Structured PhD Program
Summer Term 2015

Advanced Macroeconomics

Problem Set 3

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Deadline: 7th September 2015

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1 Exercise 1: Baxter and King (1993)

1.1 Distinguish between endogenous and exogenous variables as well as model parameters.

The exogenous variables in the model are the innovations in productivity, public consumption, public investment and the tax rate:

- ε_t^z = productivity shock
- ε_t^{GB} = shock on public consumption
- ε_t^{IB} = shock on public investment
- ε_t^{τ} = shock on tax rate

The model parameters are those variables, that are calibrated beforehand, including the steady-state levels of the target variables. Thus, in the given model, the parameters are as follows:

- β = discount rate of the household's life-time utility
- δ = depreciation rate of capital
- η = productivity of public capital
- θ = Frisch elasticity of labor supply
- α = share of capital in production

- ρ_z = smoothing parameter in productivity
- ρ_{G^B} = smoothing parameter in public consumption
- ρ_{I^B} = smoothing parameter in public investment
- ρ_τ = smoothing parameter in taxation

- $\bar{Y}$ = steady-state value of output
- $\bar{G}^B$ = steady-state value of public consumption

- $\bar{I}^B$ = steady-state value of public investment
- $\bar{TR}$ = steady-state value of transfers
- $\bar{w}$ = steady-state value of wage rate
- $\bar{N}$ = steady-state value of labor supply
- $\bar{\tau}$ = steady-state value of tax rate
- $\bar{z}$ = steady-state value of productivity
- $\bar{K}^B$ = Steady-state value of public capital
- $\bar{r}$ = Steady-state value of the interest rate
- $\bar{K}$ = Steady-state value of private capital
- $\bar{I}$ = Steady-state value of private investment
- $\bar{C}$ = Steady-state value of private consumption

Finally, the endogenous variables in the model are those variables that are determined within the system, either through analytic derivation or empirical estimation. In the given model, the endogenous variables are as follows:

- Y_t = Output
- dY_t = Change in output
- C_t = Real private consumption
- dC_t = Change in real private consumption
- I_t = real private investment
- dI_t = Change in real private investment
- r_t = Real interest rate
- dr_t = change in real interest rate
- K_t = Private capital
- N_t = real labor supply
- dN_t = Change in real labor supply
- w_t = Real wage rate

- dw_t = Change in real wage rate
- G_t^B = Public consumption
- dG_t^B = Change in public consumption
- I_t^B = Public investment
- dI_t^B = Change in public investment
- K_t^B = Public capital
- TR_t = Real lump-sum transfers
- dTR_t = Change in real lump-sum transfers
- τ_t = tax rate
- z_t = productivity

1.2 Provide a calibration for all model parameters that meet economic intuition.

The model of Baxter and King (1993) can be fully described the following model equations:

Consumption-leisure choice of the household:

$$(1-\tau_t)w_t = \theta_l \frac{C_t}{1-N_t} \tag{1}$$

Savings decision:

$$\lambda_t = \beta E_t\{\lambda_{t+1}[(1-\delta)+(1-\tau_{t+1})r_{t+1}]\} \tag{2}$$

where $\lambda_t = \frac{1}{C_t}$.

Private and public capital stocks:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{3}$$

and

$$K_t^B = (1-\delta)K_{t-1}^B + I_t^B \tag{4}$$

Production function of the firm:

$$Y_t = z_t(K^B_{t-1})^\eta(K_{t-1})^\alpha(N_t)^{1-\alpha} \tag{5}$$

Factor demands:

$$w_t(N_t) = (1-\alpha)Y_t \tag{6}$$

and

$$r(K_t) = \alpha Y_t \tag{7}$$

Budget constraint of the fiscal authority:

$$G^B_t + I^B_t + TR_t = \tau_t(w_t N_t + r_t K_{t-1}) \tag{8}$$

Market clearing:

$$Y_t = C_t + I_t + G^B_t + I^B_t \tag{9}$$

In addition, there are four equations for the exogenous shocks: Productivity:

$$log(\frac{z_t}{\bar{z}}) = \rho_z log(\frac{z_{t-1}}{\bar{z}}) + \varepsilon^z_t \tag{10}$$

Public consumption:

$$G^B_t - \bar{G}^B = \rho_{G^B}(G^B_{t-1} - \bar{G}^B)\varepsilon^{G^B}_t \tag{11}$$

Public investment:

$$I^B_t - \bar{I}^B = \rho_{I^B}(I^B_{t-1} - \bar{I}^B)\varepsilon^{I^B}_t \tag{12}$$

Taxation:

$$log(\frac{\tau_t}{\bar{\tau}}) = \rho_\tau log(\frac{\tau_{t-1}}{\bar{\tau}}) + \varepsilon^\tau_t \tag{13}$$

