Macroeconomics PhD
Problem Set 3

Simon Hoof
Masterstudent VWL
27.08.2015
Matrikelnummer: 412643

Exercise 1

1. The model consists of 13 endogenous variables $\Phi = \left[\tau, r, w, z, C, G^B, I, I^B, K, K^B, N, TR, Y\right]$, four exogenous variables $\Gamma = \left[\varepsilon^\tau, \varepsilon^z, \varepsilon^{G^B}, \varepsilon^{I^B}\right]$ and 13 parameters $\Omega = [\alpha, \beta, \delta, \eta, \theta_l, \rho_\tau, \rho_z, \rho_{G^B}, \rho_{I^B}, \bar{\tau}, \bar{z}, \bar{G}^B, \bar{I}^B]$.

---

2. As noted in the problem set persistence and standard errors shall be given by $\rho_j = 0.75$ and $\sigma_j = 0.01$, $j \in \{z, \tau, G^B, I^B\}$. I'd like to restate the steady state conditions with

$$(1-\bar{\tau})\bar{w} = \theta_l \frac{\bar{C}}{1-\bar{N}} \tag{1}$$
$$1 = \beta(1-\delta+(1-\bar{\tau})\bar{r}) \tag{2}$$
$$\delta\bar{K} = \bar{I} \tag{3}$$
$$\delta\bar{K}^B = 0.02\bar{Y} \tag{4}$$
$$\bar{Y} = \bar{z}(\bar{K}^B)^\eta \bar{K}^\alpha \bar{N}^{1-\alpha} \tag{5}$$
$$\bar{w}\bar{N} = (1-\alpha)\bar{Y} \tag{6}$$
$$\bar{r}\bar{K} = \alpha\bar{Y} \tag{7}$$
$$0 = \bar{\varepsilon}^z \tag{8}$$
$$0.2\bar{Y} + 0.02\bar{Y} + \bar{TR} = \bar{\tau}(\bar{w}\bar{N} + \bar{r}\bar{K}) \tag{9}$$
$$0 = \bar{\varepsilon}^{G^B} \tag{10}$$
$$0 = \bar{\varepsilon}^{I^B} \tag{11}$$
$$0 = \bar{\varepsilon}^\tau \tag{12}$$
$$\bar{Y} = \bar{C} + \bar{I} + 0.2\bar{Y} + 0.02\bar{Y} \tag{13}$$

Plug in the given steady state values and fix $\beta = 0.99$, $\eta = 0.05$ and $\delta = 0.02$.

$$(1-\bar{\tau})2 = 1.5\theta_l \bar{C} \tag{14}$$
$$1 = 0.99(0.98 + (1-\bar{\tau})\bar{r}) \tag{15}$$
$$0.02\bar{K} = \bar{I} \tag{16}$$
$$\bar{K}^B = 1 \tag{17}$$
$$1 = \bar{z}\bar{K}^\alpha \frac{1}{3}^{1-\alpha} \tag{18}$$
$$\frac{1}{3} = \alpha \tag{19}$$
$$\bar{r}\bar{K} = \frac{1}{3} \tag{20}$$
$$0 = \bar{\varepsilon}^z \tag{21}$$
$$0.22 = \bar{\tau}\left(\frac{2}{3} + \bar{r}\bar{K}\right) \tag{22}$$
$$0 = \bar{\varepsilon}^{G^B} \tag{23}$$
$$0 = \bar{\varepsilon}^{I^B} \tag{24}$$
$$0 = \bar{\varepsilon}^\tau \tag{25}$$
$$0.78 = \bar{C} + \bar{I} \tag{26}$$

