# PROBLEM SET No. 4

DSGE methods

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# 1 Dynare

#### 1.1 Exercise 1.1)

Replace the time series representation of at with  $a_t = \rho a_{t-1} + \epsilon_t^a$ . Consider the response of the economy to a technology shock  $\epsilon_t^a$ . Does the economy over- or underrespond to the shock relative to its natural response? How does this compare to the unit root case? Give also economic intuition for the response. Hint: Use the Matlab function plots m to get some prettier plots (put everything in one folder and write plots as the last command of your mod file.).

As one may see in the corresponding figure the natural real rate exceeds the acutal real rate (figure 1). So the policy rule does not provide the matching interest rate that the new productivity level of the economy requires. Due to a relatively low real rate the housholds demand actually more goods than the natural output would indicate (positive output gap). Since prices are sticky the firms have an incentive to produce an additional amount of goods. That means the economy over-responds to a technology shock. In contrast to the unit-root case the output declines after the initial shock. The natural output level comes back to its initial level - No permanent effect like in the unit-root case (see figure 2).

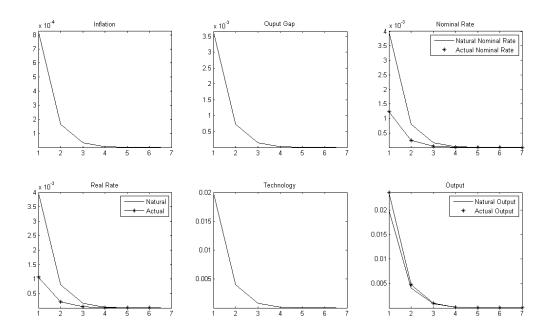


Abbildung 1: A transitory technological shock in the CGG-model(without unit-root)

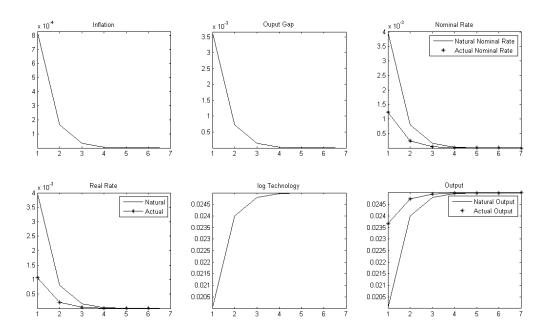


Abbildung 2: A permanent technological shock in the CGG-model (with unit-root)

# 1.2 Exercise 1.2)

Now assume that agents have advance information (news) about the future realization of the technology shock, i.e.

$$a_t = \rho a_{t-1} + \xi_t^0 + \xi_{t-1}^1$$

where  $\xi_t^0$  and  $\xi_t^1$  are both i.i.d. In economic terms agents see  $\xi_t^0$  at time t and they see  $\xi_{t-1}^1$  at time t-1. Introduce this into your code and consider a news shock to  $\xi_t^1$ . What happens with inflation and the output gap? Provide intuition behind this apparently contradictory result.

Figure 3 shows us that the inflation rate jumps to a positive level from which it decreases over time until it fades out. The output gap whereas has its peak in the second period. From t > 2 the output gap closes over time. To give an economic intuition: Due to the future technological shock the housholds anticipate a temporary income increase. According to their Euler equation they smooth their intertemporal consumption which leads to a higher demand in period 1. In period 2 the shock establishes and the productivity of the economy rises. So the natural real rate ist above the actual real rate (see figure 4). This fact explains the peak of the output gap at t = 2. The movement of the inflation rate is driven by the Calvo pricing equation. In period 2 the expected prices decline more rapidly than the short-term growth of the output gap. This explains the continuous declining inflation rate curve.