Finally, we have nine equations for the percentage (points) deviations from the steady states:

$$dY_t = 100 * (\frac{Y_t - \bar{Y}}{\bar{Y}}), \tag{14}$$

$$dC_t = 100 * (\frac{C_t - \bar{C}}{\bar{C}}), \tag{15}$$

$$dI_t = 100 * (\frac{I_t - \bar{I}}{\bar{I}}), \tag{16}$$

$$dN_t = 100 * (\frac{N_t - \bar{N}}{\bar{N}}), \tag{17}$$

$$dw_t = 100 * (\frac{w_t - \bar{w}}{\bar{w}}), \tag{18}$$

$$dr_t = 100 * (r_t - \bar{r}), \tag{19}$$

$$dTR_t = 100 * (TR_t - T\bar{R}), \tag{20}$$

$$dG_t^B = 100 * (\frac{G_t^B}{\bar{Y}} - \frac{\bar{G}^B}{\bar{Y}}), \tag{21}$$

and

$$dI_t^B = 100 * (\frac{I_t^B}{\bar{Y}} - \frac{\bar{I}^B}{\bar{Y}}). \tag{22}$$

Overall, we have 22 endogenous variables and 22 model equations. Additional information on target variables and parameters is given in the exercise:

- $\bar{Y} = 1$
- $\bar{G^B} = 0.2 * \bar{Y} = 0.2$
- $\bar{I^B} = 0.02 * \bar{Y} = 0.02$
- $T\bar{R} = 1$
- $\bar{w} = 2$
- $\bar{N} = 1/3$
- $\rho_z = \rho_{G^B} = \rho_{I^B} = \rho_\tau = 0.75$

- $\eta < \alpha$

To derive the steady states of all other endogenous variables and the implied parameter values, we first set some reasonable values for β, δ and η, following the RBC-example in the slides:

- $\beta = 0.99$
- $\delta = 0.02$
- $\eta = 0.2$

We then derive the steady states for all other endogenous variables as follows:

For public capital (K^B), eq. (4) is evaluated at the steady state[1]:

$$\bar{K^B} = (1-\delta)\bar{K^B} + \bar{I^B}. \tag{23}$$

Rearranging and inserting the values from above, this yields:

$$\bar{K^B} = \frac{\bar{I^B}}{\delta} = \frac{0.02}{0.02} = 1. \tag{24}$$

For the share of capital in production (α), eq. (6) is evaluated at the steady state:

$$\bar{w}\bar{N} = (1-\alpha)\bar{Y}. \tag{25}$$

Rearranging and inserting the values from above, this yields:

$$\bar{\alpha} = 1 - \bar{w}\frac{\bar{N}}{\bar{Y}} = \frac{1}{3}. \tag{26}$$

For private capital (K), eq. (7) is evaluated at the steady state:

$$\bar{r}\bar{K} = \alpha\bar{Y}. \tag{27}$$

Rearranging, this yields:

$$\bar{K} = \frac{\alpha}{\bar{r}}\bar{Y}. \tag{28}$$

[1]Note that the equation numbers refer to the document at hand and not to the enumeration in the exercise sheet.

For the tax rate (τ), eq. (8) is evaluated at the steady state:

$$\bar{G}^B + \bar{I}^B + \bar{TR} = \bar{\tau}(\bar{w}\bar{N} + \bar{r}\bar{K}). \tag{29}$$

Rearranging, this yields:

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \bar{r}\bar{K}}. \tag{30}$$

For the real interest rate (r), eq. (2) is evaluated at the steady state:

$$\frac{1}{\bar{C}} = \beta\frac{1}{\bar{C}}[(1-\delta) + (1-\bar{\tau})\bar{r}]. \tag{31}$$

Rearranging, this yields:

$$\bar{r} = \frac{\frac{1}{\beta} - 1 + \delta}{1-\bar{\tau}}. \tag{32}$$

Now, we can substitute eq. (19) into eq. (21):

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \bar{r}\frac{\alpha}{\bar{r}}\bar{Y}}. \tag{33}$$

Rearranging and inserting the values from above, this yields:

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \alpha\bar{Y}} = 0.22. \tag{34}$$

The value for τ can now be substituted into eq. (23). Inserting also the other steady state values, this gives:

$$\bar{r} = 0.04. \tag{35}$$

Finally, we can substitute the value for r into eq. (19) and solve for the steady state of private capital:

$$\bar{K} = 8.3. \tag{36}$$

Next, the production function is evaluated at the steady state:

$$\bar{Y} = \bar{z}(\bar{K}^B)^\eta(\bar{K})^\alpha(\bar{N})^{1-\alpha} \tag{37}$$

Solving this for z yields:

$$\bar{z} = \frac{\bar{Y}}{(\bar{K}^B)^\eta (\bar{K})^\alpha (\bar{N})^{1-\alpha}} = 1.03. \tag{38}$$

