Advanced Macroeconomics (PhD) - Problem Set 4: DSGE Methods

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Exercise 1

The solution of Exercise 1 is heavily based on the "Assignment 7" by Lawrence Christiano, precisely on its exercise 4 and especially on the solution for this exercise 4 as well as the corresponding Dynare file cggsignal.mod.[1]

The Dynare code for this exercise can be found in the file cggexamedited.mod. Running it in a folder with the matlab file plot.m creates the plots.

1.

Set exercise = 1 in the beginning of the mod file. In line with "cggsignal.mod" from Christiano, two equations are changed: First, technology now follows an AR1 in log levels. It is still named "da" to fit the structure of the file but the "da" now actually *means* "a". Second, the natural rate equation changes slightly with $-(1-\rho)$ at the beginning. In addition there has to be a third modification made in this exam: the file used in the exam contains an equation for output growth. As "da" now actually denotes "a", this output growth difference equation now also contains a first difference term of "da" rather than "a".

In the plots given by plot.m, figure 1 describes the reaction to the technology shock. The signs do switch and the economy under-responds in comparison to its natural response. In the answer to question 4 c) given in his answers file, Christiano provides intuition for this case: If the growth rate of technology is AR1 (the log difference case from the lecture), a shock in technology implies larger growth rates in the future. The wish to borrow is countered by an increase in the natural rate. In case of the log level of technology being AR1 (the exam case), a shock implies lower log levels in the future which leads to agents wishing to save. The interest rate reactions again goes against this effect.

2.

Set exercise = 2 in the beginning of the mod file. First of all, the natural rate equation is adjusted according to Christiano's "cggsignal.mod": In the Dynare file, the equation r^* now also contains the news signal. In his answer to question 4 d), Christiano also provides intuition for this result: Agents receive a signal that technology will go up in the future period and thus

[1]See `http://faculty.wcas.northwestern.edu/~lchrist/course/assignment7.htm` for the assignment's web page. The exercise sheet can be found under `http://faculty.wcas.northwestern.edu/~lchrist/course/assignment7.pdf`, the solution under `http://faculty.wcas.northwestern.edu/~lchrist/course/assignment7ans.pdf`, and the dynare file cggsignal.mod in the zip package under `http://faculty.wcas.northwestern.edu/~lchrist/course/cgg.zip`.

the natural rate will rise to go against the wish to borrow. The standard Taylor rule does not increase the interest sufficiently and thus a boom happens which can be seen at the output gap. Inflation goes down because marginal costs are expected to fall and agents thus reduce their prices.

3.

Set exercise = 3 in the beginning of the mod file. Now the fact which Christiano described in his solution to exercise 4 d) is compensated: Through the adjustment of the Taylor rule, the interest rate goes up in high enough magnitude. It thus adjusts immediately, inflation is positive and the reaction of the economy equals its natural response.

Exercise 2

1.

State variable: d_t (x_t in the lecture)
Control variable: p_t (y_t in the lecture)

2.

The standard form of the state variable given in lecture was:

$$d_{t+1} = h(d_t, \sigma) + \sigma\varepsilon_{t+1} \tag{1}$$

Using a first order Taylor expansion around the steady state (this might even be exact due to the mean value theorem):

$$h(d_t, \sigma) = h(\bar{d}, 0) + h_d(\bar{d}, 0)(d_t - \bar{d}) + h_\sigma(\bar{d}, 0)(\sigma - 0) \tag{2}$$

Plugging the equation (2) into equation (1) yields:

$$d_{t+1} = h(\bar{d}, 0) + h_d(\bar{d}, 0)(d_t - \bar{d}) + h_\sigma(\bar{d}, 0)(\sigma - 0) + \sigma\varepsilon_{t+1} \tag{3}$$

Setting $\sigma = 0$ and recognising that $h(\bar{d}, 0) = \bar{d}$ yields the decision rule:

$$\begin{aligned} d_{t+1} &= h(\bar{d}, 0) + h_d(\bar{d}, 0)(d_t - \bar{d}) + h_\sigma(\bar{d}, 0)(0 - 0) + 0\varepsilon_{t+1} \\ &= \bar{d} + h_d(\bar{d}, 0)(d_t - \bar{d}) \end{aligned} \tag{4}$$

Thus, sigma is set to zero in a non-stochastic steady state.

3.

Note that $p_t = g(d_t, \sigma)$ and $d_{t+1} = \rho d_t + e_{t+1}$. The first equation of the asset pricing model yields:

$$\begin{aligned} F(d_t, \sigma) &= u'(d_t)g(d_t, \sigma) - \beta E_t\left[u'(d_{t+1})(p_{t+1} + d_{t+1})\right] \\ &= u'(d_t)g(d_t, \sigma) - \beta u'(\rho d_t + e_{t+1})[g(\rho d_t + e_{t+1}, \sigma) + \rho d_t + e_{t+1}] = 0 \end{aligned} \tag{5}$$