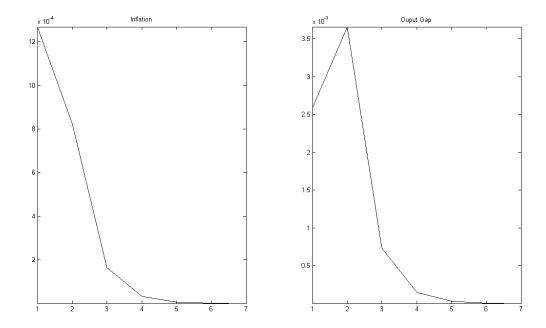


Abbildung 3: Inflation rate and output gap response to a news shock

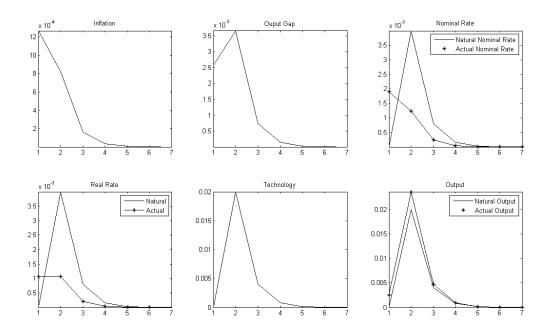


Abbildung 4: Response of the economy to an news shock

# 1.3 Exercise 1.3)

What happens with the response of the economy due to a news shock relative to its natural response, if the natural rate of interest is introduced into the policy rule, i.e.

$$r_t = r_t^* + \alpha(r_{t-1} - r_{t-1}^*) + (1 - \alpha)[\phi_{\pi}\pi_t + \phi_x x_t)]$$

Assuming an identical news shock like above, one may recognize that the monetary policy rule matches perfectly the natural and actual real rate. Because of that the inflation rate and the output gap is constantly zero (see figure 5).

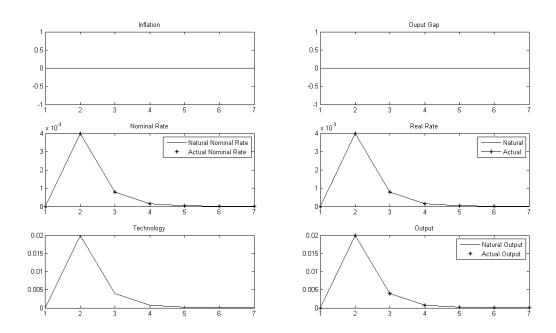


Abbildung 5: A shock with perfectly matching real and natural rate

#### 2 Solution methods

#### 2.1 Exercise 2.1)

In the notation of the lecture, which variable is the state  $(x_t)$  and which the control  $(y_t)$  variable?

The dividends  $d_t$  are generated by an AR(1)-process and so they are dependent on stochastic shocks. Therefore the households cannot fully control the variable. In terms of the lecture  $d_t$  matches with the state variable  $(x_t)$ . From their income the households control the intertemporal consumption. The intertemporal shift of income is done by purchasing assets. Therefore the price of the assets  $p_t$  corresponds with the control variable  $(y_t)$ .

# 2.2 Exercise 2.2)

Derive the decision rule for the state variable in the form of  $x_{t+1} = \overline{x} + h(x_t, \sigma) + \sigma \epsilon_{t+1}$ 

$$d_{t+1} = \overline{d} + h(d_t, \sigma) + \sigma \epsilon_{t+1} = \rho d_t + e_{t+1} \Longrightarrow d_{t+1} - \overline{d} = h(d_t, \sigma) + \sigma \epsilon_{t+1} = \rho d_t - \overline{d} + e_{t+1}$$

In this model  $h(d_t, \sigma) = \rho d_t - \overline{d}$ . Within this model  $\sigma$  reflects a measure of uncertainty. Here the model only contains one stochastic exogenous shock  $e_t$ . Hence,  $\sigma$  is equal to  $\sigma_e$  the standard deviation of  $e_t$  which is the stochastic component of the model.

# 2.3 Exercise 2.3)

Derive the decision rule for the control variable up to the second order.

 $F(x,\sigma)$  in general looks like:  $E_t f(h(x_t,\sigma) + \sigma \epsilon t + 1, g(h(x_t,\sigma) + \sigma \epsilon t + 1, \sigma), x_t, y_t)$ Rewrite  $h(x_t)$  as  $h(d_t = \rho d_t - \overline{d})$  and  $g(x_t)$  as  $g(p_t) = p(d,\sigma)$ , we may come up with the model:

$$E_t[u'(d) \cdot p(d,\sigma) - \beta \cdot u'(\rho d + \sigma \epsilon) \cdot (p(\rho d + \sigma \epsilon, \sigma) + \rho d + \sigma \epsilon)] = 0$$

