Task 1

The following explanations are based upon the lecture and assignment notes of the lecture "*Formulation, Estimation and Policy Analysis with DSGE Models"*

http://faculty.wcas.northwestern.edu/~lchrist/course/assignment7ans.pdf

(For details see: dynare code: cggexam.mod and the corresponding graphs)

1.

In the case that the growth rate of technology is AR(1) (unit root case) a jump in technology signals even greater increases in the future. This creates a desire to borrow, and so the natural rate of interest must rise to discourage this effect. The desire to borrow and the fact that the baseline Taylor rule do not incorporated the natural interest rate implies that the economy experience a positive output gap and inflation which leads to an overreaction to the technology shock. In the case that it is the level of log technology that is AR(1), a jump in technology creates the expectation that technology will be lower in the future, creating a desire to save. The higher natural rate of interest counters this desire. The desire to save and the fact that the baseline Taylor rule does not incorporated the natural interest rate either, implies that the economy in this scenario experience a negative output gap and deflation which leads to an underreaction to the technology shock.

2.

A signal that technology will rise in the future raises the natural rate of interest in order to reduce the desire to borrow which is the same as in unit root case.. Because the Taylor rule does not raise the rate of interest high enough, a boom occurs. Paradoxically, inflation falls at the same time because - although current marginal cost rises - future marginal cost is expected to fall. The latter effect explains why forward-looking price setters reduce prices.

3.

Since now the natural rate of interest is incorporated in the Taylor rule monetary policy counteracts the boom. In this scenario no inflation and output gap develops so that the natural output and the actual output is equal to the natural output. We observe no over- or under reaction to the shocks anymore.

Task 2

The equations:

$$u'(d_t) * p_t = \beta E_t[u'(d_{t+1}) * (p_{t+1} + d_{t+1})]$$
$$d_{t+1} = \rho d_t + e_{t+1}$$

1.

In this simplified asset pricing model we identify the price p_t as the control variable and the dividends d_t as the state variable.

2.

$$x_{t+1} = \bar{x} + h(x_t, \sigma) + \sigma \varepsilon_{t+1}$$

$$d_{t+1} = \bar{d} + \rho d_t + e_{t+1} + \sigma \varepsilon_{t+1}$$
$$x_{t+1} = \bar{x} + \rho x_t + e_{t+1} + \sigma \varepsilon_{t+1}$$

In this case we observe an asset pricing model where in general uncertainty matters (in form of risk premia) so we should not set σ to zero. So that unlike our results derived in the lecture uncertainty is important for the decision rule

3.

At first we have to define the function $g(x_t, \sigma)$:

$$y_t = g(x_t, \sigma)$$

Given that we could substitute

$$F(x_t, \sigma) = u'(x_t) * g(x_t, \sigma) - \beta E_t[u'(x_{t+1}) * (y_{t+1} + x_{t+1})]$$
$$F(x_t, \sigma) = u'(x_t) * g(x_t, \sigma) - \beta u'(\rho x_t + e_{t+1}) * (g(\rho x_t + e_{t+1}, \sigma) + \rho x_t + e_{t+1}) = 0$$
$$F(x_t, \sigma) = u'(x_t) * g(x_t, \sigma) - \beta u'(\rho x_t + e_{t+1}) * g(\rho x_t + e_{t+1}, \sigma) - \beta u'(\rho x_t + e_{t+1}) * (\rho x_t + e_{t+1}) = 0$$

$$F_x = \frac{dF}{dx} = u''(x_t) * g(x_t, \sigma) + u'(x_t) * g_x(x_t, \sigma) - \beta \rho u''(\rho x_t + e_{t+1}) * (g(\rho x_t + e_{t+1}, \sigma) + \rho x_t + e_{t+1}) - \beta u'(\rho x_t + e_{t+1}) * (g_x(\rho x_t + e_{t+1}, \sigma) * \rho + \rho) = 0$$

$$F_{xx} = \frac{d^2F^2}{dx^2} = u'''(x_t) * g(x_t, \sigma) + u''(x_t) * g_x(x_t, \sigma) + u''(x_t) * g_x(x_t, \sigma) + u'(x_t) * g_{xx}(x_t, \sigma) - \beta\rho^2 u'''(\rho x_t + e_{t+1}) * (g(\rho x_t + e_{t+1}, \sigma) + \rho x_t + e_{t+1}) - \beta\rho u''(\rho x_t + e_{t+1}) * (g_x(\rho x_t + e_{t+1}, \sigma) * \rho + \rho) - \beta\rho u''(\rho x_t + e_{t+1}) * (g_x(\rho x_t + e_{t+1}, \sigma) * \rho + \rho) - \beta u'(\rho x_t + e_{t+1}) * (g_{xx}(\rho x_t + e_{t+1}, \sigma) * \rho^2) = 0$$

$$F_\sigma = u'(x_t) * g_\sigma(x_t, \sigma) - \beta u'(\rho x_t + e_{t+1}) * g_\sigma(\rho x_t + e_{t+1}, \sigma) = 0$$

