Dynamic Stochastic General Equilibrium Models

Prototype DSGE

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General procedure

The general procedure involves the following steps:

- Specifying the features of the DSGE model.
- Solving the model.
- Estimating the model.

Specifying the DSGE model

The basic RBC model

- The economy consists of a large number of households and firms.
- Firms produce a homogeneous final product that can be either consumed or invested by means of capital and labor services.
- Firms own their capital stock and hire labor supplied by the households. Households own the firms.
- There are three perfectly competitive markets: the markets for consumption goods, labor services, and financial capital.
- Let c_t denote consumption, y_t output and k_t the capital stock.

- The representative agent is assumed to find the control sequences $\{c_t, h_t\}_{t=0}^{\infty}$ such that

$$maxE_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \theta \frac{h_t^{1+\psi}}{1+\psi} \right) \right] \tag{1}$$

- given the initial condition (k_0, b_0, a_0); $0 < \beta < 1$, $\theta > 0$ and $\psi > 0$.
- h_t is the fraction of total available time devoted to productive activity in period t.

- Total output is divided between consumption and investment:

$$y_t = c_t + i_t$$

- Investment is used to form physical capital, which accumulates in the standard form as $(0 < \delta < 1)$:

$$k_{t+1} = exp(b_t) i_t + (1 - \delta) k_t$$

- b_t is a shock affecting incorporated technological progress, whose properties will be defined later.
- Output is produced by means of capital and labor services, relying on a constant returns to scale technology represented by the following Cobb–Douglas production function $(0 < \alpha < 1)$:

$$y_t = exp(a_t) k_t^{\alpha} h_t^{1-\alpha}$$

- a_t represents a stochastic shock to technology or Solow residual.

The shocks

We assume that the shocks to technology, a_t, b_t are distributed with zero mean, but they are correlated:

$$a_t = \rho a_{t-1} + \tau b_{t-1} + \epsilon_t$$

$$b_t = \tau a_{t-1} + \rho b_{t-1} + \nu_t$$

The shocks

The error terms are defined as:

$$E\left[\epsilon_t\right] = 0; E\left[\nu_t\right] = 0$$

$$E\left[\epsilon_t \epsilon_s\right] = \begin{cases} \sigma_\epsilon^2 & t = s \\ 0 & t \neq s \end{cases}$$

$$E\left[\nu_t \nu_s\right] = \begin{cases} \sigma_\nu^2 & t = s \\ 0 & t \neq s \end{cases}$$

$$E\left[\epsilon_t \nu_s\right] = \begin{cases} \varphi \sigma_\nu \sigma_\epsilon & t = s \\ 0 & t \neq s \end{cases}$$

Summary of the model

The FOCs, state and exogenous equations

$$c_t \theta h_t^{1+\psi} = (1-\alpha) y_t$$

$$\beta E_t \left[\left(\frac{\exp(b_t) c_t}{\exp(b_{t+1}) c_{t+1}} \right) \left(\exp(b_{t+1}) \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] = 1$$

$$y_t = \exp(a_t) k_t^{\alpha} h_t^{1-\alpha}$$

$$k_{t+1} = \exp(b_t)(y_t - c_t) + (1-\delta) k_t$$

$$a_t = \rho a_{t-1} + \tau b_{t-1} + \epsilon_t$$

$$b_t = \tau a_{t-1} + \rho b_{t-1} + \nu_t$$

Solving a DSGE model

- Step 1. From the specification of the model, find the necessary equations characterizing the equilibrium law of motion of the system: the state equations, the exogenous equations and the first-order conditions (derived either as a Euler equation or from the Lagrangian).
- Step 2. Derive the steady state of the model. This requires first parameterizing the model and then evaluating the model at its certainty equivalence form.
- Step 3. When necessary,log-linearize the necessary equations characterizing the equilibrium law of motion of the system.
- Step 4. Solve the (log-linearized) system for the decision rule (which is also in log-linear form).