Inserting the steady-state values  $(\bar{d}, \bar{p}, \sigma = 0)$  and taking expectations lead to:

$$u'(\overline{d}) \cdot \overline{p} - \beta \cdot u'(\overline{d}) \cdot (\overline{p} + \overline{d}) = 0$$
$$\Longrightarrow \overline{p} \cdot (1 - \beta) = \beta \overline{d} \Longrightarrow \overline{p} = \frac{\beta}{1 - \beta} \cdot \overline{d}$$

In equilibrium the price of an asset is given by the dividend weighted by a term depending on the time preference. To build up the Taylor-approximation expression as pleased, we may go through several derivatives of  $F(d, \sigma)$ . The derivative of  $F(d, \sigma)$  w.r.t. d is:

$$F_d(d,\sigma) = E_t[u''(d) \cdot p(d,\sigma) + u'(d) \cdot p_d(d,\sigma) - \beta u''(\rho d + \sigma \epsilon) \cdot \rho \cdot (p(\rho d + \sigma \epsilon,\sigma) + (\rho d + \sigma \epsilon)) - \beta \rho u'(\rho d + \sigma \epsilon) \cdot (p_d(\rho d + \sigma \epsilon) + 1)] = 0$$

Applying the expectations operator and evaluating the last equation at the steady-state values  $(\overline{d}, \overline{p}, \sigma = 0)$ , one may get:

$$u''(\overline{d}) \cdot \overline{p} + u'(\overline{d}) \cdot p_d(\overline{d}, 0) - \beta u''(\overline{d})\rho(\overline{p} + \overline{d}) - \beta u'(\overline{d})(p_d(\overline{d}, 0)\rho + \rho) = 0$$

Solve the last equation for  $p_d(\overline{d})$ :

$$p_{d}(\overline{d}) \cdot (u'(\overline{d}) \cdot (1 - \beta \rho)) = -u''(\overline{d}) \cdot p(\overline{d}) + \beta \rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho \beta \cdot u'(\overline{d})$$

$$\Longrightarrow p_{d}(\overline{d}) = \frac{-u''(\overline{d}) \cdot \overline{p} + \beta \rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho \beta \cdot u'(\overline{d})}{u'(\overline{d}) \cdot (1 - \beta \rho)}$$

The next step will compute  $p_{dd}(\overline{d})$ . Therefore we take the partial derivative of  $F_d$  w.r.t. d:

$$F_{dd} = E_t[u'''(d) \cdot p(d) + u''p_d(d) + u''(d) \cdot p_d(d) + u'(d) \cdot p_{dd}(d)$$
$$-\beta \rho^2 \cdot u'''(\rho d + \sigma \epsilon)(p(\rho d + \sigma \epsilon) + \rho d + \sigma \epsilon) - \beta \rho^2 u''(\rho d + \sigma \epsilon)(p_d(\rho d + \sigma \epsilon) + 1)$$
$$-\beta \rho^2 u''(\rho d + \sigma \epsilon)(p_d(\rho d + \sigma \epsilon) + 1) + \beta \rho^2 u'(\rho d + \sigma \epsilon) \cdot (p_{dd}(\rho d + \sigma \epsilon))] = 0$$

Again taking expectations and evaluating the last equation at the steady-state values  $(\overline{d}, \overline{p}, \sigma = 0)$ :

$$u'''(\overline{d}) \cdot \overline{p} + 2 \cdot u''(\overline{d}) p_d(\overline{d}) + p_{dd}(\overline{d}) (u'(\overline{d})(1 - \beta \rho^2)$$
$$-\beta \rho^2 u'''(\overline{d}) \cdot (\overline{p} + \overline{d}) - 2\beta \rho^2 \cdot u''(\overline{d}) (p_d(\overline{d}) + 1) = 0$$

$$\Longrightarrow p_{dd}(\overline{d}) = \frac{-u'''(\overline{d}) \cdot p(\overline{d}) - 2 \cdot u''(\overline{d}) p_d(\overline{d}) + \beta \rho^2 u'''(\overline{d}) (\overline{p} + \overline{d}) + 2\beta \rho^2 \cdot u''(\overline{d}) (p_d(\overline{d}) + 1)}{u'(\overline{d}) (1 - \beta \rho^2)}$$

