DSGE Modelling and Financial Frictions

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The Course

- General Aspects of DSGE Modelling
  - Construction, Calibration and Bayesian Estimation
  - Monetary Policy
  - Informational Consistency
- Financial Frictions in DSGE Models
  - A HUGE literature!
  - Focus on three banking models embedded in an NK model
  - Build up from a RBC Core

Why DSGE Models?

- DSGE models are micro-founded with a theoretically coherent treatment of decisions of agents and market equilibrium
- The standard models assume rational, model-consistent expectations based on perfect information of shock processes and beginning of period stocks, but this assumption is widely relaxed (imperfect information, statistical learning, internal rationality, rational inattention)
- Good for 'story telling' and relatively Lucas-Critique-immune policy analysis
- Bad for forecasting - time series statistical models produce better univariate forecasts of output and inflation.
- But balance of cost-benefits have tipped towards using DSGE models for forecasting [Del Negro and Schorfheide(2012)].

From RBC to NK

The RBC Core

- Households make an intertemporal utility-maximizing choice of consumption and labour supply subject to a budget constraint. Net assets consists of capital employed by firms
- Firms (wholesale=retail) produce output according to a crt production technology and choose labour and capital inputs to minimize cost
- Investment costs and capital producers
- Labour, output and financial markets clear
- Non-zero balanced exogenous growth steady state

NK model consists of RBC core and a nominal side consisting of

- Prices set by the retail sector who convert wholesale output into differentiated goods
- Price stickiness in the form of staggered Calvo-type or Rotemberg price setting
- A nominal interest rate set by the CB in the form of simple Taylor rules

The Core RBC Model: No Investment Costs

$$
\begin{aligned}
\text{Utility} \quad &: \quad \Lambda_t = \Lambda(C_t, 1 - h_t) \\
\text{Euler} \quad &: \quad \Lambda_{C,t} = \beta E_t R_t \left[\Lambda_{C,t+1}\right] \\
\text{Labour Supply} \quad &: \quad \frac{\Lambda_{h,t}}{\Lambda_{C,t}} = -\frac{W_t}{P_t} \\
\text{Output} : Y_t \quad &= \quad (1 - c)Y_t^W = (1 - c)F(A_t, h_t, K_{t-1}) \\
\text{FOC} \quad h_t \quad &: \quad \frac{P_t^W}{P_t} F_{h,t} = \frac{W_t}{P_t} \\
\text{FOC} \quad K_t \quad &: \quad E_t \left[\frac{P_{t+1}^W}{P_{t+1}} F_{K,t+1}\right] = R_t - 1 + \delta \\
\text{FOC} \quad P_t \quad &: \quad P_t = \frac{1}{1 - \frac{1}{\zeta}} P_t^W \\
\text{Equilibrium} \quad &: \quad Y_t = C_t + G_t + I_t = C_t + G_t + K_t - (1 - \delta)K_{t-1}
\end{aligned}
$$

Note K_t is *end-of-period* capital stock.

Functional Forms and Exogenous Processes

$$
\begin{aligned}
\text{Cobb-Douglas PF} &: \quad F(A_t, h_t, K_{t-1}) = (A_t h_t)^{\alpha} K_{t-1}^{1-\alpha} \\
\text{MPh} &: \quad F_h(A_t, h_t, K_{t-1}) = \frac{\alpha Y_t^W}{h_t} \\
\text{MPK} &: \quad F_K(A_t, h_t, K_{t-1}) = \frac{(1-\alpha) Y_t^W}{K_{t-1}} \\
\text{Technology} &: \quad \ln A_t - \ln \bar{A}_t = \rho_A(\ln A_{t-1} - \ln \bar{A}_{t-1}) + \epsilon_{A,t} \\
\text{Government} &: \quad \ln G_t - \ln \bar{G}_t = \rho_G(\ln G_{t-1} - \ln \bar{G}_{t-1}) + \epsilon_{G,t} \\
\text{Lump sum Taxes} &: \quad G_t = T_t \\
\text{Non-separable CRRA} &: \quad \Lambda_t = \frac{(C_t^{(1-\varrho)}(1-h_t)^{\varrho})^{1-\sigma} - 1}{1-\sigma} \\
\text{MUC} &: \quad \Lambda_{C,t} = (1-\varrho) C_t^{(1-\varrho)(1-\sigma)-1}((1-h_t)^{\varrho(1-\sigma)}) \\
\text{MUh} &: \quad \Lambda_{h,t} = -\varrho C_t^{(1-\varrho)(1-\sigma)}(1-h_t)^{\varrho(1-\sigma)-1}
\end{aligned}
$$

Investment Costs

- Convenient to introduce capital-producing firms that convert I_t of raw output into $(1 - S(X_t))I_t$ of new capital and then sold at a real price Q_t.