$$F_{\sigma\sigma} = u'(x_t) * g_{\sigma\sigma}(x_t, \sigma) - \beta u'(\rho x_t + e_{t+1}) * g_\sigma(\rho x_t + e_{t+1}, \sigma) = 0$$

$$F_{\sigma x} = u''(x_t) * g_\sigma(x_t, \sigma) + u'(x_t) * g_{\sigma x}(x_t, \sigma) - \beta\rho u''(\rho x_t + e_{t+1}) * g_\sigma(\rho x_t + e_{t+1}, \sigma) - \beta\rho u'(\rho x_t + e_{t+1}) * g_{\sigma x}(\rho x_t + e_{t+1}, \sigma) = 0$$

$$F_{x\sigma} = u''(x_t) * g_\sigma(x_t, \sigma) + u'(x_t) * g_{x\sigma}(x_t, \sigma) - \beta\rho u''(\rho x_t + e_{t+1}) * g_\sigma(\rho x_t + e_{t+1}, \sigma) - \beta\rho u'(\rho x_t + e_{t+1}) * g_{x\sigma}(\rho x_t + e_{t+1}, \sigma) = 0$$

Now taking expectations and evaluate it at $(\bar{x}, 0)$

$$y_t = g(\bar{x}, 0) = \bar{y}$$

$$F_x = u''(\bar{x}) * \bar{y} + u'(\bar{x}) * g_x - \beta\rho u''(\bar{x}) * (\bar{y} + \bar{x}) - \beta\rho u'(\bar{x}) * g_x - \beta u'(\bar{x}) * \bar{x} = 0$$

$$F_x = (1 - \beta\rho) * u'(\bar{x}) * g_x = -u''(\bar{x}) * \bar{y} + \beta\rho u''(\bar{x}) * (\bar{y} + \bar{x}) + \beta\rho u'(\bar{x}) * g_x + \beta u'(\bar{x}) * \bar{x}$$

$$g_x = \frac{-u''(\bar{x}) * \bar{y} + \beta\rho u''(\bar{x}) * (\bar{y} + \bar{x}) + \beta u'(\bar{x}) * \bar{x}}{(1 - \beta\rho) * u'(\bar{x})}$$

$$F_{xx} = \frac{d^2F^2}{dx^2} = u'''(\bar{x}) * \bar{y} + u''(\bar{x}) * g_x + u''(x) * g_x + u'(\bar{x}) * g_{xx} - \beta\rho^2 u'''(\bar{x}) * (\bar{y} + \bar{x}) - \beta\rho u''(\bar{x}) * (g_x * \rho + \rho) - \beta\rho u''(\bar{x}) * (g_x * \rho + \rho) - \beta u'(\bar{x}) * g_{xx} * \rho^2 = 0$$

$$F_{xx} = \frac{d^2F^2}{dx^2} = u'''(\bar{x}) * \bar{y} + u''(\bar{x}) * g_x + u''(x) * g_x + u'(\bar{x}) * g_{xx} - \beta\rho^2 u'''(\bar{x}) * (\bar{y} + \bar{x})$$
$$- \beta\rho u''(\bar{x}) * \rho * (\bar{y} + 1) - \beta\rho u''(\bar{x}) * \rho * (\bar{y} + 1) - \beta u'(\bar{x}) * g_{xx} * \rho^2 = 0$$

$$(1 - \beta\rho^2)u'(\bar{x}) * g_{xx}$$
$$= -u'''(\bar{x}) * \bar{y} - u''(\bar{x}) * g_x - u''(x) * g_x + \beta\rho^2 u'''(\bar{x}) * (\bar{y} + \bar{x})$$
$$+ 2\beta\rho u''(\bar{x}) * \rho * (\bar{y} + 1)$$

$$g_{xx} = \frac{-u'''(\bar{x}) * \bar{y} - u''(\bar{x}) * g_x - u''(x) * g_x + \beta\rho^2 u'''(\bar{x}) * (\bar{y} + \bar{x}) + 2\beta\rho^2 u''(\bar{x}) * (\bar{y} + 1)}{(1 - \beta\rho^2)u'(\bar{x})}$$

$$F_\sigma = u'(\bar{x}) * g_\sigma - \beta u'(\bar{x}) * g_\sigma = 0$$

$$g_\sigma = 0$$

Since $F_{x\sigma} = F_{\sigma x}$ it must hold that $g_{x\sigma} = g_{\sigma x}$:

$$F_{x\sigma} = u''(\bar{x}) * g_\sigma + u'(\bar{x}) * g_{x\sigma} - \beta\rho u''(\bar{x}) * g_\sigma - \beta\rho u'(\bar{x}) * g_{x\sigma} = 0$$
$$F_{x\sigma} = (1 - \beta\rho)u'(\bar{x}) * g_{x\sigma} = -u''(\bar{x}) * g_\sigma + \beta\rho u''(\bar{x}) * g_\sigma$$
$$g_{x\sigma} = \frac{-(1 - \beta\rho)\, u''(\bar{x}) * g_\sigma}{(1 - \beta\rho)u'(\bar{x})}$$
$$g_{x\sigma} = -\frac{u''(\bar{x})}{u'(\bar{x})} * g_\sigma$$

Since $g_\sigma = 0$

$$g_{x\sigma} = g_{\sigma x} = 0$$

Task 3

The computations for this task are based upon the dynare code of the exercise 5 of the Lecture DSGE Modelle:

http://www1.wiwi.uni-muenster.de/oeew/studium/dsgemodels/ss2012/index.php the

(For details see the dynare code: rbcexam1.mod. and the corresponding graphs)

1.

