Advanced Macroeconomics (PhD level)

Problem Set 3

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Exercise 1:

1. Endogenous, exogenous and model parameters

- *list of endogenous variables:*

Y, C, N, K, K^B, G^B, I^B, I, TR, w, r, z, τ, dy, dc, di, dn, dr, dtr, dgb, dib

[Output of firms Y_t, consumption C_t, real labor supply N_t, private capital stock K_t and public capital stock K^B_t, public consumption G^B_t, public investment I^B_t, I_t as the gross investment, TR_t which are real lump-sum transfers, the real wage w_t, the real interest rate r_t, the total factor productivity z_t, the tax rate on output τ_t, dy etc. refer to the "observables", i.e. the deviations from the steady state of variables of interest as defined in equations 15-23 of the PSet.]

- *list of exogenous variables:*

ε^z, ε^{GB}, ε^{IB}, ε^τ [list of exogenous variables that can be shocked]

- *list of parameters:*

β, δ, η, α, θ, ϱ

[the discount rate β_t, the depreciation rate δ, productivity of public capital η, the share of capital in production α, the Frisch elasticity of labor θ_l; the smoothing parameters ϱ t]

2. Model calibration

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$\beta = 0.99$ $\quad \bar{y} = 1$ $\quad \bar{k}^B = 1$ $\quad \Theta \cdot \frac{1.56}{1.08}$

$\delta = 0.02$ $\quad \bar{G}^B = 0.2\bar{y} = 0.2$ $\quad \bar{k} = 3$ $\quad T = 0.22$

$\eta = 0.05$ $\quad \bar{I}^B = 0.02\bar{y} = 0.02$ $\quad \bar{c} =$ $\quad r = 1/9$

$\overline{TR} = 0$ $\quad \bar{I} = 0.06$ $\quad \lambda \stackrel{!}{=} 1/c$

$\bar{\omega} = 2$ $\quad z = 1.44$

$\bar{N} = 1/3$ $\quad \alpha = \frac{1}{3}$

$\rho = 0.75$

→ above steady state values $\qquad \eta < \alpha$

↓equation

(14) $y_t = C_t + I_t + G_t^B + I_t^B$

$$1 = \bar{C} + \bar{I} + 0.2 + 0.02$$

$$\underline{\bar{C} + \bar{I} = 0.78}$$

(5) $\bar{k}^B = (1-\delta)\bar{k}^B + \bar{I}^B$

$$\bar{k}^B = (1-0.02)\bar{k}^B + 0.02$$

$$0.02\bar{k}^B = 0.02$$

$$\boxed{\bar{k}^B = 1}$$

(7) $\bar{\omega}\bar{N} = (1-\bar{\alpha})\cdot\bar{y}$

$$2\,\frac{1}{3} = (1-\alpha)\ 1$$

$$\boxed{\alpha = \frac{1}{3}}$$

(8) $r\cdot\bar{k} = \alpha\cdot\bar{y}$

$$\underline{r\cdot\bar{k} = \frac{1}{3}}$$

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(10) $\bar{G}^B + \bar{I}^B + \bar{TR} = \tau(\bar{\omega}\bar{N} + \bar{r}\bar{K})$

$0.2 + 0.02 + 0 = \tau(2\cdot\frac{1}{3} + \frac{1}{3})$

$\boxed{0.22 = \tau}$

⇒ Now: K is assumed to take a value of 3

$\boxed{\bar{K} = 3}$

(4) $\bar{K} = (1-\delta)\bar{K} + \bar{I}$

$\bar{K} = (1-0.02)\bar{K} + \bar{I}$

$0.02K = \bar{I} \iff \bar{K} = 50\bar{I}$

$\bar{K} = 3 \Rightarrow \boxed{I = \frac{3}{50} = 0.06}$

- $C = 0.78 - \bar{I} = 0.78 - 0.06$

$\boxed{C = 0.72}$

- $(1-\bar{\tau})\bar{\omega} = \bar{\Theta}_1 \frac{\bar{C}}{1-\bar{N}}$

$(1-0.22)\cdot 2 = \bar{\Theta}_1 \frac{0.72}{2/3}$

$1.56 = \bar{\Theta}_1 \cdot 1.08$

$\boxed{\bar{\Theta}_1 = \frac{1.56}{1.08} = 1.\overline{44}}$

- $\bar{Y} = z\cdot(\bar{K}^B)^{\eta}\cdot(\bar{K})^{\alpha}\,(\bar{N})^{1-\alpha}$

