DSGE methods

Introduction to Dynare via Clarida, Gali, and Gertler (1999)

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Previously...

- Theory and intuition behind the Smets/Wouters' model as the workhorse DSGE model.
- Derivation of the structural form and log-linearization.

Insight

DSGE model consists of

- set of Euler equations, i.e. first-order optimality conditions,
- transition equations for structural shocks and innovations,
- observable variables and measurement errors

which can be cast into a nonlinear system of expectational difference equations.

Introduction to Dynare

Dynare

- computes the solution of deterministic models (arbitrary accuracy),
- computes first, second and third order approximation to solution of stochastic models,
- estimates (maximum likelihood or Bayesian approach) parameters of DSGE models,
- computes optimal policy (Ramsey-optimal),
- performs global sensitivity analysis and local identification of a model,
- solves problems under partial information,
- estimates BVAR and Markov-Switching Bayesian VAR models
- estimates DSGE-VAR models
- estimates Markov-Switching DSGE models
- ...

Household preferences are given by

$$\max_{C_t, N_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - exp(\tau_t) \frac{N_t^{1+\phi}}{1+\phi} \right)$$

- C_t denotes consumption, N_t denotes employment or market work,
- $\phi > 0$ denotes a labor supply parameter, $0 < \beta < 1$ is discount parameter
- τ_t is a preference shock

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^\tau, \quad \varepsilon_t^\tau \sim iidN(0, {\sigma_\tau}^2) \tag{1}$$

The budget constraint of the household is

$$P_t C_t + B_{t+1} \leq W_t N_t + R_{t-1} B_t + T_t,$$

- T_t denotes (lump-sum) taxes and profits,
- P_t denotes price level,
- W_t denotes nominal wage rate
- B_{t+1} denotes bonds purchased at time t, which deliver a non-state-contingent rate of return, R_t, in period $t+1$.

Optimality yields:

- Intratemporal optimality: Marginal cost of working (in consumption units) equals marginal benefit (the real wage)

$$exp(\tau_t) C_t N_t^{\phi} = \frac{W_t}{P_t}$$

- Intertemporal Euler equation: Utility costs of consumption foregone with corresponding benefit

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \tag{2}$$

Π_{t+1} gross inflation from t to $t+1$

Simple model: Clarida, Gali, and Gertler (1999)

Competitive firms

Represantative, competitive firm produces a homogeneous output good, Y_t, using the following Dixit-Stiglitz technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di,\ \varepsilon > 1,$$

- $Y_{i,t}$ denotes the i^{th} intermediate good, $i \in (0, 1)$.
- ε is degree of substitution between inputs (love of variety)

Competitive firm takes the price of final output good, P_t, and the prices of intermediate goods, $P_{i,t}$, as given and chooses Y_t and $Y_{i,t}$ to maximize profits. First-order condition:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \text{ with } P_t = \left(\int_0^1 P_{i,t}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

- $Y_{i,t}$ is the demand curve for the producer of $Y_{i,t}$.

Simple model: Clarida, Gali, and Gertler (1999)

The i^{th} intermediate good firm is a monopolist for $Y_{i,t}$ and uses labor, $N_{i,t}$, to produce output using the following production function:

$$\begin{aligned} Y_{i,t} &= A_t N_{i,t},\ \ a_t = log(A_t), A_0 = \bar{A} = 1 \\ \Delta a_t &= \rho_a \Delta a_{t-1} + \varepsilon_t^a, \quad \varepsilon_t^a \sim iidN(0, {\sigma_a}^2) \end{aligned} \tag{3}$$

- Δ is the first difference operator
- a_t has a unit root representation, ε_t^a is thus a shock on the growth rate of technology A_t

The i^{th} firm sets prices subject to Calvo frictions. In particular,

$$P_{i,t} = \begin{cases} \tilde{P}_t & \text{with probability } 1-\theta \\ P_{i,t-1} & \text{with probability } \theta \end{cases},$$

- $\tilde{P}_t$ denotes the price chosen by the $1-\theta$ firms that can reoptimize their price at time t.