For private investments, eq. (3) is evaluated at the steady state and solved for I. This gives:

$$\bar{I} = \delta \bar{K} = 0.17 \tag{39}$$

Next, the market clearing condition in eq. (9) is evaluated at the steady state and solved for consumption (C):

$$\bar{C} = \bar{Y} - \bar{I} - \bar{G}^B - \bar{I}^B = 0.61. \tag{40}$$

Finally, the consumption-leisure choice in eq. (1) is evaluated at the steady state and solved for the Frisch elasticity of labor (θ):

$$\theta = \frac{(1-\bar{N})(1-\bar{\tau})\bar{w}}{\bar{C}} = 1.7. \tag{41}$$

As there a no shocks in the steady state equilibrium, evaluating eq. (10) - (13) at the steady state yields that the respective shocks are zero. We now have a full set of model equations, parameter values and steady-state conditions for the endogenous variables that allows us to estimate the model in Dynare.

In the following table, a complete list of parameter values (including the steady state values) and a legend of the notation in DYNARE is given:

Variable name	Notation in DYNARE	Value
Discount rate of the household's life-time utility	beta	0.99
Depreciation rate of capital	delta	0.02
Productivity of public capital	eta	0.2
Share of capital in production	alpha	$\frac{1}{3}$
Frisch elasticity of labor supply	theta	1.7
Smoothing parameter in productivity	rhoz	0.75
Smoothing parameter in public consumption	rhogb	0.75
Smoothing parameter in public investment	rhoib	0.75
Smoothing parameter in taxation	rhotau	0.75
Steady-state value of output	ybar	1
Steady-state value of public consumption	gbbar	0.2
Steady-state value of public investment	ibbar	0.02
Steady-state value of transfers	trbar	0
Steady-state value of wage rate	wbar	2
Steady-state value of labor supply	n	$\frac{1}{3}$
Steady-state value of tax rate	taubar	0.22
Steady-state value of productivity	zbar	1.03
Steady-state value of public capital	kbbar	1
Steady-state value of the interest rate	rbar	0.04
Steady-state value of private capital	kbar	8.3
S Steady-state value of private investment	ibar	0.17
Steady-state value of private consumption	cbar	0.61

1.3 Differences, advantages and disadvantages of deterministic vs. stochastic models.

The important distinction between deterministic and stochastic models is whether future shocks are known. In deterministic models, the occurrence of all future shocks is known, whereas in stochastic models only the distribution of future shocks is known. Deterministic models thus assume full information, perfect foresight and no uncertainty around shocks. Most often, these models introduce a positive shock today and zero shocks afterwards, for instance when studying the impact of a change in regime like the introduction of a new tax regime.

Much more common in the DSGE literature (that involves already the term "stochastic" in its name) are stochastic models such as Real Business Cycle models or New Keynesian models. In these models, individuals solve their optimization problems knowing that future values of shocks are random but will have zero mean. Thus, shocks will hit at some point in time, but thereafter their expected value is zero.

For deterministic models, it is possible to find an exact numerical solution that does not require linearization. In practice however, these models are only useful to get a first idea of the model. Assuming random shocks and introducing uncertainty is much more realistic and thus stochastic models are usually more useful to answer interesting policy questions such as the reaction of real variables on nominal disturbances like monetary policy shocks. However, DSGE models often require linearization and (Taylor) approximation and are computationally more demanding than deterministic models. Overall, the stochastic model should be chosen in the present context.

1.4 DYNARE-code

Please refer to the mod-file “Gerling_PS3_Ex1.mod”.

1.5 How does (i) an unexpected temporary public consumption and (ii) an unexpected temporary public investment shock (of the same size) feed through the model? Simulate these two shocks and compare the reactions of the observables. Does it affect the real economy? Try to provide economic intuition behind the results. How do the results change if the productivity of public capital increases?

Figure (1) and (2) display how an unexpected temporary shock in public consumption feeds through the model. First, a shock to public consumption with a moderate standard error of 1% leads to an increase of public consumption of approximately five percentage points. As the IRF show, the real interest rate increases after the shock. The deviation from the steady state (dr) has a humped-shaped form as it first increases and peaks after approximately five periods. Yet, the impact on the real interest rate vanishes only slowly. An increase in the real interest rate is plausible, as the positive shock in public consumption increases the demand for money on the capital market. Accordingly, private investment drops as a reaction to the increased interest rate, but this effect is

