Advanced Macroeconomics (PhD level)

Solution to Problem Set 3

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Solution to Exercise 1

(1) Differentiation between endogenous and exogenous variables as well as model parameters.
In the Baxter and King model, all quantitative variables are endogenous. These are: output Y_t, private consumption C_t, public consumption G_t^B , private investment I_t, public investment I_t^B, private capital K_t, public capital K_t^B, public transfers TR_t, the tax rate τ_t, labor supply N_t, the real wage w_t and the real interest rate r_t. Productivity z_t, the Frisch elasticity of labor $\theta_t$and the marginal utility of consumption λ_t are also endogenously determined. Additionally we have endogenous "observables", which are the percentage deviations of Y_t, C_t, I_t and w_t from the steady state (dY_t, dC_t, dI_t and dw_t) and the percentage point deviations of N_t, r_t, TR_t, $\frac{G_t^B}{Y_t}$ and $\frac{I_t^B}{Y_t}$ (dN_t, dr_t, dTR_t, $d\,\frac{G_t^B}{Y_t}$ and $d\,\frac{I_t^B}{Y_t}$.

The exogenous variables are the four shocks, which are all normally distributed: the productivity shock ε_t^z, the public consumption shock $\varepsilon_t^{G^B}$, the public investment shock $\varepsilon_t^{I^B}$ and the tax rate shock ε_t^τ.

Finally, the model parameter are: the discount rate β, the depreciation rate δ, the productivity of public capital η, the share of capital in production α, the steady state productivity $\bar{z}$ and the smoothing parameters ρ_z, ρ_τ, ρ_{G^B} and ρ_{I^B}.

(2) Calibration
In order to calibrate the model parameters, we will first have to set reasonable values for β, δ, and η.
First of all it is important to be clear about the time frame we are working in. As with RBC models the aim is to analyze some kind of 'economic fluctuations' it is reasonable to work with shorter time periods. We will therefore assume that each period equals a quarter of the year.
Looking at the literature, we find that for the discount rate $\beta = 0.99$ is a reasonable value throughout (e.g. Kydland & Prescott (1982), p. 1363; Hansen (1985, p. 319). As the calibration of the Baxter and King model is based on King et al. (1988) and as they use $\beta = 0.988$ (p.208), we will stick to this calibration.
For the depreciation rate δ, Baxter & King (1993, p.320) use a rate of $\delta = 0.1$, which corresponds to an annual depreciation rate of 10%. The calibration follows that in King et al. (1988, p.208), where a quarterly depreciation rate of 0.025 is set. The same can be found in Kydland & Prescott (1982, p. 1361), where an annual depreciation rate of 10% is said to be a good compromise given that different types of capital depreciate at different rates. As we are also working in quarters, we will set the depreciation rate to $\delta = 0.025$. Following through with the calibration this yields a real interest rate of $r \approx 4{,}5\%$, which, from an economic background also seems to be reasonable. Hansen (1985, p. 319), Strulik & Trimborn (2011, p.14) and also Baxter and King (1993, p.320) also point out, that with their calibration they are aiming at an annual real interest rate of about 4-6% (6.5% in Baxter & King).

For the productivity of public capital η, it is given that it should be lower than the capital share in production α. α can be calculated from the given steady-state targets and is $\frac{1}{3}$. Ligthart & Suárez (2011) conduct a meta analysis and from their data find a value of $\eta = 0.14$. Baxter and King (1993, p.330) set their η to 0.05. In order to be able to raise the productivity of capital later on onto a significant different level we will also use $\eta = 0.05$.

It is noteworthy that the steady state value for the productivity $\bar{z}$ is not equal to 1 using this calibration, which would have been expected. However, the deviation is not too big, so we will nevertheless stick to the described calibration.

Using these set parameters and the targeted steady-state values from the exercise, the remaining parameters have been calibrated as follows:

(For convenience this is included as handwritten work)

Use eq. (7) in the steady state and $\bar{w} = 2$, $\bar{N} = \frac{1}{3}$ and $y = 1$ to obtain α.

