The Taylor Principle

Up to now, we have maintained the assumption that the central bank reacts to a change in inflation by implementing a bigger change in interest rates. In terms of the equation for our monetary policy rule, this means we are assuming $\beta_\pi > 1$. With this assumption, real interest rates go up when inflation rises and go down when inflation falls. For this reason, our IS-MP curve slopes downwards: Along this curve, higher inflation means lower output. Because John Taylor's original proposed rule had the feature that $\beta_\pi > 1$, the idea that monetary policy rules should have this feature has become known as *the Taylor Principle*. In these notes, we discuss why policy rules should satisfy the Taylor principle.

Three Different Cases

Recall from our last set of notes that inflation in the IS-MP-PC model is given by

$$\pi_t = \theta \pi_t^e + (1 - \theta) \pi^* + \theta \left(\gamma \epsilon_t^y + \epsilon_t^\pi \right) \tag{1}$$

where

$$\theta = \left(\frac{1}{1 + \alpha \gamma (\beta_\pi - 1)} \right) \tag{2}$$

Under adaptive expectations $\pi_t^e = \pi_{t-1}$ and the model can be re-written as

$$\pi_t = \theta \pi_{t-1} + (1 - \theta) \pi^* + \theta \left(\gamma \epsilon_t^y + \epsilon_t^\pi \right) \tag{3}$$

The value of θ turns out to be crucial to the behaviour of inflation and output in this model. We can describe three different cases depending on the value of β_π.

Case 1: $\beta_\pi > 1$

If the Taylor principle is satisfied, then $\alpha\gamma\left(\beta_\pi - 1\right) > 0$. That value being positive means that $1 + \alpha\gamma\left(\beta_\pi - 1\right) > 1$. The parameter θ is calculated by dividing 1 by this amount so this gives us a value of θ that is positive but less than one. So $\beta_\pi > 1$ translates into the case $0 < \theta < 1$.

Case 2: $\left(1 - \frac{1}{\alpha\gamma}\right) < \beta_\pi < 1$

As we reduce β_π below one, $\left(\beta_\pi - 1\right)$ becomes negative, meaning $\alpha\gamma\left(\beta_\pi - 1\right) < 0$ and $1 + \alpha\gamma\left(\beta_\pi - 1\right) < 1$. The parameter θ is calculated by dividing 1 by this amount so this gives us a value of θ that is greater than one. As β_π falls farther below one, θ gets bigger and bigger and heads towards infinity as β_π approaches $\left(1 - \frac{1}{\alpha\gamma}\right)$ (this is the value of β_π that makes the denominator in the θ formula equal zero). As long we assume that β_π stays above this level, we will get a value of θ that is positive and greater than one.

Case 3: $0 < \beta_\pi < \left(1 - \frac{1}{\alpha\gamma}\right)$

This produces a "pathological" case in which $1 + \alpha\gamma\left(\beta_\pi - 1\right) < 0$ so the value of θ becomes negative, meaning an increase in inflation expectations actually reduces inflation. We are not going to consider this case.

Macro Dynamics and Difference Equations

These calculations tell us that as long as the Taylor principle is satisfied, the value of θ lies between zero and one but that if β_π slips below one, then θ becomes greater than one. It turns out this is a very important distinction. To understand the difference between these two cases, we need to explain a little bit about difference equations.

A difference equation is a formula that generates a sequence of numbers. In economics, these sequences can be understood as a pattern over time for a variable of interest. After supplying some starting values, the difference equation provides a sequence explaining how the variable changes over time. For example, consider a case in which the first value for a series is $z_1 = 1$ and then z_t follows a difference equation

$$z_t = z_{t-1} + 2 \tag{4}$$

This will give $z_2 = 3$, $z_3 = 5$, $z_4 = 7$ and so on. More relevant to our case is the multiplicative model

$$z_t = bz_{t-1} \tag{5}$$

For a starting value of $z_1 = x$, this difference equation delivers a sequence of values that looks like this: $x, xb, xb^2, xb^3, xb^4.....$