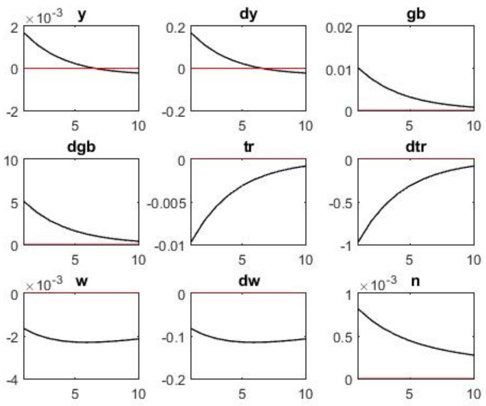

Figure 1: Shock to gb

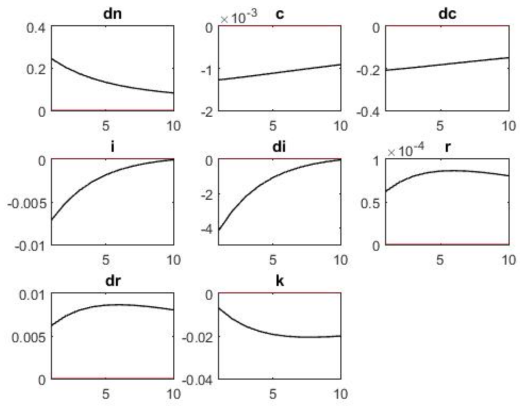

Figure 2: Shock to gb

only temporary as can be seen in the figure as di returns quickly to zero. Also, private consumption is reduced by an increase in public consumption. However, in contrast to private investment, the effect is longer-lasting (and might be permanent) as can be concluded from the very slow return of dc to zero. Hence, both, private investment and private consumption, are partly crowed out by the increase in public consumption. This effect materializes also in the reaction of output (dy). While the increase of output is quite pronounced in the short-run, dy returns quickly to the old level and drops even below the old steady state as can be concluded from the negative deviations in the figure. Finally, real labor supply as well as real wages are also affected negatively by the shock in public consumption, which can be again explained with the crowing-out of private investment and consumption. In particular, the response of dn is analogous to that of dy. Finally, public transfers decline temporarily after the shock. Also this reaction appears to be plausible given the budget constraint of the government that faces a balanced budget rule.

In contrast, a positive shock in public investment has a permanent positive impact on output (see figure (3) and (4)), even though we again observe that rising public expenditures lead to a crowing out of private investment. The crowing-out effect can be depicted most directly in the complementary reactions of private and public capital (see figure (4)). However, treating the model with a shock of the same magnitude (i.e. with a standard error of 0.0001) leads to a much more pronounced rise in public investment as compared to public consumption. Accordingly, the real interest rate increases more strongly and this effect appears to be permanent (see dr in figure (4)). Again, the rise in the interest rate leads to a partial crowding our of private investments, that is

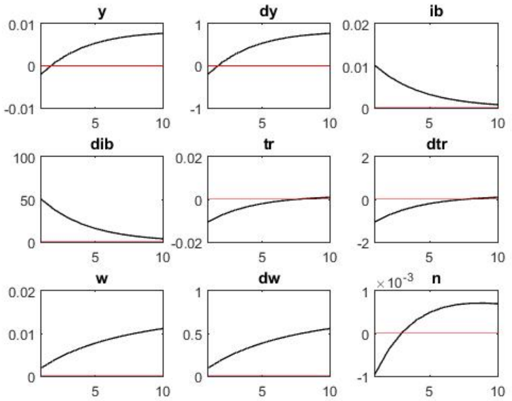

Figure 3: Shock to *ib*

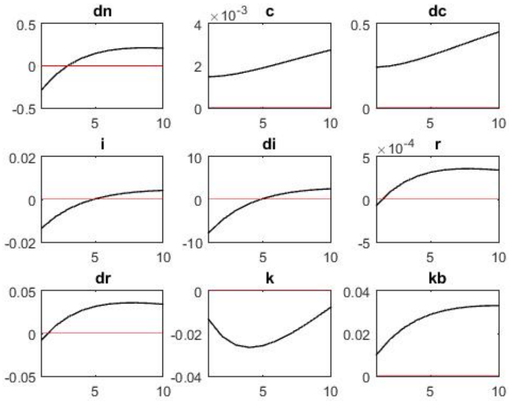

Figure 4: Shock to *ib*

also quite pronounced with a drop of almost 10 percentage points (see *di*). However, in contrast to the shock in public consumption, private consumption increases permanently (see *dc*). This effect might be explained by the impact of the Keynesian multiplier of public investment that stimulate private consumption through increased employment and wages. This explanation is supported by the positive effect of *dn* and *dw* that both rise permanently towards a new steady state level (see figure (3)). Referring back to the model equations, an increase in public investment leads to a one-to-one increase in public capital which can be used for productive activities and thus in turn increases aggregate output directly (see eq. (4) and (5)).

Hence, in the case of an unexpected shock in public investment, the stimulating effect on private consumption and the aggregate capital stock dominates the negative crowding-out effect (through higher interest rates). This is not the case with a shock in public consumption, where the positive multiplier effect seems to trickle off. Thus, the model suggests that investment-based programs aiming at stimulating aggregate output are preferable to increased government consumption.