To derive  $p_{d\sigma} = p_{\sigma d}$ , one may recognise that we are dealing with a linear homogenous system. Referring to the class, we may expect  $p_{d\sigma} = p_{\sigma d}$  to be zero. To derive this analytically, we first set up  $F_{\sigma}$ :

$$F_{\sigma}(d,\sigma) = E_{t}[u'(d) \cdot p_{\sigma}(d) - \beta u''(\rho d + \sigma \epsilon)\epsilon(p(\rho d + \sigma \epsilon) + \rho d + \sigma \epsilon)$$
$$-\beta u'(\rho d + \sigma \epsilon) \cdot (p_{d}(\rho d + \sigma \epsilon) \cdot \epsilon + p_{\sigma}(\rho d + \sigma \epsilon) + \epsilon)] = 0$$

Again taking expectations and evaluating the last equation at the steady-state values  $(\bar{d}, \bar{p}, \sigma = 0)$ , one may derive:

$$u'(\overline{d}) \cdot p_{\sigma}(\overline{d}) - 0 - 0 + \beta u'(\overline{d})p_{\sigma}(\overline{d}) + 0 = 0$$
$$\Longrightarrow u'(\overline{d}) \cdot p_{\sigma}(d) + \beta u'(\overline{d})p_{\sigma}(\overline{d}) = 0$$

The last equation will only hold if  $p_{\sigma}(\overline{d}) = 0$ , since  $\beta < 1$ . We will use this knowledge later. In the following we compute  $F_{d\sigma}$ :

$$F_{d\sigma} = E_t[u''(d) \cdot p_{\sigma}(d) + u'(d) \cdot p_{d\sigma}(d) - \beta \rho \epsilon \cdot u''(\rho d + \sigma \epsilon) \cdot (p_{\sigma}(\rho d + \sigma \epsilon) + 1)$$
$$-\beta \rho \epsilon \cdot u'''(\rho d + \sigma \epsilon) \cdot (p(\rho d + \sigma \epsilon) + \rho d + \sigma \epsilon) - \beta \rho \cdot u'(\rho d + \sigma \epsilon) \cdot (p_{d\sigma}(\rho d + \sigma \epsilon) \epsilon)$$
$$-\beta \rho \epsilon \cdot u''(\rho d + \sigma \epsilon) \cdot (p_{d}(\rho d + \sigma \epsilon) + 1)] = 0$$

And at the steady-state values  $(\overline{d}, \overline{p}, \sigma = 0)$ , we obtain:

$$E_{t}[u''(\overline{d}) \cdot p_{\sigma}(\overline{d}) - \beta \rho \epsilon \cdot u''(\overline{d}) \cdot (p_{\sigma}(\overline{d}) + 1) - \beta \rho \epsilon \cdot u'''(\overline{d}) \cdot (\overline{p} + \overline{d})$$
$$-\beta \rho \epsilon \cdot u''(\overline{d}) \cdot (p_{d}(\overline{d}) + 1)] = E_{t}[p_{d\sigma} \cdot u'(\overline{d})(1 - \beta \rho \epsilon)]$$

Taking expectations lead to:

$$u''(\overline{d}) \cdot p_{\sigma}(\overline{d}) - 0 - 0 - 0 = p_{d\sigma} \cdot u'(\overline{d})$$

And with the former derived equation  $p_{\sigma}(\overline{d}) = 0$ , we obtain:

$$p_{d\sigma} = 0 = p_{\sigma d}$$

All in all the Taylor-approximation of  $p_t$  looks like:

$$\begin{split} p_{t+1} &= \frac{\beta}{1-\beta}\overline{d} + \frac{-u''(\overline{d}) \cdot \overline{p} + \beta \rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho \beta \cdot u'(\overline{d})}{u'(\overline{d}) \cdot (1-\beta \rho)} \cdot (d_t - \overline{d}) + [g_{\sigma}]\sigma + \frac{1}{2}[g_{\sigma\sigma}]\sigma^2 \\ &+ \frac{1}{2} \frac{-u'''(\overline{d}) \cdot p(\overline{d}) - 2 \cdot u''(\overline{d})p_d(\overline{d}) + \beta \rho^2 u'''(\overline{d})(\overline{p} + \overline{d}) + 2\beta \rho^2 \cdot u''(\overline{d})(p_d(\overline{d}) + 1)}{u'(\overline{d})(1-\beta \rho^2)} \cdot (d_t - \overline{d})^2 \end{split}$$

# 3 Estimation methods

See rbcestim.mod for this model and the Bayesian estimation we did in the lecture. RUN IT ONCE TO GET SIMULATED DATA AND ESTIMATION. Now we will use the same dataset to estimate the parameters of a misspecified model. We will use the same model, however, with a small difference, i.e. technology follows a Cobb-Douglas production function. See rbcexam.mod for the new model equations and steady-state block.