Simple model: Clarida, Gali, and Gertler (1999)

Intermediate firms

- i^{th} producer is competitive in labor markets, pays $W_t(1-\nu)$ for one unit of labor.
- ν represents a subsidy which eliminates the monopoly distortion on labor in the steady state:

$$1-\nu = (\varepsilon - 1)/\varepsilon.$$

- Define real marginal costs

$$s_t = \frac{\frac{dCost}{dWorker}}{\frac{dOutput}{dWorker}} = \frac{(1-\nu)\frac{W_t}{P_t}}{exp(a_t)} = \frac{(1-\nu)C_t exp(\tau_t) N_t^{\phi}}{exp(a_t)}$$

- Each of the $1-\theta$ firms that can optimize price, choose $\tilde{P}_t$ to maximize profits:

$$E_t \sum_{j=0}^{\infty} \beta^j \theta^j \mu_{t+j} \overbrace{[\underbrace{\tilde{P}_t Y_{i,t+j}}_{\text{revenues}} - \underbrace{P_{t+j} s_{t+j} Y_{i,t+j}}_{\text{cost}}]}^{\text{period t+j profits sent to household}}$$

with:

- μ_{t+j}: marginal value of dividends to household$= u_{c,t+j}/P_{t+j}$
- θ^j firm cares only about states in which it can't reoptimize

Optimality (Calvo-Yun algebra) yields:

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} = \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(\frac{P_{t+j}}{P_t}\right)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j \left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon-1}} \equiv \frac{K_t}{F_t} = \left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} \tag{4}$$

where

$$K_t = (1-\nu)\frac{\varepsilon}{\varepsilon-1}\frac{exp(\tau_t)N_t^{\phi}C_t}{exp(a_t)} + \beta\theta E_t \Pi_{t+1}^{\varepsilon} K_{t+1}, \tag{5}$$

$$F_t = 1 + \beta\theta\Pi_{t+1}^{\varepsilon-1}F_{t+1} \tag{6}$$

Define $P_t^* = \left[\int_0^1 P_{i,t}^{-\varepsilon} di\right]^{-\frac{1}{\varepsilon}}$, law of motion of price distortion:

$$p_t^* := \left(\frac{P_t^*}{P_t}\right)^{\varepsilon} = \left[(1-\theta)\left(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\Pi_t^{\varepsilon}}{p_{t-1}^*}\right]^{-1} \tag{7}$$

Aggregate output:

$$Y_t = p_t^* exp(a_t) N_t = C_t \tag{8}$$

Steady-State

$$
\begin{aligned}
\Pi_{ss} = 1, R_{ss} = \frac{1}{\beta}, p^*_{ss} = 1,\\
F_{ss} = K_{ss} = \frac{1}{1-\beta\theta},\\
Y_{ss} = C_{ss} = N_{ss} = 1,\\
\tau_{ss} = a_{ss} = \Delta a_{ss} = 0
\end{aligned}
$$

- We have 8 equations for 10 unknowns $(C_t, p_t^*, N_t, \Pi_t, K_t, F_t, R_t, \nu)$

$\hookrightarrow$ Need to pin down policy variables ν and R_t!

- Two (in principle equivalent) ways:
  - Ramsey optimal policy (Solve Lagrangian w.r.t policy variables)
  - Taylor rule (no monopoly power, no inflation)

Ramsey equilibrium (Efficient Allocation)

The efficient equilibrium associated with the optimal monetary policy is characterized by

- zero inflation, $\Pi_t = 1$, at each date and for each realization of a_t & τ_t
- consumption and employment correspond to their first best levels
- C_t and N_t satisfy the resource constraint: $C_t = \exp(a_t) N_t$
- marginal rate of substitution between consumption and labor equals the marginal product of labor: $C_t \exp(\tau_t) N_t^{\varphi} = \exp(a_t)$

- No monopoly power:
$$1-\nu=\frac{\varepsilon-1}{\varepsilon} \tag{Eff1}$$
- No price distortion ($\forall t : p_t^* = 1$) yields no inflation
$$\Pi_t^{**}=\frac{p_t^*}{p_{t-1}^*}=1 \tag{Eff2}$$
- Efficient (first-best) allocations:
$$N_t^{**}=exp\left(\frac{-\tau_t}{1+\phi}\right) \tag{Eff3}$$
$$Y_t^{**}=C_t^{**}=exp(a_t)N_t^{**}=exp\left(a_t-\frac{\tau_t}{1+\phi}\right) \tag{Eff4}$$
- Natural interest rate back out of Euler equation:
$$R_t^{**}=\frac{1}{\beta}E_t exp\left(\Delta a_{t+1}-\frac{\tau_{t+1}-\tau_t}{1+\phi}\right) \tag{Eff5}$$

Log-linearization around steady-state yields for the efficient allocation:

$$
\begin{aligned}
\pi_t^{**} &:= log(\Pi_t^{**}) - log(\Pi_{ss}) = 0,\\
n_t^{**} &:= log(N_t^{**}) - log(N_{ss}) = -\frac{\tau_t}{1+\phi},\\
c_t^{**} &:= \log(C_t^{**}) - log(C_{ss}) = a_t - \frac{\tau_t}{1+\varphi} = \log(Y_t^{**}) - log(Y_{ss}) =: y_t^{**},\\
r_t^{**} &:= log(R_t^{**}) - log(R_{ss}) = E_t \Delta a_{t+1} - E_t \frac{\tau_{t+1} - \tau_t}{1+\varphi},
\end{aligned}
$$