Now we can derive the other values with

$$\alpha = \frac{1}{3} \tag{27}$$
$$\bar{\tau} = 0.22 \tag{28}$$
$$\bar{r} = \frac{1/\beta - (1-\delta)}{1-\bar{\tau}} \approx 0.0386 \tag{29}$$
$$\bar{K} = \frac{1}{3\bar{r}} \approx 8.6376 \tag{30}$$
$$\bar{I} = 0.02\bar{K} \approx 0.1728 \tag{31}$$
$$\bar{C} = 0.78 - \bar{I} \approx 0.6072 \tag{32}$$
$$\bar{z} = \frac{1}{\bar{K}^{1/3}\frac{1}{3}^{2/3}} \approx 1.0138 \tag{33}$$
$$\theta_l = \frac{1.56}{1.5\bar{C}} \approx 1.7126 \tag{34}$$

---

3. The key difference between stochastic and deterministic models stems from the role of uncertainty. In a deterministic world we have perfect knowledge about all future events including policy actions. Given some initial data we can derive optimal state and control trajectories leading to a steady state which generates the highest outcome, e.g. total utility flows. While in a stochastic setting there is always some randomness involved. We do

not necessarily know, if or when a shock will hit the economy. We can, however, build (mathematical) expectations if the density of those shocks is known. Thus, stochastic models are more flexible, which comes with the cost of tractability.

