Advanced Macroeconomics (PhD) - DSGE methods

Estimation methods

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Full information estimation

Idea

- Full information estimation requires a complete characterization of the data-generating-process (not only specific moments).
- Consider the linear first-order *state-space* representation of the model:

$$z_t = A(\theta)z_{t-1} + B(\theta)\varepsilon_t, \quad \text{with } E[\varepsilon_t] = 0, \quad E[\varepsilon_t\varepsilon_t'] = \Sigma_\varepsilon \tag{1}$$

$$d_t = Dz_t + \mu_t, \quad \text{with } E[\mu_t] = 0, \quad E[\mu_t\mu_t'] = \Sigma_\mu. \tag{2}$$

- $z_t = (\widehat{x}_t', \widehat{y}_t')'$ contains all model variables as deviations from steady-state, and A and B are functions of g_x and h_x.
- Matrix D combines the model variables z_t with observable data variables d_t
- Equation (1): *state-* or *transition-equation*
  - Corresponds to the solution of the model.
  - ε_t are the stochastic innovations.
- Equation (2): *observation-equation*
  - Corresponds to the measurement equations,
  - subject to possible measurement errors μ_t in the data.

Full information estimation

Idea

- Given distributional assumptions about ε_t and μ_t, one can derive the log-likelihood-function, $\log L(d|\theta)$, analytically or numerically.
- In the log-linear case and considering normally distributed variables, the Kalman-filter is used to calculate the likelihood analytically.
- In the nonlinear case the policy functions are functions of the vector of parameters θ. The particle-filter or the *efficient importance sampling* is then used to derive the likelihood numerically.
- There are two approaches for analyzing and evaluating the log-likelihood:
  1. **the classic (frequentist) Maximum-Likelihood-method**,
  2. **the bayesian method**.

Kalman-filter

Notation

We simplify and consider only the linear case and ignore possible measurement errors in the data:

- $d_t = Dz_t$
- $z_{t+1} = Az_t + B\varepsilon_{t+1}$
- $\varepsilon_i \overset{iid}{\sim} \mathcal{N}(0, \Sigma_\varepsilon),\ \Sigma_\varepsilon = E(\varepsilon_i \varepsilon_i'),\ E(\varepsilon_i \varepsilon_j') = 0$

Notation for the linear projection

$$
\begin{aligned}
\widehat{z}_{t|t-j} &= E(z_t | d_{t-j}, d_{t-j-1}, \ldots d_1) \\
\Sigma_{t|t-j} &= E(z_t - \widehat{z}_{t|t-j})(z_t - \widehat{z}_{t|t-j})' \\
\widehat{d}_{t|t-j} &= E(d_t | d_{t-j}, d_{t-j-1}, \ldots, d_1) \\
u_t &= d_t - \widehat{d}_{t|t-1} = D(z_t - \widehat{z}_{t|t-1}) \\
E(u_t u_t') &= D\Sigma_{t|t-1} D'
\end{aligned}
$$

for $t = 1, 2, \ldots, T$ and $j = 0, 1, \ldots T$.

Kalman-filter

Initialization

- Since z_t is covariance-stationary, the variance is given by:

$$\begin{aligned}
\underbrace{E(z_t z_t')}_{\equiv \Sigma_z} &= E\left[(Az_{t-1} + B\varepsilon_t)(Az_{t-1} + B\varepsilon_t)'\right] \\
&= A\underbrace{E(z_{t-1}z_{t-1}')}_{\equiv \Sigma_z}A' + B\underbrace{E(\varepsilon_t\varepsilon_t')}_{=\Sigma_\varepsilon}B' \\
\Leftrightarrow \Sigma_z &= A\Sigma_z A' + B\Sigma_\varepsilon B' \\
\Leftrightarrow vec(\Sigma_z) &= (I - A \otimes A)^{-1} vec(B\Sigma_\varepsilon B')
\end{aligned}$$