$\bar{w}\bar{N} = (1-\alpha)\bar{y}$

$\Leftrightarrow \frac{2}{3} = (1-\alpha)$

$\Leftrightarrow \alpha = \frac{1}{3}$

use eq. (8) and $\alpha = \frac{1}{3}$ and $y = 1$ to obtain $\bar{r}\bar{K}$

$\bar{r}\bar{K} = \alpha\bar{y}$

$\Leftrightarrow \bar{r}\bar{K} = \frac{1}{3}$

This can be used to obtain $\bar{\tau}$ from eq. (10) in the steady state

$\bar{G}^B + \bar{I}^B + \bar{TR} = \bar{\tau}(\bar{w}\bar{N} + \bar{r}\bar{K})$

$\Leftrightarrow 0.2 + 0.02 + 0 = \bar{\tau}\left(\frac{2}{3} + \frac{1}{3}\right)$

$\Leftrightarrow \bar{\tau} = 0.22$

Using the budget restriction yields

$\bar{C} + \bar{I} = (1-\bar{\tau})(\bar{w}\bar{N} + \bar{r}\bar{K}) + \bar{TR}$

$\Leftrightarrow \bar{C} + \bar{I} = 0.78$

Now using $\delta = 0.025$ and eq. (5) in the steady state gives

$\bar{K}^B = (1-\delta)\bar{K}^B + \bar{I}^B$

$\Leftrightarrow \bar{K}^B = 0.975\bar{K}^B + 0.02$

$\Leftrightarrow \bar{K}^B = 0.8$

With $\beta = 0.988$, $\delta = 0.025$ and $\eta = 0.05$ and using eq. (2) in the steady state yields

$\bar{\lambda} = \beta \bar{\lambda} \left((1-\delta) + (1-\bar{\tau}) \bar{r} \right) \qquad | : \bar{\lambda}$

$\Leftrightarrow 1 = 0.988 \left(0.975 + 0.78 \bar{r} \right)$

$\Leftrightarrow r = 0.0476$

Plugging this into eq. (8) in the steady state yields

$\bar{r} \bar{K} = \alpha \bar{y}$

$\Leftrightarrow 0.0476 \bar{K} = \frac{1}{3}$

$\Leftrightarrow \bar{K} = 7.0028$

Using $\bar{K}$ and $\delta = 0.025$ and eq. (4) in the steady state we can solve for $\bar{I}$

$\bar{K} = (1-\delta) \bar{K} + \bar{I}$

$\Leftrightarrow 7.0028 = 0.975 \cdot 7.0028 + \bar{I}$

$\Leftrightarrow \bar{I} = 0.1751$

We know that $\bar{C} + \bar{I} = 0.78$ so that

$\bar{C} = 0.78 - 0.1751 = 0.6049$

And using eq. (3) gives

$\bar{\lambda} = \frac{1}{\bar{C}}$

$\Leftrightarrow \bar{\lambda} = 1.6532$

All of the above now helps us to derive $\bar{\theta}$ from eq. (1) in the steady state

$(1 - \bar{\tau}) \bar{w} = \bar{\theta} \frac{\bar{C}}{1 - \bar{N}}$

$\Leftrightarrow (1 - 0.22) 2 = \bar{\theta} \frac{0.6049}{1 - \frac{1}{3}}$

$\Leftrightarrow 1.56 = \bar{\theta}\, 0.9074$

$\Leftrightarrow \bar{\theta} = 1.7192$

Finally, we can also derive $\bar{z}$ from eq. (6) in the steady state

$\bar{y} = \bar{z} \left(\bar{K}^{B} \right)^{\eta} \left(\bar{K} \right)^{\alpha} \left(\bar{N} \right)^{1-\alpha}$

$\Leftrightarrow 1 = \bar{z} \left(0.8 \right)^{0.05} \left(7.0028 \right)^{\frac{1}{3}} \left(\frac{1}{3} \right)^{\frac{2}{3}}$

$\Leftrightarrow 1 = \bar{z}\, 0.9096$

$\Leftrightarrow \bar{z} = 1.0994$

(3) Differentiation between deterministic and stochastic models.

