Westfälische Wilhelms-Universität Münster

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Answer sheet

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Advanced Macro (PhD)

Problem Set 4: **DSGE models**

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Abbildungsverzeichnis

1 task I: Clarida-Gali-Gertler model from the lecture.

1.1 task 1.1):

Replace the time series representation of a_t with $a_t = \rho a_{t-1} + \epsilon_t^{\alpha}$. Consider the response of the economy to a technology shock ϵ_t^{α}. Does the economy over- or under-respond to the shock relative to its natural response? How does this compare to the unit root case? Give also economic intuition for the response.

The original setting for the Clarida-Gali-Gertler model is now changed by the new formulation (as an AR-process) of the technology shock which is declared in the task (for the complete set of formulas see p. 10 (Foliensatz 7a)). The changed formulas can also be seen in the added .mod file. To present the corresponding responses to a technology shock one can use the additional plots.m file with respect to the definitions of the single variables of the model. The changes in this file can be seen in the added .m file. The responses of each of the variables to a technology shock ϵ_t^{α} are represented in figure 1. In this figure, one may see that the natural real rate exceeds the actual real rate (the plot on the bottom of the left hand side). Hence the policy rule does not supply an interest rate that the new productivity level of the economy requires. Furthermore the low real rate results in a positive output gap, because the households are demanding actually more goods than the natural output would supply (see the second plot at the head of figure 1). Since prices are sticky some firms have an incentive to produce an additional amount of goods. And this means that the economy over-responds to a technology shock.

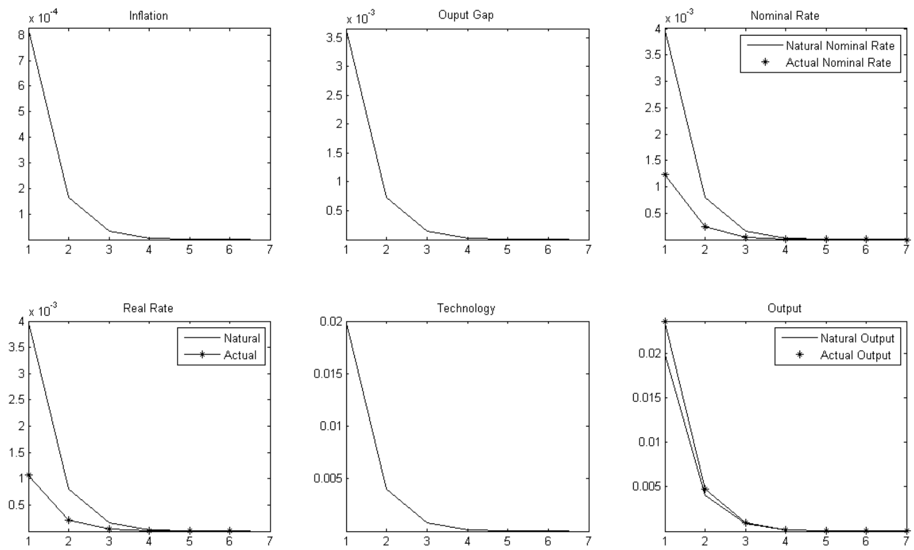

Abbildung 1: response to a technology shock ϵ_t^{α}

In contrast to the unit-root case, which represented in figure 2, the output declines after the initial shock, because of the transitory movement of the defined AR process.

And as consequence there are no permanent effects (as in contrast to the unit root case) and the natural output level comes back to its initial level, the steady state.

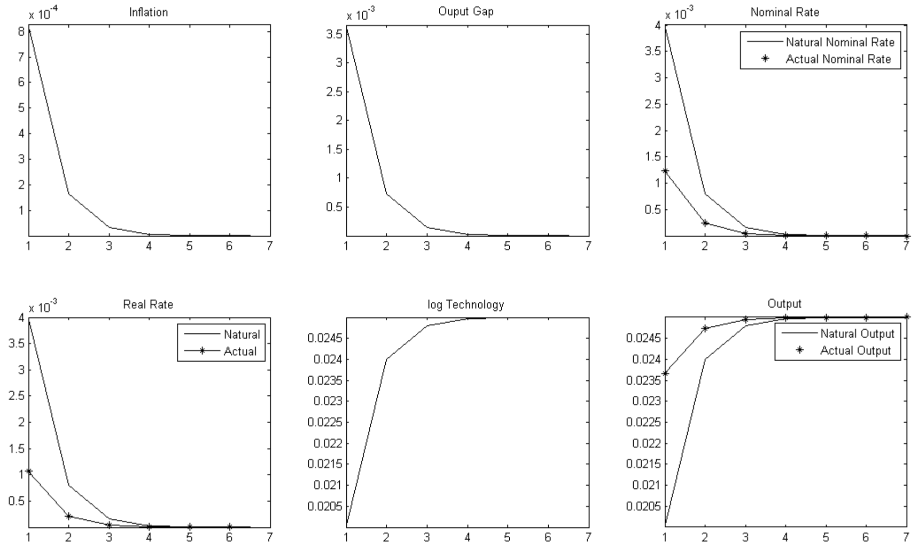

Abbildung 2: response to a technology shock ϵ_t^{α} (unit-root case)

1.2 task 1.2):

Now assume that agents have advance information (news) about the future realization of the technology shock, i.e $a_t = \rho a_{t-1} + \xi_t^0 + \xi_{t-1}^1$ where ξ_t^0 and ξ_{t-1}^1 are both iid. What happens with inflation and the output gap? Provide intuition behind this apparently contradictory result.

The responses of the economy to a technology shock are represented in figure 3. The modifications in the formulas are again represented in the added .mod file. The figure 3 represents the inflation rate (in the first plot at the head of the figure) which starts from a positive level and decreases over time until the effect fades out. In contrast to the inflation rate the output gap has its peak in the second period and is decreasing from t>2. The economic intuition could be described as follows: Due to the future technological shock the households anticipate a temporary income increase. And therefore according to the concept of the Euler equation the households smooth their intertemporal consumption. This results already in a higher demand in period 1. In period 2 the shock establishes and the productivity of the economy rises and the natural real rate is above the actual real rate. This explains the peak of the output gap at the second period. The movement of the inflation rate is driven by the Calvo pricing equation (for formulas see p. 10 (Foliensatz 7a)). In period 2 the expected prices decline more rapidly than the short-term growth of the output gap. This explains the declining movement of the inflation rate. The apparently contradictory result, the difference between the movement of the inflation rate and the output gap at the first two periods as described above, is highlighted in the figure 4.