Vectorization

The *vec*-operation stacks the rows of a $m \times n$ Matrix M into a $mn \times 1$ vector $vec(M)$. Then for arbitrary Matrices $\underset{m\times n}{A}$, $\underset{n\times p}{B}$ and $\underset{p\times k}{C}$:

$$vec(ABC) = (C' \otimes A)vec(B), \quad \text{with } \otimes : \text{Kronecker-product.}$$

Kalman-filter

Initialization

The unconditional expectation of z_1 is used for the initialization of the Kalman-filter, since there is no additional information yet:

$$\widehat{z}_1 = \underbrace{E(z_1)}_{=E(z)} = A\underbrace{E(z_0)}_{=E(z)} + B\underbrace{E(\varepsilon_1)}_{=0} \Leftrightarrow \widehat{z}_1 = 0,$$

$$vec(\Sigma_{1|0}) = E(z_1 - 0)(z_1 - 0)' = vec(\Sigma_z) = (I - A \otimes A)^{-1} vec(B\Sigma_\varepsilon B').$$

Kalman-filter

Recursion

The recursion is then given by:

$$\widehat{z}_{t+1|t} = A\widehat{z}_{t|t}$$

Formula for updating a linear projection (Hamilton (1994, S.99 und S.379))

$$\begin{aligned}
\widehat{z}_{t|t} &= \widehat{z}_{t|t-1} + \left[E(z_t - \widehat{z}_{t|t-1})(d_t - \widehat{d}_{t|t-1})'\right]\left[E(d_t - \widehat{d}_{t|t-1})(d_t - \widehat{d}_{t|t-1})'\right]^{-1} u_t \\
\Leftrightarrow \widehat{z}_{t|t} &= \widehat{z}_{t|t-1} + \Sigma_{t|t-1}D'\left(D\Sigma_{t|t-1}D'\right)^{-1} u_t \\
\Rightarrow \widehat{z}_{t+1|t} = A\widehat{z}_{t|t} &= A\widehat{z}_{t|t-1} + A\Sigma_{t|t-1}D'\left(D\Sigma_{t|t-1}D'\right)^{-1} u_t,
\end{aligned}$$

with $u_t = d_t - \widehat{d}_{t|t-1} = (d_t - D\widehat{z}_{t|t-1})$.

Kalman-filter

Recursion

- $z_{t+1} - \widehat{z}_{t+1|t} = A\left(z_t - \widehat{z}_{t|t-1}\right) + B\varepsilon_{t+1} - A\Sigma_{t|t-1}D'\left(D\Sigma_{t|t-1}D'\right)^{-1} u_t$
- The *MSE:* $\Sigma_{t+1|t} = E\left(z_{t+1} - \widehat{z}_{t+1|t}\right)\left(z_{t+1} - \widehat{z}_{t+1|t}\right)'$ is given by:

$$\Sigma_{t+1|t} =$$
$$A\Sigma_{t|t-1}A' + B\Sigma_\varepsilon B' - A\Sigma_{t|t-1}D'\underbrace{\left(D\Sigma_{t|t-1}D'\right)^{-1}\underbrace{E(u_t u_t')}_{=D\Sigma_{t|t-1}D'}\left(D\Sigma_{t|t-1}D'\right)^{-1}}_{=I} D\Sigma_{t|t-1}A'$$

Mean-Sqared-Error (MSE)

$$\Sigma_{t+1} \equiv \Sigma_{t+1|t} = A\Sigma_{t|t-1}A' + B\Sigma_\varepsilon B' - A\Sigma_{t|t-1}D'\left(D\Sigma_{t|t-1}D'\right)^{-1} D\Sigma_{t|t-1}A'$$

Kalman-filter

Summary

The Kalman-filter can be summarized as follows:

1. Initialization with
   - $\widehat{z}_1 = 0$,
   - $vec(\Sigma_{1|0}) = (I - A \otimes A)^{-1} vec(B\Sigma_\varepsilon B')$.
2. Period-t likelihood function
   - $u_t = (d_t - D\widehat{z}_{t|t-1})$
   - $d_{t|t-1} = D\widehat{z}_{t|t-1}$
   - $\Omega_{t|t-1} := E(u_t u_t') = D\Sigma_{t|t-1}D$
3. Period-t filtering density
   - $\widehat{z}_{t|t} = \widehat{z}_{t|t-1} + \Sigma_{t|t-1}D' \left(D\Sigma_{t|t-1}D'\right)^{-1} u_t$
   - $\Sigma_{t|t} = \Sigma_{t|t-1} - \Sigma_{t|t-1}D' \left(D\Sigma_{t|t-1}D'\right)^{-1} D\Sigma_{t|t-1}$
4. Period-t predictive density
   - $\widehat{z}_{t+1|t} = A\widehat{z}_{t|t-1} + A\Sigma_{t|t-1}D' \left(D\Sigma_{t|t-1}D'\right)^{-1} u_t$
   - $\Sigma_{t+1} = A\Sigma_t A' + B\Sigma_\varepsilon B' - K_t D\Sigma_t A'$.

Log-Likelihood

Given the gaussian assumption about the forecast error u_t one can derive the distribution of the data $\underset{n\times 1}{d_t}$ conditional on $(z_t, d_{t-1}, d_{t-2}, \dots)$ and set up the log-likelihood function:

Log-likelihood

$$\begin{aligned}\log \mathcal{L}(d|\theta) &= \sum_{t=1}^{T} \log \mathcal{L}(d_t|\theta) \\ &= -\frac{nT}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|\Omega_t| - \frac{1}{2}\sum_{t=1}^{T} u_t{}'\Omega_t{}^{-1}u_t.\end{aligned}$$

Maximum-Likelihood

Idea

- **Approach:** The parameters θ are fixed and the data is a random realization of this specific parametrization.
- The *Maximum-Likelihood*-estimator $\widehat{\mu}_{ML}$ is then defined as

$$\widehat{\theta}_{ML} = \underset{\theta}{\operatorname{argmax}} \left\{ \sum_{t=1}^{T} \log \mathcal{L}(d_t|\theta) \right\}.$$

- Given some regularity conditions the *ML*-estimator is consistent, asymptotically efficient and asymptotically gaussian.
- Uncertainty and inference are based upon the assumptions that **to each realization of data there corresponds a different vector of parameters that maximizes the likelihood**.
- Hint for the estimation of the parameters of a DSGE-model:
  - The dimension of d_t must be greater or equal to the dimension of the structural shocks ε_t, or otherwise the residual term has a singular variance-covariance-matrix.
  - If not: Add measurement errors or additional shocks.

Maximum-Likelihood

Discussion

- Experience shows that it can be pretty hard and tricky to estimate a DSGE model via Maximum-Likelihood.
- Data is often not sufficiently informative, i.e. the likelihood is flat in some directions (identification).
- DSGE-models are always misspecified. This can lead to absurd parameter values.

Bayesian methods

Idea

- Based upon the likelihood as well: the complete characterization of the data generating process.
- **Approach:** The parameters θ are random and data d is fixed.
- The idea is to combine known information (data) with additional believes (*prior-believes*) about the parameters and to get an expression for the conditional probability of the parameters.
- Hence, one is able to put more weight on a suspected span of the parameter space.
- Bayesian methods are a bridge between calibration and the *Maximum-Likelihood*-method:

"Bayesian Inference is a Way of Thinking, Not a Basket of Methods (Christopher Sims)"

Bayesian methods

Idea

- Likelihood-function $\mathcal{L}(d|\theta)$ is a conditional density of observed data given the parameters: $\wp(d|\theta) = \mathcal{L}(d|\theta)$.
- Denote $\wp(\theta)$ as the known prior density of the vector of parameters, then using Bayes-rule:

$$\wp(\theta|d) = \frac{\mathcal{L}(d|\theta)\wp(\theta)}{\wp(d)} = \frac{\mathcal{L}(d|\theta)\wp(\theta)}{\int \wp(\theta)\mathcal{L}(d|\theta)\ d\theta} \propto \mathcal{L}(d|\theta)\wp(\theta),$$

  with $\propto$ meaning "proportional to".
- $\wp(d)$ is the *marginal likelihood* of the data and ultimately only a constant that normalizes the expression to unity. It is independent of the parameters.
- Removing it doesn't change the form of the posterior density $\wp(\theta|d)$, it merely doesn't integrate to one.
- This non-normalized density is called *posterior-kernel* or, in logs, *log-posterior-kernel*.