In this case, the stochastic model should be estimated. Simulating a deterministic model is only useful when there is full information/ no uncertainty around shocks and when future shocks can be perfectly foreseen and are therefore exactly known regarding their size and timing at the time of computing the model. In this case however, we only know the distribution of the shocks, but not their exact time of occurrence. This can be seen from the fact that the shocks are normally distributed with a mean of zero, meaning that the future expectation of these shocks is always zero. What is also useful to know is that deterministic models do not require a linearization in order to find a solution, which makes the computation much easier. However, the information depth is not as good as in a stochastic model, which is more useful for further analysis and estimation. Here the model is estimated by log-linearizing and conducting Taylor approximations around the steady state, which is also the reason why expected future shocks or permanent changes in exogenous variables cannot be handled.

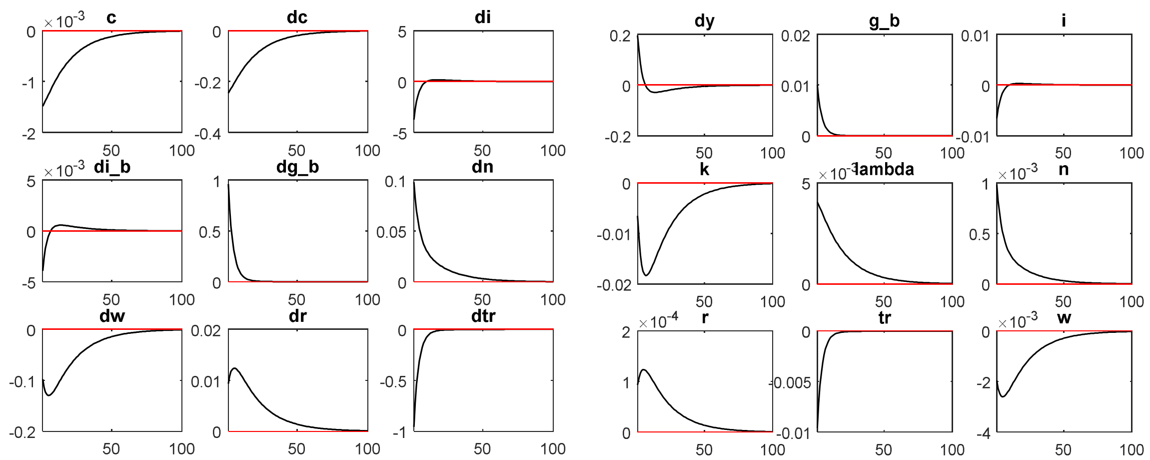
(4) DYNARE mod-file

See mod-file *baxter_king_404793.mod*!

(5) Effects of an (i) unexpected temporary public consumption and (ii) an unexpected temporary investment shock

In order to see how the two shocks feed through the model, please choose scenario 1 in the mod-file. DYNARE will then calculate both shocks of the size of 1% using a stochastic simulation (as they are unexpected shocks).

(*i*) *unexpected temporary public consumption shock*



Looking at the observables we can see that all observables are affected by the public consumption shock.

The shock of 1% can be observed in the graphical output for G^B. Here we can see that in period 0 we have the public consumption shock of 1%, which lasts approximately for 10 periods, thereby steadily declining.