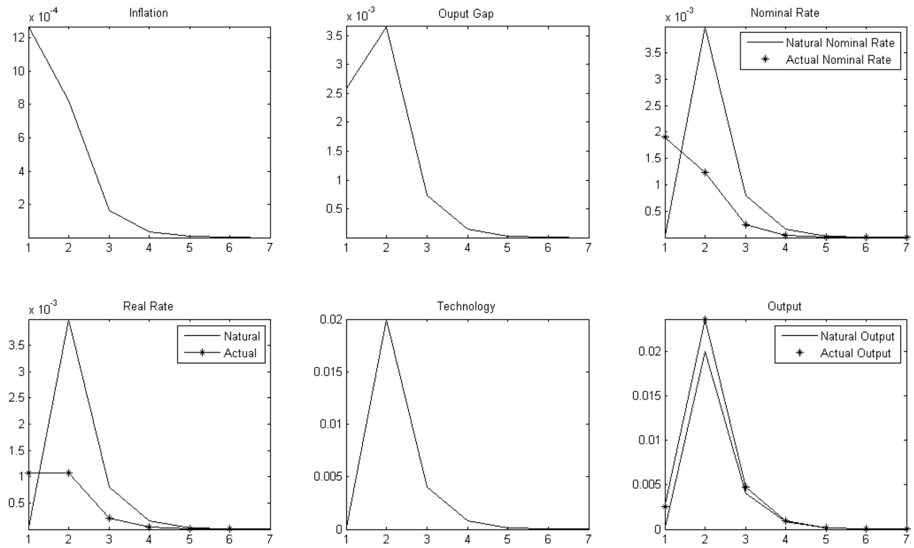

Abbildung 3: response to a technology shock ϵ_t^α which is anticipated the period before.

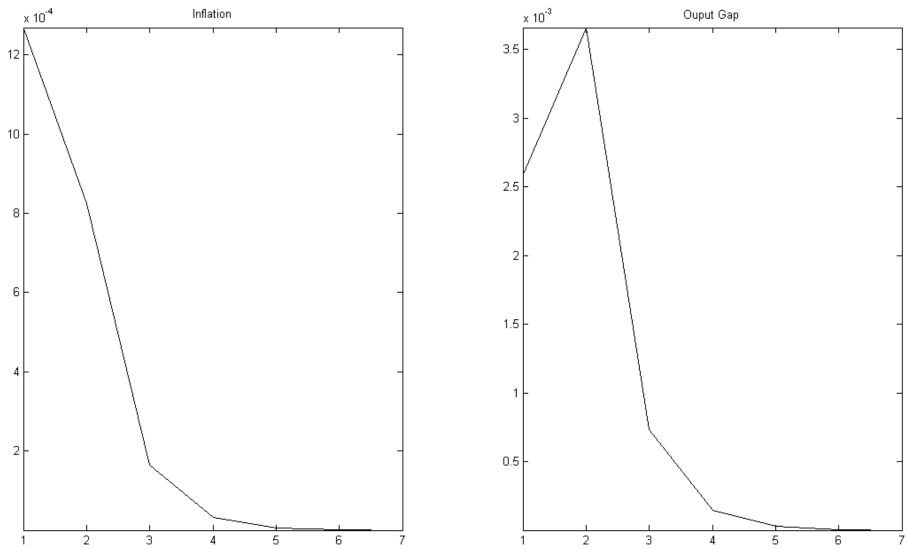

Abbildung 4: response of inflation and output gap to anticipated technology shock ϵ_t^α.

1.3 task 1.3):

What happens with the response of the economy due to a news shock relative to its natural response, if the natural rate of interest is introduced into the policy rule, i.e. $r_t = r_t^* + \alpha(r_{t-1} - r_{t-1}^*) + (1-\alpha)[\phi_\pi \pi_t + \phi_x x_t]$.

The responses of the economy due to a news shock relative to its natural response (assuming an identical news shock like in the task above) are represented in figure 5. With this figure 5 and also with the new formula and the economic intuition behind this introduction of the natural interest rate into the policy rule it is easy to observe that the monetary policy rule matches perfectly the natural and actual real rate now. Hence the inflation rate and the output gap are zero at all points in time, because of the new policy rule there cannot be any movements anymore. The figure represents as well the movements of the other variables, which are now following only the movements of the technology shock. Due to assumption of the perfect handling of the CB these results are not surprising.

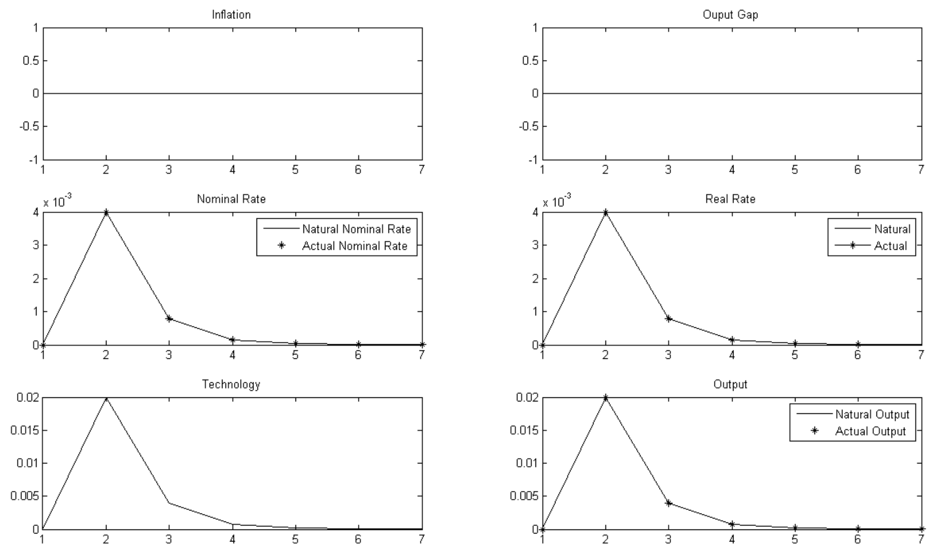

Abbildung 5: response to a technology shock ϵ_t^α with advanced policy rule

2 task II: Solution methods - a simplifed Asset-Pricing Model.

2.1 Which variable is the state and which the control variable?

In this simplified asset pricing model the dividend d_t represents the state variable and the price p_t represents the control variable. In this simplified model the households gain their whole income from the dividends of their assets. The household have to maximize their lifetime utility depending on consumption which is in this simple model equal to the dividend (because no more information are given, this is the only plausible case in this model). The dividends are defined as a simple AR (1) process which depends on stochastic shocks, the stochastic component in this model (the shocks are iid). The intertemporal consumption is controlled by their income, and more further with their demand for assets. If the first equation is rearranged a little bit one gets the following price equation:
$p_t = \beta E[\frac{u'(d_{t+1})}{u'(d_t)}(p_{t+1} + d_{t+1})]$
The price is the expectation of the future price and dividend rated with the marginal rate of substitution and discounted with the discount factor β. The control variable is

important, because it helps to derive the solution for the state variable, the optimal process of dividends (which equal the process of consumption in this model). In other words once the optimal path of the control variables is found, the solution for state variable can be derived. A control variable is characterized by the property that it is subject to the optimizing choice and an existent relationship to the state variable.