Finally, when the productivity of public capital is assumed to be higher, e.g. $\eta = 0.5$, the positive effect of a shock in public investment on the macroeconomic aggregates is even more pronounced, while the impact of a shock in public consumption is not changed substantially (see figures (5) to (8)). This is not surprising given the above argumentation that public investment directly increases public capital, which in turn raises aggregate output and this effect is stronger the higher the productivity of public capital[2].

[2]For the computation of the model with a higher η, please refer to the additional mod-file "Ger-

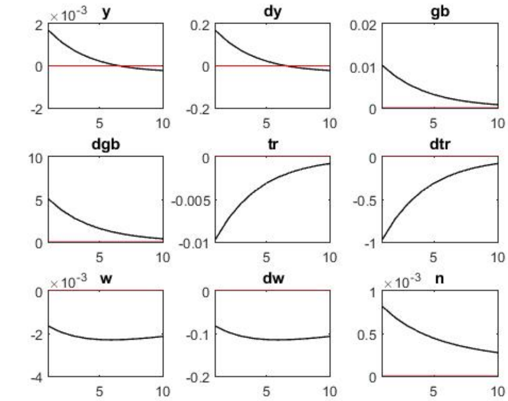

Figure 5: Shock to gb with higher η

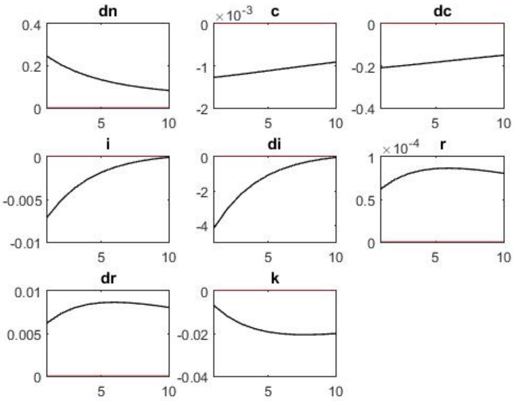

Figure 6: Shock to gb with higher η

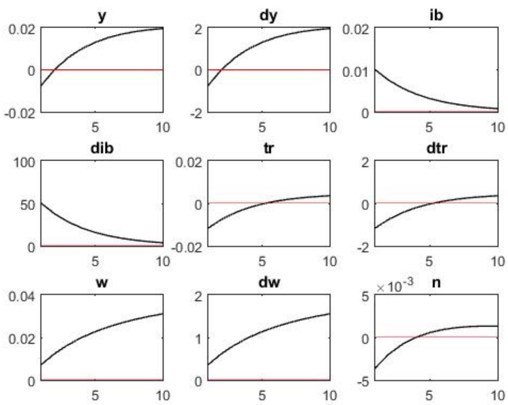

Figure 7: Shock to ib with higher η

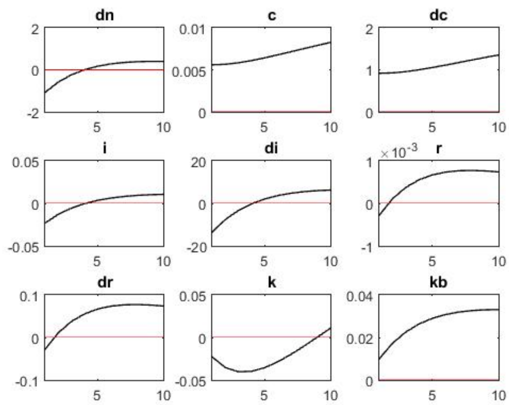

Figure 8: Shock to ib with higher η

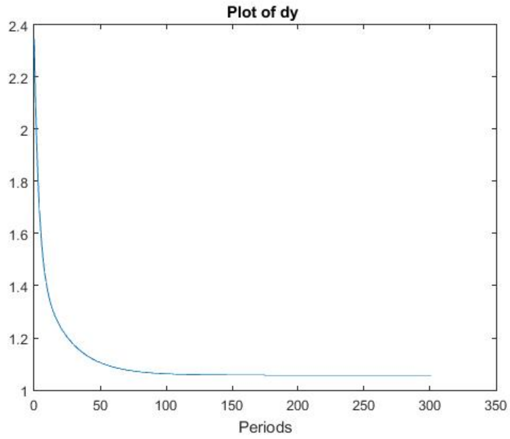

Figure 9: Effect of shock to τ on dY

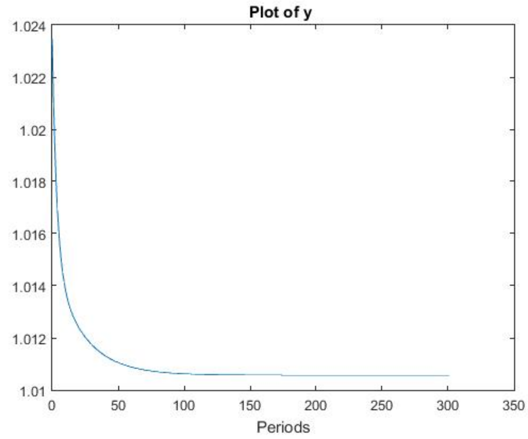

Figure 10: Effect of shock to τ on Y

1.6 How does a permanent increase in the tax rate of 1 percentage point affect the long-run equilibrium of the economy? Compute the new steady-state and compare it with the old one. Also, show the transition path from the old to the new steady state. Try to provide economic intuition.

