Structured PhD-Program (Summer Term 2015)
Advanced Macroeconomics

Problem Set 3

DSGE Models

Prof. Dr. Bernd Kempa

Institute of International Economics
Muenster School of Economics and Business
Muenster University
Supervisor: Willi Mutschler

Deadline: September 7th, 2015

Helena Helfer, M.Sc.
PhD Student at the Chair of Political Economy
Prof. Dr. Thomas Apolte
Scharnhorststrasse 100
48151 Muenster
Phone: 0251 / 83 - 25107
Email: *helena.helfer@wiwi.uni-muenster.de*

Contents

1 Baxter and King (1993)

1.1 Endogenous and exogenous variables, model parameters

The endogenous variables in the model are those that are determined within the system. In the model at hand, the endogenous variables are as follows:

- Y_t = Output
- dY_t = Change in output
- C_t = Real private consumption
- dC_t = Change in real private consumption
- I_t = real private investment
- dI_t = Change in real private investment
- r_t = Real interest rate
- dr_t = change in real interest rate
- K_t = Private capital
- N_t = real labor supply
- dN_t = Change in real labor supply
- w_t = Real wage rate
- dw_t = Change in real wage rate
- G_t^B = Public consumption
- dG_t^B = Change in public consumption
- I_t^B = Public investment
- dI_t^B = Change in public investment
- K_t^B = Public capital
- TR_t = Real lump-sum transfers
- dTR_t = Change in real lump-sum transfers

- τ_t = tax rate
- z_t = productivity

The exogenous variables are not determined within the system. In model at hand, they are the shocks in productivity, public consumption, public investment and the tax rate:

- ε_t^z = productivity shock
- $\varepsilon_t^{G^B}$ = shock on public consumption
- $\varepsilon_t^{I^B}$ = shock on public investment
- ε_t^τ = shock on tax rate

The model parameters are the variables, that we chose beforehand. In the model at hand, these parameters are calibrated as:

- β = discount rate of the household's life-time utility
- δ = depreciation rate of capital
- η = productivity of public capital
- θ = Frisch elasticity of labor supply
- α = share of capital in production

- ρ_z = smoothing parameter in productivity
- ρ_{G^B} = smoothing parameter in public consumption
- ρ_{I^B} = smoothing parameter in public investment
- ρ_τ = smoothing parameter in taxation

- $\bar{Y}$ = steady-state value of output
- $\bar{G}^B$ = steady-state value of public consumption
- $\bar{I}^B$ = steady-state value of public investment
- $T\bar{R}$ = steady-state value of transfers

- $\bar{w}$ = steady-state value of wage rate
- $\bar{N}$ = steady-state value of labor supply
- $\bar{\tau}$ = steady-state value of tax rate
- $\bar{z}$ = steady-state value of productivity
- $\bar{K}^B$ = Steady-state value of public capital
- $\bar{r}$ = Steady-state value of the interest rate
- $\bar{K}$ = Steady-state value of private capital
- $\bar{I}$ = Steady-state value of private investment
- $\bar{C}$ = Steady-state value of private consumption

1.2 Steady states

The following equations summarize the Baxter and King (1993) model:

Consumption-leisure choice of the household:

$$(1-\tau_t)w_t = \theta_l \frac{C_t}{1-N_t} \tag{1}$$

Savings decision:

$$\lambda_t = \beta E_t\{\lambda_{t+1}[(1-\delta)+(1-\tau_{t+1})r_{t+1}]\} \tag{2}$$

where $\lambda_t = \frac{1}{C_t}$.

Private and public capital stocks:

$$K_t = (1-\delta)K_{t-1} + I_t \tag{3}$$

and

$$K_t^B = (1-\delta)K_{t-1}^B + I_t^B \tag{4}$$

Production function of the firm:

$$Y_t = z_t(K_{t-1}^B)^\eta (K_{t-1})^\alpha (N_t)^{1-\alpha} \tag{5}$$

Factor demands:

$$w_t(N_t) = (1-\alpha)Y_t \tag{6}$$

and

$$r(K_t) = \alpha Y_t \tag{7}$$

Budget constraint of the fiscal authority:

$$G_t^B + I_t^B + TR_t = \tau_t(w_t N_t + r_t K_{t-1}) \tag{8}$$

Market clearing:

$$Y_t = C_t + I_t + G_t^B + I_t^B \tag{9}$$

The remaining four equations describe the exogenous shocks:

Productivity:

$$log(\frac{z_t}{\bar{z}}) = \rho_z log(\frac{z_{t-1}}{\bar{z}}) + \varepsilon_t^z \tag{10}$$

Public consumption:

$$G_t^B - \bar{G}^B = \rho_{G^B}(G_{t-1}^B - \bar{G}^B)\varepsilon_t^{G^B} \tag{11}$$

Public investment:

$$I_t^B - \bar{I}^B = \rho_{I^B}(I_{t-1}^B - \bar{I}^B)\varepsilon_t^{I^B} \tag{12}$$

Taxation:

$$log(\frac{\tau_t}{\bar{\tau}}) = \rho_\tau log(\frac{\tau_{t-1}}{\bar{\tau}}) + \varepsilon_t^\tau \tag{13}$$

Overall, we have 22 endogenous variables and 22 model equations. Additional information on target variables and parameters is given in the exercise:

- $\bar{Y} = 1$
- $\bar{G^B} = 0.2 * \bar{Y} = 0.2$
- $\bar{I^B} = 0.02 * \bar{Y} = 0.02$
- $\bar{TR} = 1$
- $\bar{w} = 2$

- $\bar{N} = 1/3$
- $\rho_z = \rho_{G^B} = \rho_{I^B} = \rho_\tau = 0.75$
- $\eta < \alpha$

To derive the steady states of all other endogenous variables and the implied parameter values, we first set some reasonable values for β, δ and η, following the RBC-example in the lecture-slides:

- $\beta = 0.99$
- $\delta = 0.02$
- $\eta = 0.2$

We then derive the steady states for all other endogenous variables as follows:

For public capital (K^B), eq. (4) is evaluated at the steady state[1]:

$$\bar{K^B} = (1-\delta)\bar{K^B} + \bar{I^B}. \tag{14}$$

Rearranging and inserting the values from above, this yields:

$$\bar{K^B} = \frac{\bar{I^B}}{\delta} = \frac{0.02}{0.02} = 1. \tag{15}$$

For the share of capital in production (α), eq. (6) is evaluated at the steady state:

$$\bar{w}\bar{N} = (1-\alpha)\bar{Y}. \tag{16}$$

Rearranging and inserting the values from above, this yields:

$$\bar{\alpha} = 1 - \bar{w}\frac{\bar{N}}{\bar{Y}} = \frac{1}{3}. \tag{17}$$

For private capital (K), eq. (7) is evaluated at the steady state:

$$\bar{r}\bar{K} = \alpha\bar{Y}. \tag{18}$$

[1] Note that the equation numbers refer to the document at hand and not to the enumeration in the exercise sheet.

Rearranging, this yields:

$$\bar{K} = \frac{\alpha}{\bar{r}}\bar{Y}. \tag{19}$$

For the tax rate (τ), eq. (8) is evaluated at the steady state:

$$\bar{G}^B + \bar{I}^B + \bar{TR} = \bar{\tau}(\bar{w}\bar{N} + \bar{r}\bar{K}). \tag{20}$$

Rearranging, this yields:

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \bar{r}\bar{K}}. \tag{21}$$

For the real interest rate (r), eq. (2) is evaluated at the steady state:

$$\frac{1}{\bar{C}} = \beta\frac{1}{\bar{C}}[(1-\delta) + (1-\bar{\tau})\bar{r}]. \tag{22}$$

Rearranging, this yields:

$$\bar{r} = \frac{\frac{1}{\beta} - 1 + \delta}{1 - \bar{\tau}}. \tag{23}$$

Now, we can substitute eq. (19) into eq. (21):

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \bar{r}\frac{\alpha}{\bar{r}}\bar{Y}}. \tag{24}$$

Rearranging and inserting the values from above, this yields:

$$\bar{\tau} = \frac{\bar{G}^B + \bar{I}^B + \bar{TR}}{\bar{w}\bar{N} + \alpha\bar{Y}} = 0.22. \tag{25}$$

The value for τ can now be substituted into eq. (23). Inserting also the other steady state values, this gives:

$$\bar{r} = 0.04. \tag{26}$$

Finally, we can substitute the value for r into eq. (19) and solve for the steady state of private capital:

$$\bar{K} = 8.3. \tag{27}$$

Next, the production function is evaluated at the steady state:

$$\bar{Y} = \bar{z}(\bar{K}^B)^\eta(\bar{K})^\alpha(\bar{N})^{1-\alpha} \tag{28}$$