$1 = z \cdot 1 \cdot 3^{\frac{1}{3}}\cdot(\frac{1}{3})^{\frac{2}{3}}$

$z = 1.44224957031$

- $r\cdot K = \frac{1}{3}$

$r = \frac{1}{3K}$

$\boxed{r = \frac{1}{9}}$

3. Deterministic versus stochastic model – maximum 10 sentences

In a deterministic model variables are determined by the model's parameters and their initial state. Shocks to the system can hit today or anytime but would be expected with perfect foresight. Therefore the model requires full information about the initial parameters, perfect foresight and no uncertainties around shocks, which doesn't depict the real world well. It is sometimes advisable, however, to start with a stochastic model first in order to get an idea of the model to be estimated as it is somewhat easier to estimate via simulation and does not need linearization.

In a stochastic model variables are determined by probability distributions of parameters, which creates randomness amongst the variables and the outcome of a model according to the underlying distributions. Today's shocks can be accounted for, they hit by surprise. Future shocks or permanent changes in exogenous variables, however, cannot be taken into account. Stochastic models require more demanding computations, but reflect better on the real world and are thus very useful for estimation.

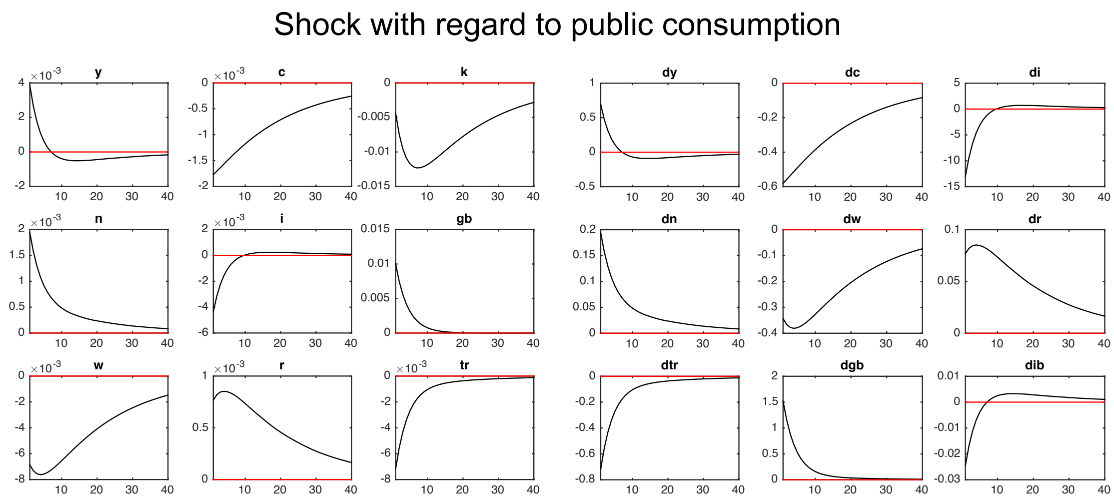
The exercise in 5) pertains to modeling an <u>unexpected</u> <u>temporary</u> public consumption and investment shock, so the stochastic model is suited for this. The exercise in 6) pertains to modeling a permanent increase in the tax rate, which requires the deterministic model.

4. .Mod file for model

See file PSet3_1_chrismal_newvalues2.mod

5. Modeling shocks

The public consumption shock yields the following figure:



As can be seen from the figure, total output (y) initially increases due to the temporary increase in public consumption (ib) but in the long-run output levels are not impacted by this temporary shock.

The increase in government spending decreases individuals' income, as negative transfers (TR) need to occur to finance the initial increase in gb (I interpret this change in TR as a lump-sum tax). Therefore, individuals reduce consumption of both goods (c) and leisure, leading to an increase in labor supply (n), which, in turn drives down real wages (w).

However, after the initial impact, the rise in aggregate demand yields an increase in labor demand and therefore real wages, allowing households to increase consumption, thereby leading to an increase in consumption from the initial shock-induced reduction.

