PROBLEM SET NO. 4

DSGE methods

Student:	Jens Middelkamp
	Bismarckallee 51
	48151 Münster
Registration number:	360749
Due date:	31.08.2013

1 Dynare

1.1 Exercise 1.1)

It can be seen from figure 1 that the natural real rate exceeds the acutal real rate, which means that the policy rule does not provide the interest rate that the new productivity level of the economy requires. Due to a relatively low real rate the housholds demand actually more goods than the natural output would indicate (positive output gap). Since prices are sticky the firms have an incentive to produce an additional amount of goods. That means the economy over-responds to a technology shock. In contrast to the unit-root case, the output declines after the initial shock. The reason for this behaviour is that a shock is always preserved in a random walk, while its impact vanishes over time in a stationary process (see figure 2).
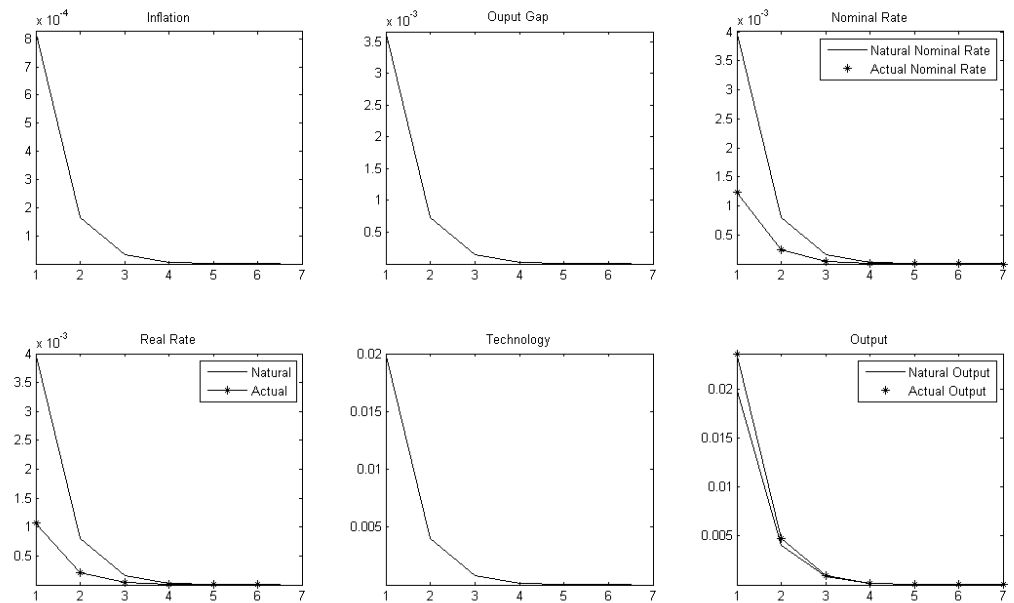

Abbildung 1: A transitory technological shock in the CGG-model(without unit-root)
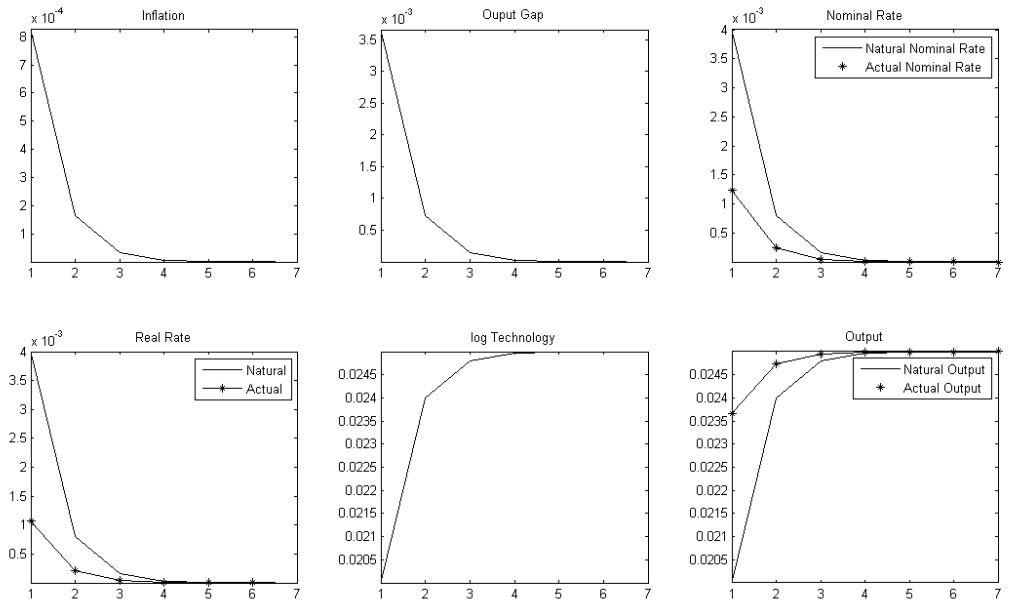
Abbildung 2: A permanent technological shock in the CGG-model(with unit-root)