Note that if b is between zero and one, then this sequence converges to zero over time no matter what value x takes but if $b > 1$, the sequence will explode off towards either plus or minus infinity depending on whether the initial value was positive or negative. The same logic prevails if we add a constant term to the difference equation. Consider this equation:

$$z_t = a + bz_{t-1} \tag{6}$$

If b is between zero and one, then no matter what the starting value is, the sequence converges over time to $\frac{a}{1-b}$ but if $b > 1$, the sequence explodes towards infinity. Similarly, if we add random shocks to the model—making it what is known as a first-order autoregressive or AR(1) model—the key thing remains the value of b. If the model is

$$z_t = a + bz_{t-1} + \epsilon_t \tag{7}$$

where ϵ_t is a series of zero-mean random shocks, then the presence of the shocks will mean the series won't simply converge to a constant or steadily explode. But as long as we have $0 < b < 1$ then the series will tend to oscillate above and below the average value of $\frac{a}{1-b}$ while if $b > 1$ the series will tend to explode to infinity over time.

The Taylor Principle and Macroeconomic Stability

These considerations explain why the Taylor principle is so important. If $\beta_\pi > 1$ then inflation dynamics in the IS-MP-PC model can be described by an AR(1) model with a coefficient on past inflation that is between zero and one (the θ in equation 3 plays the role of the coefficient b in the models just considered.) So a policy rule that satisfies the Taylor principle produces a stable time series for inflation under adaptive expectations. And because output depends on the gap between inflation expectations and the central bank's inflation target, stable inflation translates into stable output.

In contrast, once β_π falls below 1, the coefficient on past values of inflation in equation (3) becomes greater than one and the coefficient on the inflation target becomes negative. In this case, any change in inflation produces a greater change in the same direction next period and inflation ends up exploding off to either plus or minus infinity. Similarly output either collapses or explodes.

Why does β_π matter so much for macroeconomic stability? Obeying the Taylor principle means that shocks that boost inflation (whether they be supply or demand shocks) raise real interest rates (because nominal rates go up by more than inflation does) and thus reduce output, which contains the increase in inflation and keeps the economy stable. In contrast, when the β_π falls below 1, shocks that raise inflation result in lower real interest rates and

higher output which further fuels the initial increase in inflation (similarly declines in inflation are further magnified). This produces an unstable explosive spiral.

You might be tempted to think that the arguments in favour of obeying the Taylor principle as explained here depends crucially on the assumption of adaptive expectations but this isn't the case. Even before assuming adaptive expectations, from equation (1) we can see that when $\theta > 1$, the coefficient on the central bank's inflation target is negative. So if you introduced a more sophisticated model of expectations formation, the public would realise that the central bank's inflation target doesn't have its intended influence on inflation and so there would no reason to expect this value of inflation to come about. But if people know that changes in expected inflation are translated more than one-for-one into changes in actual inflation then this could produce self-fulfilling inflationary spirals, even if the public had a more sophisticated method of forming expectations than the adaptive one employed here.

Graphical Representation

We can use graphs to illustrate the properties of the IS-MP-PC model when the Taylor principle is not obeyed. Recall that the IS-MP curve is given by this equation

$$y_t = y_t^* - \alpha \left(\beta_\pi - 1\right) \left(\pi_t - \pi^*\right) + \epsilon_t^y \tag{8}$$

The slope of the curve depends on whether or not $\beta_\pi > 1$. In our previous notes, we assumed $\beta_\pi > 1$ and so the slope $-\alpha \left(\beta_\pi - 1\right) < 0$, meaning the IS-MP curve slopes down. With $\beta_\pi < 1$, the IS-MP curve slopes up. Figure 1 illustrates the IS-MP-PC model in this case under the assumption that $\pi_t^e = \pi^* = \pi_1$, i.e. that the public expects inflation to equal the central bank's target.

One technical point about this graph is worth noting. I have drawn the upward-sloping

IS-MP curve as a steeper line than the upward-sloping Phillips curve. On the graph as we've drawn it in inflation-output space, the slope of this curve is $\frac{1}{\alpha(1-\beta_\pi)}$ while the slope of the Phillips curve is γ. One can show that the condition that $\frac{1}{\alpha(1-\beta_\pi)} > \gamma$ is the same as showing that $\theta > 0$, i.e. that we are ruling out values of β_π associated with the strange third case noted above.