2.2 Derive the decision rule for the state variable

The decision rule for the state variable has the following form:
$x_{t+1} = \overline{x} + h(x_t, \sigma) + \sigma\epsilon_{t+1}$
The given formula has to equal the decision rule equation:

$$d_{t+1} = \overline{d} + h(d_t, \sigma) + \sigma\epsilon_{t+1} = \rho d_t + e_{t+1}$$

$$\Longrightarrow d_{t+1} - \overline{d} = h(d_t, \sigma) + \sigma\epsilon_{t+1} = \rho d_t - \overline{d} + e_{t+1}$$

With the result that $h(d_t, \sigma) = \rho d_t - \overline{d}$.
And as σ measures the stochastic component in the perturbation approach, it is $\sigma = \sigma_e$. Hence σ_e represents, as the standard deviation of the only exogenous shock e_t, the only stochastic component in this model.

2.3 Derive the expressions for the second order approximation for the decision rule for the control variable

$F(x, \sigma) = 0$ is defined as follows:

$$F(x, \sigma) = 0 \Rightarrow E_t f[h(x_t, \sigma) + \sigma\epsilon_{t+1}, g[h(x_t, \sigma) + \sigma\epsilon_{t+1}, \sigma], x_t, g(x_t, \sigma)] = 0$$

With x_t as the state variable, as defined above, y_t as the control variable and h, g two unknown functions which have to be approximated to solve the decision rule for the control variable (for the exact formula see the problem set).
In the following the definitions of the lecture are used to generate the solutions in the form in which they are represented in the hints. Use the defined equation of the decision rule for the state variable in the first equation of the task to derive an expression of $F(x, \sigma) = 0$:

$$E_t[u'(x_t)g(x_t, \sigma) - \beta u'(x_{t+1})(g(x_{t+1}, \sigma) + x_{t+1})] = 0$$

To derive the steady state value for y, $(\overline{y})$, one has to take the expectation at the values $(\overline{x},\ \overline{y},\ \sigma = 0)$, which yields to the following expression:

$$u'(\overline{x})\overline{y} - \beta u'(\overline{x})(\overline{y} + \overline{x}) = 0$$

$$\Longrightarrow \overline{y}(1-\beta) = \beta\,\overline{x}$$

$$\Longrightarrow \overline{y} = \frac{\beta}{1-\beta}\overline{x}$$

In the notation of the asset pricing model, the equation can written as follows:

$$\overline{p} = \frac{\beta}{1-\beta}\,\overline{d} \Longleftrightarrow \frac{(1-\beta)}{\beta}\,\overline{p} = \overline{d}$$

In the steady state the dividend is given as fraction of $\overline{p}$.
Now the first derivative of $F(x,\sigma)$, w.r.t. x, is going to be derived to calculate the expression for g_x using the derived expression for $h(x_t,\sigma)$:

$$\begin{aligned} F_x(d,\sigma) = E_t[u''(x)g(x,\sigma) + u'(x)g_x(x,\sigma) - \\ \beta u''(\rho x + \sigma\epsilon)\rho \cdot (g(\rho x + \sigma\epsilon,\sigma) + (\rho x + \sigma\epsilon)) - \\ \beta\rho u'(\rho x + \sigma\epsilon)\cdot(g_x(\rho x + \sigma\epsilon,\epsilon) + 1)] = 0 \end{aligned}$$

Now one can apply the expectation operator again and evaluate at the steady state $(\overline{x},\ \overline{y},\ \sigma = 0)$. This results in (with the use of $g(\overline{x},0) = \overline{y}$):

$$u''(\overline{x})\cdot\overline{y} + u'(\overline{y})\cdot g_x(\overline{x},0) - \beta u''(\overline{x})\rho(\overline{y}+\overline{x}) - \beta u'(\overline{x})(g_x(\overline{x},0)\rho + \rho) = 0$$

Now this equation can be solved to $g_x(\overline{x},0)$:

$$g_x(\overline{x},0)\cdot(u'(\overline{x})\cdot(1-\beta\rho)) = -u''(\overline{x})\cdot p(\overline{x}) + \beta\rho u''(\overline{x})(\overline{y}+\overline{x}) - \rho\beta\cdot u'(\overline{x})$$

With notational simplification this yields to:

$$\Longrightarrow g_x(\overline{x}) = \frac{-u''(\overline{x})\cdot\overline{y} + \beta\rho u''(\overline{x})(\overline{y}+\overline{x}) - \rho\beta\cdot u'(\overline{x})}{u'(\overline{x})\cdot(1-\beta\rho)}$$

Now the second derivative with respect to x can be derived, using the result from above:

$$\begin{aligned} F_{xx} = E_t[u'''(x)\cdot g(x) + u''g_x(x) + u''(x)\cdot g_x(x) + u'(x)\cdot g_{xx}(x) \\ -\beta\rho^2\cdot u'''(\rho x + \sigma\epsilon)(g(\rho x + \sigma\epsilon,\sigma) + \rho x + \sigma\epsilon) \\ -\beta\rho^2 u''(\rho x + \sigma\epsilon)(g_x(\rho x + \sigma\epsilon) + 1) \\ -\beta\rho^2 u''(\rho x + \sigma\epsilon)(g_x(\rho x + \sigma\epsilon) + 1) \\ +\beta\rho^2 u'(\rho x + \sigma\epsilon)\cdot(g_{xx}(\rho x + \sigma\epsilon))] = 0 \end{aligned}$$

Now one can apply the expectation operator again and evaluate at the steady state $((\overline{x},\ \overline{y},\ \sigma = 0)$. This results in:

$$
\begin{aligned}
u'''(\overline{x}) \cdot g(\overline{x}) + 2 \cdot u''(\overline{x}) g_x(\overline{x}) + g_{xx}(\overline{x})(u'(\overline{x})(1 - \beta\rho^2) \\
-\beta\rho^2 u'''(\overline{x}) \cdot (\overline{y} + \overline{x}) - 2\beta\rho u''(\overline{x})(g_x(\overline{x}) + 1) = 0
\end{aligned}
$$

Now this equation can be solved to $g_{xx}(\overline{x}, 0)$:

$$
g_{xx}(\overline{x}) = \frac{-u'''(\overline{x}) \cdot \overline{y} - 2 \cdot u''(\overline{x}) g_x(\overline{x}) + \beta\rho^2 u'''(\overline{x})(\overline{y} + \overline{x}) + 2\beta\rho u''(\overline{x})(g_x(\overline{x}) + 1)}{u'(\overline{x})(1 - \beta\rho^2)}
$$

Now the derivative $F_{d\sigma}$ should be calculated to derive $g_{x\sigma}$:

Before deriving the derivative it remains to remark that the expressions $g_{x\sigma} = g_{\sigma x}$ are zero in the setting with homogeneous equations if there exists a unique solution in the system of the unknowns (for the derivatives in the general case see p. 27, "Foliensatz 7b". In the following the analytical derivation should show that this holds also for the case of our given specific model setting.