1.2 Exercise 1.2)

Figure 3 shows us that the inflation rate jumps to a positive level from which it decreases over time until it fades out. The output gap has its peak in the second period. From $t > 2$ the output gap closes over time. This behaviour stems from the household's anticipation of the future technologcial shock. According to their Euler equation they smooth their intertemporal consumption path, which leads to a higher demand in period 1 because of the higher income in the future. In period 2 the shock kicks in and the productivity of the economy rises. This causes the natural real rate to rise above the actual real rate (see figure 4). This fact explains the peak of the output gap at $t = 2$. The movement of the inflation rate is driven by the Calvo pricing equation. In period 2 the expected prices decline more rapidly than the short-term growth of the output gap, which explains the continuous disinflation.
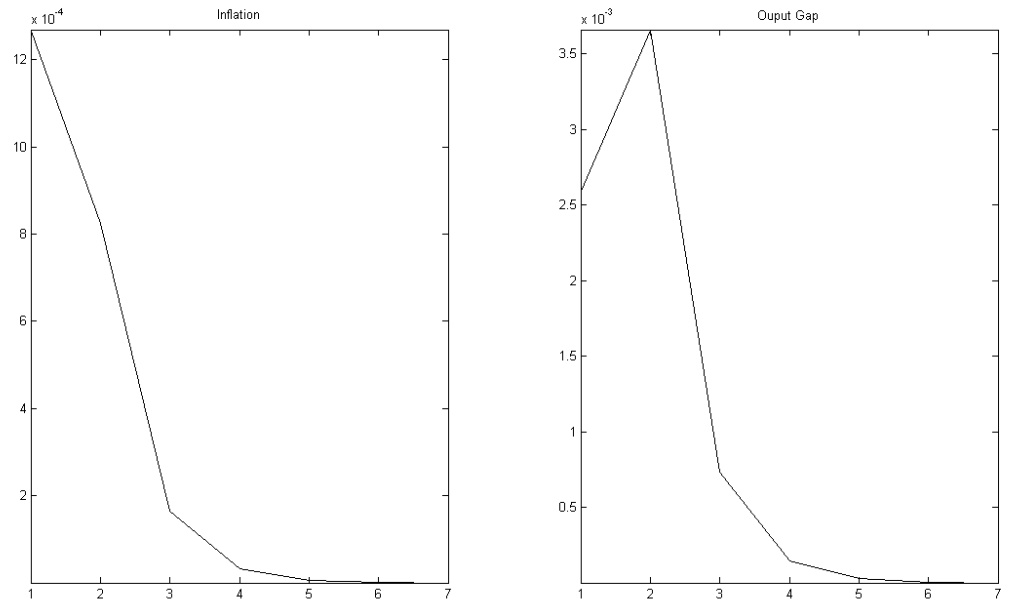
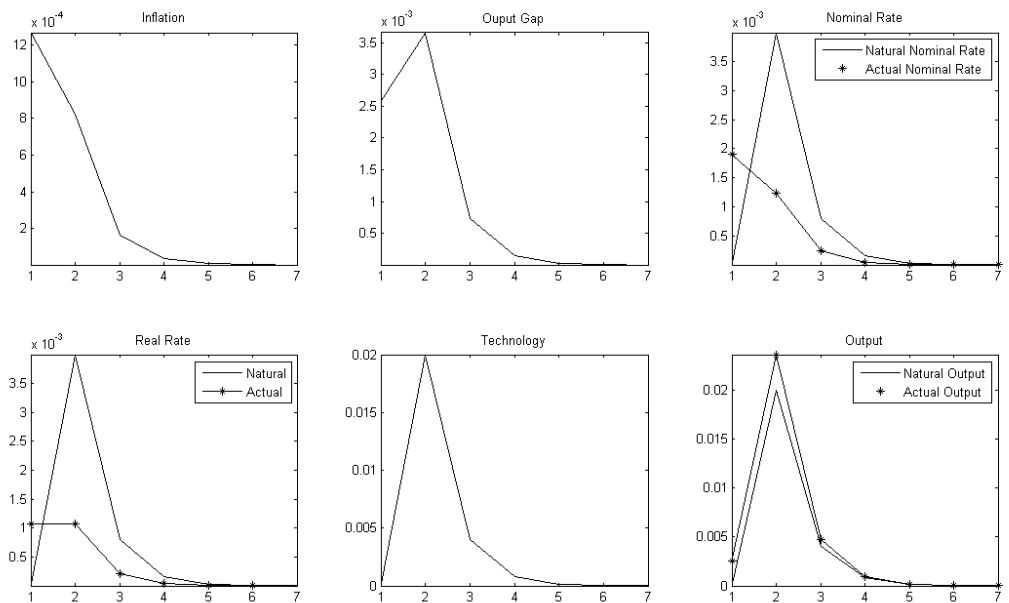Abbildung 3: Inflation rate and output gap response to a news shock

Abbildung 4: Response of the economy to an news shock

1.3 Exercise 1.3)

Assuming an identical news shock like above, one can recognize that the monetary policy rule matches the natural and actual real rate perfectly. This completely stabilizes the output gap. Inflation is also stabilized completely because it depends positively on the output gap (see figure 5).
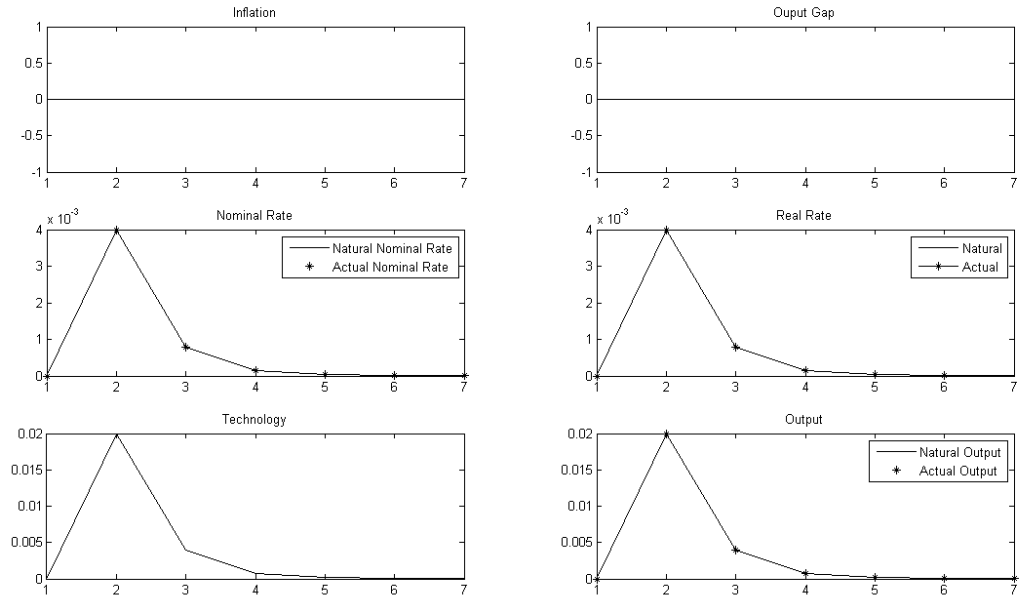
Abbildung 5: Complete sterilization of the news shock

2 Solution methods

2.1 Exercise 2.1)

The dividends d_t are generated by an AR(1)-process and so they are dependent on stochastic shocks. Therefore the households cannot fully control the variable. In terms of the lecture d_t matches with the state variable (x_t). From their income the households control the intertemporal consumption. The intertemporal shift of income is done by purchasing assets. Therefore the price of the assets p_t corresponds with the control variable (y_t).