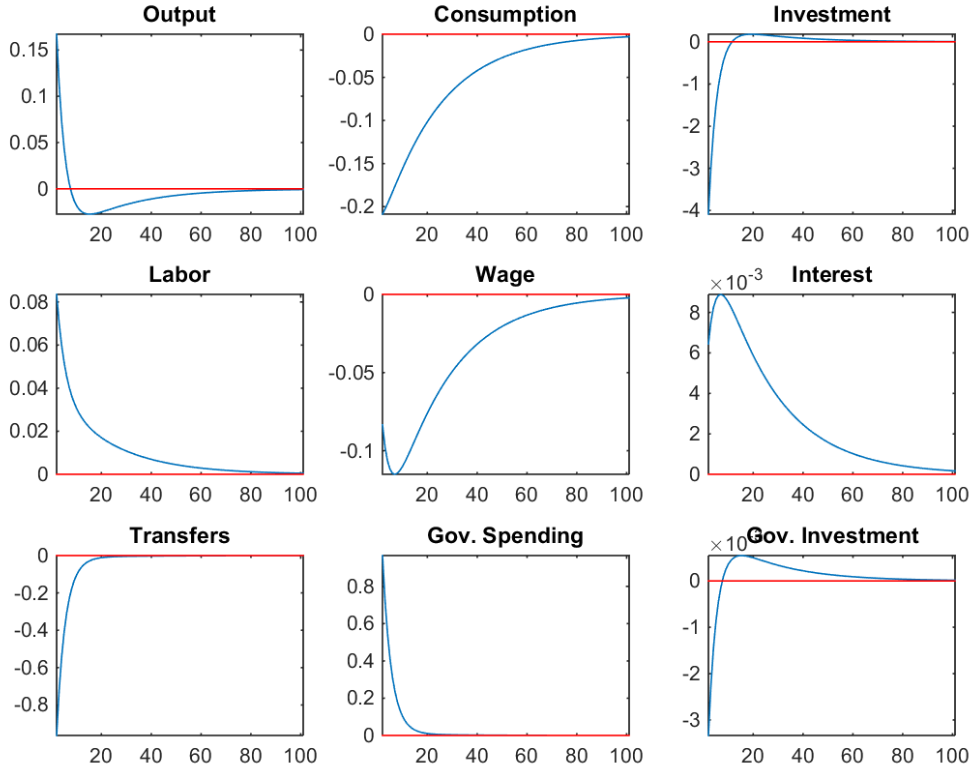
---

5. In figures (1) and (2) we report the impulse response of the observables to an unexpected shock. We distinguish four cases. Panels (1a) and (1b) show the response of a shock on public consumption and public investment respectively where government capital productivity is low $\eta = 0.05$. Panels (2a) and (2b) show the same variables for high productivity $\eta = 0.30$. In all cases the shock is of the same size, that is one percent deviation from the steady state value. I display 100 periods. The horizontal red line gives the steady state. If the red line corresponds to a horizontal axis there is no over- or undershooting, i.e. the observable successively approaches the steady state from above or below. Since all values will tend to the initial steady state there are no permanent effects on the real economy. Note that all shocks are financed by lump sum taxation, that is $\varepsilon^{G^B} = \varepsilon^{I^B} = 1\% = -\Delta TR$. It's important to point out that lump sum taxation is non-distortionary, i.e. there is no substitution effect in the labor supply decision at work. The distinction is necessary with respect to question 6. below. The model captures three important channels that determine the impacts of fiscal policy: the crowding-out effect, positive and negative wealth effects and the effect from changing the marginal product of private inputs. The last effect and the positive wealth effect only occur for a government investment shock.

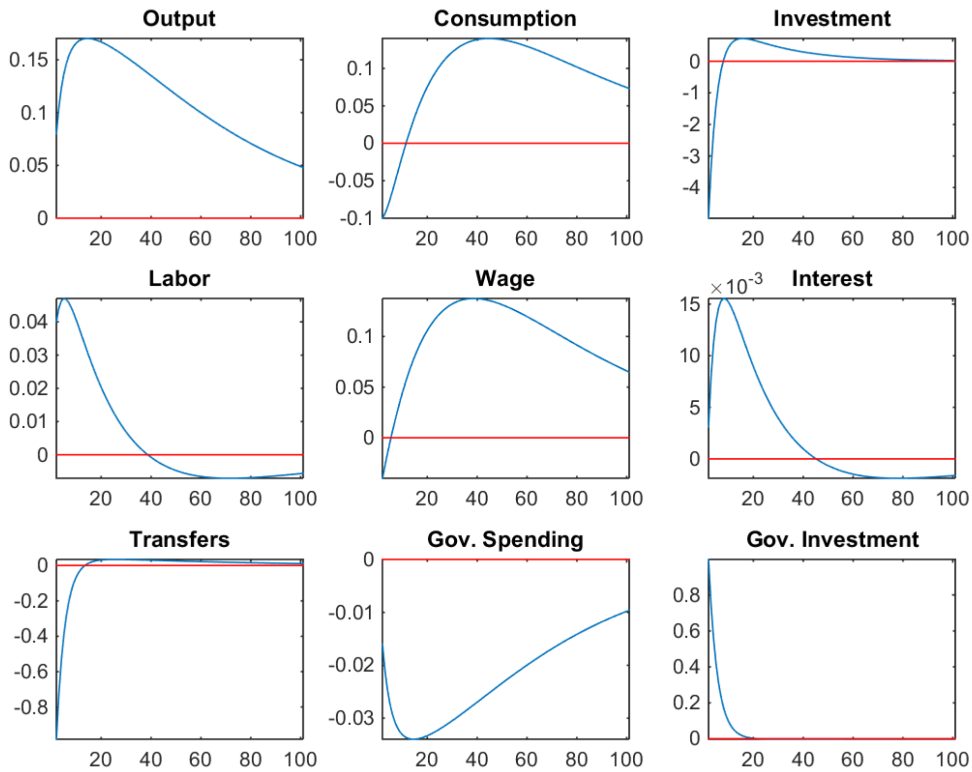
Public Spending Shock Firstly I'd like to point out that η does not effect the impact of the public spending shock, i.e. panels (1a) and (2a) are in fact identical. This is due to the fact that public spending is not productive and the marginal product of inputs do not change.
i) Crowding-out Effect: The crowding-out effect is induced by more competition for goods from higher aggregate demand. In response to a public spending shock existing resources are reallocated and there are fewer goods available for the private sector. The government thus absorbs resources and drives out private consumption and leisure as well as investment. Present goods become more valuable and the interest rate must rise.
ii) Negative Wealth Effect: If the government finances spending by lump sum taxation net income of the household decreases. This is a negative wealth effect. The household tries to compensate the wealth effect by providing more labor and increased savings (income effect). Hence higher labor input is the dominant factor for higher output. Respective prices of capital and labor thus move in opposite directions of the quantities.
On the transition path to steady state we observe overshooting in public as well as private investment and undershooting in output. With investment being negative output decreases, because capital as an input decreases. Then investment overshoots its long run level to rebuild the original capital stock. Since labor is gradually approaching the steady state from above the overshooting of investment must coincidence with an undershooting of output otherwise we cannot reach the dynamic equilibrium. As investment becomes positive output increases and approaches the steady state from below.

