Center for Quantitative Economics

Smets-Wouters (2003) Model

Implementation

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- Today we implement the model:

  Smets, Frank and Wouters, Raf
  An estimated dynamic stochastic general equilibrium model of the euro area
  Journal of the European Economic Association
  September 2003, 1(5), 1123-1175

- Any differences (scaled shocks) between the published and coded version are highlighted, and (mostly) rely on the slides of Prof. Uhlig
- We use the linearized equations derived in the first part of the lecture

$$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}\left(\hat{R}_t - E_t\hat{\pi_{t+1}}\right) + $$
$$+\frac{1-h}{(1+h)\sigma_c}\left(\hat{\varepsilon}_t^B - E_t\hat{\varepsilon}_{t+1}^B\right)$$

- Take out $E_t\hat{\varepsilon}_{t+1}^B$

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I_{t-1}} + \frac{\beta}{1+\beta}E_t\hat{I_{t+1}} + \frac{1}{S''(1)(1+\beta)}\hat{Q}_t + \frac{\beta E_t\hat{\varepsilon}^I_{t+i} - \hat{\varepsilon}^I_t}{1+\beta}$$

- Define $\frac{1}{S''(1)} = \delta$
- Take out $E_t\hat{\varepsilon}^B_{t+1}$
- Rescale $\hat{\varepsilon}^I_t$

$$\hat{Q}_t = -(\hat{R}_t - E_t\hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+r^k} E_t\hat{Q}_{t+1} + \frac{r^k}{1-\tau+r^k} E_t\hat{r}^k_{t+1} + \eta^Q_t$$

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\hat{I}_{t-1}$$

$$
\begin{aligned}
\hat{\pi}_t \quad = \quad & \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{t-1} + \\
& + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p} \left(\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p \right)
\end{aligned}
$$

- Rescale η_t^p

$$
\begin{aligned}
\hat{w}_t \quad = \quad & \frac{\beta}{1+\beta} E_t \hat{w}_{t+1} + \frac{1}{1+\beta} \hat{w}_{t-1} + \frac{\beta}{1+\beta} E_t \hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta} \hat{\pi}_t + \\
& \frac{\gamma_w}{1+\beta} \hat{\pi}_{t-1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta)\left(1+\frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \cdot \\
& \cdot \left(\hat{w}_t - \sigma_l \hat{L}_t - \frac{\sigma_c}{1-h} \left(\hat{C}_t - h \hat{C}_{t-1} \right) - \hat{\varepsilon}_t^L - \hat{\eta_t^w} \right)
\end{aligned}
$$

- Rescale η_t^w
- Change the sign of the labor supply shock $\hat{\varepsilon}_t^L$

$$\hat{L}_t = -\hat{w}_t + (1 + \frac{\psi'(1)}{\psi''(1)})\hat{r}_t^k + \hat{K}_{t-1}$$

- Define $\frac{\psi'(1)}{\psi''(1)} = \psi$

$$
\begin{aligned}
\hat{Y}_t &= (1-\tau k_y - g_y)\hat{C}_t + \tau k_y \hat{I}_t + g_y \varepsilon_t^G = \\
&= \phi \hat{\varepsilon}_t^a + \phi\alpha \hat{K}_{t-1} + \phi\alpha \frac{\psi'(1)}{\psi''(1)} \hat{r}_t^k + \phi(1-\alpha)\hat{L}_t
\end{aligned}
$$

- Define $\frac{\psi'(1)}{\psi''(1)} = \psi$
- Rescale ε_t^G

$$\begin{aligned}\hat{R}_t &= \rho\hat{R}_{t-1} + (1-\rho)\left[\bar{\pi}_t + r_\pi\left(\hat{\pi}_{t-1} - \bar{\pi}_t\right) + r_Y(\hat{Y}_t - \hat{Y}_t^f)\right] + \\ & +r_{\Delta_\pi}\left(\hat{\pi}_t - \hat{\pi}_{t-1}\right) + r_{\Delta_Y}\left((\hat{Y}_t - \hat{Y}_t^f) - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^f)\right) + \eta_t^R\end{aligned}$$

- In order to solve the model, we need to evaluate the output gap $\hat{Y} - \hat{Y}_f$
- The output gap $\hat{Y}_t$ is the difference between the actual and the potential output with perfect flexibility of prices and wages and no cost-push shocks
- Thus we augment the sticky system with a second, flexible system, where it holds that:
  - $\eta_p = 0$
  - $\eta_Q = 0$
  - $\eta_w = 0$
  - $\xi_w = 0$
  - $\xi_p = 0$
- Open the *SmetsWouters_rescale.mod* and inspect the equations!