Solving this for z yields:

$$\bar{z} = \frac{\bar{Y}}{(\bar{K}^B)^\eta(\bar{K})^\alpha(\bar{N})^{1-\alpha}} = 1.03. \tag{29}$$

For private investments, eq. (3) is evaluated at the steady state and solved for I. This gives:

$$\bar{I} = \delta\bar{K} = 0.17 \tag{30}$$

Next, the market clearing condition in eq. (9) is evaluated at the steady state and solved for consumption (C):

$$\bar{C} = \bar{Y} - \bar{I} - \bar{G}^B - \bar{I}^B = 0.61. \tag{31}$$

Finally, the consumption-leisure choice in eq. (1) is evaluated at the steady state and solved for the Frisch elasticity of labor (θ):

$$\theta = \frac{(1-\bar{N})(1-\bar{\tau})\bar{w}}{\bar{C}} = 1.7. \tag{32}$$

As there a no shocks in the steady state equilibrium, evaluating eq. (10) - (13) at the steady state yields that the respective shocks are zero. We now have a full set of model equations, parameter values and steady-state conditions for the endogenous variables that allows us to estimate the model in Dynare. The following table summarizes the calculated parameter values and provides their notation in the dynare code:

Variable name	Notation in DYNARE	Value
Discount rate of the household's life-time utility	beta	0.99
Depreciation rate of capital	delta	0.02
Productivity of public capital	eta	0.2
Share of capital in production	alpha	$\frac{1}{3}$
Frisch elasticity of labor supply	theta	1.7
Smoothing parameter in productivity	rhoz	0.75
Smoothing parameter in public consumption	rhogb	0.75
Smoothing parameter in public investment	rhoib	0.75
Smoothing parameter in taxation	rhotau	0.75
Steady-state value of output	ybar	1
Steady-state value of public consumption	gbbar	0.2
Steady-state value of public investment	ibbar	0.02
Steady-state value of transfers	trbar	0
Steady-state value of wage rate	wbar	2
Steady-state value of labor supply	n	$\frac{1}{3}$
Steady-state value of tax rate	taubar	0.22
Steady-state value of productivity	zbar	1.03
Steady-state value of public capital	kbbar	1
Steady-state value of the interest rate	rbar	0.04
Steady-state value of private capital	kbar	8.3
S Steady-state value of private investment	ibar	0.17
Steady-state value of private consumption	cbar	0.61

1.3 Deterministic vs. stochastic models

The most important difference between deterministic and stochastic models lies in the knowledge about future shocks, for the occurrence of all future shocks is exactly known at the time when the solution is computed in a deterministic environment while in a stochastic environment only the distribution of future shocks is known. In general, deterministic models imply the existence of full information, perfect foresight and no uncertainty around shocks. The latter can occur anytime and can last for one or several periods, but they will always be expected. Deterministic models do not rely on linearization, but on numerical solution techniques. Thus, when the economy is far from its steady state and linearization offers a non-sufficient approximation, deterministic models are especially useful.From a practical perspective, they provide a first assessment of the model, that will then be analyzed and estimated using a more meaningful stochastic model.

In stochastic models, shocks always come as a surprise, and afterwards, their expected value is zero. The fact that stochastic models require linearization, oftentimes with a Taylor approximation around the steady state, makes expected future shocks and permanent changes in exogenous variables impossible to manage in such a model. In the case that a stochastic model is linearized to the first order, the agents behave as though future shocks are zero - but this does not assign deterministic properties to the model. Stochastic models are more in literature than deterministic models, especially with real business cycle models or new keynesian monetary models, and they are particularly useful in estimation.

Overall, we would recommend the use of a stochastic model, since it provides more insights for analysis and when estimating than a deterministic model.

1.4 DYNARE

Cf. mod file $BaxterandKing\_Helfer$

1.5 Unexpected temporary public consumption / investment shock

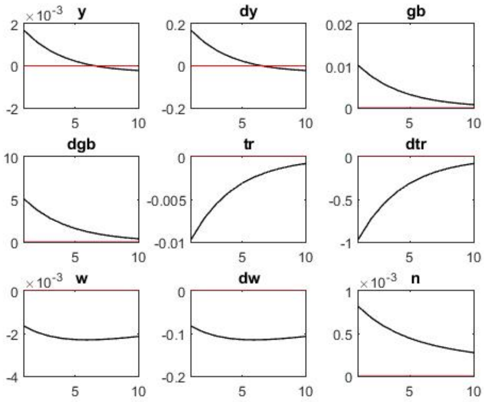

Figure 1: Shock to gb I.