It's worth noting that agents are worse off from a welfare perspective. Since consumption steadily returns to equilibrium from below which implies that it's lower in comparison to the original level. While labor from above indicating that it's higher in comparison to the original level. And since utility is an increasing function in consumption and decreasing in labor the total utility stream is lower as the situation without the shock. This implies a forgone of welfare for the sake of temporary boosting the economy.

Public Investment Shock When government spending is productive the first two effects still hold. In addition we must describe the positive wealth effect and the effect on marginal productivity.
iii) Positive Wealth Effect: People tend to save less and consume more when they expect that more goods will be available in the future. A higher stock of productive public capital implies that output will increase over time and thus acts like an increase of total factor productivity. Depending on η the positive wealth effect may dominate the other effects.
iv) Marginal Product Effect: If public capital builds up over time the marginal product of labor and private capital increase. Since inputs are paid their respective marginal products this encourages the household to work and save more due to higher wages and return to capital.
Consider panel (1b). In principle the initial response is the same as with the public spending shock, but the transition is quite different. In the short run, the crowding-out effect dominates, lowering both consumption and saving. As the productive public capital stock gradually accumulates, private investment rises in response to the resulting increase in productivity of private capital. Labor input decreases and wages increase over time due to the positive wealth effect. Note that output is gradually approaching the steady state from above.
Panel (2b) gives a somewhat unorthodox result. The wealth effect is very strong due to a high η. The household increases consumption which means less savings and investment. In addition the household lays off work. The instant impact on output becomes negative even though aggregate demand increases. Wages are positive, because of the marginal product effect, but the interest rate is actually negative, because of the decline in output. Then output increases very sharply and we observe a short run multiplier larger than unity, which is quite remarkable. We may note that the household is unambiguously better off in terms of welfare, because she consumes more and works less. Hence a public investment shock seems to be good policy if public capital is very productive.