A permanent increase in the tax rate by 1 percentage points leads to a new steady state value of aggregate output, Y, of 1.01057, which is only slightly lower than the original steady state value of 1.02348[3]. Thus, an increase in taxation only has a temporary effect on the economy. This is confirmed by figures (9) and (10) which display the reaction of aggregate output both, in absolute terms and in deviations from the steady state. As figure (11) displays, an increase in taxation is accompanied by an in crease in public transfers. Thus, the reduction in purchasing power that is caused by higher taxation is compensated by higher subsidies to the economy. This might explain the temporary effect of the tax increase. Figures (12) to (17) show the impact of a permanent increase in taxation on the other variables.

ling_PS3_Ex1_neweta.mod"

[3]Please refer to the file "Gerling_PS3_Ex1.mod" for the numbers.

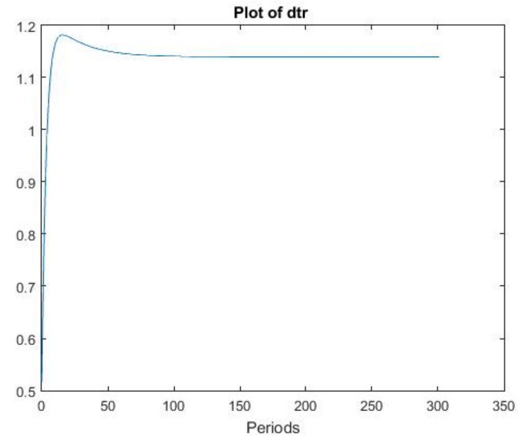

Figure 11: Effect of shock to τ on dtr

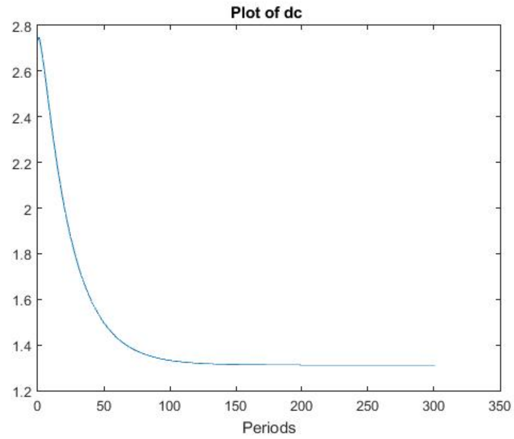

Figure 12: Effect of shock to τ on C

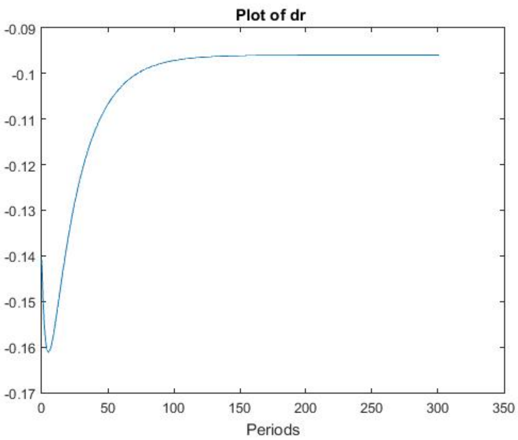

Figure 13: Effect of shock to τ on dr

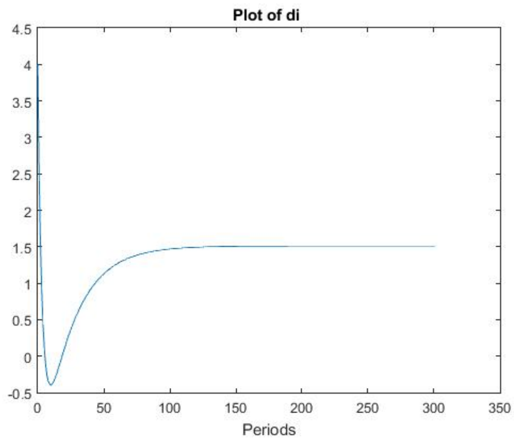

Figure 14: Effect of shock to τ on dI

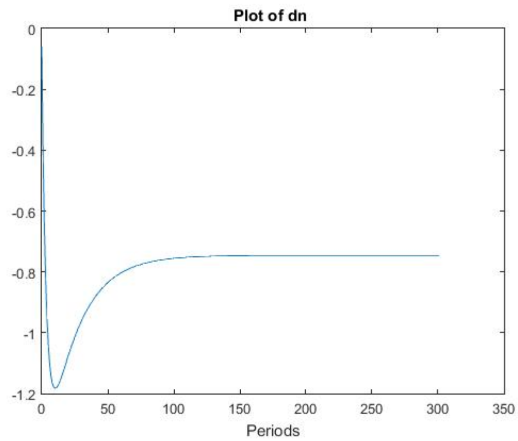

Figure 15: Effect of shock to τ on N

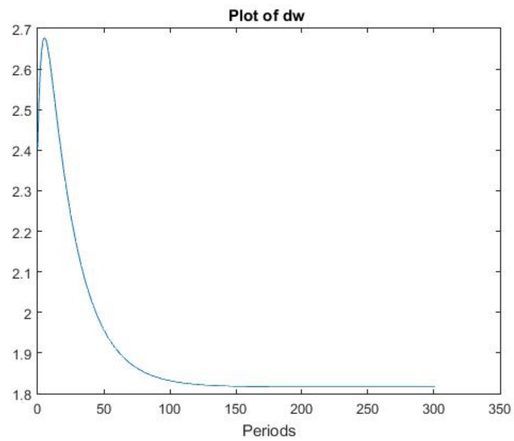

Figure 16: Effect of shock to τ on w

1.7 Willi argues that fiscal policy cannot boost the economy due to the implied crowding-out of private consumption and private investment. Do you agree given your results above? Also explain, in economic terms, the difference between private and government capital.

As depicted in exercise 1.5, there is indeed crowding-out caused by increased public activities, both in terms of consumption and investment. However, the model applications in exercise 1.5 also showed that the net-effect of an increase in public investment is not zero, but positive, whereas the net-effect of an increase in public consumption appears to be negative. Thus, the effect of public consumption does not only trickle off due to crowding out, but might even deteriorate economic welfare. In contrast, public investment can stimulate aggregate output, because it increases the aggregate capital stock in the economy through higher public capital. Still, private capital is usually assumed to be more productive than public capital, because private agents are assumed to know better than the government how to allocate resources efficiently. Therefore, we have assumed in our model that productivity of public capital is lower than private capital. Following this argumentation, the preferred policy should improve the conditions of private investment and private capital building as this is the most productive asset in the economy.