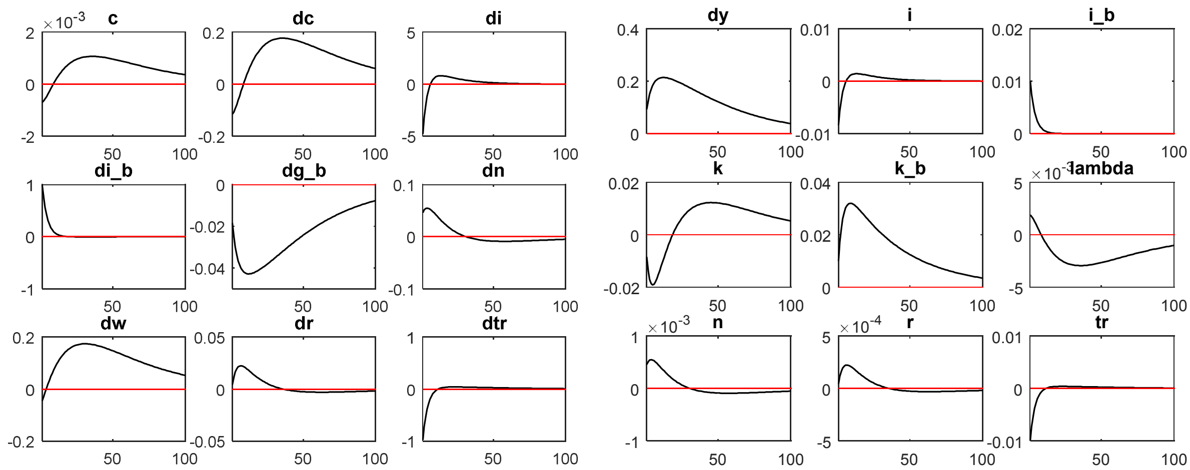
dC, dI, dY, dr and dw show the percentage deviations from their respective steady states. For dC we can see that the public consumption shock leads to a decline of private consumption by approximately 0.25% compared to the steady state and that it takes approximately 60 periods for private consumption to return to its previous steady state. Looking at dw we can see that wages first drop by approximately 1% right after the shock. Wages then keep declining for 13 more periods, until they start to return to their steady state, which takes approximately 55 periods. The adjustment process of dr is actually the same, but the other way around (interest rate first increases due to the shock). dI drops by 3.5% due to the public consumption shock, returning to its steady state quite fast (after 10 periods). Here we can also note that for a short period, investment overshoots its steady state. For output the effects are the other way around. dY rises by 0.2% due to the shock, when returning to its steady state undershoots its steady state after about 7 periods and then gradually returns to its steady state.

For dTR and dI^B the deviations can be interpreted as percentage point deviations. dI^B shows a similar reaction as dI, however we have to notice that reactions are much smaller. dI^B first drops by approximately 0.004 percentage points, then rises and overshoots its steady state level. However, what we can also see (from the DYNARE output) is that I^B only changes on a very small scale. Movements in dI^B are therefore almost exclusively driven by Y due to the specification (it gives I^B relative to GDP). dTR drops by approximately 1 percentage points and rises back to its steady state level in the same manner as the public consumption shock behaves.

From all the graphical outputs we can see that all observables return to their respective steady states after at least 100 periods, so that we can say that there are no long run effects to the real economy. In the short run however, we can see that there are effects to the real economy, especially as the shock results in movements in Y, w, C and N in the short run.

In economic terms, increased government consumption leads to an increased absorption of resources by the government. Opportunities for private consumption, leisure and investment are therefore reduced, which can also be seen by the drop in the transfers. Here government consumption has to be financed by negative transfers. The higher government consumption leads to higher goods demand and therefore also higher output, which has to be matched by a higher labor supply. We can see that labor input increases more during the initial phase where government consumption is still high, than it does afterwards. When government consumption has returned to its initial steady state, private investment rises above its steady state level in order to rebuild the capital stock for production. Looking at the relative price implications we can see that both the wage rate and the interest rate have been affected. Their movements mirror the movements in labor input and capital input.