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + (1 - S(X_t))I_t\,;\ X_t \equiv \frac{I_t}{I_{t-1}} \\ S',\, S'' &\geq 0\,;\ S(1+g) = S'(1+g) = 0 \end{aligned}$$

- With g_t a stochastic growth rate (see later), choose

$$S(X) = \phi_X(X_t - (1+g_t))^2$$

- Capital-producing firms maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[Q_{t+k}(1 - S\left(I_{t+k}/I_{t+k-1}\right))I_{t+k} - I_{t+k}\right]$$

where $\Lambda_{t,t+k} = \beta^k \left(\frac{\Lambda_{C,t+k}}{\Lambda_{C,t}}\right)$ is the real stochastic discount rate over the interval $[t, t+k]$.

- Note from the Euler equation $E_t[\Lambda_{t,t+1}] = E_t[\beta \frac{\Lambda_{C,t+1}}{\Lambda_{C,t}}] = \frac{1}{R_t}$.

Investment Costs: FOC

- This results in the first-order condition (see Appendix B)

$$Q_t(1 - S(X_t) - X_t S'(X_t)) + E_t\left[\Lambda_{t,t+1}\, Q_{t+1} S'(X_{t+1}) \frac{I_{t+1}^2}{I_t^2}\right] = 1$$

- Demand for capital by risk neutral firms must satisfy

$$R_t = \frac{E_t\left[(1-\alpha)\frac{P_{t+1}^W Y_{t+1}}{P_{t+1} K_t} + (1-\delta) Q_{t+1}\right]}{Q_t} \equiv E_t[R_{k,t+1}]$$

  where $R_{k,t} \equiv \frac{(1-\alpha)\frac{P_t^W Y_t^W}{P_t K_{t-1}} + (1-\delta) Q_t}{Q_{t-1}}$ is the the gross $[t-1, t]$ return.
- If risk-averse household own the firms this becomes $\Lambda_{t,t+1} R_t = E_t[\Lambda_{t,t+1} R_{k,t+1}]$.
- As $S = S' \to 0$ and adjustment costs disappear, $Q_t \to 1$ and we obtain $E_t\left[\frac{P_{t+1}^W}{P_{t+1}} F_{K,t+1}\right] = R_t - 1 + \delta$ as before.
- Note with this choice of $S(\cdot)$, the steady state is unchanged.

The Deterministic Steady State in Dynare

Assume a zero-growth deterministic steady state for the time being ($g = 0$), $\bar{A}_t = \bar{A}_{t-1} = A$ say and $\bar{G}_t = \bar{G}_{t-1} = G$. $K_t = K_{t-1}$, etc

Computing the steady state is the most difficult part of the set-up!! Four approaches:

1. Use Initial Guesses. For medium and large size models this approach is not recommended!!
2. Solve for n variables in n unknowns using fsolve. For the model above $n = 14$.
3. Exploit the recursive structure to solve for $1 \leq m << n$ variables with an external steady state using Matlab procedure fsolve
4. Obtain a completely recursive steady state and just use the mod file without an external steady state

A Completely Recursive Steady State

$$
\begin{aligned}
R &= \frac{1}{\beta}\,; \quad \frac{P^W}{P} = 1 - \frac{1}{\zeta} \\
\frac{K}{Y^W} &= \frac{(1-\alpha)\delta}{R-1+\delta}\frac{P^W}{P} \\
\frac{I}{Y} &= \frac{\delta K}{Y^W}\frac{Y^W}{Y} = \frac{(1-\alpha)\delta}{R-1+\delta}\frac{P^W Y^W}{PY} = \frac{(1-\alpha)\delta}{R-1+\delta} \\
\frac{C}{Y} &= 1 - \frac{I}{Y} - \frac{G}{Y} = 1 - \frac{I}{Y} - g_y \\
h &= \frac{\alpha(1-\varrho)}{\varrho C/Y + \alpha(1-\varrho)} \\
Y^W &= (Ah)(K/Y^W)^{\frac{1-\alpha}{\alpha}} \\
Y &= (1-c)Y^W\,; \quad G = g_y Y\,; \quad \frac{W}{P} = \alpha\frac{P^W}{P}\frac{Y^W}{h} \\
I &= \frac{I}{Y}Y\,; \quad C = \frac{C}{Y}Y\,; \quad C = \frac{C}{Y}Y\,; \quad C = \frac{C}{Y}Y\,; K = \frac{K}{Y^W}Y^W
\end{aligned}
$$

