2a) State variable x_t : the dividend d_t
Control variable y_t : the price p_t

b) Let x_{t+1} be:

(1) $x_{t+1} = h(x_t, \sigma(0)) + \sigma \varepsilon_{t+1}$

A first-order approximation of h around $(\bar{x}, 0)$ is:

(2) $h(x,0) = h(\bar{x},0) + h_x(\bar{x},0)(x-\bar{x})$

If we set $\sigma = 0$, we can put (2) in (1)

$$x_{t+1} = \underbrace{h(\bar{x},0)}_{=\bar{x}} + \underbrace{h_x(\bar{x},0)(x-\bar{x})}_{:= h(x_t,0)} + \sigma \cdot \varepsilon_{t+1}$$

$$= \bar{x} + h(x_t, 0) + \sigma \cdot \varepsilon_{t+1}$$

with $\sigma = 0$.

2a) State variable x_t : the dividend d_t
Control variable y_t : the price p_t

b) Let x_{t+1} be:

(1) $x_{t+1} = h(x_t, \sigma(0)) + \sigma \varepsilon_{t+1}$

A first-order approximation of h around $(\bar{x}, 0)$ is:

(2) $h(x,0) = h(\bar{x},0) + h_x(\bar{x},0)(x-\bar{x})$

If we set $\sigma = 0$, we can put (2) in (1)

$$x_{t+1} = \underbrace{h(\bar{x},0)}_{=\bar{x}} + \underbrace{h_x(\bar{x},0)(x-\bar{x})}_{:= h(x_t,0)} + \sigma \cdot \varepsilon_{t+1}$$

$$= \bar{x} + h(x_t, \sigma) + \sigma \cdot \varepsilon_{t+1}$$

with $\sigma = 0$.

Now taking Expectations and evaluate at $(\bar{x},0)$, yields:

- $y_t = g(\bar{x},0) = \bar{y}$

- $F_x = U''(\bar{x})\cdot\bar{y} + U'(\bar{x})\cdot g_x - \beta\rho U''(\bar{x})\cdot(\bar{y}+\bar{x})$
  $\quad - \beta U'(\bar{x})\cdot g_x\cdot\rho - \beta U'(\bar{x})\cdot\bar{x} \overset{!}{=} 0$

$$\Rightarrow g_x = \frac{\beta\cdot U'(x)\cdot\rho - U''(\bar{x})\cdot\bar{y} + \beta\rho U''(\bar{x})(\bar{y}+\bar{x})}{(1-\beta\rho)\cdot U'(\bar{x})}$$

- $F_{xx} = U'''(\bar{x})\cdot\bar{y} + U''(\bar{x})\cdot g_x + U''(\bar{x})\cdot g_x + U'(\bar{x})\cdot \underset{g_x}{\underline{g_{xx}}}$
  $\quad - \beta\rho^2 U'''(\bar{x})\cdot(\bar{x}+\bar{y}) - \beta\rho U''(\bar{x})\cdot\rho\cdot(g_x+1)$
  $\quad - \beta\cdot\rho^2\cdot U''(\bar{x})\cdot(\bar{y}+1) - \beta U'(\bar{x})\cdot \underline{g_{xx}}\cdot\rho^2 \overset{!}{=} 0$

$$\Rightarrow g_{xx} = \frac{2\,\beta\cdot\rho^2\cdot U''(\bar{x})(\bar{y}+1) - U'''(\bar{x})\cdot\bar{y} - U''(\bar{x})\cdot g_x\cdot 2}{U'(\bar{x})\,(1-\beta\rho^2)} + \frac{\beta\rho^2 U'''(\bar{x})(\bar{x}+\bar{y})}{U'(\bar{x})\,(1-\beta\rho^2)}$$

- $F_\sigma = U'(\bar{x})\cdot g_\sigma - \beta\cdot U'(\bar{x})\cdot g_\sigma \overset{!}{=} 0$

$$\Rightarrow g_\sigma = 0$$

- $F_{\sigma x} = U''(\bar{x})\cdot 0 + U'(\bar{x})\cdot g_{\sigma x} - \beta\cdot\rho U''(\bar{x})\cdot 0$
  $\quad - \beta\cdot U'(\bar{x})\cdot g_{\sigma x} \overset{!}{=} 0$

$$\Rightarrow \overset{\text{Schwarz}}{g_{\sigma x} = g_{x\sigma}} = 0$$

Exercise 3

4) It can be seen in this case that there is room for further convergence if the number of iterations becomes greater. It is also important to choose the correct priors to get a good estimation.
For the estimation of "tau" for example the posterior distribution is almost equal to the prior distribution.
~~But~~ ~~overall~~ The model from the lecture almost gets a posterior probability of 1, so that it is pretty good fitting.
Furthermore, the model from the lecture fits the data better than the modification in this exam.