2.2 Exercise 2.2)

Derive the decision rule for the state variable in the form of $x_{t+1} = \overline{x} + h(x_t, \sigma) + \sigma\epsilon_{t+1}$

$$d_{t+1} = \overline{d} + h(d_t, \sigma) + \sigma\epsilon_{t+1} = \rho d_t + e_{t+1} \implies d_{t+1} - \overline{d} = h(d_t, \sigma) + \sigma\epsilon_{t+1} = \rho d_t - \overline{d} + e_{t+1}$$

In this model $h(d_t, \sigma) = \rho d_t - \overline{d}$. Within this model σ reflects a measure of uncertainty. Here the model only contains one stochastic exogenous shock e_t. Hence, σ is equal to σ_e the standard deviation of e_t which is the stochastic component of the model.

2.3 Exercise 2.3)

Derive the decision rule for the control variable up to the second order.

4

In general; $F(x,\sigma)$ looks like: $E_t f(h(x_t,\sigma) + \sigma \epsilon t + 1, g(h(x_t,\sigma) + \sigma \epsilon t + 1, \sigma), x_t, y_t)$

Rewrite $h(x_t)$ as $h(d_t = \rho d_t - \overline{d}$ and $g(x_t)$ as $g(p_t) = p(d,\sigma)$, we may come up with the model:

$$E_t[u'(d) \cdot p(d,\sigma) - \beta \cdot u'(\rho d + \sigma \epsilon) \cdot (p(\rho d + \sigma \epsilon, \sigma) + \rho d + \sigma \epsilon)] = 0$$

Inserting the steady-state values $(\overline{d}, \ \overline{p}, \ \sigma = 0)$ and taking expectations lead to:

$$u'(\overline{d}) \cdot \overline{p} - \beta \cdot u'(\overline{d}) \cdot (\overline{p} + \overline{d}) = 0$$

$$\Longrightarrow \overline{p} \cdot (1 - \beta) = \beta \overline{d} \Longrightarrow \overline{p} = \frac{\beta}{1 - \beta} \cdot \overline{d}$$

In equilibrium the price of an asset is given by the dividend weighted by a term depending on the time preference. To build up the Taylor-approximation expression as pleased, we may go through several derivatives of $F(d,\sigma)$. The derivative of $F(d,\sigma)$ w.r.t. d is:

$F_d(d,\sigma) = E_t[u''(d) \cdot p(d,\sigma) + u'(d) \cdot p_d(d,\sigma) - \beta u''(\rho d + \sigma \epsilon) \cdot \rho \cdot (p(\rho d + \sigma \epsilon, \sigma) + (\rho d + \sigma \epsilon)) - \beta \rho u'(\rho d + \sigma \epsilon) \cdot (p_d(\rho d + \sigma \epsilon) + 1)] = 0$

Applying the expectations operator and evaluating the last equation at the steady-state values $(\overline{d}, \ \overline{p}, \ \sigma = 0)$, one may get:

$$u''(\overline{d}) \cdot \overline{p} + u'(\overline{d}) \cdot p_d(\overline{d}, 0) - \beta u''(\overline{d}) \rho(\overline{p} + \overline{d}) - \beta u'(\overline{d})(p_d(\overline{d}, 0)\rho + \rho) = 0$$

Solve the last equation for $p_d(\overline{d})$:

$$p_d(\overline{d}) \cdot (u'(\overline{d}) \cdot (1 - \beta \rho)) = -u''(\overline{d}) \cdot p(\overline{d}) + \beta \rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho \beta \cdot u'(\overline{d})$$

$$\Longrightarrow p_d(\overline{d}) = \frac{-u''(\overline{d}) \cdot \overline{p} + \beta \rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho \beta \cdot u'(\overline{d})}{u'(\overline{d}) \cdot (1 - \beta \rho)}$$

The next step will compute $p_{dd}(\overline{d})$. Therefore we take the partial derivative of F_d w.r.t. d:

$$F_{dd} = E_t[u'''(d) \cdot p(d) + u''p_d(d) + u''(d) \cdot p_d(d) + u'(d) \cdot p_{dd}(d)$$

$$-\beta \rho^2 \cdot u'''(\rho d + \sigma \epsilon)(p(\rho d + \sigma \epsilon) + \rho d + \sigma \epsilon) - \beta \rho^2 u''(\rho d + \sigma \epsilon)(p_d(\rho d + \sigma \epsilon) + 1)$$

$$-\beta \rho^2 u''(\rho d + \sigma \epsilon)(p_d(\rho d + \sigma \epsilon) + 1) + \beta \rho^2 u'(\rho d + \sigma \epsilon) \cdot (p_{dd}(\rho d + \sigma \epsilon))] = 0$$

Again taking expectations and evaluating the last equation at the steady-state values $(\overline{d}, \ \overline{p}, \ \sigma = 0)$:

$$u'''(\overline{d}) \cdot \overline{p} + 2 \cdot u''(\overline{d})p_d(\overline{d}) + p_{dd}(\overline{d})(u'(\overline{d})(1 - \beta \rho^2)$$

$$-\beta \rho^2 u'''(\overline{d}) \cdot (\overline{p} + \overline{d}) - 2\beta \rho^2 \cdot u''(\overline{d})(p_d(\overline{d}) + 1) = 0$$

$$\Longrightarrow p_{dd}(\overline{d}) = \frac{-u'''(\overline{d}) \cdot p(\overline{d}) - 2 \cdot u''(\overline{d})p_d(\overline{d}) + \beta \rho^2 u'''(\overline{d})(\overline{p} + \overline{d}) + 2\beta \rho^2 \cdot u''(\overline{d})(p_d(\overline{d}) + 1)}{u'(\overline{d})(1 - \beta \rho^2)}$$

To derive $p_{d\sigma} = p_{\sigma d}$, one may recognise that we are dealing with a linear homogenous system. Refering to the class, we may expect $p_{d\sigma} = p_{\sigma d}$ to be zero. To derive this analytically, we first set up F_σ:

$$F_\sigma(d, \sigma) = E_t[u'(d) \cdot p_\sigma(d) - \beta u''(\rho d + \sigma\epsilon)\epsilon(p(\rho d + \sigma\epsilon) + \rho d + \sigma\epsilon)$$

$$-\beta u'(\rho d + \sigma\epsilon) \cdot (p_d(\rho d + \sigma\epsilon) \cdot \epsilon + p_\sigma(\rho d + \sigma\epsilon) + \epsilon)] = 0$$