A Partially Recursive Steady State

$$
\begin{aligned}
R &= \frac{1}{\beta}\,; \quad \frac{P^W}{P} = 1 - \frac{1}{\zeta} \\
\frac{K}{Y^W} &= \frac{(1-\alpha)\delta}{R - 1 + \delta}\frac{P^W}{P} \\
Y^W &= (Ah)(K/Y^W)^{\frac{1-\alpha}{\alpha}} \\
Y &= (1-c)Y^W \\
K &= \frac{K}{Y^W}Y^W \\
I &= \delta K\,; \quad G = g_y Y \\
\frac{W}{P} &= \alpha\frac{P^W}{P}\frac{Y^W}{h} \\
C &= \frac{W}{P}\frac{(1-\varrho)(1-h)}{\varrho}
\end{aligned}
$$

with h given leaving the equation to solve in fsolve as

$$
Y = C + I + G
$$

Calibration

- The idea is to assume an observed baseline steady state equilibrium in the presence of some observed policy. We solve for model parameters consistent with this observation.
- Normalization: 1 unit of raw labour + 1 unit of capital gives 1 unit of output $\Rightarrow A = 1$
- Use microeconometric evidence on σ
- In RBC model with $g = 0$, suppose we observe the mark-up, R, h and long-run shares $\frac{Wh}{Y}$, $\frac{C}{Y}$, $\frac{I}{Y}$. We can then pin down α, ζ, β, ϱ and δ. See **RBC_4_RES_Course.mod** .
- In general terms, write the steady state as $\underline{X} = f(\underline{\theta})$ of outcomes where $\underline{\theta}$ is a vector of parameters. The calibration strategy is to choose a subset $\underline{X}_1$ of n observed outcomes to calibrate a subset $\underline{\theta}_1$ of n parameters. Partition $\underline{X} = [\underline{X}_1, \underline{X}_2]$ and $\underline{\theta} = [\underline{\theta}_1, \underline{\theta}_2]$. Then $\underline{\theta}_1$ is then found by solving

$$[\underline{X}_1, \underline{X}_2] = f([\underline{\theta}_1, \underline{\theta}_2])$$

for $\underline{X}_2$ and $\underline{\theta}_1$, given $\underline{X}_1$ and $\underline{\theta}_2$.

Calibration of RBC Model

Observed Equilibrium	Value
h^{obs}	0.35
wage share $= \alpha$	0.7
c_y	0.6
$i_y = iobs$	0.2
g_y	0.2
$R = Robs$	1.01
mark-up$= \frac{1}{1-\frac{1}{\zeta}}$	0.17
Calibrated Parameters	**Value**
ϱ	0.684
δ	0.020
β	0.990
ζ	7.0

Table: **Calibration of RBC Model using RBC_4_RES_Course.mod**

Dynare Code I

In the folder **modelling_no_external_ss** the set-ups without using a steady state are found.

- **RBC_RES_Course.mod** contains the model without investment costs.
- **RBC_Inv_Costs_RES_Course.mod** contains the model with investment costs
- The matlab program **graphs_rbc.m** will enable you to compare the irfs.
- The sub-folder **linear** shows the linearized set-up in Dynare - it is very easy! It is compared with the non-linear first-order solution using **graphs_.rbc**.

