Ferdinand Kuhn, Matrikelnummer 425785 (Master's student)

Advanced Macroeconomics (PhD level)
Problemset 3

Exercise 1: Baxter and King (1993)

1. From his class on introductory macroeconomics Willi remembers that as a first step it is always important to distinguish between endogenous and exogenous variables as well as model parameters. Can you help him with that?
The values for the endogenous variables are determined through the model equations in the model itself. Exogenous variables can be manually set, enter the model through the equations and affect the endogenous variables. Usually these are shocks to some endogenous variables like output or technology growth. Finally, the model is driven by the parameters i.e. the discount factor β, that are unaffected by a change in the exogenous and endogenous variables.

Can you provide a calibration for all model parameters meeting his targets and economic intuition?

Given the steady-state values from the table and Eq. (7), we can calculate the output elasicity of labor α

$$
\begin{aligned}
& wN = (1-\alpha)Y \\
\Rightarrow \quad & \alpha = \frac{1}{3}
\end{aligned}
$$

Inserting G^B and I^B into Eq. (14), we get the sum of private consumption and investment

$$
\begin{aligned}
& Y = C + I + G^B + I^B \\
\Rightarrow \quad & C + I = 0.78
\end{aligned}
$$

Substitute this and $\bar{Y}$ into the households' budget constraint yields the tax rate τ

$$\Rightarrow \quad 0.78 = (1-\tau)\bar{Y} + \bar{TR}$$
$$\Rightarrow \quad \tau = 0.22$$

Since this is an RBC model, capital depreciation rate in this case is $\delta = 0.025$ which equals a yearly depreciation rate of 10% (King, Plosser & Rebelo, 1988 p.218). Inserting this into Eq. (5) gives the steady-state value for the public capital stock K^B.

$$\Rightarrow \quad \bar{K}^B = (1-0.025)\,\bar{K}^B + \bar{I}^B$$
$$\Rightarrow \quad \bar{K}^B = 0.8$$

For the utility discount factor we use $\beta = 0.988$ as employed by King, Plosser & Rebelo (1988). Together with δ, τ and Eq. (2) we can compute the steady-state interest rate r

$$\Rightarrow \quad \lambda = 0.988 \cdot [(1-0.025) + (1-0.22)\,r]$$
$$\Rightarrow \quad \bar{r} = 0.0476227$$

Given the steady-state interest rate $\bar{r}$ and Eq. (8), we get the private capital stock in the steady state

$$\Rightarrow \quad \bar{K} = \frac{\alpha \bar{Y}}{\bar{r}}$$
$$\Rightarrow \quad \bar{K} = 6.99946$$

We can then determine with Eq. (4) the amount of private investment in the steady state

$$\Rightarrow \quad \bar{K} = (1-\delta)\bar{K} + \bar{I}$$
$$\Rightarrow \quad \bar{I} = 0.1749865$$

With private investment and Eq. (14), we get the steady-state value for private consumption

$$\Rightarrow \quad Y = C + I + G^B + I^B$$
$$\Rightarrow \quad \bar{C} = 1 - 0.1749865 - 0.2 - 0.02$$
$$\Leftrightarrow \quad \bar{C} = 0.6050135$$

Then λ can be determined as the inverse of consumption, Eq. (3)

$$\Rightarrow \quad \bar{\lambda} = \frac{1}{\bar{C}} = 1.652856$$

The Frisch elasticity of labor is computed using Eq. (1)

$$\Rightarrow \quad \theta_l = \frac{(1-\tau)\, w + (1-N)}{C}$$
$$\Rightarrow \quad \theta_l = 1.71897$$

The productivity of public capital is said to be lower than the productivity of private capital. We orient ourselves to midsize value of $\eta = 0.06$ as employed by Baxter & King (1993, p.330).
Given this assumptions, our already computed values and Eq. (6), it is possible to determine the steady-state value for the total factor productivity $\bar{z}$

$$\Rightarrow \quad 1 = \bar{z} \cdot 0.8^{0.05} \cdot 6.99949^{\frac{1}{3})} \cdot \frac{1}{3}^{\frac{2}{3}}$$
$$\Leftrightarrow \quad \bar{z} = 1.099609$$

The calibration of the parameters is summarized in the following table:

Parameter	*Description*	*Value*
α	Output elasticity w.r.t capital	$\frac{1}{3}$
β	Discount factor	0.988
δ	Depreciation rate	0.025
η	Productivity of public capital	0.05
σ	Std. dev. for shocks	0.01
θ_l	Frisch elasticity of labor	1.9123
ρ_G^B	Smoothing parameter	0.75
ρ_I^B	Smoothing parameter	0.75
ρ_τ	Smoothing parameter	0.75
ρ_z	Smoothing parameter	0.75