Again taking expectations and evaluating the last equation at the steady-state values $(\overline{d},\ \overline{p},\ \sigma = 0)$, one may derive:

$$u'(\overline{d}) \cdot p_\sigma(\overline{d}) - 0 - 0 + \beta u'(\overline{d})p_\sigma(\overline{d}) + 0 = 0$$

$$\implies u'(\overline{d}) \cdot p_\sigma(d) + \beta u'(\overline{d})p_\sigma(\overline{d}) = 0$$

The last equation will only hold if $p_\sigma(\overline{d}) = 0$, since $\beta < 1$. We will use this knowledge later. In the following we compute $F_{d\sigma}$:

$$F_{d\sigma} = E_t[u''(d) \cdot p_\sigma(d) + u'(d) \cdot p_{d\sigma}(d) - \beta\rho\epsilon \cdot u''(\rho d + \sigma\epsilon) \cdot (p_\sigma(\rho d + \sigma\epsilon) + 1)$$

$$-\beta\rho\epsilon \cdot u'''(\rho d + \sigma\epsilon) \cdot (p(\rho d + \sigma\epsilon) + \rho d + \sigma\epsilon) - \beta\rho \cdot u'(\rho d + \sigma\epsilon) \cdot (p_{d\sigma}(\rho d + \sigma\epsilon)\epsilon)$$

$$-\beta\rho\epsilon \cdot u''(\rho d + \sigma\epsilon) \cdot (p_d(\rho d + \sigma\epsilon) + 1)] = 0$$

And at the steady-state values $(\overline{d},\ \overline{p},\ \sigma = 0)$, we obtain:

$$E_t[u''(\overline{d}) \cdot p_\sigma(\overline{d}) - \beta\rho\epsilon \cdot u''(\overline{d}) \cdot (p_\sigma(\overline{d}) + 1) - \beta\rho\epsilon \cdot u'''(\overline{d}) \cdot (\overline{p} + \overline{d})$$

$$-\beta\rho\epsilon \cdot u''(\overline{d}) \cdot (p_d(\overline{d}) + 1)] = E_t[p_{d\sigma} \cdot u'(\overline{d})(1 - \beta\rho\epsilon)]$$

Taking expectations lead to:

$$u''(\overline{d}) \cdot p_\sigma(\overline{d}) - 0 - 0 - 0 = p_{d\sigma} \cdot u'(\overline{d})$$

And with the former derived equation $p_\sigma(\overline{d}) = 0$, we obtain:

$$p_{d\sigma} = 0 = p_{\sigma d}$$

All in all the Taylor-approximation of p_t looks like:

$$p_{t+1} = \frac{\beta}{1 - \beta}\overline{d} + \frac{-u''(\overline{d}) \cdot \overline{p} + \beta\rho u''(\overline{d})(\overline{p} + \overline{d}) - \rho\beta \cdot u'(\overline{d})}{u'(\overline{d}) \cdot (1 - \beta\rho)} \cdot (d_t - \overline{d}) + [g_\sigma]\sigma + \frac{1}{2}[g_{\sigma\sigma}]\sigma^2$$

$$+\frac{1}{2}\frac{-u'''(\overline{d}) \cdot p(\overline{d}) - 2 \cdot u''(\overline{d})p_d(\overline{d}) + \beta\rho^2 u'''(\overline{d})(\overline{p} + \overline{d}) + 2\beta\rho^2 \cdot u''(\overline{d})(p_d(\overline{d}) + 1)}{u'(\overline{d})(1 - \beta\rho^2)} \cdot (d_t - \overline{d})^2$$

3 Estimation methods

3.1 Exercise 3.1)

At first, variables and parameters have to be defined. Afterwards the model's equations have to be implemented. In this case, it is already given by the skript *estim.mod*. A further step is the declaration of the observable variable, a variable which is available in an

exogenous file and which dynare can use for the estimation procedure. In Bayesian estimation, the condition for undertaking estimation is that there be at least as many shocks as observables. This means for the given model which has only one shock (technology shock) an appropiate number of observables is one. For our purpose one can use the variable y (declared with the command *varobs y;*). As in the previous tasks the steady-states have to be defined due to the single steps in deriving, solving and estimating the given economic setting in the DSGE model. The next step contains the very important step of defining the priors.

Choosing the priors for the parameters is one of the most important steps in Bayesian estimation. Because of the wide range of possibilities of choosing the priors some time has to be spent in choosing and testing the robustness of the results. For the given example some different assumptions should be tested to check the sensitivity of the parameters to different priors. First of all some considerations to the domains of the parameters should be mentioned. The difference between both models is only the definition of the production function. The data is simulated in a RBC model with the following production function:

$$y = A * (\alpha * k_{t-1}^{\psi} + (1 - \alpha) * l^{\psi})^{(1/\psi)}$$

In contrast to this function the Cobb Douglas production is defined as follows:

$$y = A * (k_{t-1}^{\alpha}) * (l^{(1-\alpha)})$$

In the case of a Cobb Douglas function one knows ex ante that the parameter α is bounded in the interval $[0, 1]$ (assuming constant returns to scale). This is also case for the θ as a part of the defined preference function (see setting on p. 4 of the problem sheet). For the parameter τ one can get some problems if this parameter is too close to zero, but there are no more restrictions which could defined some a priori boundaries for this parameter.

Besides the boundaries of the parameters, there are several points to considerate. For example, it should be determined which exact prior distribution is chosen and if the boundaries mentioned before are open or closed. Because we do not have more information, we just use simple beta distributions. Since this is a fictional exercise, we do not consider information from the literature.

Due to the knowledge of the underlying data generating process one can formulate some expectations in the following estimations procedure: Because of the fact that the α should differ between both production functions (which is hard to interpret in the complete setting), the focus should lie on the estimators for the parameters τ, θ, because these

parameters have the same functions in both settings. The main question is now how the estimation is affected by misspecification and how strong the resulting bias is.