(ii) unexpected temporary investment shock



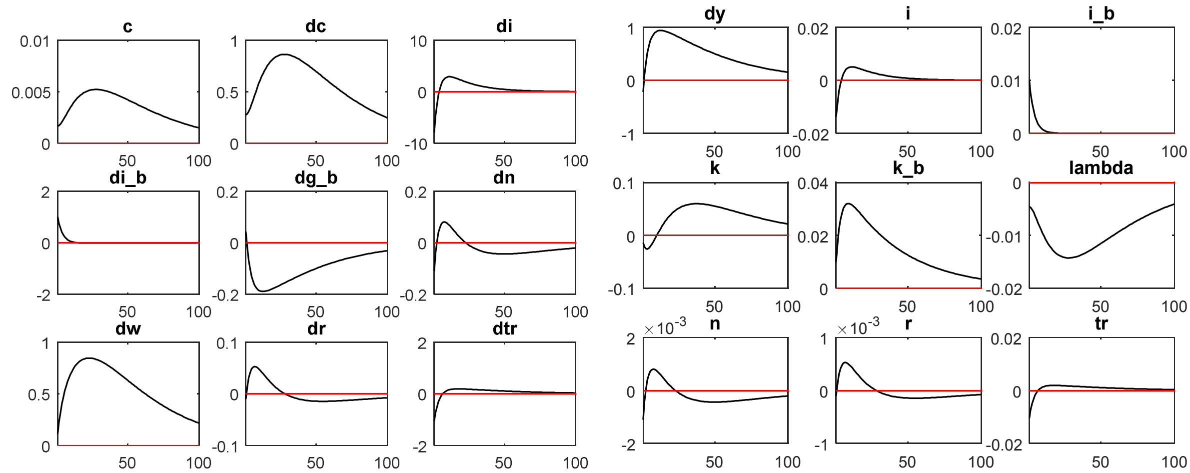
Again we can see directly that all observables are affected by the public investment shock.
This time the shock of 1% can be observed in the graphical output for I^B.

The decrease in private consumption due to the shock is less than before with only approximately 0.1% compared to the steady state. We can also see that dC rises above its steady state level in the tenth period and adjusts very slowly back to its steady state. dI drops by almost 5% due to the public investment shock, which is more than with the consumption shock. However it also rises higher above its steady state and takes longer to converge back to it. dG^B obviously shows completely different movements than before. It drops by 0.02 percentage points as an immediate reaction, decreases even further until the 30th period and only then starts to gradually increase again. However, looking at G^B in the DYNARE output we can see that this is almost exclusively driven by changes in output and therefore due to the specification of dG^B which is given relative to GDP. Looking at dY we can see that it does describe the opposite movement of dG^B, rising to 0.01% as an immediate reaction, rising even more until the 20th period and then adjusting back to its steady state. As before, dN rises, this time however by 0.05% (higher) and undershoots its steady state in the 30th period. dw first drops as an immediate reaction by 0.05%,however then immediately rises over its steady state level by 0.2%, before it gradually falls back to its steady state. dr shows the same movements as in the consumption shock, however reactions are bigger. dTR is exactly the same as before.
Again we can see that all observables converge back to steady state, meaning that there are only short run effects and no long run effects on real economy.
Movements can again be explained in economic terms. As before, increased investment is financed by negative transfers, which also has an impact on households. Also due to increased public investment again resources are absorbed by government and therefore private consumption and investment decrease and labor input increases. The different movement in output can be explained by the accumulation of public capital over time, which can be seen in the graph for K^B and which directly

yields an increased flow of output. This also influences the different movement of labor input. Additionally, due to the rising stock of public capital, marginal product schedules for private labor and capital shift over time, which also stimulate alterations in labor and private capital. We can see that, different than before, real wages rise compared to the steady state and adjust back only very slowly.

In order to analyze changes in the results if the productivity of capital increases, we will have to alter the value for η. Before we have set $\eta = 0.05$. We will now change it to $\eta = 0.2$ (this will have to be done manually). As we know from eq. (6) η is used for the calculation of $\bar{z}$, so we will also have to change $\bar{z}$ to $\bar{z} = 1.1369$ (resulting from eq. (6)). Having incorporated the changes we can analyze the graphical results again.

We can also see from eq. (6) that η only enters the model through K^B. As an unexpected temporary public consumption shock does not have any impact on K^B, changing η does not change any of the results for (i). This can also be seen when looking at the new graphs for the public consumption shock, which are exactly the same as before.
However, we do get different results when looking at the public investment shock.