Dynare Code II

In the subfolder **modelling_with_external_ ss** in the RBC folder you can find RBC codes used for various calibrations:

- **RBC_1a_RES_Course.mod** solves the steady state with an external steady-state matlab file simply using the analytical recursive steady state.
- Then **RBC_1b_RES_Course.mod** treats hours as a variable to be solved using fsolve which calls **fun_RBC.m**.
- **RBC_2_RES_Course.mod** : uses observed hours worked (*hobs*) and treats ϱ as a endogenous variable, consistent with *hobs*, found using fsolve calling **fun_RBC_2.m**.
- **RBC_3_RES_Course.mod** : uses observed hours worked and investment as a share of GDP (*hobs*, *iobs*) and treats ϱ and δ as endogenous variables, consistent with *hobs* and *iobs*, found using fsolve calling **fun_RBC_3.m**.
- **RBC_4_RES_Course.mod** : uses observed hours worked, the investment share and the interest rate (*hobs*, *iobs*, *Rss*) and treats ϱ, δ and β as endogenous variables, consistent with *hobs*, *iobs* and *Rss*, found using fsolve calling **fun_RBC_4.m**.

The New Keynesian Model: Structure

- The NK model consists of
  - An RBC real core and a nominal side consisting of
  - Prices set by a monopolistically competitive retail sector in the form of Calvo or Rotemberg price setting
  - A nominal interest rate set by the CB in the form of a simple Taylor Rule
- Derive demand for each differentiated good in terms of its relative price
- Consider optimal price setting as a dynamic optimization problem
- Special attention to a non-zero stochastic growth steady state.

Demand for Differentiated Goods

- Demand for consumption of good m is given by

$$C_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} C_t$$

Similarly for investment and government goods so in aggregate

$$Y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} Y_t \qquad (1)$$

if we assume *Dixit-Stiglitz aggregates*

$$\begin{aligned} C_t &= \left(\int_0^1 C_t(m)^{(\zeta-1)/\zeta} dm\right)^{\zeta/(\zeta-1)} \\ P_t &= \left[\int_0^1 P_t(m)^{1-\zeta} dm\right]^{\frac{1}{1-\zeta}} \end{aligned}$$

- Has property that for each differentiated good m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize C_t given total expenditure $\int_0^1 P_t(m) C_t(m) dm$.

Calvo Contracts

- There is a probability $1-\xi$ in each period that the price of each retail good m is set optimally to $P_t^0(m)$; otherwise it is held fixed.
- Each retailer m, given its real marginal cost, $MC_t(m) = MC_t$, common to all firms chooses $\{P_t^0(m)\}$ to maximize expected discounted profits

$$E_t \sum_{k=0}^{\infty} \xi^k D_{t,t+k} Y_{t+k}(m) \left[P_t^0(m) - P_{t+k} MC_{t+k}\right]$$

subject to (1), where $D_{t,t+k} \equiv \beta^k \frac{\Lambda_{C,t+k}/P_{t+k}}{\Lambda_{C,t}/P_t}$ is now the *nominal* stochastic discount factor over the interval $[t, t+k]$.
- The solution to this is

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \xi^k D_{t,t+k} Y_{t+k}(m) \left[P_t^0(m) - \frac{1}{(1-1/\zeta)} P_{t+k} MC_{t+k}\right] &= 0 \\ P_{t+1}^{1-\zeta} = \xi P_t^{1-\zeta} + (1-\xi)(P_{t+1}^0)^{1-\zeta} \end{aligned}$$

Price Dynamics

Details of algebra in Appendix E of the Notes

- *Price dynamics* can be expressed as difference equations. Defining the *nominal* stochastic discount factor by $D_{t,t+k} \equiv \beta \frac{\Lambda_{C,t+k}/P_{t+k}}{\Lambda_{C,t}/P_t}$ we can express the foc above as:

$$\begin{aligned}
\frac{P_t^0}{P_t} &= \frac{J_t}{H_t} \\
H_t - \xi\beta E_t[\Pi_{t+1}^{\zeta-1} H_{t+1}] &= Y_t \Lambda_{C,t} \\
J_t - \xi\beta E_t[\Pi_{t+1}^{\zeta} J_{t+1}] &= Y_t \Lambda_{C,t} MC_t MS_t \\
\Pi_t : \quad 1 &= \xi \Pi_t^{\zeta-1} + (1-\xi)\left(\frac{J_t}{H_t}\right)^{1-\zeta}
\end{aligned}$$

- *Real marginal costs=1/mark-up* are no longer fixed and are:

$$MC_t = \frac{P_t^W}{P_t}$$

- MS_t is a *mark-up shock* equal to $\left(\frac{1}{1-\frac{1}{\zeta}}\right)$ in the steady state.