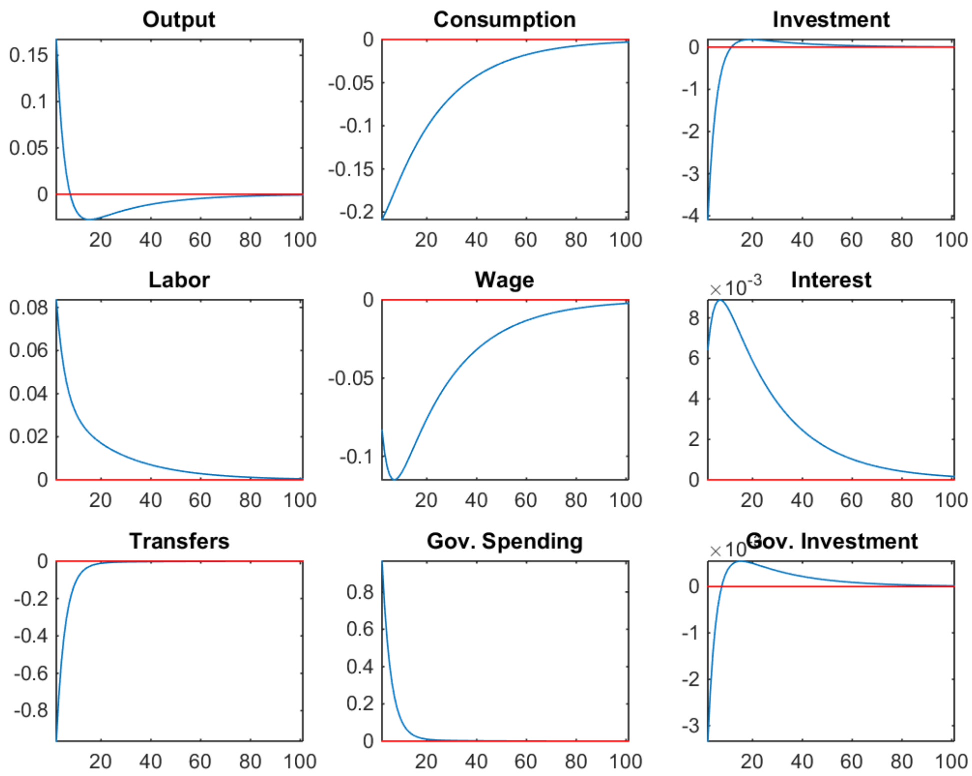


(a) Spending Shock ε^{G^B}

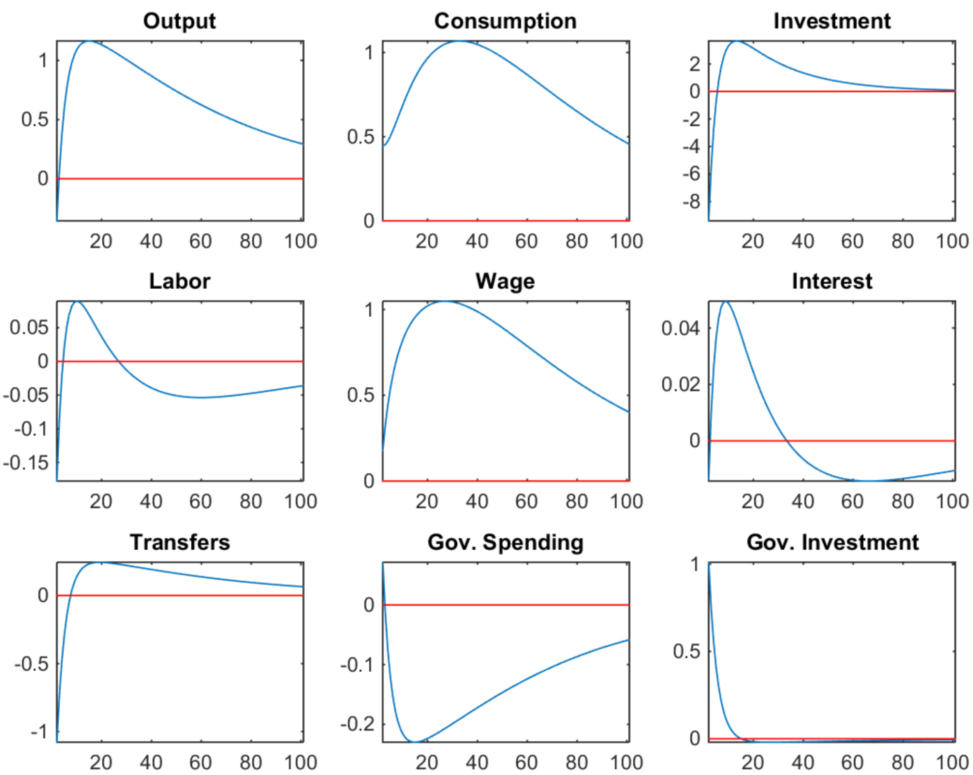


(b) Investment Shock ε^{I^B}

Figure 1: Temporary Shock on Public Spending and Investment $\eta = 0.05$



(a) Spending Shock ε^{G^B}



(b) Investment Shock ε^{I^B}

Figure 2: Temporary Shock on Public Spending and Investment $\eta = 0.30$

6. A permanent increase in the tax rate of one percent $\Delta\tau = 0.01$ means that the government will increase the income tax rate from $\bar{\tau}^0 = 0.22$ to $\bar{\tau}^1 = 0.23$. We can directly solve for ε^τ

$$\ln\left(\frac{0.23}{0.22}\right) = 0.75\ln\left(\frac{0.23}{0.22}\right) + \varepsilon^\tau \Longleftrightarrow \varepsilon^\tau = 0.0111 \tag{35}$$

to simulate a permanent tax shock. The following code snipped tells Dynare to start from the original steady state (`initval`) and where to be located at the end of the simulation (`endval`), i.e. to move towards the new $\bar{\tau}^1$ steady state.

```
initval;
    e_t = 0;
end;
steady;

endval;
    e_t = log(0.23/0.22)-rho_t*log(0.23/0.22);
end;
steady;
```

