PhD Macroeconomics – Problem Set 3

Tobias Stein
Master Student
416996

Exercise 1 – Baxter and King (1993)

1. Distinguish between endogenous and exogenous variables as well as model parameters.

Endogenous:
C Y N I K w r GB IB KB tau TR z dY dC dI dN dw dr dTR dGB dIB (22)
λ_t is replaced by equation (3); thus we need 22 endogenous variables for 22 equations.

Exogenous:
$\varepsilon_t^{GB}, \varepsilon_t^{IB}, \varepsilon_t^{\tau}, \varepsilon_t^{z}$ (4)
The exogenous variables are only the shocks.

Parameters:
$\alpha, \beta, \delta, \rho_{z,}\rho_{GB}, \rho_{IB}, \rho_{\tau}, \eta, \theta_l, \bar{z}, \bar{\tau}, \overline{I_B}, \overline{G_B}$,sig_z, sig_tau, sig_IB, sig_GB (17)

2. Reasonable values for β, δ, η and parameter calibration

In the model β is set to a value of 340/341, which is approximately 0.997. In DSGE models the discount factor usually is large and within a range of 0.9 and 0.99; in most cases higher than 0.98 (e.g. Fernández-Villaverde 2010 chose 0.999). The discount factor δ is often within a range of 0.02 and 0.03, thus the parameter is set to 0.025. As well, the public capital productivity η is set to 0.05, which is more restrictive than for example Ligthart and Suárez: "The Productivity of Public Capital: A Meta-analysis" (value of 0.14).
From equation (7) we know that the share of capital in the production is $\alpha = \frac{1}{3}$; the $\rho's$ are set to 0.75. For the Frisch elasticity of labor supply I assume a value of 1.9, which is equal to the one in Smets and Wouters (2007). As stated in Fernández-Villaverde (2010) DSGE models have been criticized for implausibly high values of the Frisch elasticity (microeconomic estimates of the Frisch elasticity lie within a range of 0.1 and 0.45).

Derive the steady-state values of all other endogenous variables:

(1) (1-$\bar{\tau}$) $* 2 = 1.9 * \frac{\bar{C}}{1-1/3}$
(10) $0.22 = \bar{\tau} * (\frac{2}{3} + \bar{r}\bar{K})$
(14) $\bar{Y} = \bar{C} + \bar{I} + \overline{G_B} + \bar{I}_B$ => $\bar{C} + \bar{I} = 0.78$

(8) $\bar{r}\bar{K} = \alpha\bar{Y} = \frac{1}{3}$
Plug (8) into (10): $0.22 = \bar{\tau}$
It follows that: => $\bar{\tau} = 0.22$

Plug this result into (1). => $\bar{C} = 52/95$
Plug this result into (14) => $\bar{I} = 221/950$

(4) $\bar{K} = (1-\delta)\bar{K} + \bar{I}$ With $\delta = 0.025, \bar{I} = 221/950$ it follows that $\bar{K} = 884/95$.
(8) $\bar{r}\bar{K} = \frac{1}{3}$ => $\bar{r} = 95/2652$
(5) $\bar{K}_B = 0.8$
(6) It follows that $\bar{z} = \frac{\bar{Y}}{\bar{K}_B^{\eta} * \bar{K}^{\alpha} * \bar{N}^{1-\alpha}} \sim 1$

Thus the steady state values for the endogenous variable are:
$\bar{Y} = 1$, $\bar{K} = 884/95$, $\bar{r} = 95/2652$, $\bar{C} = 52/95$, $\bar{I} = 221/950$, $\bar{w} = 2$, $\bar{N} = \frac{1}{3}$, $\bar{K}_B = 0.8$,
$\bar{z} \sim 1$, $\bar{\tau} = 0.22$, $\bar{I}_B = 0.02$, $\bar{G}_B = 0.2$, $\overline{TR} = 0$
As well, in the steady state the equations (15) to (23) have values of 0.

3. When to simulate the deterministic or stochastic model?

The two types of models are divided by whether future shocks are known; in case of deterministic models, the realization and emergence of all future shocks is exactly known "at the time of computing the model's solution (p. 9 Dynare User Guide (DUG)). In contrast to that, in stochastic models only the distribution is known for the future shocks, as well permanent shocks cannot "be accommodated due to the need to stationarize the model around a steady state" (DUG, p. 27). In addition to that, the agents take their decisions while having in mind that the future values of the shocks are random with zero mean.
Deterministic models are useful for the analysis of regime changes like the introduction of a new tax; as well the shocks can last one or several periods (solution does only require numerical simulation techniques). For models with full information, perfect foresight and no uncertainty about shocks the agent can specify today the decisions for all future periods. Thus, when excluding stochastic elements the whole input and output relation is conclusively determined. Contrary, in the stochastic case the agent can only specify a policy or feedback rule for the future (depending on the realization of the shocks).