Bayesian methods

Idea

- The mode is the Bayesian estimator $\widehat{\theta}_B$ of the true parameter vector:

$$\widehat{\theta}_B = \underset{\theta}{\text{argmax}} \{\log \wp(\theta|d)\} = \underset{\theta}{\text{argmax}} \{\log \mathcal{L}(d|\theta) + \log \wp(\theta)\}$$

- Procedure: Calculate the log-likelihood with the Kalman-filter and simulate the *log-posterior-kernel* through *sampling*- or *Monte-Carlo*-methods.
- In the literature – and in Dynare – the *Metropolis-Hastings-algorithm* is commonly used.
- Inference can then be conducted via the properties of the posterior-distribution.

An and Schorfheide (2007, S. 132)

The algorithm constructs a Gaussian approximation around the posterior mode and uses a scaled version of the asymptotic covariance matrix as the covariance matrix for the proposal distribution. This allows for an efficient exploration of the posterior distribution at least in the neighborhood of the mode.

- The algorithm uses the fact that under very general regularity conditions the moments of a distribution are asymptotically normal.
- It constructs a sequence of draws (Markov-chains) from a proposal density.
- This does not need to be identical with the posterior density. It is only required that the algorithm can draw samples from the whole range of the posterior density.

Bayesian methods

Metropolis-Hastings-algorithm

- The current candidate (draw) θ^* is dependent on the previous candidate $\theta^{(s-1)}$.
- Weights for all candidates are the same, however, they are only accepted with a certain probability α, calculated as the ratio of the *posterior-kernel* of the current to the one of the previous candidate.
- Due to this construct the algorithm tends to shift the draws from areas of low posterior probability to areas of high probability.
  - If $\theta^{(s-1)}$ is in an area of high posterior probability, it is likely that only candidates in the same area are accepted.
  - If $\theta^{(s-1)}$ is in an area of low posterior probability, it is very likely that new candidates are accepted.
- The covariance-matrix of the proposal distribution plays a major role, since it is important to set α neither too large nor to small.
- Current practice uses the covariance matrix of the mode $\widehat{\theta}_B$ and scales it with a factor c such that the average acceptance probability is between 20% and 30%.

Bayesian methods

Metropolis-Hastings-algorithm

1. Specify c_0, c and S.
2. Maximize $\log \mathcal{L}(d|\theta) + \log \wp(\theta)$ using numerical methods. $\widehat{\theta}_B$ denotes the mode.
3. Calculate the inverse of the Hessian evaluated at the mode, denote it with Σ_B.
4. Specify an initial value $\theta^{(0)}$ or draw it from $\mathcal{N}(\widehat{\theta}_B, c_0^2 \Sigma_B)$.

5. For $s = 1, \ldots, S$:
   - Draw θ^* from the candidate-generating distribution (proposal density) $\mathcal{N}(\mu^{(s-1)}, c^2 \Sigma_B)$.
   - Calculate the acceptance probability α:

$$\alpha \equiv \alpha\left(\theta^{(s-1)}, \theta^*\right) = \frac{\mathcal{L}\left(\theta^* | d\right) \ \wp\left(\theta^*\right)}{\mathcal{L}\left(\theta^{(s-1)} | d\right) \ \wp\left(\theta^{(s-1)}\right)}$$

   - With probability $\min\{\alpha, 1\}$ accept the jump from $\theta^{(s-1)}$ to θ^*. In other words: If $\alpha \geq 1$, set $\theta^{(s)} = \theta^*$.
   - With complementary probability don't accept the jump, i.e. draw a uniformly distributed random variable r between 0 and 1:
     - If $r \leq \alpha$ set $\theta^{(s)} = \theta^*$ (jump).
     - If $r > \alpha$ set $\theta^{(s)} = \theta^{(s-1)}$ (don't jump).