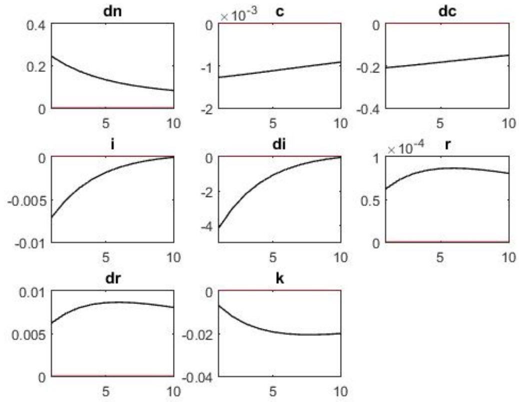

Figure 2: Shock to gb II.

Figures 1 and 2 display the shock in public consumption and how it feeds through our model. As expected, the positive shock induces an increase in public consumption of roughly 5 percentage points. Among the most interesting impulse response functions to analyze is private consumption, which is reduced by the rise in public consumption, and does not return to zero after ten periods, therefore the effect might be permanent. This can be explained by a partial crowding out effect. It is also interesting to study the interest rate, which increases after the shock and then peaks after roughly five periods, to decrease slightly - approximately the same response is visible in the deviation from the steady state of the interest rate. Since an increase in public consumption increases the need for capital, which dies not multiply with the shock, an increase in the interest rate is intuitive. As a reaction to the increasing interest rate, private investment drops - albeit only temporarily, and returns to zero after 10 periods. The negative effect of the shock on private investment (temporarily) and on private consumption (maybe permanently, or at least longer lasting) is also visible in the output. Even though it increases immediately after the shock, it drops below its old steady state after ten periods. Real labor supply and real wages are also affected negatively by the shock in public consumption, which can be again explained with the crowing-out of private investment and consumption. Also, public transfers decrease temporarily following the shock. Given the balanced budget constraint of the government this response is intuitive.

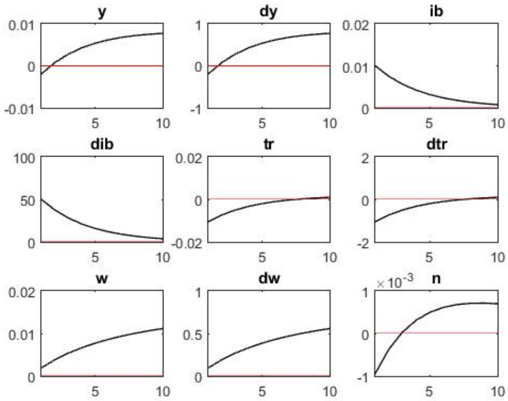

Figure 3: Shock to ib I.

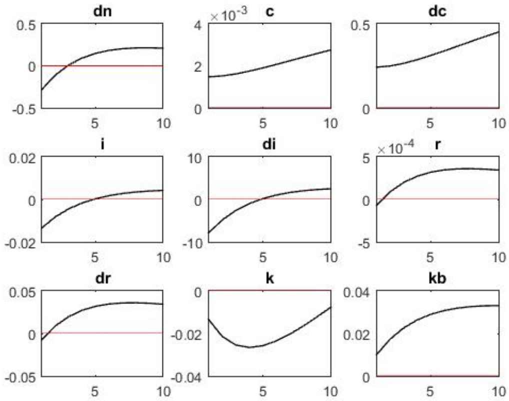

Figure 4: Shock to ibII.

Figures 3 and 4 display the shock in public investment and how it feeds through our model. The shock is of the same size (i.e. with a standard error of 0.0001) as before. Unlike before, we observe a positive effect of the shock on the output. Even though output decreases ever so slightly in the aftermath of the shock, it increases permanently to a level above its old steady state after ten periods. Also, we observe a permanent increase in private consumption. A possible explanation lies in the Keynesian multiplier of public investment that enhances private consumption through increased employment and wages. This theoretical evidence is supported by the positive effect of dn and dw that both increase. Still, the effect of the shock is not outright positive since again we observe a crowding out: rising public expenditures lead to a crowing out of private investment. The shock leads to a much more pronounced rise in public investment as compared to public consumption. Accordingly, the real interest rate increases more strongly and this effect appears to be permanent after 10 periods, unlike before. As observed before, the rise in the interest rate leads to a partial crowding our of private investments.