#### 3.1 Exercise 3.1)

Define priors for  $\alpha, \theta$  and  $\tau$ . How many observable variables do you need? Choose an appropriate number of observables.

before estimating parameters, some basic steps have to be done: Starting with declaring variables and parameters. Afterwards the model's equations have to be implemented (Here this is already given by the skript: estim.mod). A further step is the declaration of the observable variable, a variable which is available in an exogenous file and which dynare can use for the estimation procedure. In Bayesian estimation, the condition for undertaking estimation is that there be at least as many shocks as observables. This means for the given model which has only one shock (technology shock) an appropriate number of observables is one.

For our purpose one can use the variable y (declared with the command  $varobs\ y$ ;). As in the previous tasks the steady-states have to be defined due to the single steps in deriving, solving and estimating the given economic setting in the DSGE model. The next step contains the very important step of defining the priors.

Define priors: Choosing the priors for the parameters is one of the most important steps in Bayesian estimation. Because of the wide range of possibilities of choosing the priors some time has to be spent in choosing and testing the robustness of the results. For the given example some different assumptions should be tested to check the sensitivity of the parameters to different priors - the robustness. First of all some considerations to the domains of the parameters should be mentioned.

To demonstrate the difference between both models which differ only in the definition of the production function, these functions are represented in the following:

The data is simulated in a RBC model with the following production function:  $y = A * (\alpha * k_{t-1}^{\psi} + (1 - \alpha) * l^{\psi})^{(1/\psi)}$ .

In contrast to this function the Cobb Douglas production is defined as follows:

$$y = A * (k_{t-1}^{\alpha}) * (l^{(1-\alpha)})$$

Due to the case of a Cobb Douglas function one knows ex ante that the parameter  $\alpha$  is bounded in the interval: [0,1]. This is also case for the  $\theta$  as a part of the defined function of preference (see setting on p. 4 of the problem sheet). For the parameter  $\tau$  one can get some problems if this parameter is too close to zero, but there are no more restrictions which could defined some a priori boundaries for this parameter. Besides the boundaries of the parameters, one has to think about the following points: should those boundaries be open on both sides, the shape of the prior distributions, especially about the symmetry, the skewness. Hence one can choose and build up an appropriate distribution. Because we do not have more information (information from the literature could be involved) we just use simple beta distributions. Due to the knowledge of the underlying data generating process one can formulate some expectations in the following estimations procedure: Because of the fact that the  $\alpha$  should differ between both production functions which is hardly to interpret in the complete setting, the focus should lie on the estimators for the parameters  $\tau$ ,  $\theta$ , because these parameters have the same functions in both settings. Therefore the question remains how the misspecified model would estimate the parameters and in which direction they will be biased in contrast to the true parameters. This question should be answered with the following approaches.

For the following estimation procedures we use different possibilities in defining the priors to pursue two aims: Firstly, to check how sensitive (or robust) the Bayesian estimation procedure (with the Metropolis Hastings algorithm) react on different choices of priors. And secondly to evaluate and to get an impression of the bias between the original parameters and the estimated ones. For this purpose we choose priors which are greater than the mean of the original parameter, priors which are smaller and priors which differ in the direction of bias. In contrast to the announced purposes we also apply the original parameters as priors. Because there are no information for the used distribution we consider different derivations (in defining other kinds of distributions than simple beta definitions dynare often displays that some problems occur during the procedure. This is not known before applying the posterior mode or complete estimation via Metropolis Hastings algorithm, but should be already mentioned at this point). All used priors and the results which belong to these priors are saved in the additional textfile. A further selection of the the mentioned priors for applying the estimation procedure in task 3.3 is made in task 3.2 with the diagnostic tool mode check which helps to evaluate the decision of the priors and to indicate some problems.

#### 3.2 Exercise 3.2)

Estimate the posterior mode using the estimation command and a limited sample with 200 observations. Check the posterior mode using *mode check*. If you get errors due to a non-positive definite Hessian, try a different optimization algorithm or change the initial values.