Table 1: Parameter calibration

The steady-state values for the model calculation are as follows:

$Variable$	$Steady-state\ value$
$\bar{C}$	0.6050135
$\bar{G^B}$	0.2
$\bar{I}$	0.1749865
$\bar{I^B}$	0.02
$\bar{K}$	6.99946
$\bar{K^B}$	0.8
$\bar{\lambda}$	1.652856
$\bar{N}$	$\frac{1}{3}$
$\bar{r}$	0.0476227
$\bar{\tau}$	0.22
$\bar{TR}$	0
$\bar{w}$	2
$\bar{Y}$	1
$\bar{z}$	1.099609

Table 2: Steady-state values

Willi is not sure if he wants to simulate the deterministic or stochastic model. Can you provide some guidance when to use which one? Since Willi is impatient, please be brief and try to explain it in a maximum of 10 sentences.

To decide whether he should model a deterministic or stochastic model it is useful to differentiate the key characteristics of both methods. In a deterministic model, we assume full information, perfect foresight and no uncertainty around shocks. In the moment of the model's solution, it is exactly known when and how future shocks will occur. Even the duration of the shocks are known because of perfect foresight. In contrast to that is the stochastic model, in which the shock hits with a suprise but after that has an expected value of zero. In our case we only have assumptions about the mean and the standard deviation of the shocks. This implies an application of a stochastic model.

Willi has heard of the powerful toolbox DYNARE, so he asks you to help him set up this model in DYNARE. Write a DYNARE mod-file for this model, commenting each step such that Willi clearly understands each block.
See attached file `bk.mod`.

How does (i) an unexpected temporary public consumption and (ii) an unexpected temporary public investment shock (of the same size) feed through the model? Simulate these two shocks and compare the reactions of the observables. Does it affect the real economy? Try to provide economic intuition behind the results. How do your results change if the productivity of capital increases?

Unexpected temporary shock to public consumption

The rise in government consumption leads to a temporary increase in output that is followed by a modest decline until it slowly reaches the steady state again. Private consumption, private investment and the wages decrease sharply. The reason for this is that higher government consumption drives out private consumption and investment. As a consequence, higher demand for capital by the government to finance its consumption augment the interest rate. The higher consumption happens at the expense of lower government investment and is financed by higher transfers from the households.

From the households' view, lower private investment results in a lower capital stock that has somehow to be compensated to explain the increase in output. To smoothen its consumption over time, the households will supply more labor which then leads to a drop in the wages because of oversupply. Also to rebuild the lost capital stock, households will overinvest lightly in subsequent periods. As the shock fastly subsides and government consumption returns to its steady state, the households have not adjusted to the full extent. This might explain the modest decline in output after the initial increase.

Unexpected temporary shock to public investment A shock to public investment directly enlarges the government's capital stock. As the public capital stock is part of the output function, output increases. This happens not as sharp as with the other shock described above. The shock to public investment is financed by higher transfers from the households while government consumption lightly decreases. Greater transfers again result in adjustment actions by the households. They will consume and invest less which reduces their capital stock. They will supply more labor to compensate for their lost income. Wages rise thereafter

As time passes, the effect of public investment on output and increasing labor supply are reflected in the peak in output. Households will consume less while they again start rebuilding their capital stock by investing more. After the shock has already subsided, output growth is merely driven by increased private labor and capital supply. Less government investment is followed by lower interest rates, making it easier to substitute expensive labor for capital. This forces downward pressure on wages and thus households will supply less labor.

The adjustment to the steady state is quite fast for public investment because there is only one shock. All other variables return to their state state in about 200 periods. This is a hint that higher pulic investment represents a more sustainable way to boost economic growth.

Increase in productivity of public capital

An increase in the public capital productivity is modelled by a higher value of η. Baxter & King (1993, p.330) provide a table for different values of η. For this task, a computation with $\eta = 0.2$ instead of $\eta = 0.05$ will be conducted.