6. Estimate the posterior expectation of a function $\hbar(\theta)$ with $\frac{1}{S}\sum_{s=1}^{S}\hbar\left(\theta^{(s)}\right)$.
7. If the average acceptance probability does not yield a desirable value (typically between $20\% - 30\%$) or the algorithm does not converge, change c_0, c or S.

Bayesian methods

Remarks

- Bayesian estimation of a DSGE-model requires that the number of shocks is equivalent to the numbers of observable variables.
- Common choice for priors: gaussian, (normal, shifted or inverse) Gamma, Beta or the uniform distribution.
- Choosing a proper prior one has to consider lower and upper bounds as well as the skewness and kurtosis of the distribution.
- The results can vary due to the choice of priors and their parametrization.
- Therefore one has to check the robustness of the results:
  - Try a different parametrization.
  - Try more general priors.
  - Noninformative priors.
  - Sensitivity analysis.

Properties of the Posterior-distribution

- The posterior density combines all information about θ: information after the data is observed as well as information prior to the data.
- Bayesian estimation works for every sample size, however, it has also the following asymptotic properties:
  1. The priors become irrelevant for the determination of the posterior.
  2. The posterior converges to a degenerate distribution around the true value.
  3. The posterior is approximately gaussian.
- Using the posterior distribution one can
  - set up Bayesian confidence intervals (credibility sets),
  - calculate forecasts using the predictive-density: $\mathcal{L}(d_f|d) = \int \mathcal{L}(d_f|(\theta|d))d\theta = \int \mathcal{L}(d_f|\theta, d)\wp(\theta|d)d\theta$
  - compare models.

Model comparison

- Models can differ in their prior distribution, the likelihood and the parameters.
- Bayesian approach: Calculate the probability that model i is the true model, given the data.
- Suppose there are $i = 1, 2$ models M_i with prior probability $p_i = P(M_i)$ that model M_i is the true model.
- Each model has a set of parameters θ_i with a prior distribution $\wp_i(\theta_i)$ and a likelihood $\mathcal{L}_i(d|\theta)$.
- Then the probability of model 1 being the true model given the data, is given by:

$$P(M_1|d) = \frac{P(M_1)\mathcal{L}_1(d|M_1)}{\mathcal{L}(d)} = \frac{p_1 \int \mathcal{L}_1(d, \theta_1|M_1)d\theta_1}{\mathcal{L}(d)}$$
$$= \frac{p_1 \int \mathcal{L}_1(d|\theta_1, M_1)\wp_1(\theta_1|M_1)d\theta_1}{\mathcal{L}(d)}$$
$$\text{with } \mathcal{L}(d) = p_1 \int \mathcal{L}_1(d|\theta_1, M_1)\wp_1(\theta_1|M_1)d\theta_1 + p_2 \int \mathcal{L}_2(d|\theta_2, M_2)\wp_2(\theta_2|M_2)d\theta_2$$

Model comparison

- The expected value of the likelihood given the prior distribution is the so-called *marginal-likelihood* for model i:

$$m_i(d) = \int \mathcal{L}_i(d|\theta_i, M_i)\wp_i(\theta_i|M_i)d\theta_i$$

- Using this, one can calculate the *posterior-odds*:

$$PO_{12} = \frac{P(M_1|d)}{P(M_2|d)} = \underbrace{\frac{p_1}{p_2}}_{\text{Prior-Odds-Ratio}} \cdot \underbrace{\frac{m_1(d)}{m_2(d)}}_{Bayes-factor}$$

- Together with $P(M_1|d) + P(M_2|d) = 1$ one gets the *posterior-model-probabilities*:

$$P(M_1|d) = \frac{PO_{12}}{1 + PO_{12}}, \qquad P(M_2|d) = 1 - P(M_1|d).$$

Model comparison

- The *marginal likelihood* measures the quality of a model to characterize data.
- The *Posterior-Odds* don't hint to the true model. They solely describe which model, compared to the other, has the highest conditional probability.
- A $PO_{12} >> 1$ is an indication that the data as well as the priors prefer model 1.
- Guidelines of Jeffrey (1961):
  - $1:1-3:1$ weak evidence for model 1,
  - $10:1-100:1$ strong evidence for model 1,
  - $>100:1$ decisive evidence for model 1.
- Implementation and calculation of the integrals is done by numerical MCMC- and sampling-methods, as well as Laplace- or Harmonic-Mean-approximation.