To estimate the given model one can use the command estimation(...) while in the brackets the options for the procedure is defined (the full command is announced in the additional .mod file which could be changed with the announced commands to achieve results with changed options). First the datafile is the option to announce the name of the file with the observations used for estimation. Here one has to use the data which is simulated with the RBC model and was saved in a matlab .m file (This file is also added to the other files, and has to be used for all of the estimations to have a common basis. The command nobs is used to declare the number of observations in the file which should be used for the procedure. This is the command which is needed for the task and which has to be set to 200. To check the posterior mode one can use the command mode check. If this command is announced in the estimation command, dynare is going to plot the posterior density for values and for every estimated parameter around the given mode. This diagnostic tool should help to identify problems with the optimization procedure. "A clear indication of a problem would be that the mode is not at the trough (bottom of the minus) of the posterior distribution."

Now some of the mentioned priors can be evaluated with the *mode check* command (due to the large number of possibilities in choosing the priors, only some main detections are described). The represented diagnostic plot which is given by the command *mode check* (see figure: 6) indicates that there is no problem existent.

In this plot the vertical line should coincide with the blue curve (the log-post). A very flat green curve (log-lik Kernel) (as in the part of the plot for the third parameter:  $\tau$  in the figure 7) means that one can only extract less information for this parameter for the data (this fact is later also shown in the comparison of the prior and the posterior distribution - in this case the posterior distribution shape is quite similar to the prior distribution shape). For the case that the prior values are set as true values, these diagnostic shows that there are some problems with the prior distribution assumption. Especially the check plot which are represented in the figure 7 seems not to be appropriate priors.

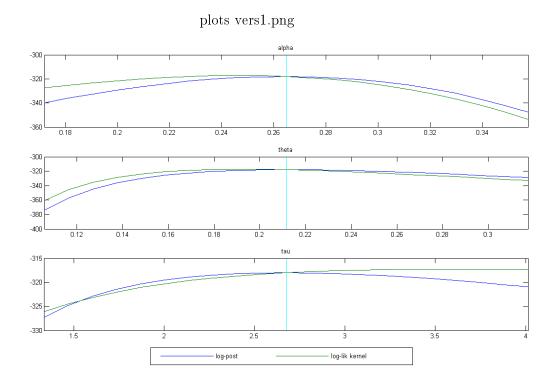


Abbildung 6: plot of the check plots for priors of version 1 (see priors txt.)

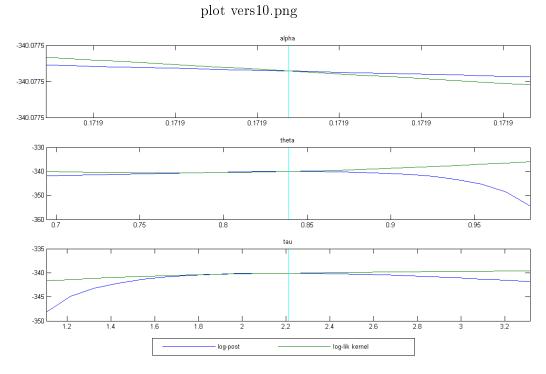


Abbildung 7: plot of the check plots for priors of version 1 (see priors txt.)

# 3.3 Exercise 3.3)

If you are satisfied with the posterior mode, approximate the posterior distribution using the Metropolis-Hastings-Algorithm with  $3 \times 5000$  iterations. How large is your acceptance rate (change *jh scale* if you're not satisfied). Provide also the diagnostic plots. If the algorithm does not converge to the (ergodic)

10

# posterior-distribution, repeat the algorithm with 1000 more iterations without discarding the previous draws.

Metropolis-Hastings algorithm with  $3 \times 5000$  iterations: could be generated with the command estimation and the additional option mh replic is used to declare the number of replications of the Metropolis Hastings algorithm - here initialized with 5000 iterations. The command mh nblocks defines the number of chains for the Metropolis Hastings alorithm. In this case three chains are used (to check the robustness also the suggestion, the default setting, of five chains is used. See comments in task 3.4).

The commands  $mode\ compute=0$ ,  $mode\ file=rbcexam\ mode$  are used for the following purposes: In the case of setting  $mode\ compute=0$ , the mode is not computed at the following Metropolis Hastings algorithm. In this case the procedure uses the stored mode (stored and called with the following name structure: MODEL NAME mode). This option is not necessary for the estimation procedure but helps to speed up the estimation process.