The relevant derivatives of this functions are for d_t:

$$\begin{aligned} F_d = {} & u''(d_t)g(d_t, \sigma) + u'(d_t)g_d(d_t, \sigma) - \beta\rho u''(\rho d_t + e_{t+1})[g(\rho d_t + e_{t+1}, \sigma) + \rho d_t + e_{t+1}] \\ & - \beta u'(\rho d_t + e_{t+1})[g_d(\rho d_t + e_{t+1}, \sigma)\rho + \rho] \end{aligned} \tag{6}$$

$$\begin{aligned} F_{dd} = {} & u'''(d_t)g(d_t, \sigma) + u''(d_t)g_d(d_t, \sigma) + u''(d_t)g_d(d_t, \sigma) + u'(d_t)g_{dd}(d_t, \sigma) \\ & - \beta\rho^2 u'''(\rho d_t + e_{t+1})[g(\rho d_t + e_{t+1}, \sigma) + \rho d_t + e_{t+1}] \\ & - \beta\rho u''(\rho d_t + e_{t+1})[g_d(\rho d_t + e_{t+1}, \sigma)\rho + \rho] \\ & - \beta\rho u''(\rho d_t + e_{t+1})[g_d(\rho d_t + e_{t+1}, \sigma)\rho + \rho] \\ & - \beta u'(\rho d_t + e_{t+1})[g_{dd}(\rho d_t + e_{t+1}, \sigma)\rho^2] \end{aligned} \tag{7}$$

and for σ:

$$F_\sigma = u'(d_t)g_\sigma(d_t, \sigma) - \beta u'(\rho d_t + e_{t+1})g_\sigma(\rho d_t + e_{t+1}, \sigma) \tag{8}$$

$$F_{\sigma\sigma} = u'(d_t)g_{\sigma\sigma}(d_t, \sigma) - \beta u'(\rho d_t + e_{t+1})g_{\sigma\sigma}(\rho d_t + e_{t+1}, \sigma) \tag{9}$$

And lastly the cross derivative (which has to have the equal value in either order due to $f_{xy} = f_{yx}$ under fairly general conditions):

$$\begin{aligned} F_{\sigma d} = {} & u''(d_t)g_\sigma(d_t, \sigma) + u'(d_t)g_{\sigma d}(d_t, \sigma) \\ & - \beta\rho u''(\rho d_t + e_{t+1})g_\sigma(\rho d_t + e_{t+1}, \sigma) \\ & - \beta u'(\rho d_t + e_{t+1})g_{\sigma d}(\rho d_t + e_{t+1}, \sigma) \end{aligned} \tag{10}$$

In the non stochastic steady state, $\bar{p}$ is given by:

$$\bar{p} = g(\bar{d}, 0) \tag{11}$$

Taking expectations of F_d, evaluating at $(\bar{d}, 0)$, and setting this to zero yields:

$$\begin{aligned}
&\bar{u}''(\bar{d})\bar{p} + u'(\bar{d})g_d - \beta\rho u''(\bar{d})[\bar{p} + \bar{d}] - \beta\rho u'(\bar{d})g_d - \beta\rho u'(\bar{d}) = 0 \\
\Leftrightarrow &u'(\bar{d})g_d - \beta\rho u'(\bar{d})g_d = -\bar{u}''(\bar{d})\bar{p} + \beta\rho u''(\bar{d})[\bar{p} + \bar{d}] + \beta\rho u'(\bar{d}) \\
\Leftrightarrow &g_d = \frac{-\bar{u}''(\bar{d})\bar{p} + \beta\rho u''(\bar{d})[\bar{p} + \bar{d}] + \beta u'(\bar{d})\rho}{(1 - \beta\rho)u'(\bar{d})}
\end{aligned} \tag{12}$$

And the same for F_{dd}:

$$\begin{aligned}
&\rho^2\beta u'(\bar{d})g_{dd} - u'(\bar{d})g_{dd} = u'''(\bar{d})\bar{p} + 2u''(\bar{d})g_d - \beta\rho^2 u'''(\bar{d})[\bar{p} + \bar{d}] - 2\beta\rho^2 u''(\bar{d})[g_d + 1] \\
\Leftrightarrow &g_{dd}(\rho^2\beta u'(\bar{d}) - u'(\bar{d})) = u'''(\bar{d})\bar{p} + 2u''(\bar{d})g_d - \beta\rho^2 u'''(\bar{d})[\bar{p} + \bar{d}] - 2\beta\rho^2 u''(\bar{d})[g_d + 1] \\
\Leftrightarrow & - g_{dd}u'(\bar{d})(1 - \rho^2\beta) = u'''(\bar{d})\bar{p} + 2u''(\bar{d})g_d - \beta\rho^2 u'''(\bar{d})[\bar{p} + \bar{d}] - 2\beta\rho^2 u''(\bar{d})[g_d + 1] \\
\Leftrightarrow &g_{dd} = \frac{-u'''(\bar{d})\bar{p} - 2u''(\bar{d})g_d + \beta\rho^2 u'''(\bar{d})[\bar{p} + \bar{d}] + 2\beta\rho^2 u''(\bar{d})[g_d + 1]}{u'(\bar{d})(1 - \rho^2\beta)}
\end{aligned} \tag{13}$$