Identification

General problem

For a mathematical expression with many conditions and parameters, but only a limited sample, there can exist different combinations of parameters that yield the same result and a similar dataset.

- Consider two vectors of parameters θ_1 and θ_2 for which

$$\mathcal{L}(d|\theta_1) = \mathcal{L}(d|\theta_2).$$

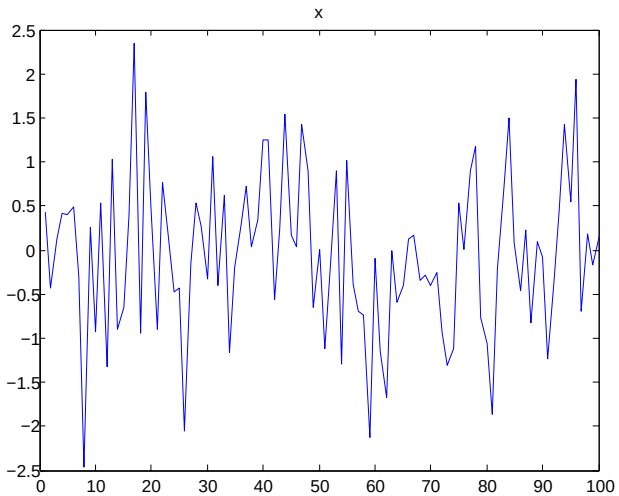
- If $\theta_1 = \theta_2$, then there is identification. If, however, $\theta_1 \neq \theta_2$, then one does not know which vector of parameters has generated the data.

Identification

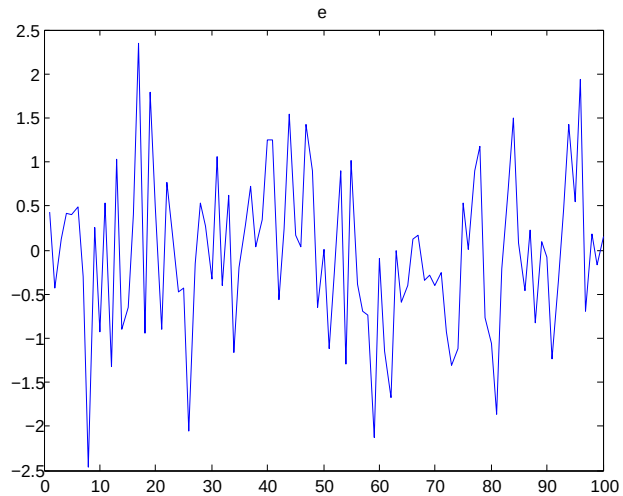
- Simple example: Consider the following ARMA(1,1)-process

$$x_t = \phi_1 x_{t-1} + \varepsilon_t - \phi_2 \varepsilon_{t-1}, \text{ with } \varepsilon \sim N(0, \sigma^2)$$

with parameter vector $\theta = (\phi_1, \phi_2, \sigma)'$:



(a) $\theta_0 = (0.4, 0.4, 1)'$



(b) $\theta_1 = (0, 0, 1)'$

- Obviously both models generate the same data (as long as the shocks ε_t are the same). θ_0 and θ_1 are observationally equivalent.
- Note $\sigma = 1$ is **partially identifiable**.

Identification

- Identification is a mathematical problem (dependencies).
- Identification problems arise if distinct parameter values do not lead to distinct probability distributions of the data.
- Drawing inferences from the probability distribution leads thus to wrong conclusions from estimation and inference.
- When identification fails, properties of estimators change.
- Even with an infinite sample it is not possible to pin down some parameters, no matter what estimation procedure one uses.
- Identification can be studied prior to estimation.
- Identification tests: Order and rank conditions, via autocovariances, spectral densities, information matrix, imposing restrictions, Bayesian methods ...