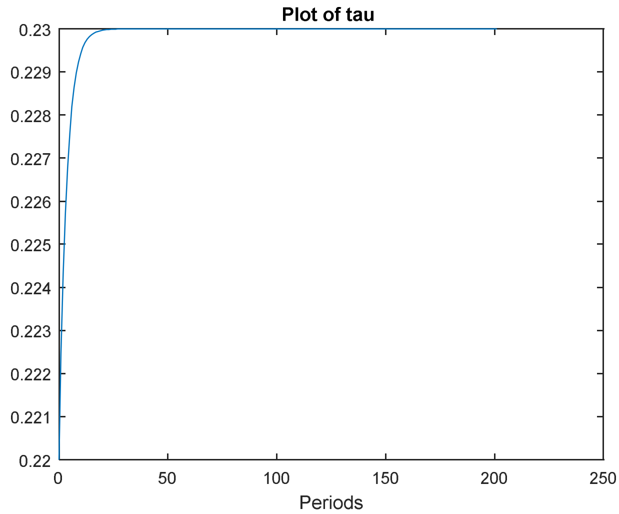


Looking at the graphs we can see that the general movements of the observables have not changed, but that effects (in either directions) are much greater than before (notice the different scaling!). Especially the effect on private consumption and output is noteworthy.

η describes the productivity of public capital. From equation (6) and also from economic intuition we know that the higher the productivity of public capital, the more output can be produced, holding constant the amount of public capital. This explains the much higher increase in dy as a reaction to increasing η.We can also see from the graphs, that due to the increased η there is no crowding out of public consumption. The crowding out of public investment however is greater than before.

(6) Effects of a permanent increase in the tax rate of 1 percentage point

In order to calculate the effect of a permanent increase in the tax rate of 1% we will have to simulate a deterministic shock. For this, we need to calculate the steady state which will be reached after the shock. Knowing that the old steady state for $\tau = 0.22$, a 1% increase in τ means that we will have to reach a new steady state of $\tau = 0.23$. However, in DYNARE we need to enter the value for ε_t^τ in order to calculate the shock correctly. This value can be calculated by using eq (13), so that: $\varepsilon_t^\tau = \log\left(\frac{0.23}{0.22}\right) - 0.75 \log\left(\frac{0.23}{0.22}\right)$. This has been programmed in DYNARE and can be chosen by scenario 2. The command *rplot* then gives us a graph showing the transition path of τ (and other variables) from the old to the new steady state.



First of all it is important to notice that as we are now dealing with a permanent shock, the observables do not converge back to their steady state but rather converge to a new steady state.

Due to the increased tax rates, individuals' incentives to work and to invest decrease, which can also be seen from the graphs for dN and dI. As a result also output and private consumption decrease and converge to a lower new steady state. What we can also see from the graphs is that dTR increases and converges to a higher new steady state. We can see from the household's budget constraint that an increase in transfers has a positive effect on consumption and investment. So this might be the reason why consumption and investment first undershoot their new long run steady state and then increase slightly before reaching the new steady state, which is still lower than before.

(7) Effects of fiscal policy on the economy

As we have seen from exercise 5, fiscal policy does imply a crowding out of public consumption and public investment. In the case of the public consumption shock, I would agree with Willi that the short boost in output is negligible. However looking back at the public investment scenario, we have seen that due to increased public investment, the public capital stock builds up which in the beginning leads to a crowding out of public consumption and investment, however along the path actually increases consumption and investment above their steady state, which is also reflected in output. This effect is bigger, the greater η is. We have seen that with a greater η crowding out of consumption completely vanishes and the increase in output is noteworthy. I would therefore conclude that a temporary investment shock can boost the economy temporarily (obviously only short term, as variables return to steady state) and that the positive impact on the economy is greater, the greater is η. In the case of the permanent increase in the tax rate, we have seen that private consumption and investment is permanently crowded out, resulting in a permanent decrease of output. The economy is therefore not boosted and again we can agree with Willi. Obviously, a decrease in the tax rates would also have the opposite effect, boosting the economy long term.

The difference between public and private capital can again be seen from eq. (6). The impact of private capital on output is restricted by the share of capital in production, which is denoted by α. Government capital however, 'comes on top' and its impact is only dependent on its own productivity. The described interaction of capital and labor can therefore generate a multiplier effect, with government capital this is not possible.