Turning to the derivatives of σ and applying the above on F_σ yields:

$$\begin{aligned}
&u'(\bar{d})g_\sigma = \beta u'(\bar{d})g_\sigma \\
\Leftrightarrow &g_\sigma[u'(\bar{d}) - \beta u'(\bar{d})] = 0 \\
\Rightarrow &g_\sigma = 0
\end{aligned} \tag{14}$$

And on $F_{\sigma d}$ using the result $g_\sigma = 0$:

$$\begin{aligned}
&u''(\bar{d})g_\sigma + u'(\bar{d})g_{\sigma d} - \beta\rho u''(\bar{d})g_\sigma - \beta u'(\bar{d})g_{\sigma d} = 0 \\
\Rightarrow &u''(\bar{d})0 + u'(\bar{d})g_{\sigma d} - \beta\rho u''(\bar{d})0 - \beta u'(\bar{d})g_{\sigma d} = 0 \\
\Leftrightarrow &g_{\sigma d}[u'(\bar{d}) - \beta u'(\bar{d})] = 0 \\
\Rightarrow &g_{\sigma d} = 0
\end{aligned} \tag{15}$$

And thus also:

$$\Rightarrow g_{d\sigma} = 0 \tag{16}$$

Exercise 3

The mod file corresponding to this Exercise 3 is named “rbcexamedited.mod”. Most of the code was copied from the example from the lecture “rbcestim.mod”.

1.

The priors for α, θ, and τ are set to the same distributions and parameter values as in the lecture example to allow maximum comparability of the results from the miss-specified model (miss-specified because basically the parameter psi has been erased in the exam example). The posterior probabilities introduced later on are a formal Bayesian approach of comparing and assessing different models. In regard to the observables: we have simulated data for y, c, and l in the lecture but defined only y as observable variable. In the following exercise, I will as well define the output y as observable variable.

2.

See the Dynare code for this part.

3.

Also see the Dynare code for this part. The convergence diagnostics show that especially for theta and tau, further draws from the MH algorithm are needed. The additional 1000 draws make these diagnostics graphs look more supporting for convergence. The acceptance rate looks well in line with what can e.g. be found on the internet regarding Bayesian estimation for DSGE models (see e.g. Wouter den Haan’s notes).

4.

In terms of computing time, the 5000 + 1000 iterations of the MH algorithm allow a fairly quick comparison of results. To achieve a higher accuracy, one could increase the number of draws to 10,000 and beyond. It can still be seen in the univariate as well as multivariate diagnostics, that there is room for further convergence. In addition, to check the robustness of such a Bayesian estimation, one would need to test different priors and compare the robustness of the outcomes in relation to the choice of those priors. The choice of correct priors also has a positive effect on possible identification issues: priors that exclude absurd parameter values can make it less likely that posterior distributions look almost exactly like prior distributions as a result of identification problems (see slide 30 from lecture #3 for further details). What is interesting in this regard is the case of the variable tau. In the correctly specified model in the example from the lecture, it can be seen in the prior/posterior plots that there is hardly any additional information to estimate tau. The posterior distribution is almost equal to the prior distribution. Yet, in the miss-specified model and the resulting plot from "rbcexamedited.mod", the posterior distribution of tau is slightly more distinct from the prior distribution. However, the model from the lecture almost gets a posterior probability of one in part 5 below and is (correctly) identified as the better fitting model. Thus, although it is important to note that the estimation hardly seems to generate new information to identify a parameter (in this case the estimation of tau in rbcestim.mod), this does not necessarily mean that the result does not fit the data generating process. Overall, the model from the lecture fits the data far better than the modification in this exam. Naturally, this is a technical example as the model from the lecture is the data generating process itself and the exam model has been deliberately miss-specified. Yet, model selection by posterior probabilities or other criteria can be of distinct importance as the estimated parameter values in both cases are very similar. In a real world example, a Bayesian estimation would also increase its credibility if one tests many specifications of utility functions, production functions etc. All of these different models can than be compared by their posterior probabilities and would yield a richer picture about what functional forms fit the data best. As mentioned above, the acceptance rate looks in line with recommendations from the literature. Jump and target distribution seem to fit fairly well.

5.

The prior probability of 0.5 implies that the posterior probabilities are simply a weighted average of the normalising constants (these constants are the integrals of likelihood times prior over the entire parameter space). More precisely, the posterior probability is computed by dividing the normalising constants from the posterior distribution of one model by the sum of both normalising constants. There are no prior probabilities in this fraction left as they are equal for each model and cancel out. Now the posterior probabilities highly point to the correctly specified model (its posterior probability is almost 1). Such a high weight would probably be somewhat unrealistic in a real world example. The correctly specified model exactly fits the mathematical structure with which the data was created. Thus, its posterior probability should in fact be very close to one. As no model would resemble reality in such a strong fashion, the posterior probability would likely be lower with real world data. However, this example shows that the model selection with Bayesian criteria can work very well. The posterior probabilities can also be used for forecasting issues in combination with methods such as Bayesian model averaging (BMA).