Macroeconomics PhD Problem Set 2

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Exercise 1

1. Endogenous and exogenous variables and model parameters

Endogenous variables: C_t, N_t, G_t^B, I_t^B, w_t, r_t, K_t, TR_t, τ_t, λ_t, K_t^B, I_t, Y_t, z_t, dY_t, dC_t, dI_t, dN_t, dw_t, dr_t, dTR_t, dG_t^B, dI_t^B.
Exogenous variables: ϵ_t^z, $\epsilon_t^{G^B}$, $\epsilon_t^{I^B}$, ϵ_t^τ.
Model parameters: β, θ_l, δ, η, α, ρ_z, ρ_{G^B}, ρ_{I^B}, ρ_τ, σ_z, σ_{G^B}, σ_{I^B}, σ_τ, $\bar{Y}$, $\bar{G}^B$, $\bar{I}^B$, $\bar{K}$, $\bar{K}^B$, $\bar{N}$, $\bar{w}$, $\bar{r}$, $\bar{TR}$, $\bar{I}$, $\bar{C}$, $\bar{\tau}$, $\bar{z}$, $\bar{\lambda}$.

2. Calibration of the model

I calibrate the model using the following equations:

$$\bar{K}^B = \frac{\bar{I}}{\delta} = 0.2$$
$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{Y}} = 0.22$$
$$\alpha = 1 - \frac{\bar{w} \cdot \bar{N}}{\bar{Y}} = 0.3333$$
$$\bar{r} = \frac{\frac{1}{\beta} - (1-\delta)}{1-\bar{\tau}} = 0.1412$$
$$\bar{K} = \frac{\alpha \cdot \bar{Y}}{\bar{r}} = 2.3615$$
$$\bar{z} = \frac{\bar{Y}}{(\bar{K}^B)^\eta \cdot \bar{K}^\alpha \cdot \bar{N}^{1-\alpha}} = 1.6929$$
$$\bar{I} = \bar{K} \cdot \delta = 0.2361$$
$$\bar{C} = \bar{Y} - \bar{I} - \bar{I}^B - \bar{G}^B = 0.5439$$
$$\bar{\lambda} = \frac{1}{\bar{\tau}} = 1.8387$$
$$\theta_l = (1-\bar{\tau}) \cdot \bar{w} \cdot \frac{1-\bar{N}}{\bar{C}} = 1.9123$$

For increased η only the value of $\bar{z}$ changes.

3. Deterministic vs. stochastic model

Deterministic models are mainly used to analyze impacts of regime changes in DSGE models. Agents are assumed to have full information and perfect foresight and there is no uncertainty around shocks. Shocks can either be modeled as unexpected shocks hitting the economy today or as shocks expected with perfect foresight, that are modeled as shocks in the future.

For a model solution numerical simulation is applied to fit the model equations of the endogenous variables as well as the first order conditions and shock structures. Steady state and linearization aren't needed and therefore the method is also applicable as the economy is far away from the steady state. Stochastic models are often used for solving RBC models or new keynesian monetary models. Shocks are unexpected and hit today with zero expectation afterwards. A linearization is needed for the solution, which is why future shocks or permanent changes in exogenous variables cannot be calculated. Stochastic models are not deterministic but agents behave according to the certainty equivalence property as if future shocks were equal to zero.

4. mod-file

See bk.mod.

5. An unexpected shock to public consumption and public investment

Figure 1 shows the effect of an unexpected temporary decrease in public consumption. The decrease in public consumption triggers several temporary effects. In the short run on the one hand there is a sharp fall in output, labor supply and the real interest rate and on the other hand there occurs to be an increase in private investment and consumption, real lump-sum transfers and the real wage rate. All variables eventually return to their steady state values. Output, private investments and transfers show a little faster recovery than the other variables while output and investment slightly overshoot their steady state value.

The decrease in public consumption has a direct effect on the governmental budget constraint. Since the government is spending less, there is more money to spend for transfers as investments are held constant. However, the decrease in public consumption requires output to fall since markets have to clear. With higher income due to transfers private households reoptimize. They can effort to raise consumption and investment and at the same time reduce labor supply. Since the fall in output is larger than the fall in labor supply the real wage rate rises. The stock of capital is constant for the previous period and increases afterwards due to increased private consumption. This leads to a decrease in the real interest rate as output falls. Due to the lower drop in output compared to public consumption, there is a gradual recovery because the fiscal authority faces a lower income due to a lower tax base. This is followed by reverse adjustments in public consumption and transfers triggering adverse effects that push the economy back to its steady state.