Discussion of full information estimators

- More restrictive assumptions are needed compared to the limited information estimation: specification of the distribution of the schocks, i.e. the likelihood.
- Advantages of a *Maximum-Likelihood*-estimation lie in the full characterization of the data-generating-process and the exact, consistent and efficient estimation of the parameters.
- "Dilemma of absurd parameter estimates": Problem of the ML-estimation due to wrong distributional assumptions, problems in the optimization algorithm or non-separable identifiable parameters.
- Even transformations, upper and lower bounds, etc. are only limited to help overcome this problem, when the likelihood is flat.

Discussion of full information estimators

- This is where Bayesian methods come in and bridge the gap between calibration and the *ML-principle*.
- Considering priors one can incorporate additional information into a model.
- "Dilemma of absurd parameter estimates": Even with Bayesian means it is not possible to estimate these parameters (the posterior looks almost the same as the prior), but one can assign probability such that these parameters are very unlikely.

$\Rightarrow$ Using priors one can exclude these absurd parameter estimates.

- Nevertheless the point of robustness and identification of the parameters remains a critical topic.

Discussion of full information estimators

An und Schorfheide (2006, S.124)

Once one acknowledges that the DSGE model provides merely an approximation to the law of motion of the time series (...), then it seems reasonable to assume that there need not exist a single parameter vector (...), that delivers, say, the "true" intertemporal substitution elasticity or price adjustment costs and, simultaneously, the most precise impulse responses to a technology or monetary policy shock. Each estimation method is associated with a particular measure of discrepancy between the "true" law of motion and the class of approximating models.

Exercise: Estimation with Bayesian methods

Consider the following simplified RBC-model (social planer problem);

$$\max_{\{c_{t+j},l_{t+j},k_{t+j}\}_{j=0}^{\infty}} W_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j})$$

$$\begin{aligned} s.t. \quad y_t &= c_t + i_t, & A_t &= Ae^{a_t}, \\ y_t &= A_t f(k_{t-1}, l_t), & a_t &= \rho a_{t-1} + \varepsilon_t, \\ k_t &= i_t + (1-\delta)k_{t-1}, & \varepsilon_t &\sim N(0, \sigma_\varepsilon^2), \end{aligned}$$

where preferences and technology follow:

$$u(c_t, l_t) = \frac{\left[c_t^\theta (1-l_t)^{1-\theta}\right]^{1-\tau}}{1-\tau}, \qquad f(k_{t-1}, l_t) = \left[\alpha k_{t-1}^\psi + (1-\alpha) l_t^\psi\right]^{1/\psi}.$$

Optimality is given by:

$$\begin{aligned} u_c(c_t, l_t) - \beta E_t \left\{u_c(c_{t+1}, l_{t+1}) \left[A_{t+1} f_k(k_t, l_{t+1}) + 1 - \delta\right]\right\} &= 0, \\ -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} - A_t f_l(k_{t-1}, l_t) &= 0, \\ c_t + k_t - A_t f(k_{t-1}, l_t) - (1-\delta)k_{t-1} &= 0. \end{aligned}$$

Exercise: Estimation with Bayesian methods

(a) Write a mod-file for this model (with a sensible calibration and a steady-state block).

(b) Simulate a sample of 10000 observations for c_t, l_t and y_t using `stoch_simul` and save it in a mat-file.

(c) Define priors for α, θ and τ (or a different set of parameters).

(d) Estimate the posterior mode using the `estimation` command and a limited sample with 100 observations. How man observable variables do you need? Check the posterior mode using `mode_check`. If you get errors due to a non-positive definite Hessian, try a different optimization algorithm or change the initial values.

Exercise: Estimation with Bayesian methods

(e) If you are satisfied with the posterior mode, approximate the posterior distribution using the the Metropolis- Hastings-Algorithmus with 3×5000 iterations. If it does not converge to the (ergodic) posterior-distribution, repeat the algorithm without discarding the previous draws.

(f) How robust are the results regarding the specification of the priors? Repeat the estimation of the posterior-mode for different priors.