Parameters	**Pdf's**
alpha	Beta
theta	Beta
tau	Gamma

The number of observables should always match the number of shocks. Which means that we have one observable variable y.

2.

MODE CHECK – right model

RESULTS FROM POSTERIOR MAXIMIZATION

parameter	Prior mean	Mode	s.d	t-stat	Prior	pstdev
alpha	0.450	0.4565	0.0294	15.5291	beta	0.0500
theta	0,350	0,3529	0,0435	8,1189	Beta	0,0500
tau	2,207	2,0094	0,4522	4,4437	Gamma	0,5000

Log data density [Laplace approximation] is -342.768462.

MODE CHECK - Missspecified model

parameter	Prior mean	Mode	s.d	t-stat	Prior	pstdev
alpha	0.450	0.2806	0.0253	11.1012	beta	0.0500
theta	0,350	0,2423	0,0404	6.0022	Beta	0,0500
tau	2,207	2,5826	0,5538	4,6639	Gamma	0,5000

Log data density [Laplace approximation] is -352.900747.

3, 4.

For the diagnostic plots see: uni_Diagnosis_plot_rbcexam.pdf and multi_Diagnosis_plot_rbcexam.pdf and

ESTIMATION RESULTS– right model

MH- Algorithm-1

parameter	Prior mean	Post mean	Conf. interval		Prior	pstdev
alpha	0.450	0.4557	0.4131	0.5084	beta	0.0500
theta	0,350	0,3598	0,2902	0,4305	Beta	0,0500
tau	2,207	2,2278	1,4312	2,9702	Gamma	0,5000

ESTIMATION RESULTS– right model

MH- Algorithm-2

parameter	Prior mean	Post mean	Conf. interval		prior	pstdev
alpha	0.450	0.4543	0.4057	0.5006	beta	0.0500
theta	0,350	0,3607	0,2938	0,4311	Beta	0,0500
tau	2,207	2,2128	1,4678	2,9730	Gamma	0,5000

Log data density is -342.789956.

Missspecified model:

ESTIMATION RESULTS

MH- Algorithm-1

parameter	Prior mean	Post mean	Conf. interval		prior	pstdev
alpha	0.450	0.2797	0.2390	0.3198	beta	0.0500
theta	0,350	0,2517	0,1911	0,3203	Beta	0,0500
tau	2,207	2,7970	1,8964	3,7217	Gamma	0,5000

Log data density is -352.896575.

acceptance .rate: 0.2535 0.2494 0.2593

ESTIMATION RESULTS

MH- Algorithm-2

parameter	Prior mean	Post mean	Conf. interval		prior	pstdev
alpha	0.450	0.2790	0.2394	0.3189	beta	0.0500
theta	0,350	0,2519	0,1934	0,3220	Beta	0,0500
tau	2,207	2,7895	1,8964	3,7734	Gamma	0,5000

Log data density is -352.892583.

acc.rate 0.2408   0.2817   0.2707

If we compare the parameter estimates of the two models we see the influence of the missspecification concerning the Cobb-Douglas production function. We observe that the posterior modes from the mode check for both models differ much and so the estimates of the missspecified model are much less accurate compared to the estimate of the right model. This tendency exists also in the MH-algorithm estimates.
Besides this inaccuracy which stems from the fact of our missspecification the diagnostic plots show that the within and between measures of the different chains converge in most cases. (This result becomes more decisive if expand the algorithm to 10000 iterations). Most important the acceptance rate averages around 0.254 and 0.2644 with is good since we try to set up the algorithm with acceptance rate between 0.2-0.3 respectively. The priors and posteriors distribution are also quite different and the posterior mode of theta is different form the numerical determined posterior mode (green line).
A way to improve our estimation is to conduct a sensitivity analysis with different priors. Another approach could be to undertake more MH-simulations in order to improve the posterior modes estimates. Nonetheless it seems improbable that this analysis could improve our estimation in such a way that the effect of the missspecification is gone.
After this analysis it seems reasonable, that if the estimates remaining unsatisfactory, the missspecification should be tackle and we might try to improve our model.

5.

Model Comparison

Model	rbcestim	rbcexam1
Priors	0,5	0,5
Log marginal density	-342.768462	-352.900747
Bayes ratio	1	0,00004
Posterior Model Probability	0,999960	0,000040

Posterior-Odds are the ratio of the Posterior Model Probabilities

The model comparison with the posterior odds indicate that our first model(the right model) is much better in characterizing the data.