Estimation of DSGE models

Methods of Limited Information

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- Estimation of a DSGE-model is hampered by
  - Expectations about future variables
  - Non-linearities
  - Stochastic processes
  - ...

→ *Limited-information-estimators*:
  - *Generalized Method of Moments*
  - *Indirect Inference*

Generalized Method of Moments

- Idea: Represent DSGE-model as *moment-* or *orthogonality-conditions*:

$$\mathbb{E}\left[f(\boldsymbol{\theta}, \boldsymbol{\Upsilon_t})\right] = \mathbb{E}\begin{bmatrix} f_1(\mathbf{w_t}, \boldsymbol{\theta})\mathbf{u_t} \\ \vdots \\ f_m\left(\mathbf{w_t}, \boldsymbol{\theta}\right)\mathbf{u_t} \end{bmatrix} = \mathbf{0}$$

- $\boldsymbol{\theta} \in \mathbb{R}^k$ is the true vector of parameters, $\mathbf{w_t} \in \mathbb{R}^d$ a vector of exogenous variables, $\mathbf{u_t} \in \mathbb{R}^l$ a vector of instruments and $\boldsymbol{\Upsilon_t} = [\mathbf{w_t}' \, \mathbf{u_t}']'$.
- Vector-valued functions: $f : r \times 1$ and $f_i : m \times 1$
- Number of orthogonality-conditions is equal to $r = m \times l$

Moment Conditions

- Moment-conditions are derived from
  - first-order-conditions
  - steady-states
  - expected values/ variances
  - properties of the stochastic processes

→ Solving the model is only a sufficient condition!

- Replacing the theoretic conditions by their empirical counterparts (estimation equations) gives the GMM-estimator.
- Developed by Hansen (1982), first to use it for DSGE-models were Christiano and Eichenbaum (1992) and Burnside, Eichenbaum and Rebelo (1993).

GMM-Estimator

$$\hat{\boldsymbol{\theta}} = \underset{\theta}{\text{argmin}} \left\{ \left[\frac{1}{T} \sum_{t=0}^{T} f(\boldsymbol{\theta}, \boldsymbol{\Upsilon}) \right]' \times \boldsymbol{\Omega} \times \left[\frac{1}{T} \sum_{t=0}^{T} f(\boldsymbol{\theta}, \boldsymbol{\Upsilon}) \right] \right\}.$$

- If we set the weighting matrix $\boldsymbol{\Omega}$ equal to the identity matrix I, we get a least square problem (if $r > k$)

$\rightarrow$ If we do not use instruments, we are conceptually close to our calibration exercise!

Identification

- If $r < k$, then the model is *under-identified*
  $\rightarrow$ find additional instruments or moment conditions
- If $r = k$, then the model is *exactly-identified*
  $\rightarrow$ the weighting matrix does not play any role, since there is a unique solution to the quadratic form
- If $r > k$, then the model is *over-identified*
  $\rightarrow$ No solution, only a minimum depending on the weighting matrix

Question: Is there an optimal weighting matrix?

Hansen (1982) shows, that the optimal (= smallest standard errors) weighting matrix is given by the inverse of the variance-covariance-matrix of the empirical analogous

- Given some regularity conditions one can show that $\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta})$ is gaussian
- The optimal weighting matrix Ω^* minimizes the variances of $\hat{\theta}$
- In the over-identified case we can formally test the hypothesis, that the model is able to describe the data generating process (*J/Overidentification-Test*)
- If we cannot express moment conditions analytically, we can use simulated moments yielding the Simulated Method of Moments (SMM)
- For more details see the course **Econometrics PhD**

Example: Estimating the Euler-equation with GMM

Simple Euler-equation:

$$\beta E_t \left\{ c_{t+1}^{-\tau}(1 + r_{t+1} - \delta) \right\} = c_t^{-\tau}$$

$$\Leftrightarrow E_t \left\{ \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\tau} (1 + r_{t+1} - \delta) \right\} = 1$$

$$\Rightarrow E_t \left\{ \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\tau} (1 + r_{t+1} - \delta) - 1 \right] \begin{bmatrix} 1 \\ \frac{c_t}{c_{t-1}} \\ r_t \end{bmatrix} \right\} = \mathbf{0}$$

with $\boldsymbol{\theta} = (\beta, \delta, \tau)'$ parameters to be estimated, exogenous variables (data) $\mathbf{w_t} = \left(\frac{c_{t+1}}{c_t}, r_{t+1} \right)'$ and instruments e.g. $\mathbf{u_t} = (1, \frac{c_t}{c_{t-1}}, r_t)'$.