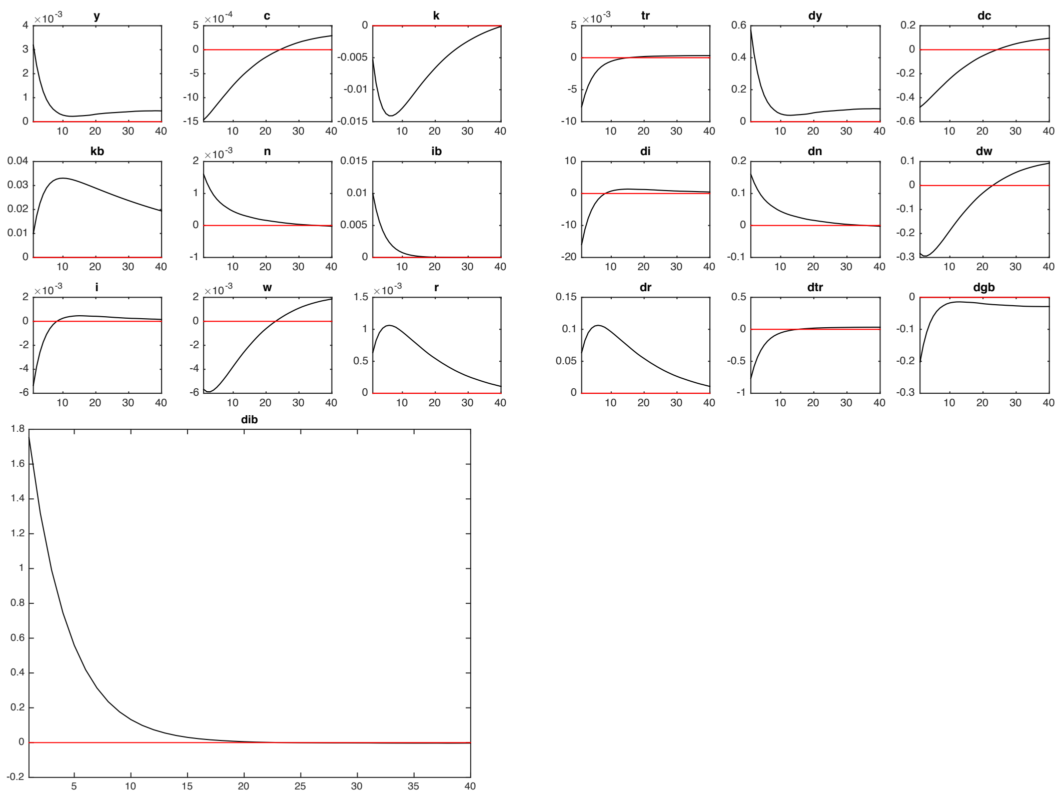
In terms of the changes in real wages and real interest rate (r) one can see that they move in opposite direction after the initial shock of increased public spending. Wages initially decrease due to increased labor supply and then recover while interest rates increase due to increased scarcity of capital and then reduce over time.

In terms of private investment (i), the increase in real interest rate initially decreases private investment, which in turn reduces private capital stock (k).

Overall, the real economy is not impacted by the temporary change in public consumption in the long-run.

The public investment shock yields the following figures:

Shock with regard to public investment



Contrary to the case of a temporary unanticipated increase in public consumption, the increase in public investment increases long-run output. I explain this using the intuition provided in the Baxter/King paper that was referenced in the PSet: Whereas government purchases simply remove resources from the economy without altering steady-state prices, public investment increases the public capital stock and makes private factors (labor and capital) more productive.

The productivity of private capital has no impact on the long-run results for a public consumption shock. In case of the public investment shock, since the capital stock has a larger productivity the long-term output effects will become larger, as well.

6) Tax rate increase by 1 percentage point

I ran my .mod file as a deterministic model using the calculated steady-state values from the stochastic simulation as initial guess values (file is called PSet3_1chrismaldeter.mod). I then use the same values (but for tau) in an endval block that I add to the mod file. This endval block contains tau=0.739091 (1% percentage point increase compared to the old-stead-state rate). Tau is defined as an exogenous variable now (varexo command). I used this approach as it was suggested in the Dynare documentation. However, when I do so, the model is unable to find the new steady-state. I also tried treating tau as a parameter (instead of a variable) set to 0.739091, used the calculated steady-state values for the other variables from question 5 (without a shock) and ran this revised model. The idea here was that if the model converges to a steady-state then I could compare this steady-state to the old one. While this would not tell me the transition, at least I would have been able to compare the long-run equilibria. Again, Dynare tells me that it cannot find a steady-state. Finally, instead of using an endval block, I defined an additional exogenous variable tau1, defined an additional equation tau=0.729091 +tau1 (with 0.729091 the initial steady-state tax rate), and introduced a shock in period 1 that is constant to the final period, specifying that tau1 should change to 0.1 (from its initial value of 0). When I run this model, I get the error message: "Error using lnsrch1 (line 71 Some element of Newton direction isn't finite. Jacobian maybe singular or there is a problem with initial values." I do believe that the initial values are correct given that they are identical to the simulated equilibrium from before.