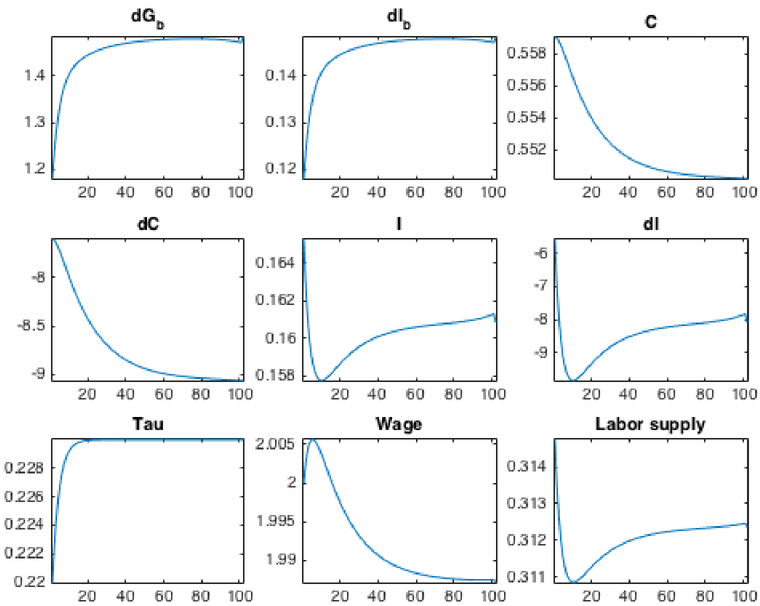
With regard to the first shock, the results do not change since a higher η does not affect adjustment behaviour of the households to compensate for the higher transfers. This is different for the second shock. More public investment increases the government's capital stock. Since η defines the productivity of this capital stock in the production function, a higher output must be obtained. The example computation with $\eta = 0.05$ provides a peak value of $Y = 2$ whereas $\eta = 0.2$ results in $Y = 8$. Also all other variables (except I^B and K^B because they are affected in the same way as before) are higher. It can be said that a higher productivity of the governments' capital stock even more boosts the output growth that occurs from higher government investment.

How does a permanent increase in the tax rate of 1 percentage point affect the long run equilibrium of the economy? Compute the new steady-state and compare it with the old one. Also, show the transition path from the old to the new steady state. Try to provide economic intuition.

A 1 percentage point increase in the tax rate is implemented as a deterministic shock due to the fact that a tax increased is usually announced by the government. Given Eq. (13), setting $\tau_t = \tau_{t-1} = 0.23$ (as this is can be approximated from the AR-1 process) and $\bar{\tau} = 0.22$ we get

$$\Rightarrow \epsilon^{\tau} = ln\left(\frac{0.23}{0.22}\right) - 0.75 \cdot ln\left(\frac{0.23}{0.22}\right)$$
$$\Leftrightarrow \quad \epsilon^{\tau} = 0.01111$$

The following graphic shows the effects of the computation with $\epsilon^{\tau} = 0.01111$



The down left graph shows the adjustment of the tax rate from 0.22 to 0.23 as part of the AR(1) process in Eq.(13). Government investment and government consumption rise due to the fact that the government collects more taxes. In contrast, output as well as

private investment and consumption decline during the 20 adjustment periods and arrive at a lower steady state. From around period 15, private investment and labor supply start to increase, possibly to mitigate the effect of decreasing output and to smooth consumption for the households. The wages at first increase sharply but therafter fall rapidly and adjust to a new steady state.
The economic intuition behind that is, that the tax is constructed as a lump sum tax on the overall production. Households know that there will be a tax increase, so they reduce consumption as well as investment. In the long run, the tax increase results in lower levels for output, private consumption and investment and labor supply by the households while government consumption and investment have risen.

Willi argues that fiscal policy cannot boost the economy due to the implied crowding-out of private consumption and private investment. Do you agree given your results above? Also explain, in economic terms, the difference between private and government capital.
As seen in the previous tasks, a mere increase in government consumption leads to a temporary higher output in the economy but happens at the expense of crowding out private consumption and private investment. This is due to the construction of the model, in which households finance government consumption through higher transfers.
A different approach is to increase the tax rate. This allows the government to finance more consumption and investment. Eventually this leads to a lower overall output, consumption and investment and therefore is not a good solution.
A good alternative is the increase of government investment. This increases the capital stock without crowding out private consumption and investment. It also boosts the economy but rather sustainable than with more government consumption. So Willi is not right with his saying that fiscal policy cannot boost the economy as this solution shows.

Exercise 2: An and Schorfheide (2007)

Consider a version of the An and Schorfheide (2007) model. The mod-file AnScho.mod contains (incomplete code to estimate the An and Schorfheide model with Bayesian methods.

1. How many observable variables do you need for a Bayesian estimation? Include `varobs` into the modfile.
To perform a Bayesian estimation you need as many observable variables as there are shocks in the model. I therefore include the `YGR` (output growth), `INFL` (inflation) and `INT` (interest rate) in `varobs`.

2. Simulate data for your observable variables and save these into a matfile called `simdat.mat`.

See attached file `simdat.mat`.

3. Briefly explain the intuition behind prior information in a Bayesian estimation (maximum 10 sentences).

The prior information about parameters is based on economic considerations or is derived from empirical data. Depending on how certain you are about the value of a parameter, you assume a probability distribution with mean and standard deviation. You can also get estimates for your parameters from data you have simulated or collected. The objective of a Bayesian estimation is now to improve the choice of parameters based on both the subjective prior information and the estimates given by the data. This kind of estimation can help to mitigate the "dilemma of absurd parameter estimates". This happens in the way that it assigns low probabilities to parameter values suggested by the data that seem unlikely and don't match your prior assumptions.