Indirect Inference

- Introduced to econometrics by Gourieroux, Monfort and Renault (1993) and Smith (1993) for nonlinear time-series models.
- Idea:
  1. Simulate data using the DSGE-model
  2. Estimate an auxiliary model using
     a) the true dataset
     b) the simulated dataset
  3. Choose the parameters of the DSGE-Model which minimize the difference between the parameters from a) and b).

Indirect Inference

- In practice one often uses VAR-models as auxiliary models
- The solution of a DSGE-model in its state-space form corresponds closely to a VAR-model.
- Methods of estimation:
  1. Parameters of the VAR-model: Ruge-Marcia (2007).
  2. Impulse-Response-Matching: Christiano, Eichenbaum and Evans (2005).
- The second method enables one to incorporate the dynamic properties of the VAR-model into the DSGE-model.
- Identification issues: different combinations of parameters can generate the same impulse-responses.

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Indirect Inference Estimator (Impulse Response)

$$\widehat{\boldsymbol{\theta}} = \underset{\theta}{\text{argmin}} \left\{ [\boldsymbol{\Xi} - \boldsymbol{\Xi}(\boldsymbol{\theta})]' \times \boldsymbol{\Omega} \times [\boldsymbol{\Xi} - \boldsymbol{\Xi}(\boldsymbol{\theta})] \right\}.$$

- $\boldsymbol{\Xi}$ is the impulse-response of the estimated VAR using the true dataset, $\boldsymbol{\Xi}(\boldsymbol{\theta})$ the analogous with the simulated datasets, $\boldsymbol{\Omega}$ a weighting matrix
- *Method-of-Moments* interpretation, since the impulse-responses are functions of the covariances and autocovariances of the variables of the VAR-model
- *Indirect Inference* interpretation, since the auxiliary model is a misspecified version of the true state-space representation

- Useful if the derivation of a specific criteria, like the likelihood, is analytically not possible or the evaluation too difficult
- You only need a few assumptions about the first and second moments of the shocks (no distribution)
- Advantage compared to pure calibration:
  $\Rightarrow$ Statistical inference is possible (standard errors)
- Limiting to only relevant characteristics (distance function between theoretical and empirical moment) leads to robust estimators.
- J-Test of overidentification is a formal statistical test of the validity of your model
- But: Rejection of the null gives no hint on what is wrong with the model

- GMM is robust towards misspecification, especially if you restrict to only a few conditions
- Explicit solution or approximation of the DSGE-model is not necessary
- GMM-estimators are reliable, however less efficient than the estimators you obtain using methods of full information
- Choosing the right moment-conditions, instruments and algorithms for calculating the weight-matrix and numerical optimization are very complex branches of research

- *Small-Sample-Bias*: Favorable properties of *GMM* are only valid asymptotically (worse for *SMM* and *Indirect Inference*)
- Monte-Carlo-experiments show that for DSGE-models you need at least $T = 300$ observations for the asymptotics to kick in
- For quarterly data that means about 75 years of data!
- The relevant data for DSGE-models includes only the last 30-40 years
- Further issue: How to find good and **time-homogenous** data for output-gap, technology, . . . ?

- Pros and cons of *GMM* are also true for *impulse-response-matching*
- Main advantage of this form of *Indirect Inference*: Limitation to only a few time-series
- Further advantages: Auxiliary model needs not to be specified correctly
- Opposed to GMM the DSGE-model needs to be solved explicitly, since otherwise one is not able to simulate data and impulse-responses from the model