The Log-linear Approximation Method

- Solving nonlinear dynamic optimization model with log-linear approximation has been proposed in particular by King et al. (1988) and Campbell (1994) in the context of Real Business Cycle models.
- The general idea of this method is to replace all the necessary equations in the model by approximations, which are linear in the log-deviation form.
- Given the approximate log-linear system, we can use several (linear and non-linear) methods to solve for the decision rule, which is also in the form of log-linear deviations.

- Formally, let X_t be a variable of interest and $\bar{X}$ the corresponding steady state. Then,

$$x_t \equiv lnX_t - ln\bar{X}$$

is regarded to be the log-deviation of X_t. In particular, $100x_t$ is the percentage of X_t that deviates from $\bar{X}$.

- The following building block for such log-linearization can be used:

$$X_t = \bar{X}e^{x_t} \tag{2}$$

$$e^{x_t + ay_t} \approx 1 + x_t + ay_t \tag{3}$$

$$x_t y_t \approx 0 \tag{4}$$

Estimation

We will consider:

- Calibration;
- GMM;
- Maximum-likelihood method;
- Bayesian estimation method.

Estimation

Calibration

- find values of structural parameter in micro- and macro-references;
- solve the DSGE model and generate *artificial* data;
- collect the (relevant) *actual* data;
- detrend these data and estimate an atheoretical VAR;
- calculate the relevant moments (variances, covariances, autocovariances) for both the *artificial* and *actual* data;
- compare these moments and (qualitatively) evaluate the difference;
- draw your conclusion about the goodness of your DSGE model.

Dynare

In this context, Dynare can be used for the following purposes:

- solving the model;
- generating the artificial data;
- calculating the relevant moments and impulse response functions;

Dynare codes to solve the model

Preamble

- The preamble consists of declarations to setup the endogenous and exogenous variables, the parameters and their assigned values.

```
var y, c, k, h, a, b;
varexo e, u;
parameters beta, alpha, delta, theta, psi, rho, tau;
alpha = 0.36;
rho = 0.95;
tau = 0.025;
beta = 0.99;
delta = 0.025;
psi =0;
theta = 2.95;
phi = 0.1;
```

Dynare codes to solve the model

Declaring the model in levels

```
model;
c*theta*h^(1+psi)=(1-alpha)*y;
k =beta*(((exp(b)*c)/(exp(b(+1))*c(+1)))*
     (exp(b(+1))*alpha*y(+1)+(1-delta)*k));
y = exp(a)*(k(-1)^alpha)*(h^(1-alpha));
k = exp(b)*(y-c)+(1-delta)*k(-1);
a = rho*a(-1)+tau*b(-1) + e;
b = tau*a(-1)+rho*b(-1) + u;
end;
```

The Dynare code

Declaring the model in logarithms

```
model;
exp(c)*theta*exp(h)^(1+psi)=(1-alpha)*exp(y);
exp(k) = beta*(((exp(b)*exp(c))/(exp(b(+1))*exp(c(+1))))
        *(exp(b(+1))*alpha*exp(y(+1))+(1-delta)*exp(k)));
exp(y) = exp(a)*(exp(k(-1))^alpha)*(exp(h)^(1-alpha));
exp(k) = exp(b)*(exp(y)-exp(c))+(1-delta)*exp(k(-1));
a = rho*a(-1)+tau*b(-1) + e;
b = tau*a(-1)+rho*b(-1) + u;
end;
```

The Dynare code

Declaring initial values (for the model in levels)

```
initval;
y = 1.08068253095672;
c = 0.80359242014163;
h = 0.29175631001732;
k = 11.08360443260358;
a = 0;
b = 0;
e = 0;
u = 0;
end;
```

The Dynare code

Declaring the shocks

```
shocks;
var e; stderr 0.009;
var u; stderr 0.009;
var e, u = phi*0.009*0.009;
end;
```

Note that it is possible to shut down a shock by assigning it a zero variance.

The model is solved and simulated by the *stoch_simul;* command. By default, the following quantities are reported:

- the coefficients of the approximated decision rules;
- the moments of the variables;
- correlations and autocorrelations;
- the impulse response functions (IRFs) for each exogenous shocks.