4. Estimate the model with your simulated data and Bayesian methods.

See scenario 2 in the attached file `AnScho_FK.mod`.

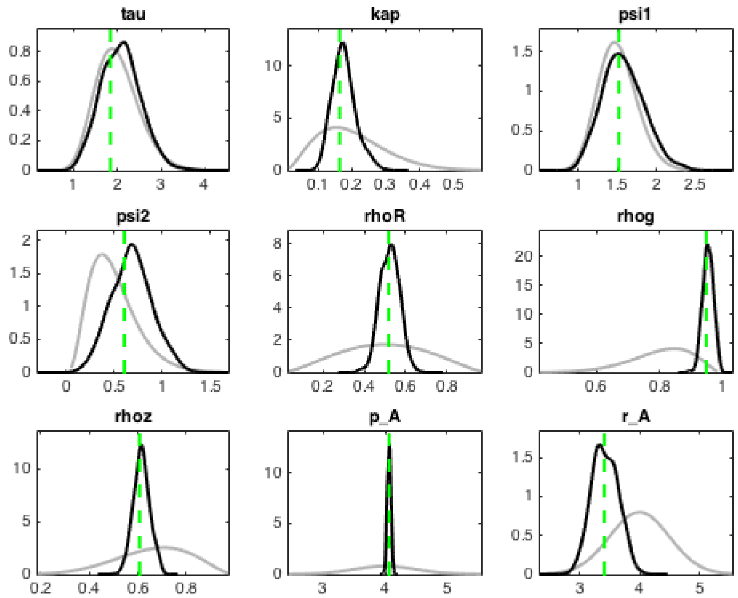
5. Consider the figure *Priors and Posteriors* (ignore the other output). How would you assess the quality of this estimation exercise?

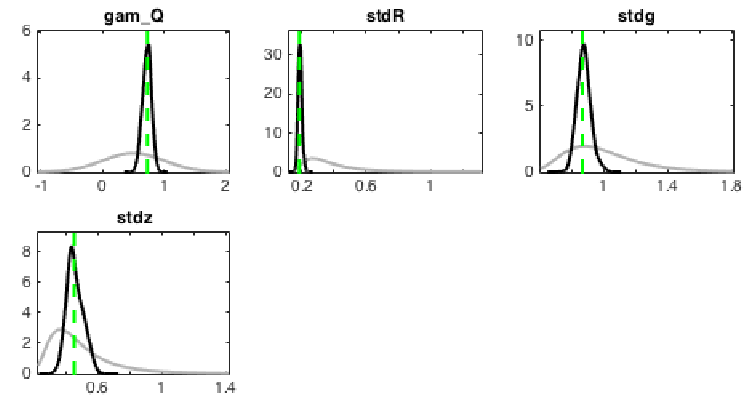
In the case of τ and ψ_1, the posterior density function seems to match the density function based on the prior information quite well. We can say that the model is nicely specified with the parameter values of τ and ψ_1, which are included to simulate the data.

With regard to κ, ρ_R, p_A, $stdg$, the means of the posterior and the prior distribution are very likely to match each other. However, the chosen probability distributions are not in accordance. This may result, because of uncertainty about the prior parameter value that is reflected in a high standard deviation. Take for example κ, here the mean is more or less the same but the prior distribution has bigger tails. For our Bayesian estimation this means, that the parameters included into the model (and therefore also included into the simulated data) are alright with our prior economic beliefs.

A special case is p_A and to some extent $stdR$. The prior information does not really provide helpful input for the Bayesian estimation. We suppose the mean to be located somehow around a certain value but other values are still very likely. Yet, the data posterior density function clearly suggests that the parameter value lies within a small range and thus only supports our insecure prior beliefs.

Somewhat different are the cases where the prior and posterior density function clearly lie apart. For ρ_G and r_A, our prior assumptions suggest a certain mean for the paramter value but the posterior function clearly deviates from this assumption. As the posterior function already includes our prior information, the likelihood function of our model must lie even farer away. This is a hint for absurd parameter estimates that were used to simulate the data. Data was simulated for a calibrated value of $\rho_G 0.95$ whereas our economic beliefs suggested a mean value of 0.8 with a standard deviation of 0.10. As a consequence, we should consider re-calbrating the model with respect to some parameters, i.e. ρ_G and r_A.


tau
kap
psi1
psi2
rhoR
rhog
rhoz
p_A
r_A


gam_Q
stdR
stdg
stdz