Mark-up, Price Dispersion and Indexing

- Note that mark-up is now counter-cyclical (previously was fixed) which gives the NK model a Keynesian demand effect.
- The remaining change is that *price dispersion* Δ_t reduces output which, as shown in Appendix E, is now given by

$$
\begin{aligned}
Y_t^W &= \frac{(A_t h_t)^\alpha K_{t-1}^{1-\alpha}}{\Delta_t} \\
\Delta_t &\equiv \frac{1}{n}\sum_{j=1}^{n}(P_t(j)/P_t)^{-\zeta} = \xi \Pi_t^{\zeta}\Delta_{t-1} + (1-\xi)\left(\frac{J_t}{H_t}\right)^{-\zeta}
\end{aligned}
$$

- However Δ_t is of second order so for a first-order approximation we can put $\Delta = 1$, its steady state value.
- Easy to add *indexing* - Π_t becomes $\tilde{\Pi}_t \equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma}}$

Rotemberg Contracts

- Retail firms face quadratic price adjustment costs given by $\frac{\xi^R}{2}\left(\frac{P_{it}}{P_{it-1}}-1\right)^2 Y_t$ where parameter ξ^R measures the degree of price stickiness.
- For each producer m, given its real marginal cost common to all firms $MC_t(m)=MC_t$, the objective is at time t to choose $\{P_t(m)\}$ to maximize discounted real profits

$$
\begin{aligned}
E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \Bigg[\frac{P_{t+k}(m) Y_{t+k}(m)}{P_{t+k}} & - & MC_{t+k} \\
& - & \frac{\xi^R}{2}\left(\frac{P_{t+k(m)}}{P_{t+k-1(m)}}-1\right)^2 Y_{t+k} \Bigg]
\end{aligned}
$$

subject to (1),

- The solution to this is

$$
1-\zeta+\zeta MC_t-\xi^R(\Pi_t-1)\Pi_t+\xi^R E_t\left[\Lambda_{t,t+1}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=0
$$

Rotemberg and Cavo Contracts Compared

- Only other change is to the resource constraint:

$$\begin{aligned}\text{Rotemberg} &: \quad Y_t = C_t + G_t + I_t + \frac{1}{2}\xi^R(\Pi_t - 1)^2 Y_t \\ \text{Calvo} &: \quad Y_t = C_t + G_t + I_t\end{aligned}$$

- These are much easier to model. In some contexts (e.g., deep habit) essential.
- For zero inflation gives the same first order dynamics as Calvo (without indexing). To get this result use the linearized formes of the Phillips curve to impose:

$$\xi^R = \frac{(1-\zeta)\xi}{(1-\xi)(1-\beta\xi)}$$

Monetary Policy

- *Fischer equation* for *ex post* real interest rate

$$R_t^{ex} = \left[\frac{R_{n,t-1}}{\Pi_t}\right]$$

- Two possible forms of Taylor rule: 'implementable' and 'conventional':

$$\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \\
&+ \theta_y \log\left(\frac{Y_t}{Y}\right) + \log MPS_t \\
\textit{or} \ \log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) \\
&+ \theta_y \log\left(\frac{Y_t}{Y_t^F}\right) + \log MPS_t
\end{aligned}$$

where Y_t^F is the flexi-price level of output and MPS_t is a monetary policy shock process.

The Euler Eqn and Demand for Capital

- The Euler equation now takes the form

$$\Lambda_{C,t} = E_t \left[\frac{R_{n,t}}{\Pi_{t+1}} \Lambda_{C,t+1} \right] \Rightarrow E_t[\Lambda_{t,t+1} R^{ex}_{t+1}] = 1$$

  because now the existence of a price level variable (absent in the RBC model) creates a distinction between the ex ante and ex post real interest rate.

- Demand for capital is now given by

$$\begin{aligned} E_t[R^{ex}_{t+1}] = E_t \left[\frac{R_{n,t}}{\Pi_{t+1}} \right] &= \frac{E_t \left[(1-\alpha) \frac{P^W_{t+1}}{P_{t+1}} \frac{Y^W_{t+1}}{K_t} + (1-\delta) Q_{t+1} \right]}{Q_t} \\ &\equiv E_t[R^{ex}_{k,t+1}] \end{aligned}$$

  if firms are risk-neutral or

$$E_t[\Lambda_{t,t+1} R^{ex}_{t+1}] = \; \equiv \; E_t[\Lambda_{t,t+1} R^{ex}_{k,t+1}]$$

  if they are owned by households who are risk-averse.