For this purpose firstly one has to derive the derivative F_σ:

$$
\begin{aligned}
F_\sigma(x, \sigma) = E_t[u'(x) \cdot g_\sigma(x) - \beta u''(\rho x + \sigma\epsilon)\epsilon(g(\rho x + \sigma\epsilon) + \rho x + \sigma\epsilon) \\
-\beta u'(\rho x + \sigma\epsilon) \cdot (g_x(\rho x + \sigma\epsilon) \cdot \epsilon + g_\sigma(\rho x + \sigma\epsilon) + \epsilon)] = 0
\end{aligned}
$$

Using the expectation operator and evaluating at the steady-state values $(\overline{x},\ \overline{y},\ \sigma = 0)$, this leads to:

$$
\begin{aligned}
u'(\overline{x}) \cdot g_\sigma(\overline{x}) - 0 - 0 + \beta u'(\overline{x}) g_\sigma(\overline{x}) + 0 = 0 \\
\Longrightarrow u'(\overline{x}) \cdot g_\sigma(x) + \beta u'(\overline{x}) g_\sigma(\overline{x}) = 0
\end{aligned}
$$

The latter only holds if $g_\sigma(\overline{x}) = 0$, because one can assume that β is lying in the interval $[0, 1]$, and therefore it is smaller than one. Now the derivative $F_{x\sigma}$ is going to be derived:

$$
\begin{aligned}
F_{x\sigma} = E_t[u''(x) \cdot g_\sigma(x) + u'(x) \cdot g_{x\sigma}(x) - \beta\rho\epsilon \cdot u''(\rho x + \sigma\epsilon) \cdot (g_\sigma(\rho x + \sigma\epsilon) + 1) \\
-\beta\rho\epsilon \cdot u'''(\rho x + \sigma\epsilon) \cdot (g(\rho x + \sigma\epsilon) + \rho x + \sigma\epsilon) - \beta\rho \cdot u'(\rho x + \sigma\epsilon) \cdot (g_{x\sigma}(\rho x + \sigma\epsilon)\epsilon) \\
-\beta\rho\epsilon \cdot u''(\rho x + \sigma\epsilon) \cdot (g_x(\rho x + \sigma\epsilon) + 1)] = 0
\end{aligned}
$$

Evaluating at the steady-state values $(\overline{x},\ \overline{y},\ \sigma = 0)$, yields to the following equation:

$$E_t[u''(\overline{x})\ g_\sigma(\overline{g}) - \beta\rho\epsilon \cdot u''(\overline{x}) \cdot (g_\sigma(\overline{x})+1) - \beta\rho\epsilon \cdot u'''(\overline{x}) \cdot (\overline{y}+\overline{x})$$
$$-\beta\rho\epsilon \cdot u''(\overline{x}) \cdot (p_x(\overline{x})+1)] = E_t[g_{x\sigma} u'(\overline{x})(1-\beta\rho\epsilon)]$$

At least the expectations operator has to be applied again, which leads to the following expression:

$$u''(\overline{x})\ g_\sigma(\overline{x}) - 0 - 0 - 0 = g_{x\sigma}\ u'(\overline{x})$$

Now the derived information, that $g_\sigma(\overline{x}) = 0$, can be used for the latter equation. All in all this results in: $g_{x\sigma} = g_{\sigma x} = 0$. And this is exactly what one has expected from the general case.

3 task III: Bayesian Estimation methods with Dynare

See rbcestim.mod for this model and the Bayesian estimation we did in the lecture. RUN IT ONCE TO GET SIMULATED DATA AND ESTIMATION. Now we will use the same dataset to estimate the parameters of a misspecified model. We will use the same model, however, with a small difference, i.e. technology follows a Cobb-Douglas production function. See rbcexam.mod for the new model equations and steady-state block.

3.1 task 3.1)

Define priors for α, θ and τ. How many observable variables do you need? Choose an appropriate number of observables.

Some general comments to estimate DSGE models with Bayesian methods: Some basic steps have to be done before we can apply parameter estimation. As in the other tasks firstly the full model has to be declared. For this purpose on has to start with declaring variables and parameters. In the following the model has to be defined (in this task this step is already given by the script: *estim.mod*). A further step is the declaration of the observable variable, a variable which has to be available in a exogenous file and which the package dynare can use for the estimation procedure.
How many observables variables do one need:
In Bayesian estimation the condition for undertaking estimation is that there be at least as many shocks as observables. This means for the given model which has only one shock (technology shock) that one has to declare one observable variable. In this case one can use the variable y (which is going to be declared with the command *varobs y;*). As in the previous tasks the steady states have to be defined due to the single steps in deriving, solving and estimating the given economic setting in the

DSGE model. The next step contains the step of defining the priors.
Define priors:
Choosing the priors for the parameters is one of the most important steps in Bayesian estimation, but also very tricky and it requires some experiences in handling. Because of the wide range of possibilities of choosing the priors some time has to be spent in choosing and testing the robustness of the results. For the given task some different assumptions should be tested to check the sensitivity of the parameters to different priors - the robustness. First of all some considerations to the domains of the parameters should be mentioned.
To demonstrate the difference between both models which differ only in the definition of the production function, these functions are represented in the following:
The data is simulated in a RBC model with the following production function: $y = A * (\alpha * k_{t-1}^{\psi} + (1-\alpha) * l^{\psi})^{(1/\psi)}$.
In contrast to this function the Cobb Douglas production is defined as follows:
$y = A * (k_{t-1}^{\alpha}) * (l^{(1-\alpha)})$
Due to the case of a Cobb Douglas function one knows ex ante that the parameter α is bounded in the interval: $[0, 1]$. This is also case for the θ as a part of the defined function of preference (see setting on p. 4 of the problem sheet). For the parameter τ one can get some problems if this parameter is too close to zero, but there are no more restrictions which can be used to define some a priori boundaries for this parameter. Beside the boundaries of the parameters, one has to think about the following points: should those boundaries be open on both sides, how should one choose the shape of the prior distributions, the latter contains especially the characteristics of symmetry, skewness and kurtosis. Depending on the information one can choose and build up an appropriate distribution. Because we do not have more information than the one which has been already mentioned (further approaches would use information from the literature) we just use simple beta distributions. Due to the knowledge of the underlying data generating process one can formulate some expectations in the following estimations procedure: because of the fact that the α should differ between both production functions which is hardly to interpret in the complete setting, the focus should lie on the estimators for the parameters τ, θ, because these parameters have the same functions in both settings. Therefore the question remains how the misspecified model would estimate these parameters and also in which direction these estimators will be biased in contrast to the true parameters. This question should be answered with the following approaches.