While I cannot present actual simulation results to show how the permanent increase in tax rate (for labor and capital) impacts on the long-run equilibrium, I would still like to provide some verbal insight on this from the paper by Baxter and King. As no debt exists in the PSet model, the increase in the tax rate necessarily leads to additional government purchases, while at the same, incentives for individuals to work and to invest decrease due to the higher tax rate, lowering output ceteris paribus. The overall effect of an increase in the tax-rate on long-run output is dependent on the type of purchases the government is doing, i.e. increased government spending versus increased government investment. As shown under 5) if the government increases investment, it increases public capital stocks over time, which increases productivity of private factors, as well. If government simply increases consumption, long-run output is not positively impacted.

7. Impact of fiscal policy on the economy

Baxter and King argue that additional government investment can increase long-term output as it makes labor and private capital more productive over time. This can overcompensate the initial crowding-out of private investment and consumption. This is also what is shown in the figures above in case of a temporary increase of government investment.

Exercise 2:

1. Number of observable variables in Bayesian estimation

We need as many observable variables as the number of shocks modeled. Here, 3 shocks are modeled, so 3 observable variables are required: YGR: quarter to quarter per capita GDP growth rates, INFL: annualized quarter-to-quarter inflation rates, INT: annualized interest rates

2. Simulation of observables

I simulate the observable variables YGR, INFL and INT by augmenting the AnScho.mod file. I use the information on parameter calibration contained in the file on the learnweb site and solve the model using the calibrated model (see file AnScho.simulation.mod) for 5;000 observations. I save the observations for the three variables in the file simdat2.mat und use the file as observation data to estimate the parameters of the model through Bayesian estimation in the file AnScho_chrismal.mod.

3. Briefly explain the intuition behind prior information in a Bayesian estimation (maximum 10 sentences).

In many cases a modeler will have “prior” “belief” (information) about the probability that an unknown parameter lies in a certain section of the parameter space. “Prior” means that this information is based on data obtained before the (experimental) data of the current research being employed. This prior belief can be used to construct a prior probability distribution for the parameter, that is the relative likelihood that the true value of the parameter lies in any of the regions of the parameter space prior to observing any values from the experiment.

On the other hand, the posterior probability distribution represents the relative likelihood that the parameter lies in any of the regions of the parameter space after the new observations have been included as well. The posterior probability distribution therefore represents the updated “beliefs” about the parameter after having seen the data. If new data is abundant, the posterior distribution will strongly reflect this new data. When new data is limited, the posterior will be similar to the prior distribution.

4. Estimate the model with your simulated data and Bayesian methods

I saved AnScho.mod as a new file called AnScho_chrismal.mod. As explained in question 2, I added the three observable variables YGR, INFL and INT using