Habit

- Introducing *external habit* ('keeping up with the Jones') for household j we now have

$$\begin{aligned}
\Lambda_t^j &= \frac{(C_t^j - \chi C_{t-1})^{(1-\varrho)} L_t^{\varrho})^{1-\sigma_c} - 1}{1-\sigma_c} \\
\Lambda_{C,t}^j &= (1-\varrho)(C_t^j - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)-1}((1-h_t)^{\varrho(1-\sigma_c)}) \\
\Lambda_{h,t}^j &= -\varrho(C_t^j - \chi C_{t-1})^{(1-\varrho)(1-\sigma_c)}(1-h_t)^{\varrho(1-\sigma_c)-1}
\end{aligned}$$

  where aggregate per capita consumption C_{t-1} is taken as given.
- Then in an equilibrium of identical households, $C_t^j = C_t$, $\Lambda_t^j = \Lambda_t$, $\Lambda_{C,t}^j = \Lambda_{C,t}$ and $\Lambda_{h,t}^j = \Lambda_{h,t}$
- Other formulations are *internal habit*, *deep habit* and habit also in *leisure*.

A Balanced-Positive-Growth Steady State

- Much of the literature imposes a *zero-inflation* ($\Pi = 1$) and *zero-deterministic growth* ($g_t = 0$). This is not data consistent, but we can easily set up models with a balanced-exogenous-growth positive inflation steady state.
- Considering trend growth first, write the process for A_t as

$$
\begin{aligned}
A_t &= \bar{A}_t a_t \\
\bar{A}_t &= (1+g)\bar{A}_{t-1} \exp(\epsilon_{A,t}) \\
\Rightarrow \log \bar{A}_t &= \mu + \log \bar{A}_{t-1} + \epsilon_{A,t} \\
\log a_t - \log a &= \rho_a(\log a_{t-1} - \log a) + \epsilon_{a,t}
\end{aligned}
$$

  where $\mu = \log(1+g) \approx g$ and A_t is a labour-augmenting technical progress parameter which we decompose into a temporary AR1 process, a_t and a stochastic trend, a random walk with drift, $\bar{A}_t$.
- Thus the balanced growth deterministic steady state path (bgp) is driven by labour-augmenting technical change growing at a net rate g.

Stationarizing Variables

- Now stationarize variables by defining cyclical, stationary components:

$$
\begin{aligned}
Y_t^c &\equiv \frac{Y_t}{\bar{A}_t} = (1-c)\frac{(a_t h_t)^{\alpha}\left(\frac{K_{t-1}}{\bar{A}_t}\right)^{1-\alpha}}{\Delta_t} \\
&= (1-c)\frac{(a_t h_t)^{\alpha}\left(\frac{K_{t-1}^c}{(1+g_t)}\right)^{1-\alpha}}{\Delta_t} \\
K_t^c &\equiv \frac{K_t}{\bar{A}_t}; \quad C_t^c \equiv \frac{C_t}{\bar{A}_t} \\
I_t^c &\equiv \frac{I_t}{\bar{A}_t}; \quad \left(\frac{W_t}{P_t}\right)^c \equiv \frac{W_t}{A_t P_t}
\end{aligned}
$$

for all non-stationary variables.

Stationarizing Variables (cont.)

Dynamic equation in cyclical components involving a lead or lag needs modifying as follows:

$$\Lambda^c_{C,t} \equiv \frac{\Lambda_{C,t}}{\bar{A}_t^{(1-\varrho)(1-\sigma_c)-1}} = (1-\varrho)(C^c_t - \chi C^c_{t-1}/(1+g_t))^{(1-\varrho)(1-\sigma_c)-1}$$

$$\Lambda^c_{t,t+1} = \beta(1+g_{t+1})^{(1-\varrho)(1-\sigma_c)-1}\frac{\Lambda^c_{C,t+1}}{\Lambda^c_{C,t}}$$

$$K^c_t = (1-\delta)\frac{K^c_{t-1}}{1+g_t} + (1-S(X^c_t))I^c_t$$

$$X^c_t = (1+g_t)\frac{I^c_t}{I^c_{t-1}}$$

and similarly for H^c_t and J^c_t, where

$$g_t \equiv \frac{(\bar{A}_t - \bar{A}_{t-1})}{\bar{A}_{t-1}} = (1+g)\exp(\epsilon_{A,t}) - 1$$

is the stochastic trend.