Now consider what happens when there is an increase in inflation expectations when β_π falls below one. Figure 2 shows a shift in the Phillips curve due to inflation expectations increasing from π_1 to π_h (You can see that the value of inflation on the red Phillips curve when $y_t = y_t^*$ is $\pi_t = \pi^h$). Notice now that, because the IS-MP curve is steeper than the Phillips curve, inflation increases above π_h to take the higher value of π_2. Inflation overshoots the public's expected value.

Figure 3 shows what happends next if the public have adaptive expectations. In this next period, we have $\pi_t^e = \pi_2$ and inflation jumps all the way up to the even higher value of π_3. We won't show any more graphs but the process would continue with inflation increasing every period. These figures thus show graphically what we've already demonstrated with equations. The IS-MP-PC model generates explosive dynamics when the monetary policy rule fails to obey the Taylor principle.

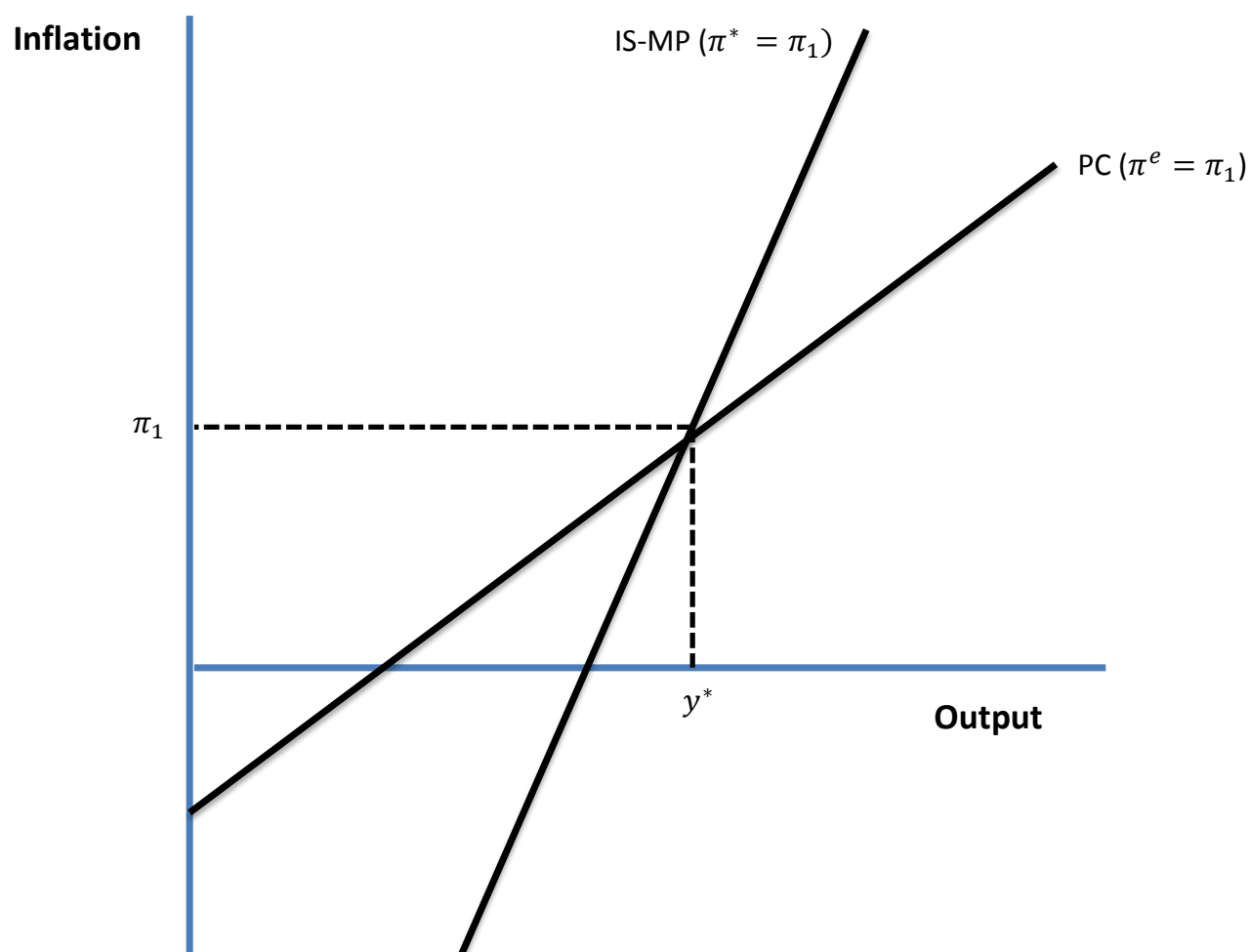## Figure 1: The IS-MP-PC Model when $\left(1 - \frac{1}{\alpha\gamma}\right) < \beta_\pi < 1$