For the following estimation procedures we use different possibilities in defining the priors to pursue two aims: Firstly to check how sensitive (or robust) the Bayesian estimation procedure (with the Metropolis Hastings algorithm) react on different choices of priors. And secondly to evaluate and to get an impression of the bias between the original parameters and the estimated ones. For this purpose we choose priors which are greater than the mean of the original parameter, priors which are smaller and priors which differ in the direction of bias. In contrast to the announced purposes we also apply the original parameters as priors. Because there are no information for the used distributions we consider different derivations (in defining other kinds of distributions than simple beta definitions dynare often displays that some problems occur during the procedure. This is not known before applying the posterior mode or complete estimation via Metropolis Hastings algorithm, but should be already mentioned at this point). All used priors and the results which belong to these priors are saved in the additional textfile. A further selection of the mentioned priors for applying the estimation procedure in task 3.3 is made in task 3.2 with the diagnostic tool *mode check* which helps to evaluate the decision of the priors and to indicate some problems.

3.2 task 3.2)

Estimate the posterior mode using the estimation command and a limited sample with 200 observations. Check the posterior mode using *mode check*. If you get errors due to a non-positive definite Hessian, try a different optimization algorithm or change the initial values.

To estimate the given model one can use the command *estimation(...)* while in the brackets the options for the procedure is defined (the full command is announced in the additional .mod file which could be changed with the announced commands to achieve results with those changed options). First the *datafile* is the option to announce the name of the file with the observations used for estimation. Here one has to use the data which is simulated with the RBC model and was saved in a matlab .m file (This file is also added to the other files, and has to be used for all of the estimations to have a common basis). The command *nobs* is used to declare the number of observations in the file which should be used for the procedure. This is the command which is needed for the task and which has to be set to *200*. To check the posterior mode one can use the command mode check. If this command is announced in the *estimation* command, Dynare is going to plot the posterior density for values and for every estimated parameter around the given *mode*. This diagnostic tool should help to identify problems with the optimization procedure. " A clear indication of a problem would be that the mode is not at the trough (bottom of the minus) of the posterior distribution." (cf. Dynare user guide.)

Now some of the mentioned priors can be evaluated with the *mode check* command (due to the large number of possibilities in choosing the priors, only some main detections are described). The represented diagnostic plot which is given by the command *mode check* (see figure: 6) indicates that there is no problem existent.

plots vers1.png

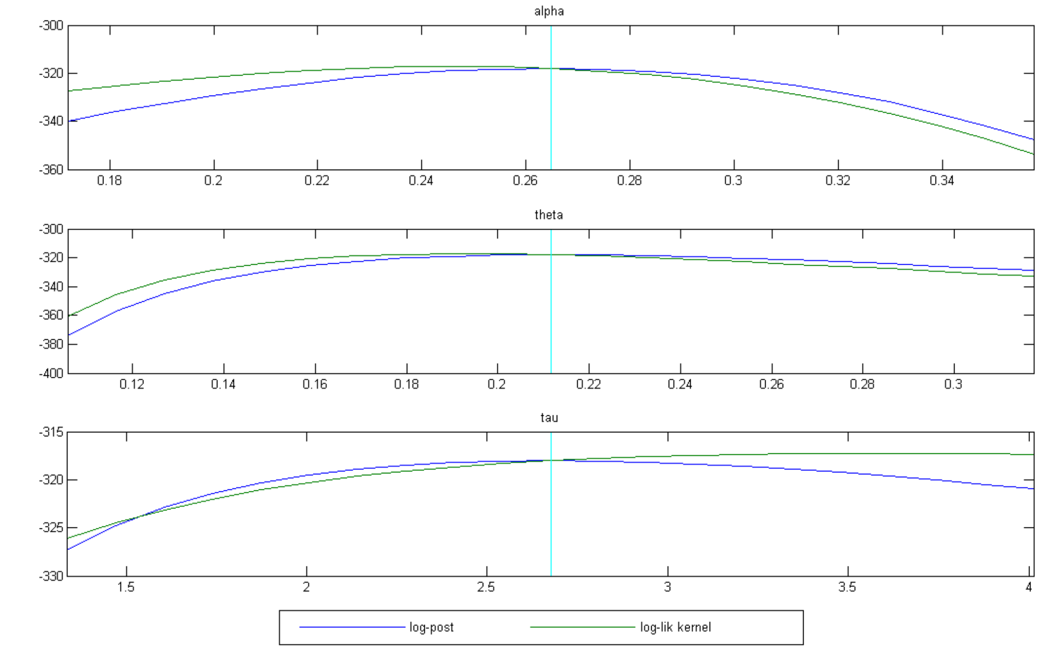

Abbildung 6: plot of the check plots for priors of version 1 (see priors txt.)

In this plot the vertical line should coincide with the blue curve (the log-post). A very flat green curve (log-lik Kernel) (as in the part of the plot for the third parameter: τ in the figure 7) means that one can only extract less information for this parameter for the data (this fact is later also shown in the comparison of the prior and the posterior distribution - in this case the posterior distribution shape is quite similar to the prior distribution shape).