2 Exercise 2: An and Schorfheide (2007)

2.1 How many observable variables do you need for a Bayesian estimation?

The condition for conducting a Bayesian estimation is that there are at least as many shocks as there are observables. Thus, the number of observable variables must be smaller or equal to the number of shocks. In our case, we should thus declare up to 3 observable variables for which data needs to be simulated. As observables, quarter-to-quarter GDP p.c. growth YGR, annualized quarter-to-quarter inflation rates $INFL$ and annualized nominal interest rates INT are chosen.

2.2 Simulate data for your observables and save these into a matfile called simdat.mat.

Please refer to the mod-file “Gerling_PS3_Ex2.mod” and the simdat.mat-file for this exercise.

2.3 Briefly explain the intuition behind prior information in a Bayesian information (max. 10 sentences).

In Maximum Likelihood estimation, we assume that there exists a true value of the parameter of interest that we seek to estimate by randomly drawing a sample from the population. In contrast, with a Bayesian estimation approach, our parameter of interest is assumed to be a random variable with a probability distribution, whereas the drawn sample is assumed to be fix or given. The distribution of the parameter has an important function in Bayesian estimation techniques, as it summarizes knowledge from 2 sources, the prior information and the sample information. The prior information is the researcher’s subjective belief about the distribution of the parameter and can be justified by economic theory. Prior information influences the shape of the estimated posterior distribution that combines both, the data information from the sample and the non-data information from the prior. In general, the Bayesian estimator reflects the (posterior) distribution of a (random) parameter conditional on the data and the prior density.

The reasoning for using prior information is the general view that more information is better than less and the problem of weak or under-identification of many (macro-)models. This implies, that - depending on the sample size and the quality of the definition of the parameters in the data - the choice of the prior distribution will have more (small sample size; poorly defined parameters) or less (large sample size; well defined parameters) effect on the posterior inference. Common distribution choices for priors include gaussian, Gamma, Beta or uniform distributions.

2.4 Estimate the model with your simulated data and Bayesian methods.

Please refer to the mod-file “Gerling_PS3_Ex2.mod”.