The acceptance rate can be set with the command *mh jscale* and has a default value of 0.2 while the literature mostly uses values between 0.2 and 0.4. Choosing an accurate acceptance rate is also an important issue in Bayesian estimating procedure, because in the case of very high acceptance rate the candidate parameters are rejected too often and in the case of a very low acceptance rate they are accepted too often. The aim is to choose an acceptance rate which is located between both mentioned extremes. Therefore the option of applying different values for the acceptance rate is an important practical issues (to check the robustness of the estimators three different acceptance rates are used for the priors with the true parameters).

As asked in the task one has to provide diagnostic plots as well. These diagnostics are displayed automatically because the command nodiagnostics is not set in the estimation command, hence the default setting is used: dynare is computing and displaying tabular and graphical diagnostics, for example the convergence diagnostics for the Metropolis Hastings algorithm, which could be used to check if the blocks are converging.

To repeat the algorithm without discarding the previous draws one can use the following command in the estimation command: mh drop and one could define the fraction of the initially generated parameter vectors which should be dropped. If the command is set to zero none of the previous draws will be discarded.

#### 3.4 Exercise 3.4)

Take a stand on your Bayesian estimation. What is good, what can be improved and how? Compare the estimation of the common parameters of the true model with the misspecified model.

Take a stand on your Bayesian estimation: By the estimation procedure one gets the following diagnostic graphical plots: First one gets a representation of the priors for each parameter. Which is not important for the purpose of evaluating the applied estimation procedure.

Figure 8 represents the MCMC univariate diagnostics. The plot in figure 8 (priors version 1) shows clearly that there is a quite similar movement of the chains and that they are lying quite closely at the end of the iterations which can be interpreted as a sign of convergence. Here three measures for the three parameters are plotted. The interval: being constructed from an 80 percent confidence interval around the parameter mean. The m2: measure of the variance and the m3: based on third moments. The red and blue lines differ between parameter vectors for the between and the within chains.

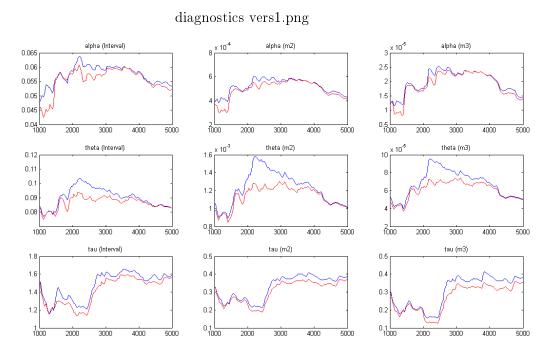


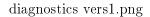
Abbildung 8: plot of univariate diagnostics for the priors of version 1 (see priors txt.)

The plot MCMC multivariate diagnostics represents an aggregate measure with analog differentiations as in the univariate case (see above). In the figure 9 it is possible to consider a kind of a movement to convergence. But this is as well a question of the demanded precision.

In the case of the priors from version 1 with larger deviations the plot shows a significant movement of divergence (see figure 10).

In contrast to the figure 10, the figure 11, the multivariate diagnostics for the priors of version 3, seems to have an obvious movement of convergence.

In the case of instability or existing movements which are suffering from divergence it is



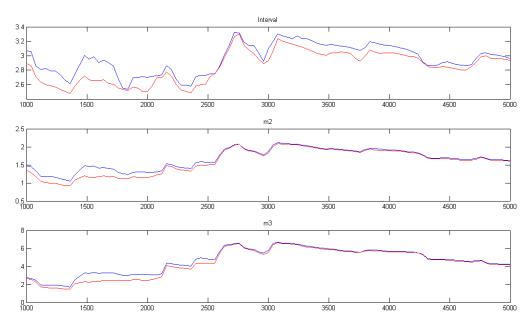


Abbildung 9: plot of multivariate diagnostics for the priors of version 1 (see priors txt.)

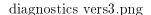
diagnostics vers10.png

#### Interval m2 0.5

Abbildung 10: plot of multivariate diagnostics for the priors of version 10 (see priors txt.)

possible to improve the findings by using some different priors (as an indication of poor priors: this has been already done with the figures above). Another opportunity is to choose a greater number of Metropolis Hastings iterations. This should be done for the priors version1 with 5 chains and 10000 iterations. The results in the multivariate diagnostics are presented in the figure 12.