The Observation (Measurement) Equation

- Thus we have for output

$$\log Y_t \equiv \underbrace{\log Y_t^c}_{\text{Stationarized DSGE model}} + \log \bar{A}_t \qquad (2)$$

  and similarly for all the remaining non-stationary variables.
- This is our *theoretical DSGE model* in stationary form in which output, consumption, investment and the real wage all have a common stochastic trend.
- Taking first differences

$$\Delta \log Y_t = \Delta \log Y_t^c + \Delta \log \bar{A}_t = \Delta \log Y_t^c + \log(1+g) + \epsilon_{A,t}$$

- Defining $y_t^c \equiv \log \frac{Y_t^c}{Y^c}$ in deviation form (defined in dynare for irfs) and approximating $\log(1+g) \simeq g$ we arrive at the *measurement equation* used in estimation

$$\Delta \log Y_t^{obs} = \Delta y_t^c + g + \epsilon_{A,t}$$

- Can add a *measurement error* in output, $\epsilon_{Y,t}$ giving:

$$\Delta \log Y_t^{obs} = \Delta y_t^c + g + \epsilon_{A,t} + \Delta \epsilon_{Y,t}$$

Remaining Non-Zero-Inflation Steady State

- The steady state for the rest of the system is the same as for the RBC model except for the following:

$$
\begin{aligned}
(R^{ex})^c &= \frac{(1+g)^{1+(\sigma_c-1)(1-\varrho)}}{\beta} = \frac{R_n^c}{\Pi} \\
\Lambda^c &= \frac{1}{(R^{ex})^c}; \qquad I^c = \frac{(\delta+g)K^c}{1+g}; \qquad X^c = 1+g \\
\frac{J}{H} &= \left(\frac{1-\xi\Pi^{\zeta-1}}{1-\xi}\right)^{\frac{1}{1-\zeta}} \\
MC^c &= \left(1-\frac{1}{\zeta}\right)\frac{J(1-\Lambda^c\xi\Pi^{\zeta})}{H(1-\Lambda^c\xi\Pi^{\zeta-1})} \qquad (3)
\end{aligned}
$$

where R^c and R_n are the real and nominal steady state interest rates and Π is inflation.

- For Rotemberg price contracts (3) is replaced with

$$
MC^c = 1 - \frac{1-(1-\xi(\Pi-1)\Pi(1-\Lambda^c(1+g))}{\zeta}
$$

The Smets-Wouters Model and Beyond

- [Smets and Wouters(2007)] and [Smets and Wouters(2003)] have two further ingredients: capital utilization and sticky wages. Regarding the former, owners of physical capital can control the intensity at which capital is utilized in production.
- Sticky wages can be modelled in an analogous fashion to sticky prices with Rotemberg or Calvo contracts, expressing wage dynamics as difference equations using the same technique we employed for price dynamics (see [Cantore *et al.*(2012)] which extends the Smets-Wouters set-up to CES wholesale production.)
- [Schmitt-Grohe and Uribe(2011)] adds a further non-stationary shock to the technology converting homogeneous output into investment leading to a relative price of investment trend. The TFP-investment trend shock then accounts for most of business cycle fluctuations.
- [Lafourcade and Wind(2012)] carries out a similar analysis for a open-economy NK model and introduce a third stochastic trend to labour supply. See also presentation by Kai Liu.

Dynare Code I

- Dynare mod files **NK_RES_Course.mod** and **NK_Rotemberg_RES_Course.mod** for Calvo and Rotemberg contracts respectively, are to be found in subfolder **NK**.
- Both allow options for a conventional Taylor or implementable rule. The parameter ϱ in the model is calibrated to hit hours worked $h = 0.35$ (as for the RBC model).
- In addition we now calibrate the discount factor β, to target the nominal interest rate, which in turn depends on an observed inflation rate.
- Matlab files to compare models and do *stability-indeterminacy analysis* are also included and for the latter some lines of code are commented out.

Dynare Code II

- Since we are using the steady state to calibrate β and ϱ, in order to compare *different steady states*, for example with different long-run inflation rates, it is necessary for calibrated parameters to remain constant. See dynare mod files **NK_free_parameters_RES_Course.mod** and **NK__free_parameters_Rotemberg_RES_Course.mod**.
- It is instructive to compare the NK and RBC models (with investment costs) using **graphs_rbc_nk**.

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