The Dynare code

Solving the model, some options

- ar = Integer: Order of autocorrelation coefficients to compute and to print (default = 5)
- periods = Integer: If different from zero, the model will be simulated and empirical moments will be computed instead of theoretical moments. The value of the option specifies the number of periods to use in the simulations (default = 0)
- drop = Integer: Number of points dropped at the beginning of simulation before computing the summary statistics (default = 100)
- irf = Integer: Number of periods on which to compute the IRFs (default = 40)
- order = [1,2,3]: Order of Taylor approximation (default = 2)
- replic = Integer: Number of simulated series used to compute the IRFs (default = 1, if order = 1, and 50 otherwise)

The Dynare solution

Decision rules (coefficients)

	y	c	k	a	h	b
Constant	1.080953	0.803479	11.083988	0	0.29187	0
(correction)	0.00027	-0.000113	0.000383	0	0.000114	0
k(-1)	0.005358	0.038542	0.941817	0	-0.012547	0
a(-1)	1.836717	0.424583	1.419062	0.95	0.341715	0.025
b(-1)	0.837086	-0.31874	1.419062	0.025	0.341715	0.95
e	1.911522	0.456074	1.455448	1	0.350477	0
u	0.83084	-0.347518	1.455448	0	0.350477	1
k(-1),k(-1)	-0.000693	-0.00062	-0.000072	0	0.00064	0
a(-1),k(-1)	0.031013	0.011201	0.018983	0	-0.005454	0
a(-1),a(-1)	1.35484	0.198327	1.191903	0	0.113219	0
b(-1),k(-1)	0.026057	-0.02445	0.018983	0	-0.005454	0
b(-1),a(-1)	1.010716	0.003915	2.383805	0	0.226438	0
b(-1),b(-1)	0.118206	0.149375	1.191903	0	0.113219	0
e,e	1.473867	0.220058	1.253809	0	0.1191	0
u,e	1.036212	-0.015958	2.507619	0	0.238199	0
u,u	0.102686	0.16578	1.253809	0	0.1191	0
k(-1),e	0.031946	0.012476	0.01947	0	-0.005594	0
k(-1),u	0.026588	-0.026065	0.01947	0	-0.005594	0
a(-1),e	2.826253	0.417711	2.444928	0	0.232244	0
a(-1),u	0.989536	-0.006871	2.444928	0	0.232244	0
b(-1),e	1.058095	-0.004158	2.444928	0	0.232244	0
b(-1),u	0.221009	0.314583	2.444928	0	0.232244	0

The Dynare solution

Moments' variables

APROXIMATED THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
y	1.0847	0.0897	0.008
c	0.8065	0.0529	0.0028
k	11.187	1.2603	1.5883
a	0	0.034	0.0012
h	0.2917	0.0119	0.0001
b	0	0.034	0.0012