plot vers10.png

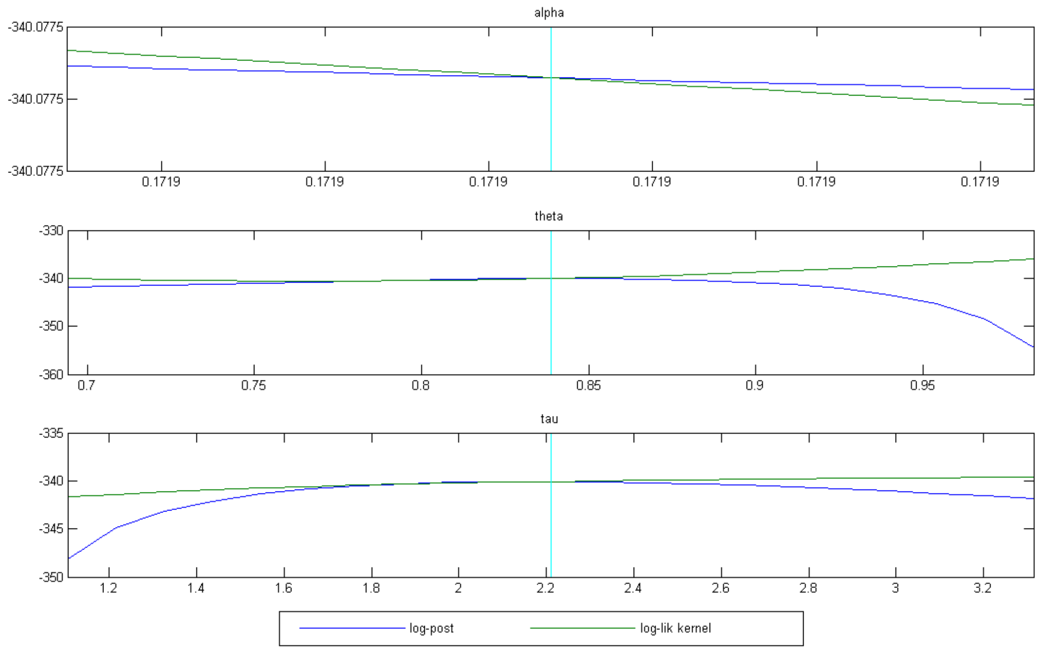

Abbildung 7: plot of the check plots for priors of version 1 (see priors txt.)

Especially the check plot which is represented in the figure 7 seem not contain appropriate priors.

3.3 task 3.3)

If you are satisfied with the posterior mode, approximate the posterior distribution using the Metropolis-Hastings-Algorithm with 3 $\times$ 5000 iterations. How large is your acceptance rate (change *jh scale* if you're not satisfied). Provide also the diagnostic plots. If the algorithm does not converge to the (ergodic) posterior-distribution, repeat the algorithm with 1000 more iterations without discarding the previous draws.

Metropolis-Hastings algorithm with 3 $\times$ 5000 iterations:

Could be generated with the command *estimation* and the additional option *mh replic* which is used to declare the number of replications of the Metropolis Hastings algorithm. Here defined by 5000 iterations. The command *mh nblocks* defines the number of chains for the Metropolis Hastings algorithm. In this case three chains are used (to check the robustness also the suggestion, the default setting, of five chains is used. See comments in task 3.4).

The commands *mode compute=0, mode file=rbcexam mode* are used for the following purposes: In the case of setting *mode compute = 0*, the mode is not computed at the following Metropolis Hastings algorithm. In this case the procedure uses the stored mode (stored and called with the following name structure: MODEL NAME mode). This option is not necessary for the estimation procedure but helps to speed up the estimation process.

The acceptance rate can be set with the command *mh jscale* and has a default value of 0.2 while the literature mostly uses values between 0.2 and 0.4. Choosing an accurate acceptance rate is also an important issue in Bayesian estimating procedure, because in the case of very high acceptance rate the candidate parameters are rejected too often and in the case of a very low acceptance rate they are accepted too often. The aim is to choose an acceptance rate which is located between both mentioned extremes. Therefore the option of applying different values for the acceptance rate is an important practical issues (to check the robustness of the estimators three different acceptance rates are used for the priors with the true parameters. To see those acceptance rates see the added textfile with the different priors and estimation results).

As asked in the task one has to provide diagnostic plots as well. These diagnostics are displayed automatically because the command *nodiagnostics* is not set in the *estimation* command, hence the default setting is used: dynare is computing and displaying tabular and graphical diagnostics, for example the convergence diagnostics for the Metropolis Hastings algorithm, which could be used to check if the blocks are converging.

To repeat the algorithm without discarding the previous draws one can use the following command in the estimation command: *mh drop* and one could define the fraction of the initially generated parameter vectors which should be dropped. If the command is set to zero none of the previous draws is discarded.

3.4 task 3.4)

Take a stand on your Bayesian estimation. What is good, what can be improved and how? Compare the estimation of the common parameters of the true model with the misspecified model.

Take a stand on your Bayesian estimation:

By the estimation procedure one gets the following diagnostic graphical plots: First one gets a representation of the priors for each parameter. Which is not important for the purpose of evaluating the applied estimation procedure.

The second plot represents the *MCMC univariate diagnostics*. The plot in figure 8 (priors version 1) shows clearly that there is a quite similar movement of the chains and that they are lying quite closely at the end of the iterations which can be interpreted as a sign of convergence. Here three measures for the three parameters are plotted. The *interval*: being constructed from an 80 percent confidence interval around the parameter mean. The *m2*: measure of the variance and the *m3*: based on third moments. The red and blue lines differ between parameter vectors for the between and the within chains.

diagnostics vers1.png

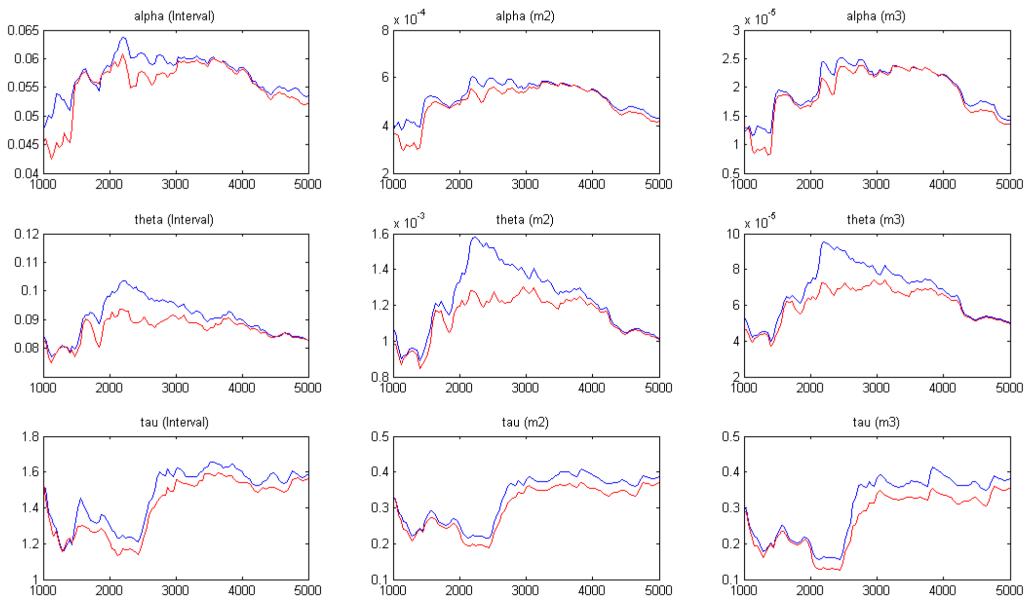

Abbildung 8: plot of univariate diagnostics for the priors of version 1 (see priors txt.)

