Macro 1: Numerical Methods Assignment 2

Precautionary Savings

Assignment Due: 24/11/2013

Question 1

- Consider an individual who lives two periods. In the first period he has income A, and in the second period his income is stochastic and denoted by y
- His income can only take two values; either $y = x > 0$ (with probability $\frac{1}{2}$) or $y = -x$ (with probability $\frac{1}{2}$)
- The discount rate and interest rate are both exogenous and equal to zero
- The individual chooses consumption in period 1 c_1 knowing A, but not the realisation of y [let c_2 denote consumption in period 2]
- It is always the case that $A - x > 0$
- Let E denote the expectations operator
- The objective function of the individual is $V = u(c_1) + Eu(c_2)$
- You can assume that $u(.)$ is concave and satisfies the Inada conditions

(a)

Write out the individual's inter-temporal budget constraint. **(5 marks)**

(b)

Write out the Euler Equation. **(5 marks)**

(c)

If $x = 0$, what is c_1? **(2 marks)**

(d)

If u(c) is a quadratic utility function, how does consumption in the first period respond to setting $x \neq 0$. Explain your answer. **(3 marks)**

(e)

Now consider the utility function:

$$u(c) = 1 - e^{-\lambda c} \tag{1}$$

Find $u'(c)$, and define functions in an python notebook of the utility function and the marginal utility. **(5 marks)**

(f)

Find $u'''(c)$, and the Coefficients of Relative and Absolute Risk Aversion (possibly as functions of c). What type of function is this? **(5 marks)**

(g)

Plot $u'(c)$, with multiple values of λ on the same figure, and interpret. **(5 marks)**

(h)

For the case where $x \neq 0$, find c_1. **(10 marks)**

(i)

Now define D as precautionary savings. i.e. Amount of savings made with risky income minus the level of savings made without risk.

Analyse how D responds to a change in λ, A or x, where a rise in x is equivalent to an increase in the variance of income.

Hint: You could possibly create a function in your ipython notebook called $D(A,x)$, graph this in (x,y) space, and anaylse for different values of the inputs and parameters, keeping the others fixed. **(15 marks)**

Question 2

Now consider the utility function:

$$u(c) = \frac{c^{1-\lambda} - 1}{1-\lambda} \tag{2}$$

(a)

Repeat parts (e) to (i) of question 1 for this new utility function. **(35 marks)**

(b)

Compare your results to part(i) of question 1 to your equivalent results for the new utility function. [Hint: It is not necessary to plot the two precautionary savings functions for the different utility functions to compare]**(10 marks)**