Solution to Exercise 2

(1) Observable variables in a Bayesian estimation

For a Bayesian estimation the number of observable variables has to equal the number of structural shocks. As we have three structural shocks given in this model, we also need three observable variables, namely output growth YGR, inflation $INFL$ and interest rates INT (see An & Schorfheide (2007), p. 124).

(2) Simulation of the data

See scenario 1 in mod-file *An-Schorfheide_404793.mod* and mat-file *simdat.mat*!

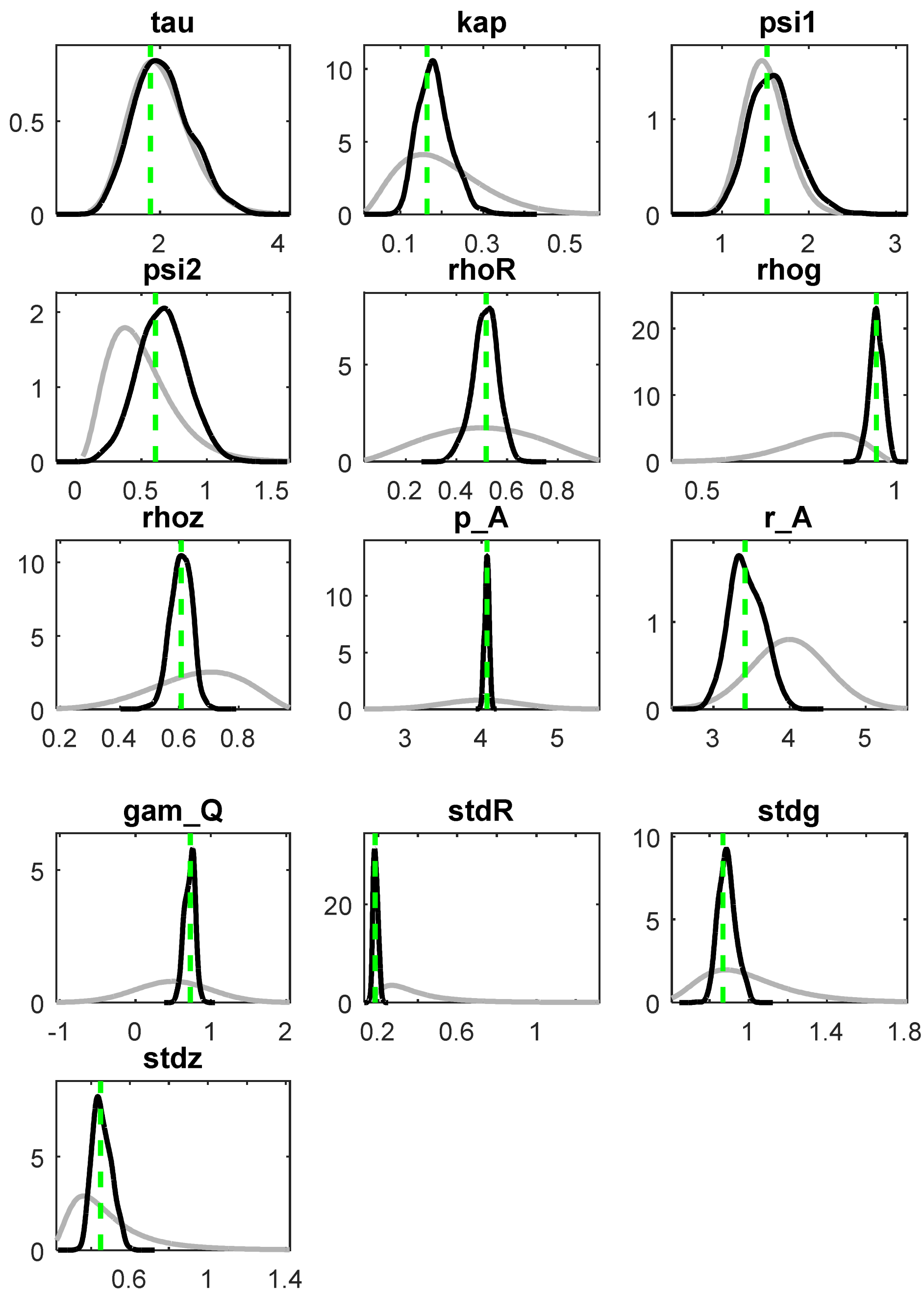
(3) Intuition behind prior information in a Bayesian estimation

