# DSGE-Models

## Limited Information Estimation
## General Method of Moments and Indirect Inference

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Summer 2012

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# Limited Information Estimation

- Estimating the parameters of a DSGE-models one has to cope with several difficulties: Expectations about future variables, non-linearities, stochastic processes. . .
- For such cases there exist very general econometric methods like the *General Method of Moments (GMM)* and the method of *Indirect Inference*.
- *Limited-information-estimators*, since there is no likelihood, but only specific moments of interest that are adjusted to data *(Matching-Moments)*.

# General Method of Moments

- Developed by Hansen (1982), first to use it for DSGE-models were Christiano and Eichenbaum (1992) and Burnside, Eichenbaum and Rebelo (1993).
- Idea: Represent DSGE-model as *moment-* or *orthogonality-conditions*:

$$E\left[\mathfrak{g}(\underset{k\times 1}{\boldsymbol{\mu}}, \underset{a\times 1}{\boldsymbol{\Upsilon}_{\mathbf{t}}}\right] = E\begin{bmatrix} \mathfrak{f}_1(\underset{d\times 1}{\mathbf{w_t}}, \underset{k\times 1}{\boldsymbol{\mu}})\underset{l\times 1}{\mathbf{u_t}} \\ \vdots \\ \mathfrak{f}_{\mathfrak{m}}(\mathbf{w_t}, \boldsymbol{\mu})\,\mathbf{u_t} \end{bmatrix} = \mathbf{0},$$

- $\boldsymbol{\mu}$ is the true vector of parameters, $\mathbf{w_t}$ a matrix of exogenous variables, $\mathbf{u_t}$ a matrix of instruments and $\boldsymbol{\Upsilon}_{\mathbf{t}} = (\mathbf{w_t}', \mathbf{u_t}')'$.
- Vector-valued functions: $\mathfrak{g} : r \times 1$ and $\mathfrak{f}_{\mathfrak{i}} : l \times 1$.
- Number of orthogonality-conditions is equal to $r = ml$.

# General Method of Moments

- Orthogonality-conditions are derived from the first-order-conditions, the *steady-state* relations and the properties of the stochastic processes.
- Find the estimator $\widehat{\boldsymbol{\mu}}_{\mathbf{G}}$, that solves the empirical analogous of the orthogonality-conditions as „close“ as possible, with the weight-matrix $\boldsymbol{\Omega}$ defining, what „close“ means.

**GMM-estimator**

$$\widehat{\boldsymbol{\mu}}_{\mathbf{G}} = \min_{\mu} \left\{ \left( \frac{1}{T} \sum_{t=0}^{T} \mathfrak{g}(\boldsymbol{\mu}, \boldsymbol{\Upsilon}) \right)' \times \boldsymbol{\Omega} \times \left( \frac{1}{T} \sum_{t=0}^{T} \mathfrak{g}(\boldsymbol{\mu}, \boldsymbol{\Upsilon}) \right) \right\}.$$

# General Method of Moments

- If $r < k$, then the model is under-identified $\Rightarrow$ find additional instruments: lagged variables or use the fact that the shocks are not correlated.
- If $r = k$, then the model is exactly-identified: The weight-matrix does not play any role, since there is a unique solution to the quadratic form.
- If $r > k$, then the model is over-identified $\Rightarrow$ several solutions that can be derived either analytically or numerically. The weight-matrix picks those moment-conditions that lead to a more precise estimation.
  - Hansen (1982) shows, that the optimal weight matrix is given by the inverse of the variance-covariance-matrix of the empirical analogous.
  - Given some regularity conditions one can show that $\sqrt{T}(\widehat{\boldsymbol{\mu}} - \boldsymbol{\mu})$ is gaussian.
  - In the over-identified case this enables one to formally test the hypothesis, that the model is able to describe the data generating process (*J-Test*).

Simple Euler-equation:

$$
\begin{aligned}
\beta E_t\left[c_{t+1}^{-\tau}(1+r_{t+1}-\delta)\right] &= c_t^{-\tau} \\
\Leftrightarrow E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\tau}(1+r_{t+1}-\delta)\right] &= 1 \\
\Leftrightarrow E_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\tau}(1+r_{t+1}-\delta)-1\right] &= 0 \\
\Rightarrow E_t\left\{\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\tau}(1+r_{t+1}-\delta)-1\right]\begin{pmatrix}1\\ \frac{c_t}{c_{t-1}}\\ r_t\end{pmatrix}\right\} &= \mathbf{0}
\end{aligned}
$$

with $\boldsymbol{\mu}=(\beta,\delta,\tau)'$ parameters to be estimated, exogenous variables (data) $\mathbf{w_t}=\left(\frac{c_{t+1}}{c_t}, r_{t+1}\right)'$ and instruments e.g. $\mathbf{u_t}=(1,\frac{c_t}{c_{t-1}},r_t)'$.