2.5 Consider the figure *Priors and Posteriors.* How would you assess the quality of this estimation exercise?

Looking at the distributions of the priors (grey line) and the posteriors (black line) in figure (17) and (18), the quality of of the estimation - as well as of the prior - appears to be generally good. For the estimated parameters of tau, $psi1$ and p_A, prior and posterior are almost perfectly aligned, implying that the data does not add much information to the posterior. In contrast, the estimation of the other variables, especially with regard to $rhoR$, $rhoG$, $rhoz$ or gam_Q, yields posteriors that differ substantially from the prior information. This implies that the data adds new information to the model and/or that our prior beliefs did not match well.

References

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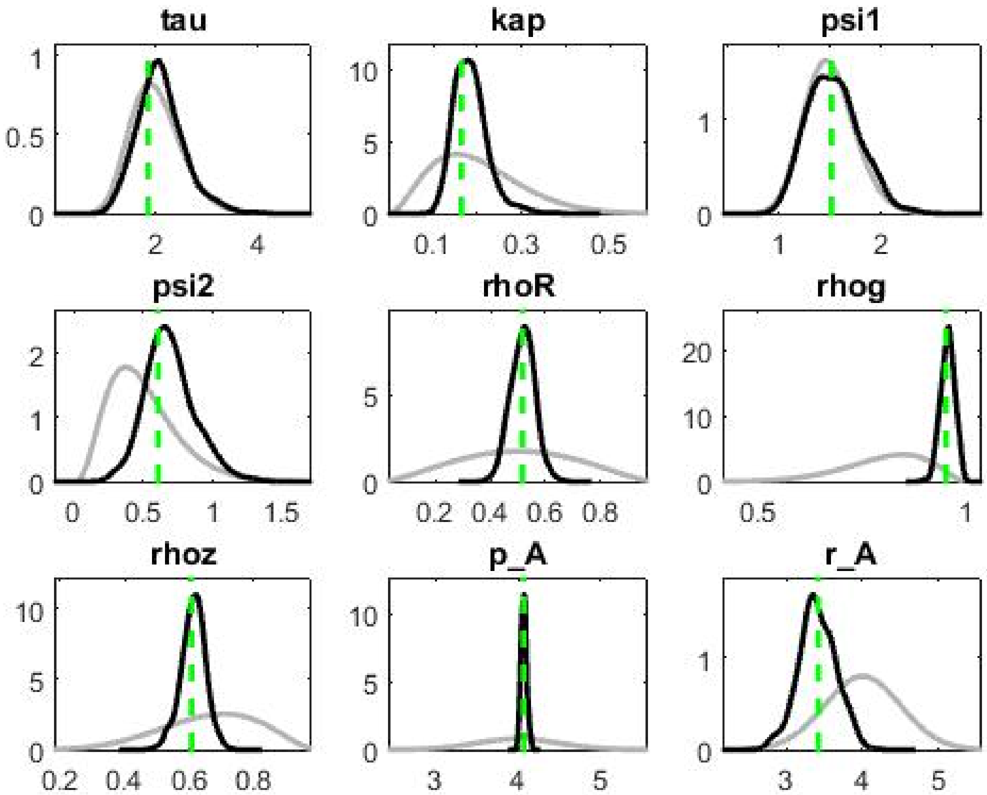

Figure 17: Priors and Posteriors.

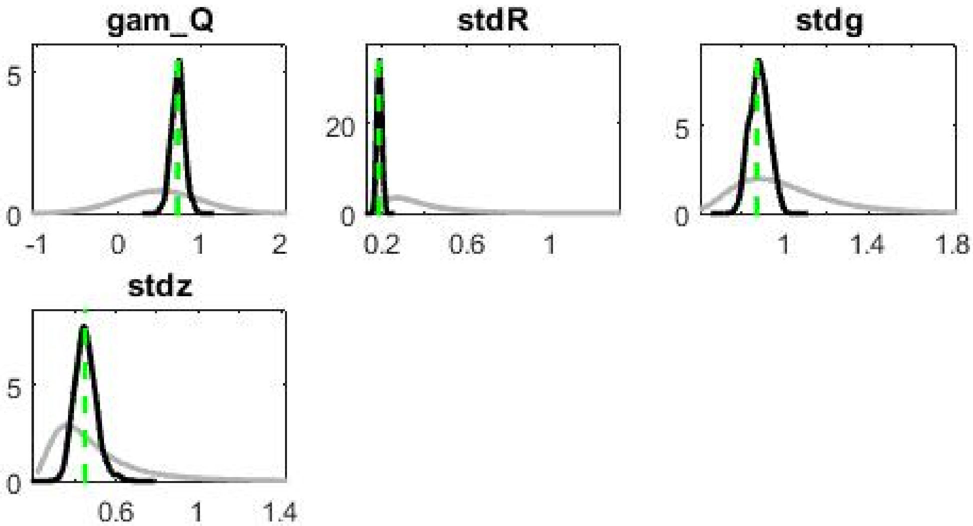

Figure 18: Priors and Posteriors (continued).