diagnostics vers1.png

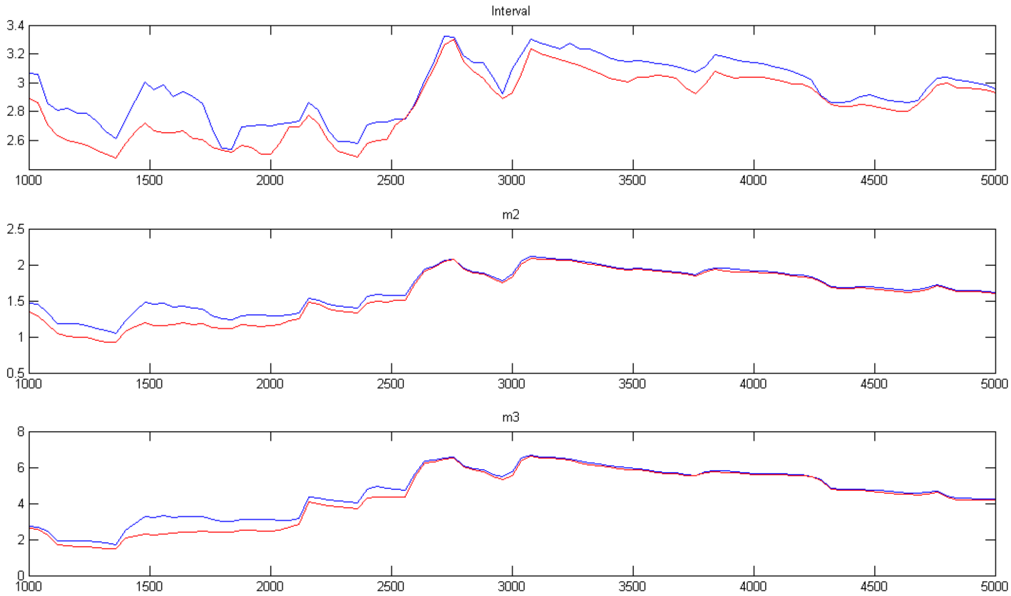

Abbildung 9: plot of multivariate diagnostics for the priors of version 1 (see priors txt.)

The plot *MCMC multivariate diagnostic represents* represents an aggregate measure with analog differentiations as in the univariate case (see above). In the figure 9 it is possible to consider a kind of a movement to convergence. But this is as well a question of the demanded precision.

In the case of the priors from version 1 with larger deviations the plot shows a significant movement of divergence (see figure 10).

diagnostics vers10.png

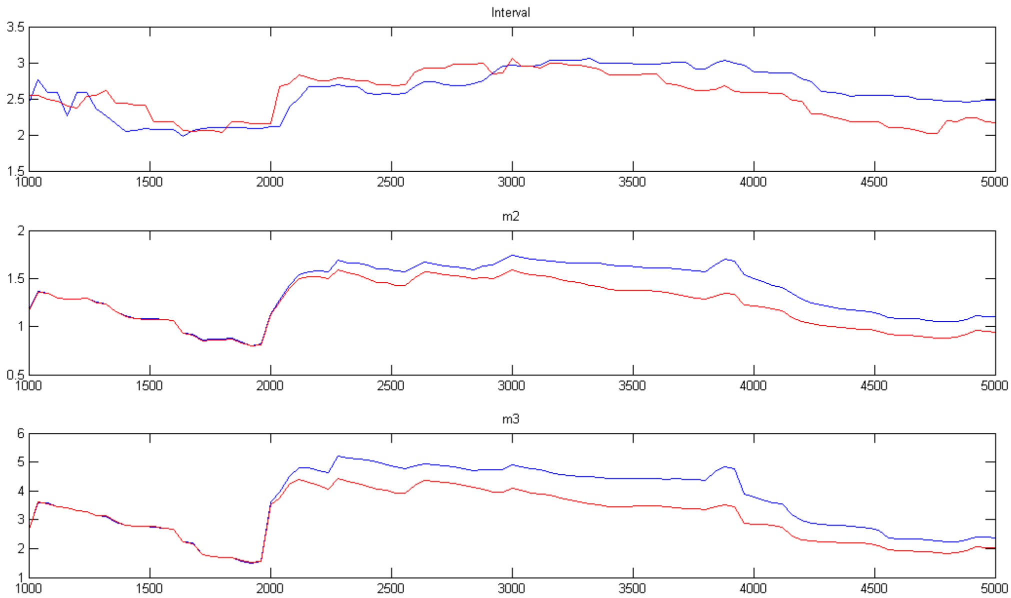

Abbildung 10: plot of multivariate diagnostics for the priors of version 10 (see priors txt.)

In contrast to the figure 10, the figure 11, the multivariate diagnostics for the priors of version 3 (see textfile for the defined priors), seem to have an obvious movement of convergence.

diagnostics vers3.png

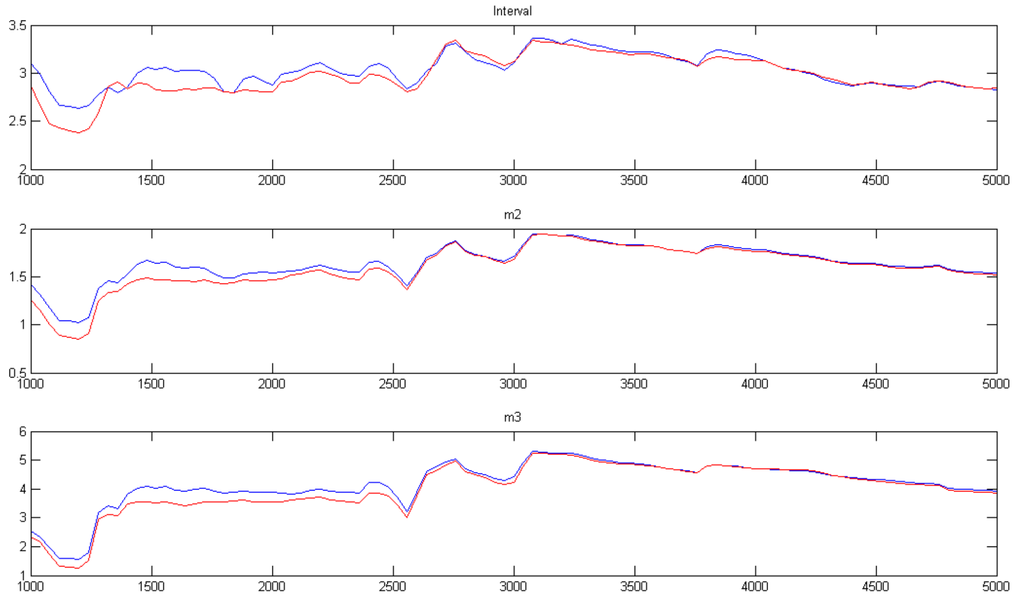

Abbildung 11: plot of multivariate diagnostics for the priors of version 3 (see priors txt.)