Figure (3) shows the transition path for the observables. Here we display 200 periods to make sure that the economy is in the new steady state. In contrast to the temporary shock a permanent shock has lasting effects on the real economy, i.e. the economy will not return to the original steady state. The new steady state values are given by

$$\begin{array}{lllll}
\overline{\tau} = 0.23 & \overline{r} = 0.0391 & \overline{w} = 1.9871 & \overline{z} = 1.0138 & \overline{C} = 0.5979 \\
\overline{G}^B = 0.2 & \overline{I} = 0.1682 & \overline{I}^B = 0.02 & \overline{K} = 8.4077 & \overline{K}^B = 1 \\
\overline{N} = 0.3308 & \overline{TR} = 0.0068 & \overline{Y} = 0.9860 & &
\end{array}$$

We may observe that output, consumption, investment, labor input and wages are lower in the new steady state whereas the remaining variables increase.
An increase in the income tax rate is distortionary, because it changes the labor supply as well as the consumption decision. A tax increase reduces the net income of the household. Since working and investing becomes less attractive, the household tend to substitute leisure for labor and consumption for investment (substitution effect). Less labor supply and less investment result in less output. The decline in output makes current goods less valuable and thus the interest rate decreases. However, since savings are crowded-out the interest rate must increase over time which stops the shrinking process of investment. The decline in labor supply puts upward pressure on wages in the short run. There is however an income effect at work too. To stabilize consumption the household increases labor supply and thus wages tend to fall over time. But the substitution effect outweighs the income effect and labor input is lower in the new steady state. In addition the government uses the additional tax