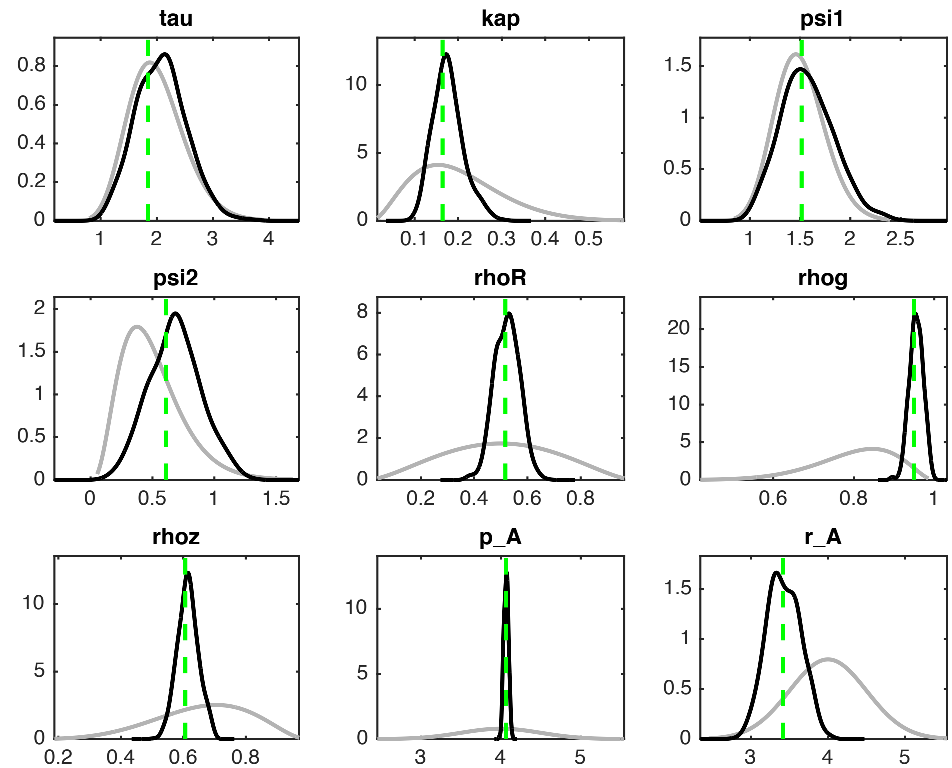
```
varobs YGR, INFL, INT;
```

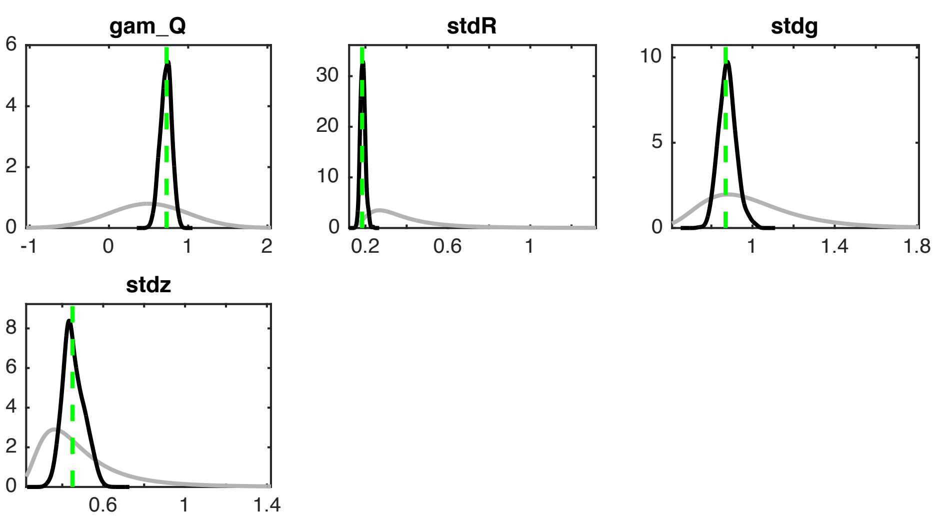
I then use the command provided in the PSet to estimate the parameters:
(Note that I saved the simulated observed variables to simdat2.mat):

```
estimation(order=1,datafile=simdat2,first_obs=401,nobs=200,mh_replic=2500,mh_nblo
cks=3,mh_jscale=0.65);
```

5. Quality of the estimation exercise

This is the figure "priors and posteriors" from the Dynare output:





For each of the parameters Dynare shows one figure for the prior and posterior distribution. The grey line is the prior distribution; the black line is the posterior distribution. The dotted green line is the posterior mode.

Looking at the figure, we can see that the probability distribution has changed drastically for most parameters (exceptions are tau and psi1). We can see two core differences: For some parameters (psi2, rhog, rhoz and r_A) the mode of the distribution has changed "significantly" (used in a non-technical meaning here). For most parameters (all but tau, psi1, psi2) the kurtosis of the distribution has changed, moving to a higher degree of peakedness of the distribution for each of these parameters (moving from a platykurtic or mesokurtic distribution to a leptokurtic distribution). What this tells me is that the estimation of the parameter has improved the belief of the distribution of the parameters. Essentially there is a higher probability that the actual parameter values are in a narrower parameter space than before.