In the case of instability or existing movements which are suffering from divergence it is possible to improve the findings by using some different priors (as an indication of poor priors: this has been already done with the figures above). Another opportunity is to choose a greater number of Metropolis Hastings iterations. This should be done for the priors of version 1 with 5 chains and 10000 iterations. The results in the multivariate diagnostics are presented in the figure 12.

diagnostics vers9.png

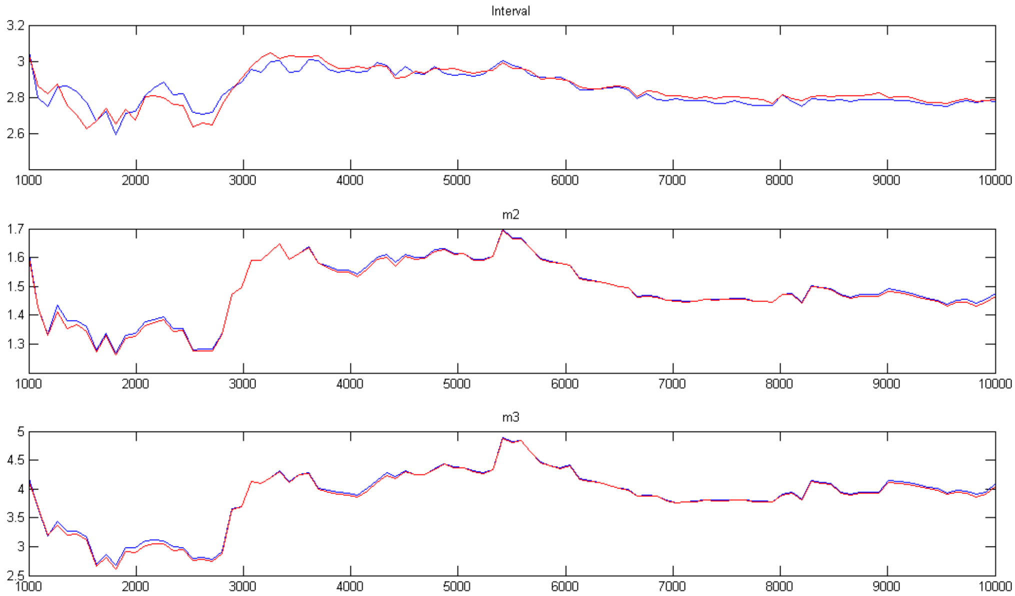

Abbildung 12: plot of multivariate diagnostics for the priors of version 9 (see priors txt.)

Now the most important diagnostic plot should be discussed, the comparison of the posterior and the prior distributions - which is the main result of the estimation algorithm and as well a graphical tool to indicate existing problems.
The figure which compares the prior with the posterior distributions. Here represented by the grey lines (the prior distributions) and the black lines (the posterior distributions). The green line represents the posterior mode.

posteriors vers1.png

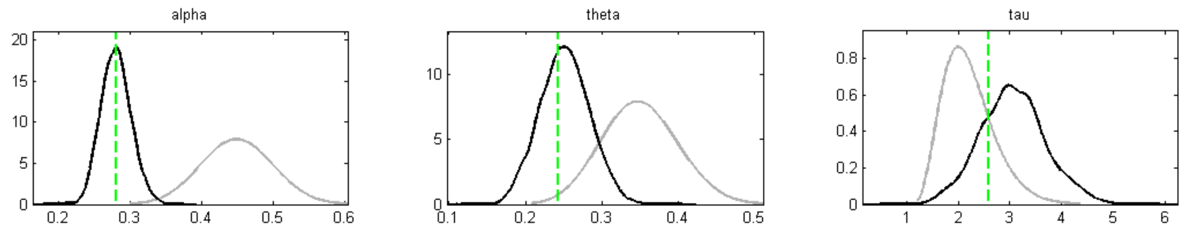

Abbildung 13: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

Figure 13 represents the plot of the comparison between the priors and posteriors, version 1. More of these plots can be generated by using the priors, which are added to the textfile and the .mod file. Figure 14 shows another plot of priors and posteriors which suffer from computational and identification problems.

posteriors vers10.png

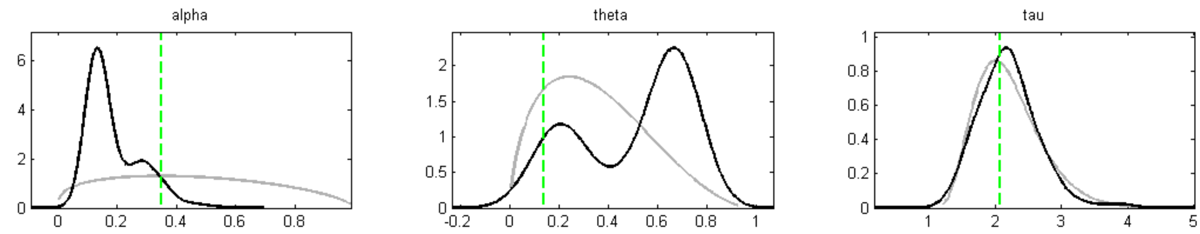

Abbildung 14: plot of the comparison between the priors and posteriors, version 1 (see priors txt.)

The plot of the smoothed shocks is important for checking the movement and the frequency of the shocks. This plot shows that the shock is centered around zero which is also assumed.

Compare the estimation of the common parameters of the true model with the misspecified model:

If the estimators for the true model are compared with the misspecified model, there are several observations which have to be announced: If the priors of the parameters are not chosen too close to their boundaries (in the underlying, overlying and also for the opposed case) the estimator of the θ parameter is always smaller than the true value and the τ value is always larger than the true value (see the additional textfile for results of the estimations). If the priors are chosen around the true parameters and not too close to their boundaries, the estimation results are quite robust, but are biased in the described form. The results could change if the priors are set to extreme scenarios. But these results are not surprising and these problematic priors can be detected by the diagnostic graphics, for example with the check plots (see tasks above) or by comparing the different models with the commands applied in task 3.5.

3.5 task 3.5)

Calculate the posterior-odds and the posterior-model-probabilities weighting each model by 0.5 a prior.

The command *model comparison* can used to compare two models with own vectors of parameters which are estimated by using the same sample. This is exactly the case in this task with the original RBC model and the "misspecified"model with Cobb Douglas production function. Only few words to the model comparison: The important values are the ones of the marginal density (here in log version). So the comparison between two models works with the following decision rule: the model with the higher liklihood fits the data better. Hence for the given models the true model estimation fits the data better than the so called misspecified model (see 3.5.txt for comparison results and use the added .mod file for replication). This is again not a surprising result and makes clear how in an unknown model setting one can choose between two available production functions or in general differ between two model settings.