Inflation

IS-MP ($\pi^* = \pi_1$)

PC ($\pi^e = \pi_1$)

π_1

y^*

Output

Figure 2: An Increase in π^e when $\left(1 - \frac{1}{\alpha\gamma}\right) < \beta_\pi < 1$
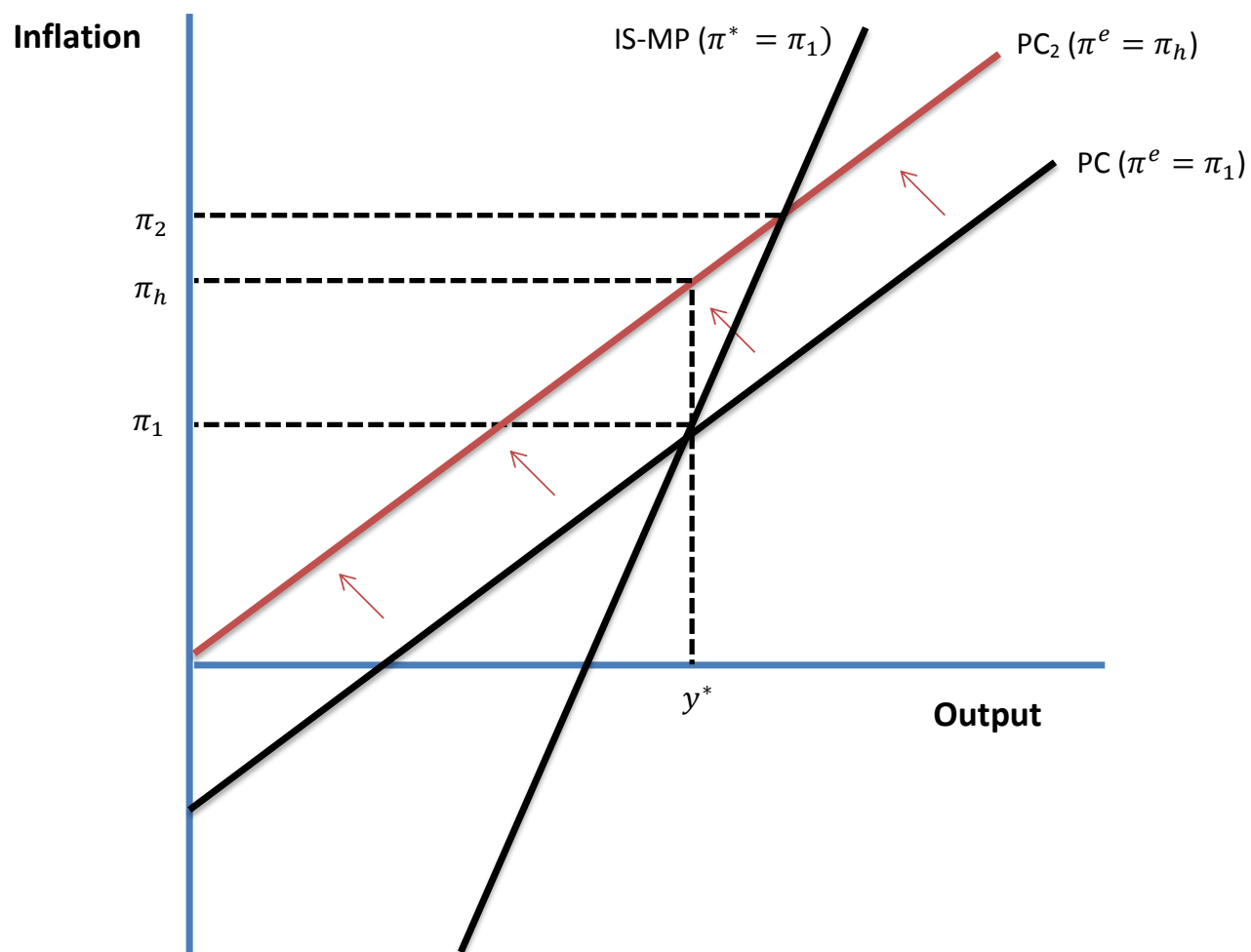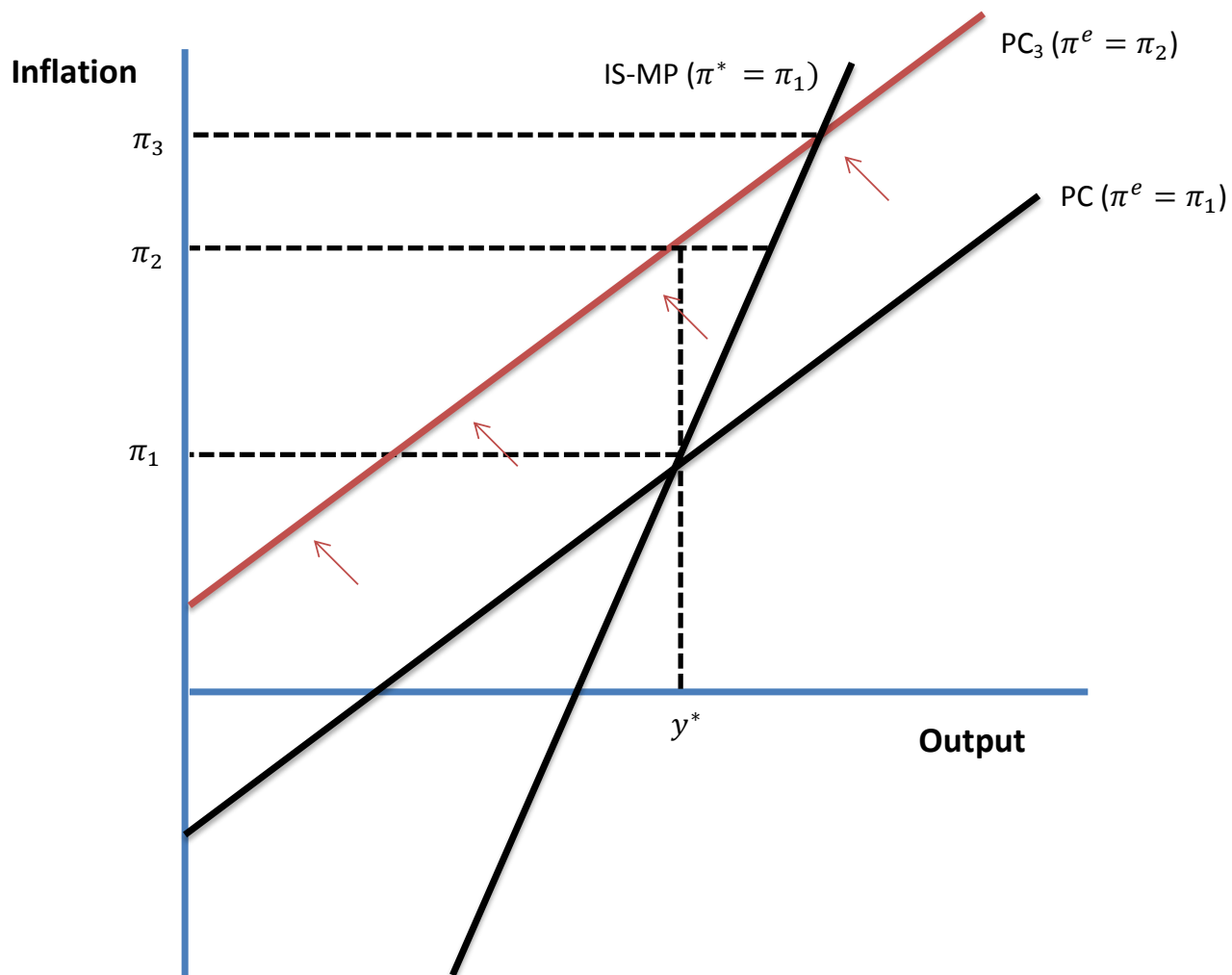
Figure 3: Explosive Dynamics when $\left(1 - \frac{1}{\alpha\gamma}\right) < \beta_\pi < 1$