For the following estimation procedures we use different priors to pursue two aims: First, to check how sensitive (or robust) the Bayesian estimation procedure (with the Metropolis Hastings algorithm) reacts to different choices of priors. And second, to evaluate and to get an impression of the bias of the parameter estimates. For this purpose, we choose priors which are larger than the mean of the original parameter, priors which are smaller and priors which differ in the direction of bias. In contrast to the announced purposes we also apply the original parameters as priors. Because there are no information for the used distribution we consider different derivations (in defining other kinds of distributions than simple beta definitions dynare often displays that some problems occur during the procedure. This is not known before applying the posterior mode or complete estimation via Metropolis Hastings algorithm, but should be already mentioned at this point). All used priors and the results which belong to these priors are saved in the additional textfile. A further selection of the the mentioned priors for applying the estimation procedure in task 3.3 is made in task 3.2 with the diagnostic tool *mode check* which helps to evaluate the decision of the priors and to indicate some problems.

3.2 Exercise 3.2)

To estimate the given model one can use the command *estimation(...)* while in the brackets the options for the procedure is defined (the full command is announced in the additional .mod file which could be changed with the announced commands to achieve results with changed options). First the *datafile* is the option to announce the name of the file with the observations used for estimation. Here one has to use the data which is simulated with the RBC model and was saved in a matlab .m file (This file is also added to the other files, and has to be used for all of the estimations to have a common basis. The command *nobs* is used to declare the number of observations in the file which should be used for the procedure. This is the command which is needed for the task and which has to be set to *200*. To check the posterior mode one can use the command mode check. If this command is announced in the *estimation* command, dynare is going to plot the posterior density for values and for every estimated parameter around the given *mode*. This diagnostic tool should help to identify problems with the optimization procedure. A clear indication of a problem would be that the mode is not at the trough (bottom of the minus) of the posterior distribution. Now some of the mentioned priors can be evaluated with the *mode check* command (due to the large number of possibilities in choosing the priors, only some main detections are

described). The represented diagnostic plot which is given by the command *mode check* (see figure: 6) indicates that there is no problem existent.
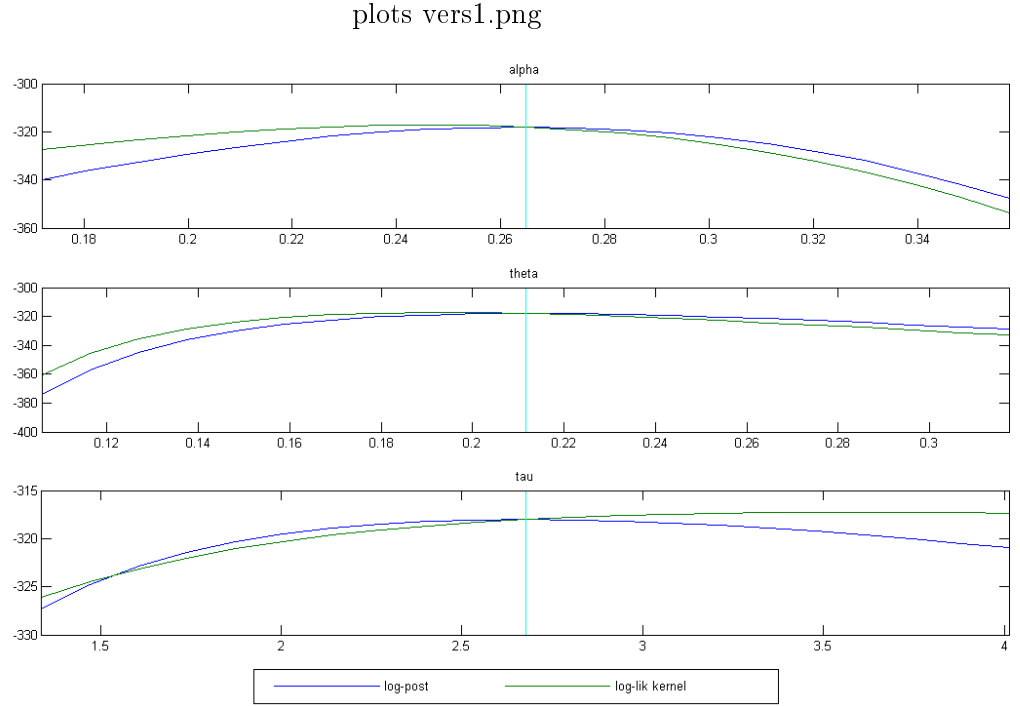
plots vers1.png

Abbildung 6: plot of the check plots for priors of version 1 (see priors txt.)

In this plot the vertical line should coincide with the blue curve (the log-post). A very flat green curve (log-lik Kernel) (as in the part of the plot for the third parameter: τ in the figure 7) means that one can only extract less information for this parameter for the data. This fact is later also shown in the comparison of the prior and the posterior distribution - in that case the posterior distribution shape is quite similar to the prior distribution shape. For the case that the prior values are set as true values, these diagnostic shows that there are some problems with the prior distribution assumption. Especially the check plot which are represented in the figure 7 seems not to be appropriate priors.

3.3 Exercise 3.3)

Metropolis-Hastings algorithm with 3 × 5000 iterations: could be generated with the command *estimation* and the additional option *mh replic* is used to declare the number of replications of the Metropolis Hastings algorithm - here initialized with 5000 iterations. The command *mh nblocks* defines the number of chains for the Metropolis Hastings alorithm. In this case three chains are used (to check the robustness also the suggestion, the default setting, of five chains is used. See comments in task 3.4).