Figures 5 to 8 display the effect of a higher productivity of public capital, e.g. $\eta = 0.5$ (Cf. mod file $BaxterandKing2\_Helfer$). Interestingly, the changes in responses to the shock in public consumption are minor, while they are more pronounced for the shock in public investment, especially in the output.

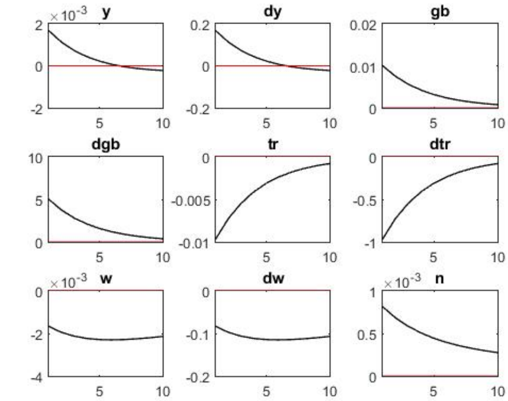

Figure 5: Shock to gb with higher η I.

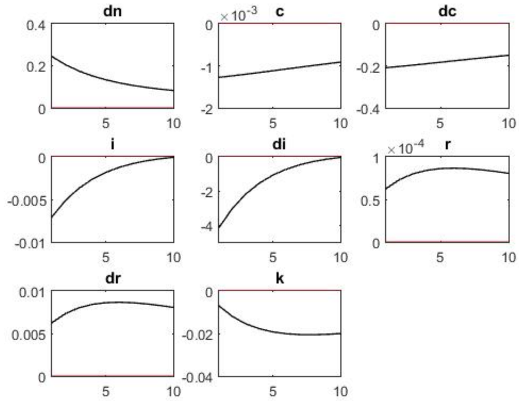

Figure 6: Shock to gb with higher η II.

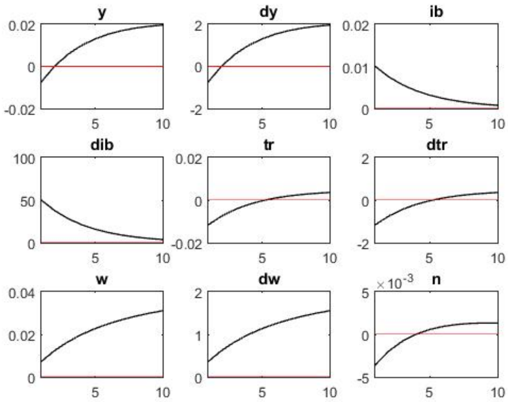

Figure 7: Shock to ib with higher η I.

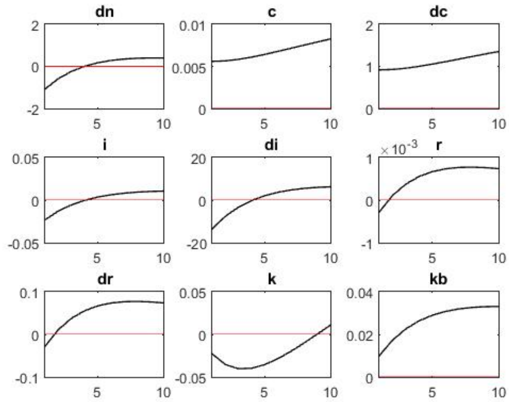

Figure 8: Shock to ib with higher η II.

1.6 Increase in the tax rate by one percentage point

A permanent increase in the tax rate by one percentage point does not damage the economy permanently, it only has a temporary effect and thus hardly affects the long-run equilibrium. Figures 9 and 10 display the response of the absolute output and of the output in deviations from its steady state. The new steady state value of the aggregate output is at 1.01057 only slightly lower than the previous steady state value at 1.02348. Figures 11 shows how the increase in the tax rate leads to a rise in transfers, which can be interpreted as means of subsidizing private households, which suffer a decrease in public consumption following the increase in taxation. Please refer to the DYNARE code for the responses of the other variables.