There is a trade-off between realism (real world is stochastic) and the easiness to understand the model results. In the deterministic environment the assumptions and equations that were selected "determine" the results whereas in the stochastic model, elements of randomness are introduced; thus each time you run the model you will get different results. To get a first impression of the model it is sufficient to use a deterministic specification; the stochastic model can be implemented for a deeper understanding and for further estimation.

5.
Economic Intuition – Compare Scenario 2 with Scenario 4:
In case of a positive government spending shock from 0.2 to 0.21 for one period there can be observed a negative effect on consumption and investment as an initial response. Nonetheless output increases by 0.15 percent in the short run and from thereon converges against its initial value of one; therefore the crowding out effect is surpassed by the positive effect of an expansionary government spending. An increase in government spending has a negative

impact on the wealth of private individuals, therefore the labor supply increases and the real wage decreases (Baxter and King 1993). Private capital decreases in the short run because of the negative investment response. Due to the fact that the model implies a balanced budget the increase in government spending goes hand in hand with a decrease in lump-sum transfers. Thus, the driving factor for a positive short-run effect in output is the negative welfare effect for the individuals, which causes an increase in the labor supply. After the peak value of 1.0015 output falls below its initial value of one because of the strong negative response in private capital; investment reaches a peak after roughly 25 periods to restore private capital. In the long run all variables return to their steady state values.

For a positive government investment shock output responds in a hump-shaped manner; the increase in output is longer lasting compared to the former shock. The temporary increase in public capital causes changes in the accumulation of private capital; while public capital increases private capital instantly decreases. Firstly, consumption decreases because of the crowding out effect caused by the governmental absorption of resources (direct effect; TR decreases) and the balanced budget condition (Baxter and King 1993). Secondly, the negative welfare effect leads to an increase in the real labor supply and a decrease in the real wage. The negative investment response turns after some periods into a positive response because private capital expands until it equals its new marginal product (changed due to public capital). Baxter and King assume that public capital alters private marginal products. An increase in investment goes hand in hand with an increase in output and consumption. Thus, the real wage increases and the real labor supply decreases.

In the short run both shocks affect the real economy. While the government spending shock only enters the model via a welfare shock the government investment shock also changes the marginal product of private capital. Thus, in the former case consumption and investment decrease while in the latter case both variables firstly decrease and after some periods overshoot its initial values (and afterwards slowly converge against the steady state).

How do the results change if the productivity of public capital increases?
An increase in η has no effect on the government spending shock (can be seen in the graphs of the percent deviations from steady state). Increasing the productivity of public capital leads to stronger direct effects for the government investment shock (Baxter and King 1993). As well, the response of private capital increases and therefore the amplitude of the variables increases. Stronger reactions in output and consumption go hand in hand with stronger reactions in the real wage and real interest rate; the real labor supply already decreases on impact and afterwards responds in a hump-shaped manner.

6.
Permanent increase in the tax rate; provide economic intuition:
As can be seen in Figure 2 of Scenario 7 output, consumption and private capital decrease if the tax rate increases because the net returns are influenced negatively (obviously private capital decreases). The plots for the percent deviations highlight that the real wage is above its new steady state value and steadily decreases; thus the tax increase has an overall negative welfare effect on the individuals. The lump-sum transfers peak approximately after periods 10-15. Investment and real labor supply reach their minimal turning points in the periods where the lump-sum transfer reaches its maximum. Afterwards the lump-sum transfers decrease and converge against the new steady state value, whereas the real labor supply and investment increase again (but they do not reach their initial values). With lower output the

tax revenues decrease, which influence the transfers negatively and therefore the labor supply increases again. The rise of the two variables is determined by the decrease in the lump-sum transfers, thus individuals are willing to work at a lower real wage to out cancel at least partly the negative welfare effect. Due to an increasing government spending (absolute value is fixed but the relative value increases; can be seen in the Figure of percent deviations) the real interest rate and investment (after periods 10-15) increase (like in Baxter and King 1993). Thus, in the new steady state consumption, output, investment, private capital, real labor supply and the real wage are below the starting values of the steady state.

7.
Can fiscal policy boost the economy?
For both temporary shocks (government investment & spending) the output rose above its initial value. Thus, the crowding-out effect was surpassed by the negative welfare effect, which causes an increase in the labor supply plus the direct effects of government activities. Moreover, as explained in task 5 fiscal policy can boost the economy in the short run; therefore the fiscal multiplier exceeds 1.