The commands *mode compute=0, mode file=rbcexam mode* are used for the following purposes: In the case of setting *mode compute = 0*, the mode is not computed at the following
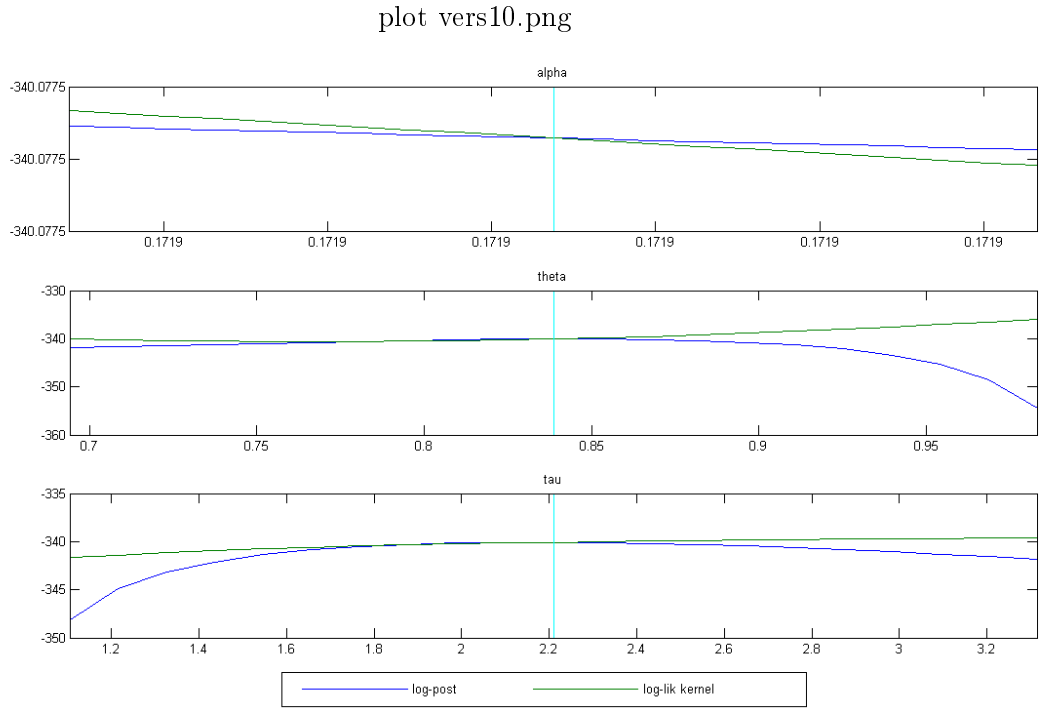
plot vers10.png

Abbildung 7: plot of the check plots for priors of version 1 (see priors txt.)

Metropolis Hastings algorithm. In this case the procedure uses the stored mode (stored and called with the following name structure: MODEL NAME mode). This option is not necessary for the estimation procedure but helps to speed up the estimation process.

The acceptance rate can be set with the command *mh jscale* and has a default value of 0.2 while the literature mostly uses values between 0.2 and 0.4. Choosing an accurate acceptance rate is also an important issue in Bayesian estimating procedure, because in the case of very high acceptance rate the candidate parameters are rejected too often and in the case of a very low acceptance rate they are accepted too often. The aim is to choose an acceptance rate which is located between both mentioned extremes. Therefore the option of applying different values for the acceptance rate is an important practical issues (to check the robustness of the estimators three different acceptance rates are used for the priors with the true parameters).

As asked in the task one has to provide diagnostic plots as well. These diagnostics are displayed automatically because the command *nodiagnostics* is not set in the *estimation* command, hence the default setting is used: dynare is computing and displaying tabular and graphical diagnostics, for example the convergence diagnostics for the Metropolis Hastings algorithm, which could be used to check if the blocks are converging.

To repeat the algorithm without discarding the previous draws one can use *mh drop* in the

estimation command and one could define the fraction of the initially generated parameter vectors which should be dropped. If the command is set to zero none of the previous draws will be discarded.

3.4 Exercise 3.4)

First, one gets a representation of the priors for each parameter, which is not important for the purpose of evaluating the applied estimation procedure. Figure 8 represents the *MCMC univariate diagnostics*. The plot in figure 8 (priors version 1) shows clearly that there is a quite similar movement of the chains and that they are lying quite closely at the end of the iterations which can be interpreted as a sign of convergence. Here three measures for the three parameters are plotted. The *interval*: being constructed from an 80 percent confidence interval around the parameter mean. The *m2*: measure of the variance and the *m3*: based on third moments. The red and blue lines differ between parameter vectors for the between and the within chains.
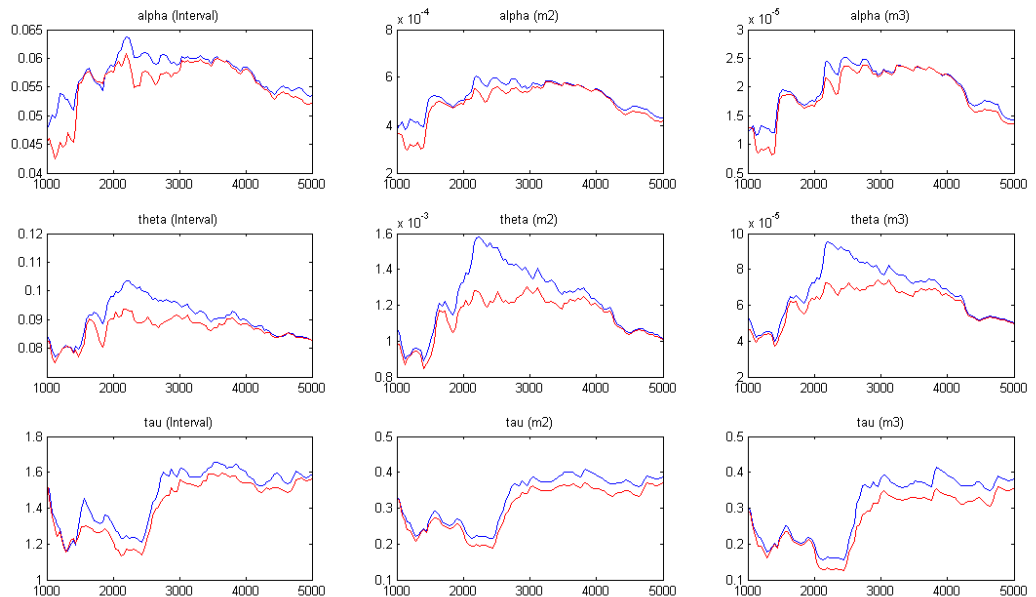
diagnostics vers1.png

Abbildung 8: plot of univariate diagnostics for the priors of version 1 (see priors txt.)

The plot *MCMC multivariate diagnostics* represents an aggregate measure with analog differentiations as in the univariate case (see above). In the figure 9 it is possible to consider a kind of a movement to convergence. But this is as well a question of the demanded precision.

