boldsymbol{I_t^B}$: Public investment	
	$\boldsymbol{K_t^B}$: Public capital	
	$\boldsymbol{z_t}$: Total factor productivity / technological change	

Of course the observables from equations (15) – (23) are endogenous as well

2) Calibration of the model

First of all we choose reasonable values for the parameters β, η and δ. The time preference rate is set to 0.99, following the general procedure. This is also done for the depreciation rate δ, where we calibrate with a value of 0.10, i.e. 10 % per year. From the given values we know that in the steady-state the wage ratio is $\frac{2}{3}$ and GDP equals one. Plugging this into the relation from (7),

$$wN = (1 - \alpha)Y, \tag{1}$$

dropping the time indices for the steady-state, we obtain a value of $\frac{2}{3}$ for $1 - \alpha$ and therfore $\frac{1}{3}$ for the capital share which can also be simply derived from the distribution-side GDP. Subsequently the public capital productivity η has to be smaller than $\frac{1}{3}$. Following Baxter and King we choose it to be 0.05.

The next step is to derive the steady-state of the tax rate τ. This can be done by using equation (10), the government budget constraint which is

$$G^B + I^B + TR = \tau(wN + rK). \tag{2}$$

Plugging in the given steady-state values leads us to a tax rate of 0.22.

This result can now be used to calculate the steady-state real interest rate r by means of equation (2), i.e.

$$\lambda = \beta\lambda[(1 - \delta) + (1 - \tau)r]. \tag{3}$$

Note that the expectation term drops out since we are in a non-stochastic steady-state. Furthermore assuming that λ is unequal to zero, which is reasonable as it is the marginal utility (see Eq. (3)), we can divide by λ and gain an interest rate of about 0.141155, i.e. 14.12 %. This value is clearly too high and could be changed by varying β and δ but for our purposes this is of minor importance.

Having the steady-state for r we can calculate the equilibrium private capital stock from equation (8) which yields a value of 2.3615 for K.

Plugging this into the law of motion for the private capital stock,

$$K = (1 - \delta)K + I, \quad (4)$$

we get a value of 0.23615 for steady-state private investment I.

The law of motion for public capital can be used to obtain 0.20 for the public capital stock K^B.

The steady-state consumption can simply be calculated from the expenditure-side GDP-equation, leading to a value of 0.54385 for C and its marginal utility λ of 1.8387 as the inverse (see Eq. (3)).

The values derived before imply a Frisch elasticity of labor of 1.9123, according to equation (1), i.e.

$$(1 - \tau)w = \theta_l \frac{C}{(1 - N)}. \quad (5)$$

This result is not in line with the empirical literature where the Frisch elasticity of labor is found to range between 0.2 and 0.5. Nevertheless since we want to analyse for general effects in this simple model this will not be a problem.

Finally we can plug our values into the steady-state production function,

$$Y = z(K^B)^\eta (K)^\alpha (N)^{1-\alpha}, \quad (6)$$

in order to obtain the steady-state TFP z of 1.6929.

All in all we have gained the following Parameters and steady-state values which will be needed later on for the calibration and the initial values of the model.

$\boldsymbol{\theta_l}$	1.9123	$\overline{\boldsymbol{I^B}}$	0.02
$\boldsymbol{\delta}$	0.10	$\overline{\boldsymbol{G^B}}$	0.2
$\boldsymbol{\beta}$	0.99	$\overline{\boldsymbol{K}}$	2.3615
η	0.05	$\overline{\boldsymbol{K^B}}$	0.2
$\boldsymbol{\alpha}$	$1/3$	$\overline{\boldsymbol{N}}$	$1/3$
$\overline{\boldsymbol{Y}}$	1	$\overline{\boldsymbol{w}}$	2
$\overline{\boldsymbol{C}}$	0.54385	$\overline{\boldsymbol{r}}$	0.141155
$\overline{\boldsymbol{I}}$	0.23615	$\overline{\boldsymbol{z}}$	1.6929
$\overline{\boldsymbol{TR}}$	0	$\overline{\boldsymbol{\lambda}}$	1.8387

3)

The main difference between deterministic and stochastic models is the anticipation of shocks. In a deterministic setting, future and present shocks are known by the agents without any uncertainty, i.e. there is perfect foresight. Furthermore the shocks can last more than one period. This type of model is preferably used for analyzing regime changes.
In a stochastic model, only the distribution of the shocks is known, so the agents don’t foresee a shock and its impact. The shock occurs only for one period and thereafter the expected value is zero again.
Therefore in this case we want to simulate a stochastic model since all that is known about the shocks is that they are normally distributed with mean zero and standard errors of 1%. Note that though the expected value is zero, the model is still stochastic. These models are useful for RBC and DSGE models where the economy is frequently hit by shocks like technology or policy shocks.

4)

See the .mod file ‘baxter_king.mod’

5)

A shock on G_B means that government spending increases surprisingly in period zero. As there is no debt in this model, the transfers TR directly become negative by nearly the same amount. This leads to a negative income effect for the consumption-smoothing household. Since it is anticipated that the spending today will have to be paid back in the future the households decrease their consumption and savings/investment. At the same time the labor offered is increased. One can imagine it, following Baxter and King who argue with war times, that today a large effort is made in order to finance temporary necessary government spending so the public sector acquires resources which could have been used in a productive and utility gaining way. As the supply of labor increases, the real wage decreases. Furthermore the private capital stock also falls beyond its steady-state value as a consequence of diminished investment. The smaller capital stock subsequently has a higher marginal productivity which leads to an increase in the real interest rate. All in all what happens is the so-called crowding out, i.e. the additional demand by the government extrudes private demand for investment and consumption. This is also called the Ricardian equivalence which says that money taken via lump-sum taxes from the private sector has no substantial effect if it flows back. So we see an initial boost of GDP but this is only due to the increased labor supply. There is not such as a Keynesian multiplier effect in this model. As government spending strictly follows an AR(1) process, it declines gradually back to its steady state. This is also the case for the other endogenous variables. For the interest rate r, the capital stock K and the real wage w we thereby see a short further increase / decline before the adjustment goes towards the steady-state again. This stems from the fact that capital enters the production function with a lag. When it is rebuild by investment, GDP actually falls below the steady state for some periods which can be seen as the effort of rebuilding by additional investment which can also be observed during this time. An increased productivity of public capital has no effect on the results discussed above. The explanation is that the government does not invest in order to increase its capital but rather consumes the resources.