Now the most important diagnostic plot should be discussed, the comparison of the poste-



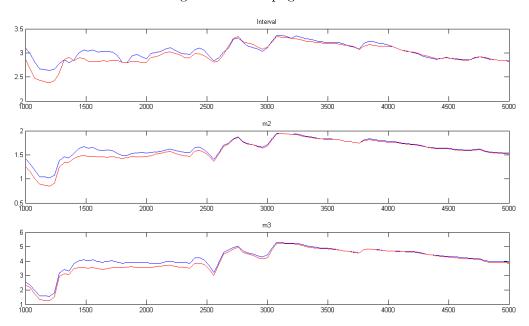


Abbildung 11: plot of multivariate diagnostics for the priors of version 3 (see priors txt.)

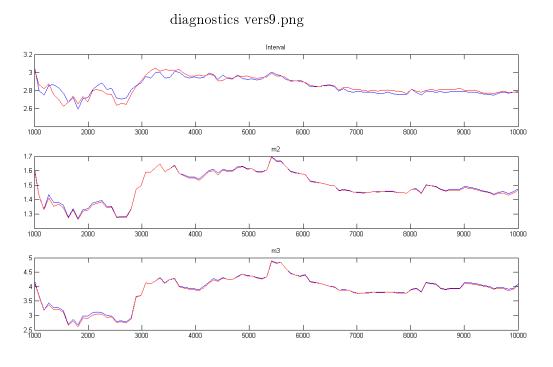


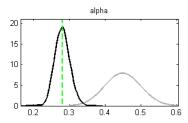
Abbildung 12: plot of multivariate diagnostics for the priors of version 9 (see priors txt.)

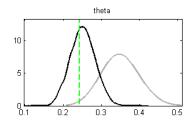
rior and the prior distributions - which is the main result of the estimation algorithm and as well a graphical tool to indicate existing problems.

The figure which compares the prior with the posterior distributions. Here represented by the grey lines (the prior distributions) and the black lines (the posterior distributions). The green line represents the posterior mode.

Figure 13 represents the plot of the comparison between the priors and posteriors, version 1.

#### posteriors vers1.png





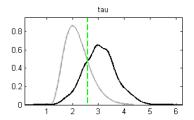


Abbildung 13: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

More of these plots are added to this textfile. To show another plot of priors and posteriors which suffer from computational problems.

posteriors vers10.png

alpha

6
4
2
0
0
0
0.2
0.4
0.6
0.8



0.2 0.4 0.6

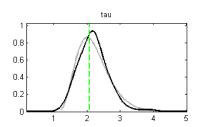


Abbildung 14: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

The plot of the smoothed shocks is important for checking the movement and the frequency of the shocks. The plot shows that the shock is centered around zero which is also assumed. Compare the estimation of the common parameters of the true model with the misspecified model:

If the estimators for the true model are compared with the misspecified model, there are several observations which have to be announced: If the priors of the parameters are not chosen too close to their boundaries (in the underlying, overlying and also for the opposed case) the estimator of the  $\theta$  parameter is always smaller than the true value and the  $\tau$  value is always larger than the true value (see the additional textfile for results of the estimations). If the priors are chosen around the true parameters and not too close to their boundaries, the estimation results are quite robust, but are biased in the described form. The results could change if the priors are set to extreme scenarios. But these results are not surprising and problematic priors can be detected by the diagnostic graphics, for example with the check plots (see tasks above).

# 3.5 Exercise 3.5)

Calculate the posterior-odds and the posterior-model-probabilities weighting each model by 0.5 a prior.

The command *model comparison* can used to compare two models with own vectors of parameters which are estimated by using the same sample. This is exactly the case in this task with the original RBC model and the "misspecified"model with Cobb Douglas production function. Only few words to the model comparison: The important values are the ones of the marginal density (here in log version). So the comparison between two models works with the following decision rule: the model with the higher liklihood fits the data better. Hence for the given models the true model estimation fits the data better than the so called misspecified model (see 3.5.txt for comparison results). This is again not a surprising result and makes clear how in an unknown "real life scenario" one can choose between two available production functions.