In the case of the priors from version 1 with larger deviations the plot shows a significant movement of divergence (see figure 10).
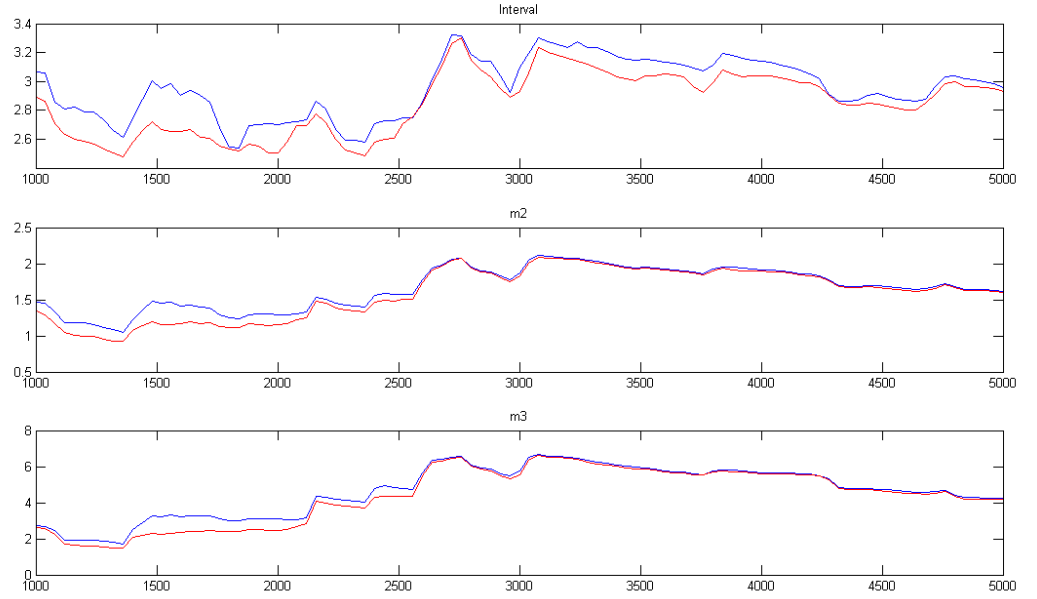
diagnostics vers1.png

Abbildung 9: plot of multivariate diagnostics for the priors of version 1 (see priors txt.)
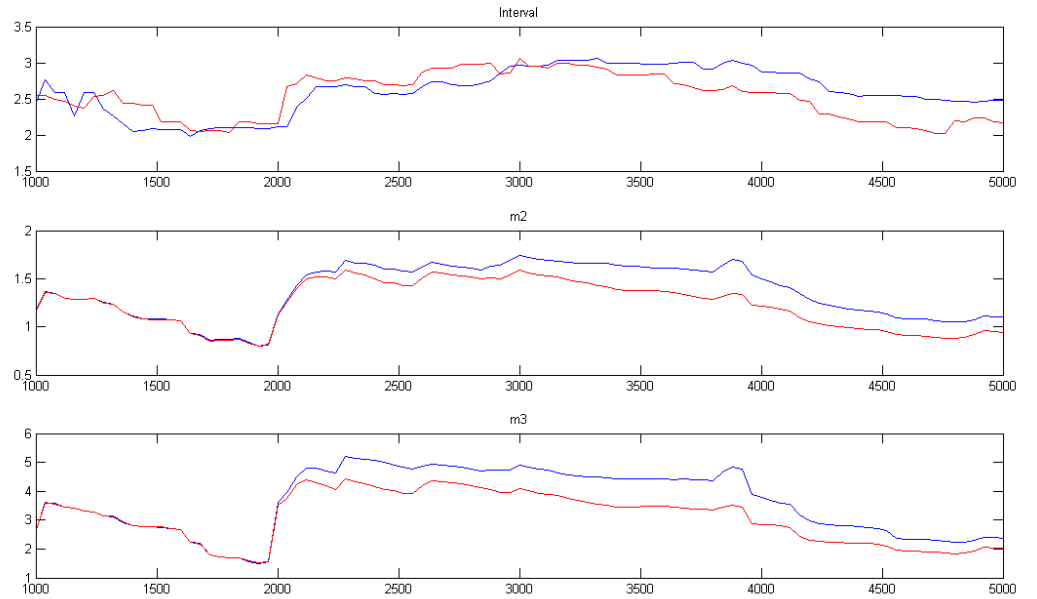
diagnostics vers10.png

Abbildung 10: plot of multivariate diagnostics for the priors of version 10 (see priors txt.)

In contrast to the figure 10, figure 11 (the multivariate diagnostics for the priors of version 3) seems to have an obvious movement of convergence.

In the case of instability or divergent movements it is possible to improve the findings by using different priors (as an indication of poor priors: this has been already done with the figures above). Another opportunity is to choose a greater number of Metropolis Hastings iterations. This should be done for the priors version1 with 5 chains and 10000 iterations.
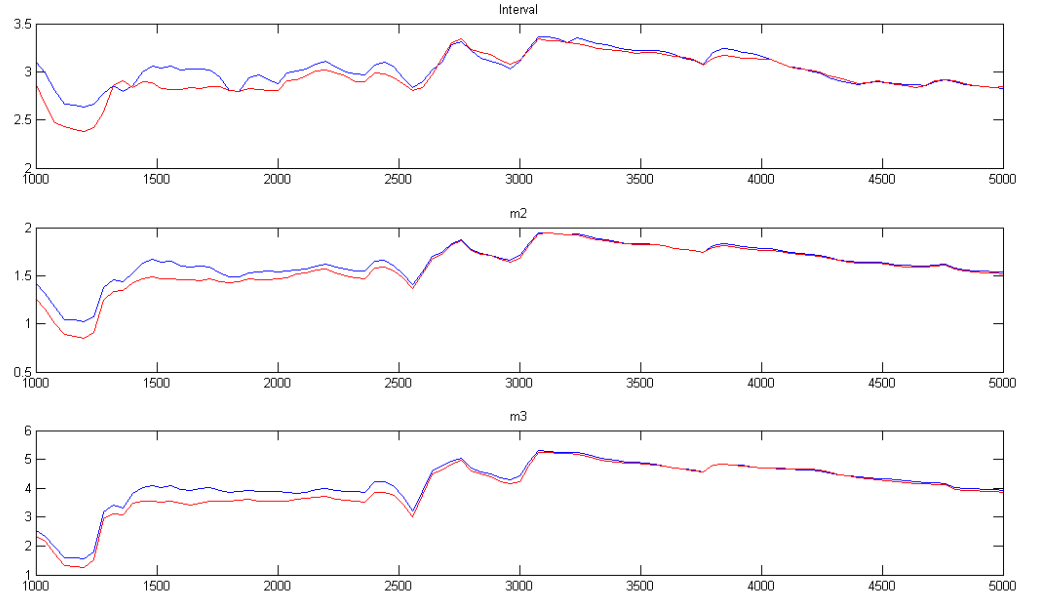
diagnostics vers3.png

Abbildung 11: plot of multivariate diagnostics for the priors of version 3 (see priors txt.)