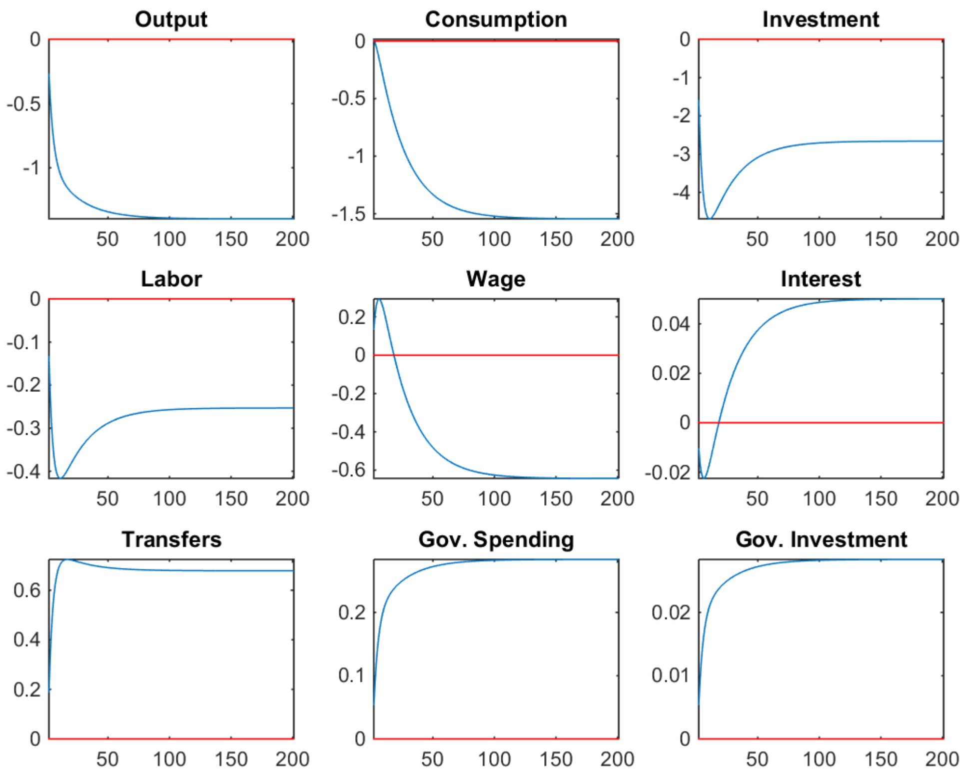


Figure 3: Permanent Tax Shock ε^{τ}

revenue to finance transfers, public spending and public investment. Note that in the very instant of the shock consumption actually *increases*[1] which indicates that the crowding-out effect is outweighed by substituting consumption for investment and the positive wealth effect of the additional transfers. In the aftermath the long run multiplier on output is larger than unity, i.e. $|\Delta Y/\Delta\tau| = |(1-0.986)/(0.22-0.23)| = |-1.4| > 1$.

---

7. Willi's argument relies on prefect Ricardian equivalence, which is: One dollar taken from households to finance one dollar of governmental purchases does not change aggregate demand. The argument may hold for temporary shocks, since the economy will definitely move back to the old equilibrium. We may argue, however, that a government investment shock is still useful, because the short run multiplier on output is larger than unity if public capital productivity is sufficiently large and the household is unambiguously better off in terms of welfare. In addition the shock might be temporary, but lasting for a long time horizon. All in all panel (2b) reports good public policy.
If we consider permanent changes in fiscal policy the economy can definitely be boosted or depressed for the sake of completeness. As noted in figure (2) the economy moves towards a lower plateau. Reverse the argument and we would see that a decrease in tax rates will boost the economy and the effect is lasting. If the tax multiplier is in fact larger than unity

[1] I admit it's hard to see, but consumption is marginally above the red line at $t = 0$. Which can be confirmed by the output and investment panel which look slightly different a the at the top x-axis.

then the fiscal policy is self financing. Caveat: we need to carefully distinguish between lump sum taxation and distortionary income taxes. It might be useful to investigate a permanent transfer shock to evaluate the difference. In addition the results hinge crucially on the labor supply elasticity.
Concerning the difference between private and government capital we may argue that government capital is any kind of capital that the private sector either cannot or will not build. Hence government capital is in principal a public good, because it's a non rival production input. Consider things like roads, electrical grids, airports or infrastructure in general. Education and national security could also count.

Exercise 2

1. For a proper model specification one needs as many observables as exogenous shocks. Here we consider three structural shocks (monetary policy, government spending, technology growth) and three observables (output growth, inflation, interest rates).

---

3. The idea of Bayesian estimation is to combine subjective beliefs with objective data to estimate a parameter. The subjective belief is the prior with some distribution. You update your prior with gathered data to obtain a posterior distribution of the parameter. If the prior is perfectly uninformative the estimation solely relies on the data. If it is perfectly informative than the posterior is the prior. In general we would argue that the posterior is a combination of the prior and the data. So in principle if your prior is very informative, you'll need a lot of data to change your mind about the prior. And if you have a lot of data, the data will presumably dominate the posterior distribution.

---

5. Figure (4) shows the prior (grey) and posterior (black) distribution of the estimated parameters. If prior and posterior are indistinguishable and overlap just perfectly, then the subjective prior does not hold extra information already given by the data. We may argue that this is the case for τ (`tau`), ψ_1 (`psi1`) and roughly for ψ_2 (`psi2`). In all other cases they differ substantially, i.e. the prior adds information which is not given in the data. To asses the quality of the estimation we need a criterion. We can tell that the estimation is of high quality if the variance of the posterior distribution is low. That is, the closer the black line surrounds the dashed green line the truer the estimated value. High quality is clearly given for $\pi^{(A)}$ (`p_A`) and σ_R (`stdR`). Nevertheless the other parameters are well approximated too, since even though the distribution of the prior is wide the posterior distribution is quite narrow.