# Indirect Inference

- Introduced to econometrics by Gourieroux, Monfort and Renault (1993) and Smith (1993) for nonlinear time-series models.
- Indirect: estimation is based on the simulation of data.
- Important requirement: One has to be able to simulate data with the model for different values of the parameters.
- These simulated datasets are estimated by an auxiliary model and compared to an estimation of the true data.
- Idea of *Indirect Inference*: Estimate the true dataset with the auxiliary model. Simulate data using your DSGE-model for different values of the parameters. Estimate these simulated datasets with the auxiliary model and compare the estimators to the ones obtained with the true dataset until you find estimators that are *almost* the same. Choose the parameters that generated that dataset.

# Indirect Inference

- In practice one often uses VAR-models.
- Great empirical forecast-performance („*work-horse*").
- The solution of a DSGE-model in its state-space form corresponds closely to a VAR-model.
- Generally two possible methods of estimation:
  1. Parameters of the VAR-model: Ruge-Marcia (2007).
  2. Impulse-Response-Matching: Christiano, Eichenbaum and Evans (2005).
- Very similar: Impulse-responses are functions of the parameters.
- The second method enables one to incorporate the dynamic properties of the VAR-model into the DSGE-model.
- However, identification issues: different combinations of parameters can generate the same impulse-responses.

## Indirect Inference Estimator

$$\widehat{\boldsymbol{\mu}}_{\mathbf{I}} = \min_{\mu} \left\{ (\boldsymbol{\Xi} - \boldsymbol{\Xi}(\boldsymbol{\mu}))' \times \boldsymbol{\Omega} \times (\boldsymbol{\Xi} - \boldsymbol{\Xi}(\boldsymbol{\mu})) \right\}.$$

- $\boldsymbol{\Xi}$: impulse-responses of the estimated VAR using the true dataset, $\boldsymbol{\Xi}(\boldsymbol{\mu})$ the analogous with the simulated datasets, $\boldsymbol{\Omega}$ a weight-matrix.
- Smith (1993) shows, that using the inverse of the variance-covariance-matrix of $\boldsymbol{\Xi}$ for the weighting matrix, $\sqrt{T}(\widehat{\boldsymbol{\mu}} - \boldsymbol{\mu})$ is gaussian.
- *Method-of-Moments* interpretation, since the impulse-responses are functions of the covariances and autocovariances of the variables of the VAR-model.
- *Indirect Inference* interpretation, since the auxiliary model is a misspecified version of the true state-space representation.

- Methods are useful if the derivation of a specific criteria, like the likelihood, is analytically not possible or the evaluation too difficult.
- You only need a few assumptions about the first and second moments of the shocks (no distribution).
- Great advantage compared to calibration: Standard errors $\Rightarrow$ statistical inference is possible!
- Limiting to only relevant characteristics (distance function between theoretical and empirical moment) leads to robust estimators.
- J-Test of overidentification is a formal statistical test of the validity of your model.
- Rejection of the null, however, gives no hint on what is wrong with the model.

- GMM is robust towards misspecification, especially if you restrict to only a few conditions.
- There is no need for an explicit solution or approximation of the DSGE-model.
- GMM-estimators are reliable, however less efficient than the estimators you obtain using methods of full information.
- Often one losses efficiency and there are identification issues, if one tries to exploit interdependencies between the blocks.
- Choosing the right moment-conditions, instruments and algorithms for calculating the weight-matrix and numerical optimization are a very complex branch of research.

- *Small-Sample-Bias*: Favorable properties of *GMM* are only valid asymptotically.
- Monte-Carlo-experiments show that for DSGE-models you need at least $T = 300$ observations for the asymptotics to kick in.
- For quarterly data that means about 75 years of data!
- The relevant data for DSGE-models includes only the last 30-40 years.
- Further issue: How to find good and **time-homogenous** data for output-gap, technology, ...?

- Pros and cons of *GMM* are also true for *impulse-response-matching*.
- Main advantage of this form of *Indirect Inference*: Limitation to only a few time-series.
- Further advantages: Auxiliary model needs not to be specified correctly.
- Opposed to GMM the DSGE-model needs to be solved explicitly, since otherwise one is not able to simulate data and impulse-responses from the model.