An Increase in the Inflation Target

Figure 4 illustrates what happens in the IS-MP-PC model when the central bank changes its inflation target. The increase in the inflation target shifts the IS-MP curve upwards i.e. each level of output is associated with a higher level of inflation. However, because the IS-MP curve is steeper than the Phillips curve, the outcome is a reduction in inflation. Output also falls.

Even though this is exactly what our earlier equations predicted (the coefficient on the inflation target is $1 - \theta$ which is negative in this case) this seems like a very strange outcome. The central bank sets a higher inflation target and then inflation falls. Why is this?

The answer turns out to reflect the particular form of the monetary policy rule that we are using. This rule is as follows:

$$i_t = r^* + \pi^* + \beta_\pi \left(\pi_t - \pi^* \right) \tag{9}$$

You might expect that a higher inflation target would lead to the central bank setting a lower interest rate, i.e. they ease up to allow the economy to expand and let inflation move higher. However, if you look closely at this formula, you can see that an increase in the inflation target actually leads to a *higher* interest rates when $\beta_\pi < 1$.

This can be explained as follows. The inflation target appears twice in equation (9). It appears in brackets as part of the "'inflation gap" term $\pi_t - \pi^*$ which is muliplied by β_π. If this was the only place that it appeared, then indeed a higher inflation target would lead to lower interest rates. However, the first part of rule relates to setting the interest rate so that when inflation equalled its target, real interest rates would equal their "natural rate" r^*. The rule is set on the basis that if inflation is going to be higher on average, then the nominal interest rate also needs to be higher if real interest rates are to remain unchanged (this is

commonly called the "Fisher effect" of inflation on interest rates).

Putting these two effects together, we see that an increases of x in the inflation target raises the nominal interest rate by x due to the real interest rate component and reduces it by $\beta_\pi x$ due to inflation now falling below target. If $\beta_\pi < 1$ then the higher inflation target results in higher interest rates and thus lower output. This is the pattern shown in Figure 4.