APPROXIMATED VARIANCE DECOMPOSITION

	e	u
y	70.30	29.7
c	65.16	34.84
k	55.00	45
a	88.20	11.8
h	55.00	45
b	17.43	82.57

The Dynare solution

Correlations and autocorrelations

APPROXIMATED MATRIX OF CORRELATIONS

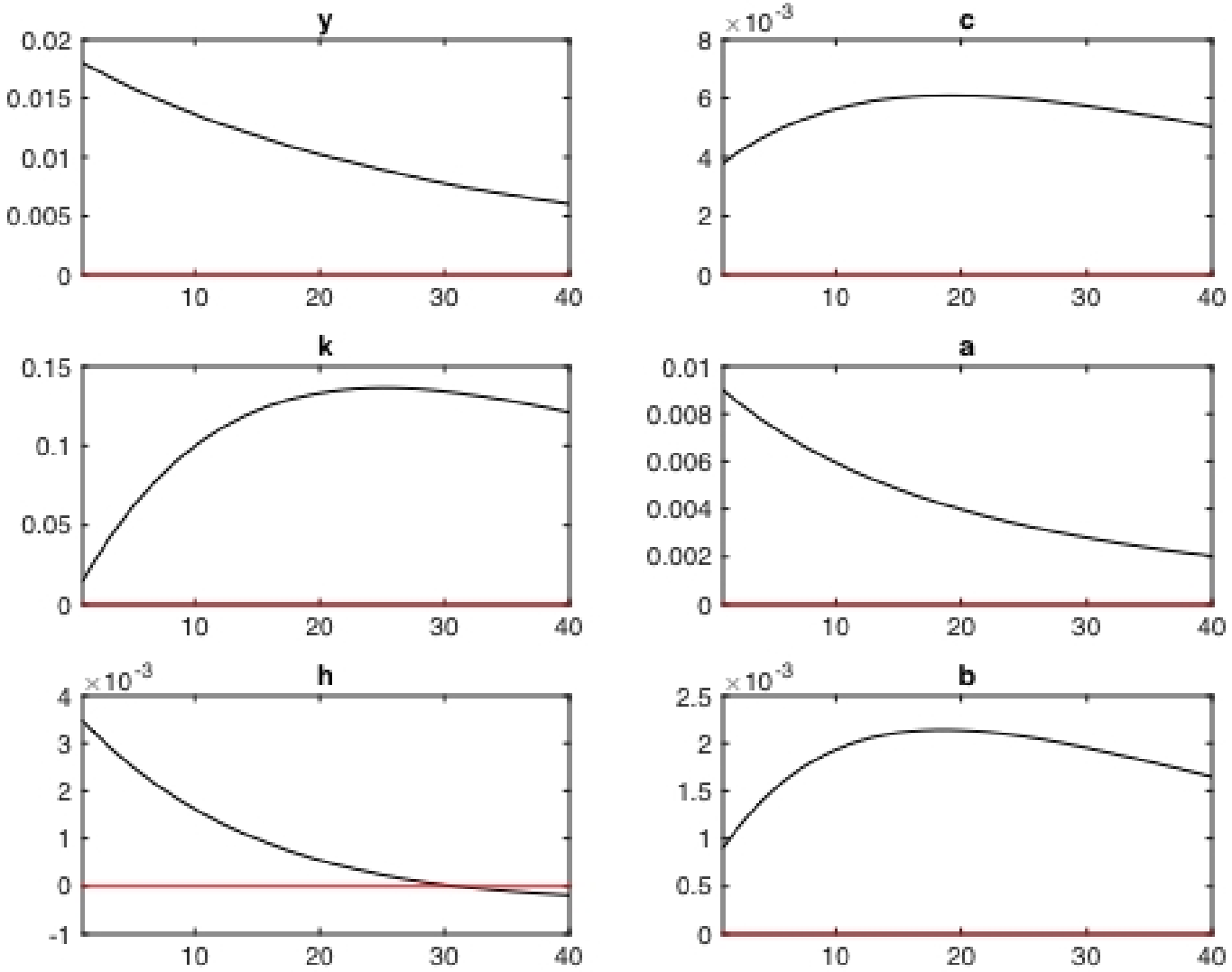
Variables	y	c	k	a	h	b
y	1	0.8742	0.8548	0.9563	0.6237	0.7773
c	0.8742	1	0.9704	0.8396	0.1656	0.6138
k	0.8548	0.9704	1	0.7504	0.1739	0.7504
a	0.9563	0.8396	0.7504	1	0.5906	0.5627
h	0.6237	0.1656	0.1739	0.5906	1	0.5906
b	0.7773	0.6138	0.7504	0.5627	0.5906	1

APPROXIMATED COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
y	0.9762	0.953	0.9303	0.9081	0.8864
c	0.9949	0.9889	0.9819	0.9741	0.9656
k	0.9992	0.9971	0.9937	0.9891	0.9834
a	0.9641	0.9299	0.8973	0.8662	0.8365
h	0.9195	0.8442	0.7739	0.7082	0.6468
b	0.9641	0.9299	0.8973	0.8662	0.8365

The Dynare solution

The IRFs: orthogonalized shocks to e



The Dynare solution

The IRFs: orthogonalized shocks to u