Explain the difference between private and government capital:

Government capital can either be used for current expenditures (e.g. wages for employees) or consumable goods like drugs. As well, government capital can be used to finance hospitals, build new streets/bridges or airports. Expenditures for salaries differ in that way from spending's for roads that the latter ones have a lasting impact on the growth of the economy and can increase the effectiveness of private capital (e.g. used to buy new trucks). Moreover, physical government capital (like harbors) can increase private production possibilities[1].

Private capital can make use of public capital by using roads, bridges, state-run harbors or canals. Private capital can be spend for ships, cars etc. thus government capital can help to attract capital flows or foreign direct investments. Government capital, when used for education, can increase human capital, which might be a key factor for economic growth.

Both terms therefore are closely related because of the interdependence between the effectiveness of each other.

[1] http://www.sciencedirect.com/science/article/pii/0165176583900885

Exercise 2: An and Schorfheide (2007)

1. How many observable variables do you need for Bayesian estimation?

As stated on slide 51 (chapter on DSGE methods) the estimation of a DSGE model requires "that the number of shocks is equivalent to the numbers of observable variables." In addition to that this condition is "less stringent […] than for maximum likelihood estimation" (DUG, p. 46). Nonetheless it can be the case that not all parameters can be identified, hence the posterior distribution and the prior distribution are identical (DUG, p. 46).
In case that the observable variables are real world data, it is important to consider data with a good "signal-to-noise ratio"; otherwise data may be measured by error or noise and thus are imprecise (Pfeifer 2014).

3. Explain the intuition behind prior information in Bayesian estimation (10 sentences).

Prior information contains upper and lower bounds for the parameters as well as knowledge on the kurtosis and skewness of the distribution (e.g. inverse gamma distribution or beta distribution). Thus, considering prior information works like "weights in the estimation process" because the posterior distribution leaves out "strange points where the likelihood peaks" (DUG, p. 78). Therefore priors can help to reduce the "dilemma of absurd parameter estimates"; which arises (as explained in the User Guide) due to a flat posterior distribution. In this case priors add curvature to nearly flat likelihood functions and thus strongly influence the posterior distribution (An and Schorfheide, p. 14). Non-technically, Bayes rule combines prior believes and the likelihood function (given the data) to obtain the posterior distribution; thus it combines "internal" information (the data) and "external" information (prior distribution). Relevant prior information increases the chance not to waste any relevant facts and thus to strengthen inferences about the true value[2].
As stated on slide 44 of the lecture notes Bayesian estimation makes use of Bayes-rule, as well as taking into consideration that the likelihood function is a "conditional density of observed data given the parameters":

$$p(\theta|\mathrm{d}) = \frac{L(d|\theta)\mathrm{p}(\theta)}{p(d)} = \frac{\mathrm{L}(\mathrm{d}|\theta)\mathrm{p}(\theta)}{\int p(\theta)L(d|\theta)\mathrm{d}\theta} \propto L(d|\theta)\mathrm{p}(\theta).$$

Prior information can either be informative or non-informative; in case of non-informative priors the posterior distribution is barely effected by external information. On one side it can be argued that the use of informative and strict priors is subjective and can lead to desired results; on the other side prior information can strengthen inference and adds important pre-knowledge[3]. To sum up, priors more or less increase the importance of a predefined "parameter subspace" by giving "weights on the likelihood function" (DUG, p. 79).

5. Consider the figure Priors and Posteriors. How would you assess the quality of this estimation exercise?

Directly comparing the priors and posteriors gives valuable insights about the relevance of the information in the data for the parameters of interest (An and Schorfheide 2007, p. 13). Thus, in case that the figure for the priors and posteriors is the same then the data do not contain any

[2] http://www.medicine.ox.ac.uk/bandolier/painres/download/whatis/What_is_Bay_stats.pdf (p. 2)
[3] http://www.medicine.ox.ac.uk/bandolier/painres/download/whatis/What_is_Bay_stats.pdf (p. 5)

additional information. The figures for priors and posteriors can be interpreted as follows: The grey line represents the priors; whereas the black line represents the posterior (dotted green line is the value for the posterior mode)[4]. In most cases of the Figures 8 and 9 the prior and posterior distributions are different which should be the case for identifying the parameters. The only problematic parameters are *tau* und *psi1* because the priors and posteriors differ barely from their priors (varobs are ‘YGR’,’ INFL’, ‘INT’); nonetheless there is still some informative value in the data for these two parameters.

For example when changing varobs to ‘y’, ‘p’ , ‘INT’ the parameters for ‘r_A’ and ‘p_A’ have nearly identical prior and posterior means.

Therefore the former choice for varobs leads to a relatively adequate specification of the model parameters because the data add information to all prior believes (prior means and posterior means differ for each parameter).

[4] http://www.wiwi.hu-berlin.de/de/professuren/vwl/wipo/team/former/kriwoluzky/intro_dynare_handout.pdf (p. 9)