A different story comes up if we shock public investment I_B. The initial effects are basically the same but they are not as large in impact as they were when the government spending shock happened. There is again a crowding-out effect since resources for investment are absorbed by the government which leads to the same effects as described before. But as the government now spends the resources in a productive way the households expect more consumption in the future. This increases the public capital stock which is productive, leading to a peak of the GDP boost after some periods. On the other hand the marginal products of private capital and labor increase relatively which leads to investment. So as the private capital stock grows again, the interest rate decreases. All the variables tend backwards to the steady-state. Thereby labor supply and the interest rate even fall below it for several periods as the private capital stock is above its steady-state being more productive than public capital. So there is a positive income effect for the households. The impact of the public investment shock becomes even stronger if the productivity of public capital is calibrated to be larger. The explanation is simply that in this case public capital creates absolutely and relatively to the other factors more income than before. So we see that the boost of the economy lasts longer if the resources are used in a productive way.

6)

In order to increase the tax rate permanently by 1% the government has to shock it appropriately in the initial period which stems from the AR(1) process, τ, the tax rate follows. To calculate the desired size of the shock we have to insert the old steady-state of 0.22 and the new one of 0.23 into equation (13). Doing this we obtain a shock of about 0.01111. A gradual process is induced. The higher tax rate causes consumption and the labor supply to decrease (and subsequently real wages to rise temporarily) which can be seen looking at the optimal consumption-leisure choice of the household. The same happens for investment as the tax rate is also taken for capital incomes. So the interest rate increases as the capital stock falls below its initial steady-state. Since the higher tax rate does not affect government consumption and investment there is no boost for the economy and GDP immediately decreases. All the variables head towards a new steady-state. The effects are overall negative. Investment, employment, real wages, consumption and GDP converge to a lower steady-state. Transfers end up higher, as the government has a permanent surplus which is not spend but there is no further income effect.

7)

First of all we have to state that we have modelled an RBC model without nominal rigidities like sticky prices and wages. In a DSGE framework the government has better chances to boost the economy by increased spending even if there are Ricardian households. Nevertheless also adopting the context of this model the statement is not completely correct. Government spending can actually influence the economy for a short time but there is no multiplier effect, so the term ‘boost’ would be exaggerated. Nevertheless there are positive effects even though followed by negative ones.

What is central to the model of Baxter and King is the role of public capital. Using the absorbed resources in a productive way, i.e. investing, the government can maintain the positive effects for a while, but it has to be said again that a recession follows. The size of the impact depends crucially on the productivity of public capital. As can be seen in equation (6), public capital enters the production function in a non-rivalry way, i.e. the factor share is not connected with the private capital share and the wage ratio. So it follows that public capital does not extrude private factors. Examples for public capital are public goods like infrastructure, education or health care. Though it can reasonably be assumed, that private capital is more productive as a consequence of negative incentives for public capital management.

As taxation also belongs to fiscal policy we have to additionally say that a permanent positive boost can be induced by lowering the tax rate.

Exercise 2: An and Schorfheide (2007)

1)

For a Bayesian estimation we need as much observables as there are shocks. In this case the number is three. For the rest: see .mod file ‘AnScho.mod’

2)

See .mod file ‘AnScho.mod’

3)

In a Bayesian estimation we regard the data as fixed while the parameters are random variables. We also compute a Maximum Likelihood estimation but augment it with prior information on the parameters. This information comes from empirical evidence, experience, intuition and knowledge on the parameter values. From this information a prior distribution of the parameter value is constructed, i.e. a calibration. Thereby we have to specify a mean and a variance given by a reasonable bandwith as well as the skewness and kurtosis which is mostly done by assuming a specific distribution function. What now a Bayesian estimation does is to push the Maximum Likelihood estimation towards our prior distribution. The result is the posterior which can be seen as a compromise between the results obtained from the data and our prior knowledge. In the context of a model simulation we can assess the quality of the model, i.e. if it fits our prior evidence well. This is because the observables are simulated while our prior information is based on real evidence.

4)

See .mod file ‘AnScho.mod’

5)

In order to assess the quality of the estimation we can compare the fit of the prior and the posterior. This allows us to assess the quality of the model as well, because the data are simulated.

For tau and psi1 we see a very good fit of the density functions which can also be stated for psi2 to lesser extent. The mean of the parameter value is captured very well by the posterior of rhoR, p_A and also gam_Q. But here the posterior density functions are quite differently shaped regarding the variance and kurtosis, leading to large tails and therefore a bigger uncertainty. In general it can be stated that the mean (indicated as a green dashed line) of the posterior seems to lie within a reasonable confidence interval of the prior density function. Thus the null of the posterior expected value being within the bandwith of our prior will hold for most of the parameters which can be seen as an indication for a good estimation. Though there are cases like rhog or stdr where the fit is not satisfying. All in all the prior information obviously is useful for the esimation.