Bayesian estimation is basically a combination between parameter calibration and the Maximum-Likelihood method. In Maximum-Likelihood-estimation we aim at finding point estimates for the parameters that maximize the likelihood given some data. The problem is that data is often not sufficiently informative or that DSGE-models are misspecified, which leads to absurd parameter values. With Bayesian methods, known information from simulated data is combined with additional believes about the parameters (*priors*), which are derived from economic theory or empirical considerations. This has the advantage that we are able to put more weight on a suspected span of the parameter space and less weight on "absurd" realizations from the data. In mathematical terms, the estimated parameter vector $\widehat{\theta_B}$ is the maximum of the sum of the likelihood function of parameters given the data and the prior beliefs about the parameter distribution. The parameters can then be estimated using for example the Metropolis-Hastings-algorithm. Having estimated the parameters the resulting posterior distribution can then be compared with the prior assumptions.

(4) Estimation of the model

See scenario 2 in mod-file *An-Schorfheide_404793.mod!*

(5) Assessment of the quality of the Bayesian estimation



The grey line represents the prior distribution and the black line the posterior distribution. For a good estimate, the prior and posterior distributions should not be excessively different. This is a point that needs further attention. Also, the posterior distributions should be close to normal, which here is the case for all parameters. The dotted green line represents the value at the posterior mode. Ideally the mode should be in the centre of the posterior distribution, which again is the case for all parameters.

Looking at the parameters in more detail, we can see that the prior and posterior distribution using the simulated data are quite a good fit for τ, ψ_1 and ψ_2. For all three parameters, the density functions are almost congruent, only for ψ_2 the posterior distribution does not pick up the skewness to the left which can be seen in the prior distribution. From the DYNARE output we can also see that the prior and posterior mean are almost the same and the prior mean also lies in the 90% HDI confidence interval for all three parameters. We can therefore say that the model is well specified in regard to the parameter values of τ, ψ_1 and ψ_2, which are used to simulate the data.

For the parameters κ, ρ_R and $\sigma_z$we can also observe that the prior mean lies in the respective 90% HDI confidence interval, however from the graphical output we can see that both the prior distributions are much wider (displaying a greater parameter variance) than the posterior distributions. Probably due to uncertainty about the parameter value, the prior belief about the distribution was probably not too informative, so that the posterior distribution is to a great extent driven by the data.

For ρ_G, ρ_Z, π^A, r^A, γ^Q, σ_R and σ_G the prior mean does not lie in the 90% HDI confidence interval. We can see that for all of these variables the prior distribution is mostly very flat and wide, meaning that a high variance has been assumed probably again due to high uncertainty about the parameter. Especially for σ_R and π^A the posterior density function are very tight, displaying less variance of possible parameter values, whereas the prior distribution is very wide.

The parameters $\rho_G$and r^A are also special as their density functions clearly lie apart. The prior believes suggest a certain mean for the parameter value, however the posterior distribution clearly deviates from this. What is important to keep in mind in this case is that the posterior distribution already includes the prior assumptions, so the likelihood function must lie even further away. This might be due to absurd parameter estimates that were used to simulate the data. For ρ_G data was simulated for a calibrated value of 0.95, whereas our prior beliefs were such that the mean was 0.8 with a standard deviation of 0.1. Recalibrating the model with respect to $\rho_G$and r^A might therefore yield a better fit of the prior and posterior distribution.

Overall it seems that the quality of the estimation is not too good, as for most parameters, the distributions differ a lot. As already touched on, this might be due to the fact, that the prior distribution is often not very informative – the density functions are flat and display a high variance. It might therefore be helpful to gather more information about the parameters, so that the prior distribution can be given with more confidence. This would result in an estimation that is less data driven and would include more characteristics of the priors.

In general, for interpretation purposes it would also have been helpful if the graphical output by DYNARE would have also shown the likelihood.