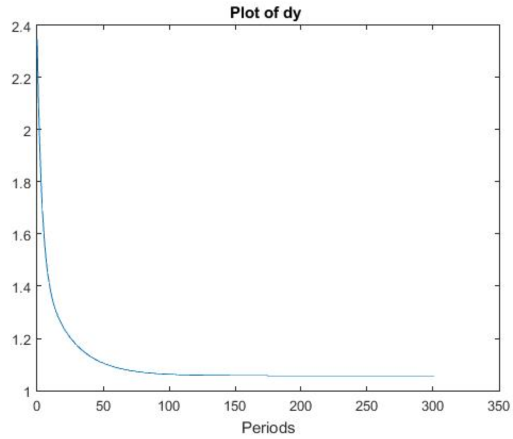

Figure 9: Effect of shock to τ on dY.

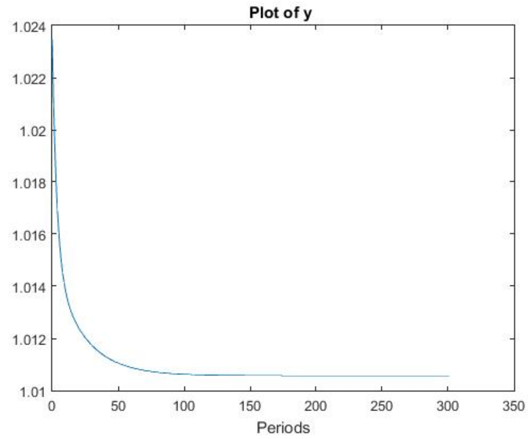

Figure 10: Effect of shock to τ on Y.

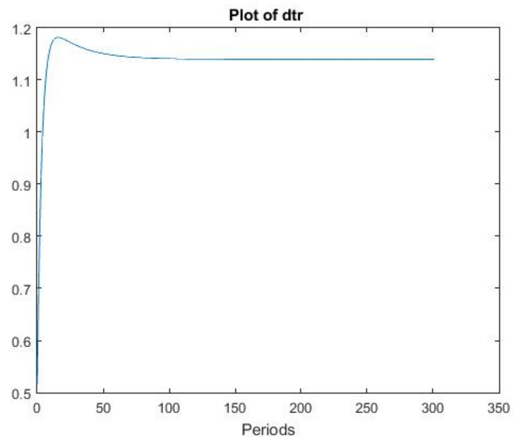

Figure 11: Effect of shock to τ on dtr.

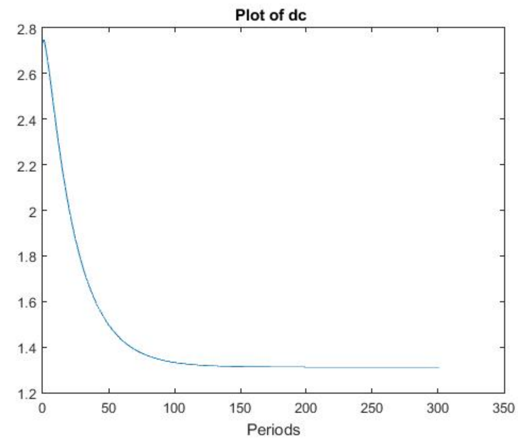

Figure 12: Effect of shock to τ on C.

1.7 Crowding out?

As we have seen above, a shock in public consumption leads to a crowding out of private consumption, and a shock in public investment leads to a crowding out in private investment. This seems to confirm the crowding out hypothesis that Willi states. But since an economy is made of many more variables than just consumption and investment, the more relevant variable might be the overall output. We have seen that the aggregate out did respond favorably to a shock in public investment while it did not react favorably to a shock in public consumption. The intuitive reason for this observation is that public investment adds to the public capital stock, which in turn increases the capital stock of the economy as a whole. The increased capital stock can be put to productive uses and thus increase economic welfare. Since private capital is in general and in our model assumed to be ore productive, because private agents are closer to the market than the government, policies should be aimed at both kinds of capital. focus on both kinds of capital.

2 An and Schorfheide (2007)

2.1 Number of observable variables

In Bayesian estimation, the number of shocks needs to be less or equal to the number of observable variables, because otherwise stochastic singularity occurs. Since three shocks are predefined in the model, we need three observable variables. In the given model, the observable variables are quarter-to-quarter per capita GDP growth YGR, annualized quarter-to-quarter inflation rates $INFL$ and annualized nominal interest rates INT. They are included into the mod file using the *varobs* command.