Figure 2 shows the effect of an unexpected decrease in public investment. Compared to figure 1 it can be seen easily that a drop in public investment has a longer lasting impact on most variables. Nevertheless the economy does also move back to its steady state value and there is no long run effect on the real economy. Obviously the effects in the very short run are comparable to effects in figure 1. Equation 5 reveals where the differences come from. The drop in public investment has a negative effect on the public capital stock, one of the input factors for production that have a lagged influence on production and therefore output. So these effects materialize with a lag of one period. The economy's capital stock now decreases since the fall in public investment is larger than the increase in private investments. The drop in output is amplified by a decrease in production. Since there is less demand for labor the wage rate falls, too. Households now face a lower income and decrease investment and consumption, again triggering negative effects on aggregate demand and on future production. Like for the decrease in public consumption the fiscal authority faces a lower tax base and lowers transfers relative to investments. Since the initial

disturbances are stronger and have a negative impact on both the private and the public stock of capital it takes longer for the economy to recover. A comparison of figure 1 and 3 reveals that the effects of a shock in public consumption do not depend on the productivity of public capital. This makes sense, since public capital remains unaffected. For a shock to public investment the effects still are the same qualitatively. But as figure 4 shows the negative lagged effects triggered by the decrease in the public stock of capital have a larger impact. However, the time needed for the economy to recover seems to remain the same.

6. A permanent increase in the tax rate

The effects of a permanent increase in the tax rate on income from labor and capital are shown by figure 5. In the long run output, private consumption and investment, labor supply and the real wage rate decrease and the real interest rate increases. Table 1 shows a comparison of the initial steady state and the new one. Output and the private stock of capital respond strongest to the increased tax rate. All variables show less reaction in percentage terms than the tax rate. This is due to the increase in transfers that partially offsets the negative impacts. Overall the tax rate hike leads to a lower steady state and seems to have unfavorable effects on the economy. As the tax rate increases the fiscal authority is able to pay higher transfers. However, since the tax base decreases transfers show only a minor increase that is not able to offset the effects of the tax rate hike. Households income is lowered and due to the higher tax rate on income from capital and labor, households have less incentives to work. Labor supply decreases leading to a slight increase of the real wage rate in the short term. Households give more weight to consumption than to investments while both decrease, as figure 5 shows that the drop in investment is much sharper than the drop in consumption. With a time lag of one period the negative effects of a decreasing stock of private capital lead to lower production and less labor demand triggering a further drop in income and therefore private consumption, investment, the real wage rate and output. The real interest rate rises because the fall in the private capital stock is stronger than the fall in output. The increased interest rate eventually leads to a slight adjustment in investments of private households accompanied by a slight increase in labor supply. The overall negative effect is mainly due to the drop in investments, as in exercise 1.5 for the shock to public investments. Any changes to the fiscal authorities budget constraint due to public consumption, investment and the tax rate are balanced by adjustments of transfer payments. The negative effects could better be offset by a permanent increase in public investment instead of transfers while the success would depend on the productivity of public capital, as seen in exercise 1.5.

7. Crowding-out and the difference between private and government capital

The results of exercise 1.5 show on first sight effects comparable to a crowding-in effect on private consumption and investment. So for adverse shocks there should be something like a crowding-out effect. The initial reduction in either public consumption or public investment trigger an increase in private consumption and investment. This effect is however not induced by the decreasing interest rate, as mostly assumed for crowding-out or crowding-in effects, but by an increase in transfer payments. There is no possibility for debt in the model, which is why a decrease in interest rates does even lower the incentives for investments since the income from capital decreases. In this model fiscal policy would be able to boost the economy in the short run, since consumption and investments respond less than proportional to changes in public consumption or investment.

Private capital is held by private agents for generating returns by use in the production process. The stock of private capital emerges due to the investments of private agents. Public capital is provided by the government or public agents to enhance the productivity of private capital. That includes for example infrastructure, public administration, education and health care. An increase in public capital can possibly increase the marginal product of private capital. As seen in exercise 1.5 public investments have an effect on output and a permanent increase in public investment could possibly boost the economy. An increasing public stock of capital can improve economic growth depending on the productivity of public capital, although the productivity of public capital is often assumed to be much lower than private capital productivity.