- $**$ indicates a variable corresponding to the Ramsey equilibrium, i.e. natural rates,
- lower case letters are log of the corresponding variable minus log of corresponding steady-state, e.g. $x_t = log(X_t) - log(X_{ss})$

Simple model: Clarida, Gali, and Gertler (1999)

Closing the model: Taylor rule

- Most models are extended by a Taylor rule that is designed such that in steady-state inflation is zero, e.g.

$$r_t = \alpha r_{t-1} + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right]$$

- If the Taylor rule includes the Ramsey equilibrium, we call it optimal monetary policy:

$$r_t = r_t^{**} + \alpha(r_{t-1} - r_{t-1}^{**}) + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right]$$

Taylor-principle

The monetary authority reacts to a change in inflation by implementing a bigger change in interest rates ($(1-\alpha)\phi_\pi > 1$).

- Technical requirement for stable and convergent solution (Blanchard-Khan)
- If $(1-\alpha)\phi_\pi \leq 1$: shocks that raise inflation result in lower real interest rates and higher output, which further fuels the initial increase in inflation
  $\hookrightarrow$ unstable explosive spiral
  $\hookrightarrow$ indeterminacy

Simple model: Clarida, Gali, and Gertler (1999)

Log-linearized model and Taylor-rule

Linearizing about the steady-state the model equations are given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \qquad \text{(Phillips curve)}$$

$$x_t = E_t x_{t+1} - (r_t - E_t \pi_{t+1} - r_t^{**}) \qquad \text{(IS equation)}$$

$$r_t = \alpha r_{t-1} + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right] \qquad \text{(baseline Taylor-rule)}$$

$$\Delta a_t = \rho_a \Delta a_{t-1} + \varepsilon_t^a \qquad \text{(technological shock)}$$

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^\tau \qquad \text{(preference shock)}$$

$$r_t^{**} = \rho_a \Delta a_t + \frac{1-\rho_\tau}{1+\varphi}\tau_t \qquad \text{(natural interest rate)}$$

$$\Delta y_t = x_t - x_{t-1} + \Delta a_t - \frac{\tau_t - \tau_{t-1}}{1+\phi} \qquad \text{(output growth)}$$

with

$$x_t = y_t - y_t^{**} \qquad \text{(output gap)}$$

$$y_t^{**} = a_t - \frac{1}{1+\phi}\tau_t \qquad \text{(natural output)}$$

$$\kappa = \frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\theta} \qquad \text{(auxiliary parameter)}$$

Exercise: CGG-Model with Dynare

1. Install Matlab and Dynare, open cgg.mod, try to understand the code, run it. Interpret the Dynare output.
2. Compute the impulse response function of the model to a technology shock and a preference shock for the next 7 periods.
3. Given the IRF, indicate whether the economy over- or under-responds due to the shocks, relative to their natural response. What is the economic intuition in each case? (Hint: Use plots.m)
4. Do the calculations with $\phi_\pi = 0.99$. Explain the error message and give economic intuition behind this.

Return to $\phi_\pi = 1.5$.

5. Explain why it is that when the monetary policy rule is replaced by the natural equilibrium, i.e. $r_t = r_t^{**}$, the solution is indeterminate.
6. Now replace the monetary policy rule by

$$r_t = r_t^{**} + \alpha(r_{t-1} - r_{t-1}^{**}) + (1-\alpha)\left[\phi_\pi \pi_t + \phi_x x_t\right]$$

   Explain why this Taylor rule uniquely supports the natural equilibrium.

Calibrate the model to a more empirically relevant parametrization:
$\phi_x = 0.15, \alpha = 0.8, \rho = 0.9$

7. Simulate the model for 1000 periods. Save the middle 100 observations of Δy_t and π_t into an Excel-file as well as into a mat-file. Plot the path of output growth.

Now let's estimate the coefficients as well as the standard errors of the stochastic process $(\rho_a, \rho_\tau, \sigma_a, \sigma_\tau)$

8. via maximum likelihood: Use 1000 observations to verify that everything works fine and start far from the true values.
9. via maximum likelihood: Use only 50 observations and start at the true values. Do the results change?
10. via Bayesian methods: Use the inverted gamma distribution (mean to true values, standard deviation 10) as the prior on the two standard errors and the beta distribution (mean to true values, standard deviation to 0.4) as the prior on the two autocorrelations. Use 50 observations for the estimation, 1000 MCMC replications, one MCMC chain, and 1.5 for the scale parameter.