2.2 Data simulation

Cf. mod file $AnSchoSolution\_Helfer.mod$

2.3 Prior information in Bayesian estimation

Prior probability distribution ("prior" for short) in Bayesian inference means refers to information of uncertain quality about an unknown parameter or about a latent variable. The existence of a prior for any problem is generally justifiable with axioms from decision theory. Two key issues arise when setting up prior distributions, firstly regarding the information is going into the prior distribution and secondly regarding the properties of the resulting posterior probability distribution, which is obtained by multiplying the prior by the likelihood function and then normalizing. Depending on the sample size and the quality of the definition of the parameters in the data, the choice of the prior distribution will have more (small sample -size and poorly defined parameters) or less (large sample size and well defined parameters) effect on the posterior inference. Common distribution choices for priors include gaussian, Gamma, Beta or uniform distributions. In choosing an appropriate prior, one has to consider lower and upper bounds as well as skewness and curtosis of a distribution. Since results may vary due to the choice of the priors and their parametrization, one should try a different paramterization and more general priors to check for the robustness of the results. overall, the idea can be summarized as matching known information (data) with additional believes regarding the probability distribution of the parameters (priors) in order to obtain an expression for the conditional probability of these parameters.

2.4 Model estimation

Cf. mod file $AnSchoSolution\_Helfer.mod$

2.5 Figures "Priors and Posteriors"

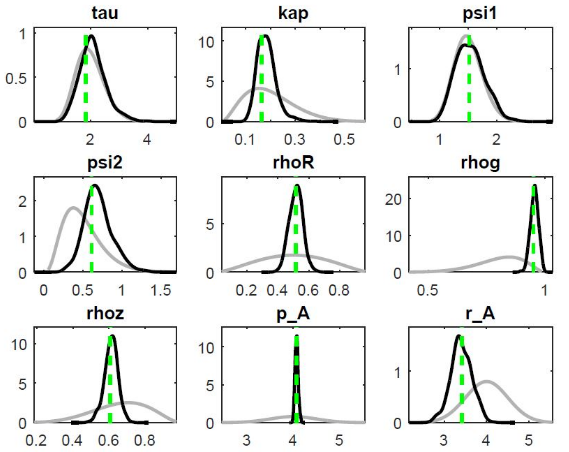

Figure 13: Priors and Posteriors I.

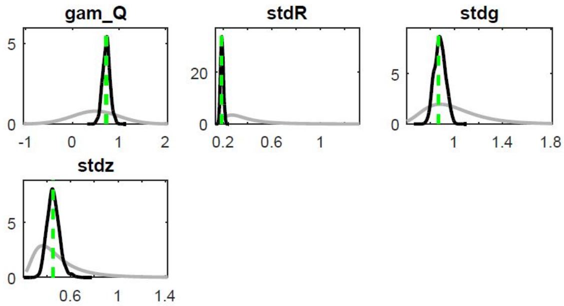

Figure 14: Priors and Posteriors II.

Looking at the distributions of the priors (grey lines) and the posteriors (black lines), it is apparent that the quality of the estimation is good overall. In the cases of tau, $psi1$ and p_A, prior and posterior are are almost perfectly aligned, which slightly reduces the quality of the estimation. In case of the other parameters, especially when considering e.g. $rhoR$, $rhoG$, $rhoz$ or gam_Q it is obvious that prior and posterior are distinctly different from one another. In general, when prior and posterior are not aligned, it implies that the estimation added a lot of new information to the model, which is a sign of its quality.

References

An, Sungbae and Frank Schorfheide (2007): Bayesian Analysis of DSGE Models, in: Econometric Reviews, Vol. 26, No. 2-4, pp.113-172.

Baxter, Marianne and Robert G. King (1993): Fiscal Policy in General Equilibrium, in: American Economic Review, Vol. 83, No. 3, pp. 315-334.

Gelman, Andrew (2002): Prior distribution, in: Encyclopedia of Environmetrics, Edited by Abdel H. El-Shaarawi and Walter W. Piegorsch, Vol. 3, pp. 1634-1637.

Griffoli, Thomas Mancini (2013): Dynare User Guide. An introduction to the solution and estimation of DSGE models, Public beta version.

Mutschler, Willi (2015): PhD Macroeconomics, DSGE models, Lecture slides.

Ruge-Murcia, Francisco J. (2003): Methods to Estimate Dynamic Stochastic General Equilibrium Models, Centre interuniversitaire de recherche en économique quantitative CIREQ, Cahier 17-2003.