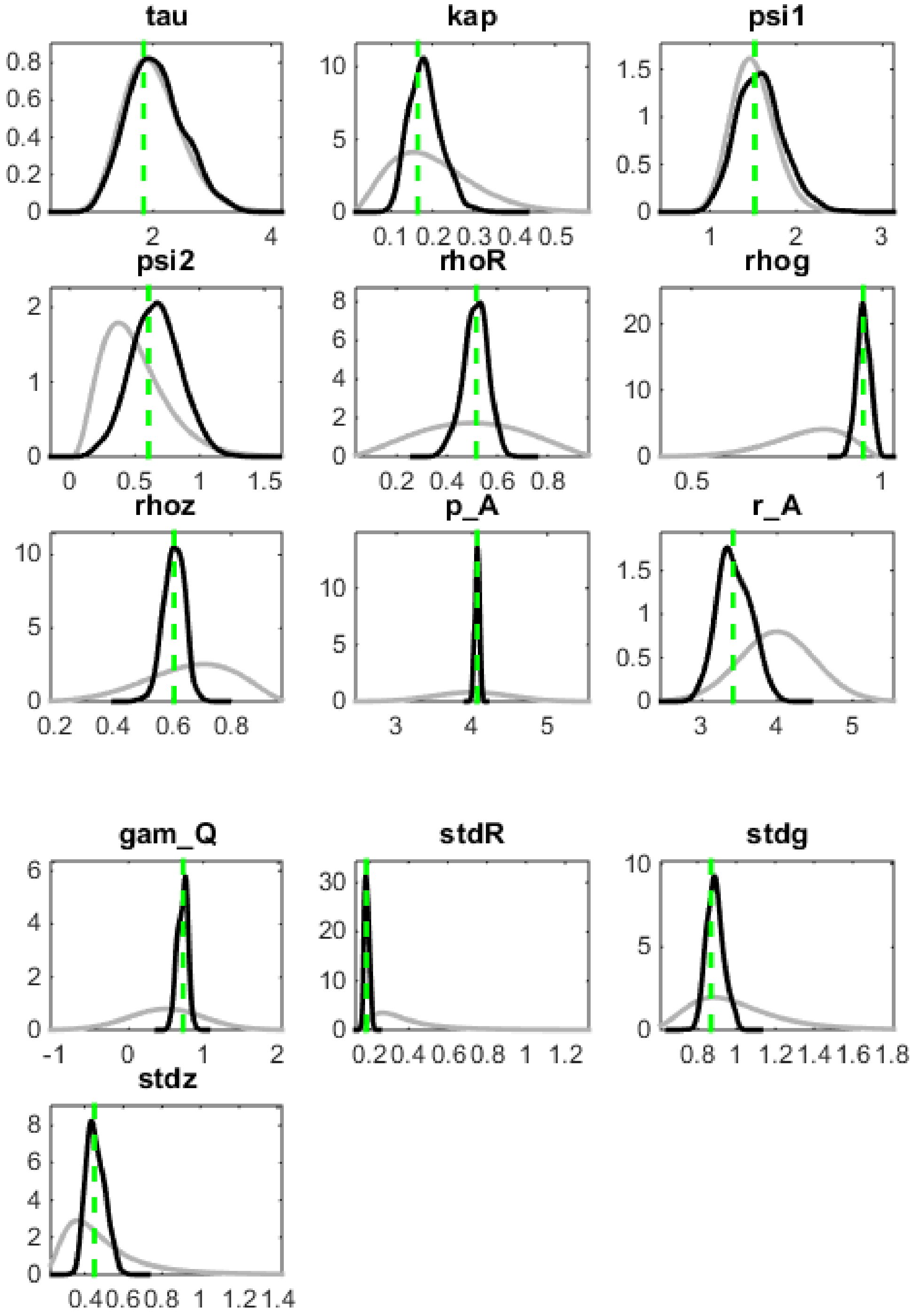


Figure 4: Priors and Posteriors