Figure 4: An Increase in π^* when $\left(1 - \frac{1}{\alpha\gamma}\right) < \beta_\pi < 1$
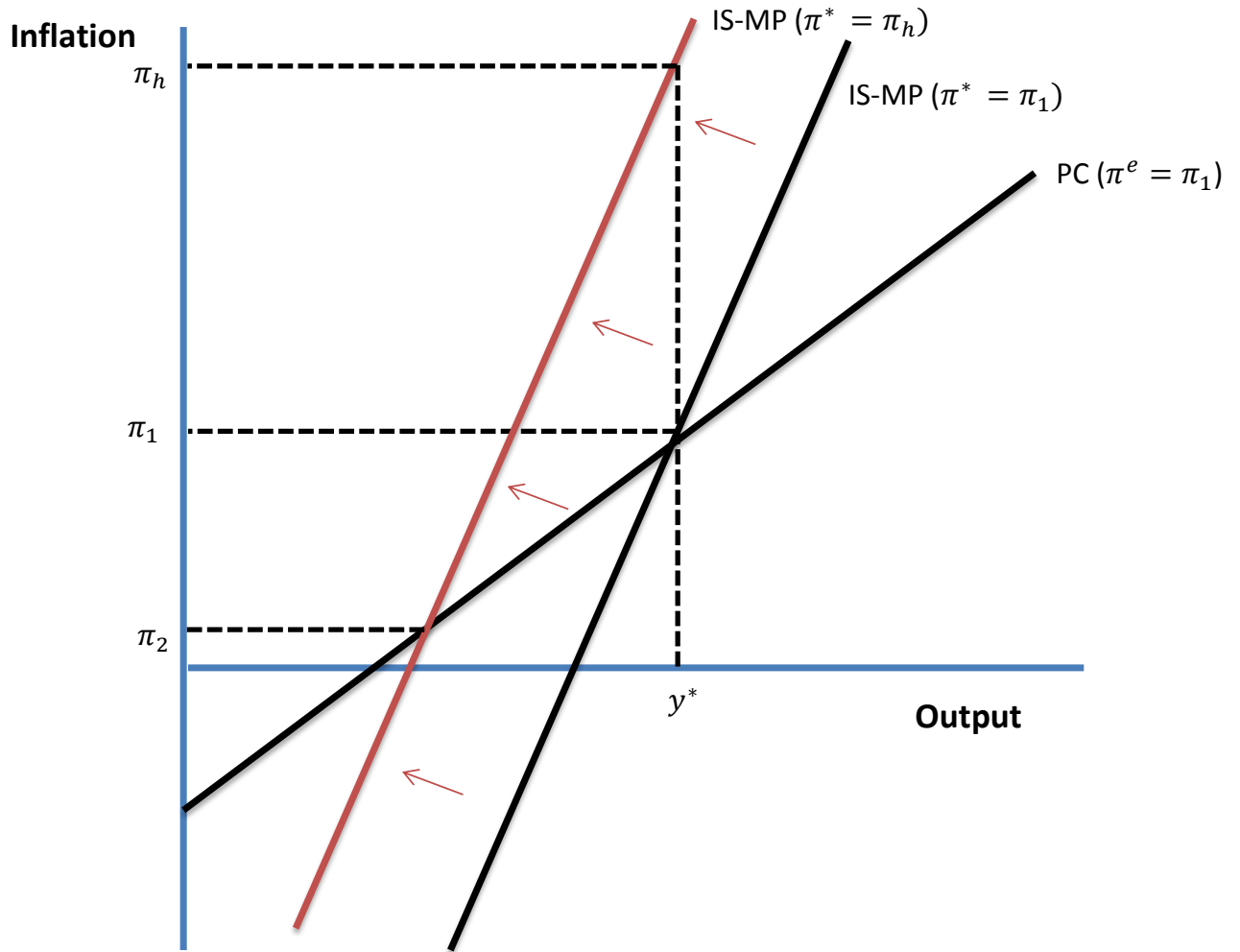
Evidence on Monetary Policy Rules and Macroeconomic Stability

Is there any evidence that obeying the Taylor principle provides greater macroeconomic stability? Some economists believe there is.

The website links to a paper titled "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory" by Richard Clarida, Jordi Gali and Mark Gertler. These economists reported that estimated policy rules for the Federal Reserve appeared to show a change after Paul Volcker was appointed Chairman in 1979. They estimated that the post-1979 monetary policy appeared consistent with a rule in which the coefficient on inflation that was greater than one while the pre-1979 policy seemed consistent with a rule in which this coefficient was less than one. They also introduce a small model in which the public adopts rational expectations (more on what this means later) and show that failure to obey the Taylor principle can lead to the economy generating cycles based on self-fulfilling fluctuations. They argue that failure to obey the Taylor principle could have been responsible for the poor macroeconomic performance, featuring the stagflation combination of high inflation and recession, during the 1970s in the US.

There are a number of differences between the approach taken in Clarida, Gali, Gertler paper and these notes (in particular, their estimated policy rule is a "forward-looking" one in which policy reacts to expected future values of inflation and output) and the econometrics are perhaps more advanced than you have seen but it's still a pretty readable paper and a nice example of policy-relevant macroeconomic research.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. Definition of the Taylor principle.

2. How variations in β_π affect θ: The three different cases.

3. Difference equations and conditions for stability.

4. Rationale for why obeying the Taylor principle stabilises the economy.

5. How the three cases are represented on graphs.

6. How to graph the explosive dynamics when Taylor principle is not satisfied.

7. Impact of a change in the inflation target when Taylor principle is not satisfied.

8. Evidence on monetary policy rules and macroeconomic stability.