The results in the multivariate diagnostics are presented in the figure 12.
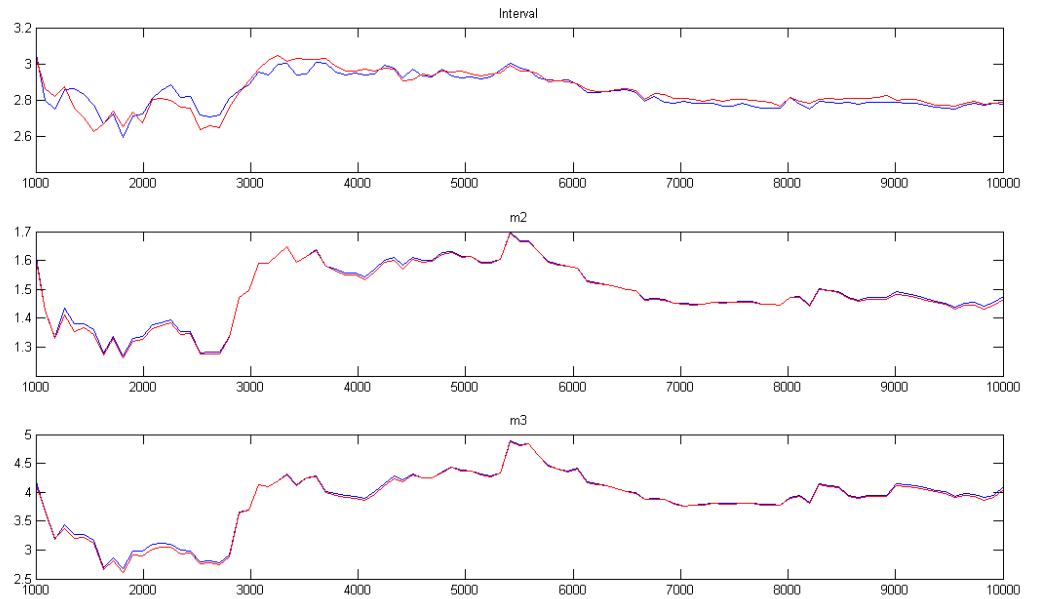
diagnostics vers9.png

Abbildung 12: plot of multivariate diagnostics for the priors of version 9 (see priors txt.)

Now the most important diagnostic plot should be discussed, the comparison of the posterior and the prior distributions - which is the main result of the estimation algorithm and as well a graphical tool to indicate existing problems. The figure which compares the prior with the posterior distributions. Here represented by the grey lines (the prior distributions)

and the black lines (the posterior distributions). The green line represents the posterior mode.
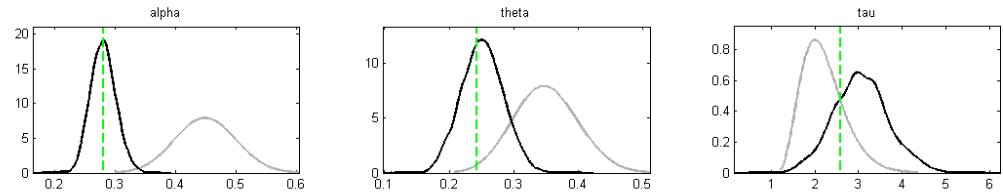
posteriors vers1.png

Abbildung 13: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

Figure 13 represents the plot of the comparison between the priors and posteriors of version 1. More of these plots are added to this textfile. To show another plot of priors and posteriors which suffer from computational problems.
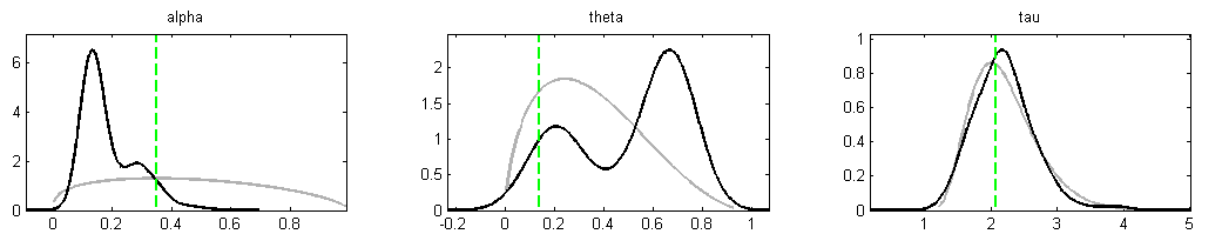
posteriors vers10.png

Abbildung 14: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

The plot of the smoothed shocks is important for checking the movement and the frequency of the shocks. The plot shows that the shock is centered around zero, which was also assumed.

If the estimators for the true model are compared with the misspecified model, several important points can be observed: If the priors of the parameters are not chosen too close to their boundaries (in the underlying, overlying and also for the opposed case) the estimator of the θ parameter is always smaller than the true value and the τ value is always larger than the true value (see the additional textfile for results of the estimations). If the priors are chosen around the true parameters and not too close to their boundaries, the estimation results are quite robust, but are biased in the described form. The results could change if the priors are set to extreme scenarios. But these results are not surprising and problematic priors can be detected by the diagnostic graphics, for example with the check plots (see exercises above).

3.5 Exercise 3.5)

The command *model comparison* can used to compare two models with own vectors of parameters which are estimated by using the same sample. This is exactly the case in this task with the original RBC model and the "misspecified"model with Cobb Douglas production function. The important values are the ones of the marginal density (here in log version). So the comparison between two models works with the following decision rule: The model with the higher liklihood fits the data better. Hence for the given models the true model estimation fits the data better than the so called misspecified model (see 3.5.txt for comparison results).