DSGE Modelling and Financial Frictions
Data Consistent Estimation of the NK Model

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Data Consistent Estimation of the NK Model

- We can estimate the NK model using first differences with measurement equations

$$
\begin{aligned}
\Delta \log Y_t^{ob} &= \Delta y_t^c + g + \epsilon_{A,t} + \Delta \epsilon_{Y,t} \\
\log \Pi_t^{ob} &= \pi_t^c + \log \Pi + \epsilon_{\Pi,t} = \pi_t + \log \Pi + \epsilon_{\Pi,t} \\
\log R_{n,t}^{ob} &= r_{n,t}^c + \log R_n + \epsilon_{R_n,t} = r_{n,t} + \log R_n + \epsilon_{R_n,t}
\end{aligned}
$$

- where ϵ_Y, etc incorporate measurement errors and $\epsilon_{A,t}$ is the trend shock.
- parameters g, Π and R_n *must have the same values that appear in the steady state of the cyclical DSGE model.*

Data Consistent Estimation of the NK Model

- Same measurement equations used in [Smets and Wouters(2007)] with the addition of measurement errors.
- The advantage of estimating the model using growth variables is that the measurements, like the variables in the model, are non-trended, and one can avoid potential difficulties in the likelihood function due to unit roots.
- The usual procedure is to linearize around a zero-inflation zero-growth steady state but if we do so our theoretical model is inconsistent with the measurement equation. [Smets and Wouters(2003)] do exactly the same thing and [Smets and Wouters(2007)] linearize around a positive growth path, but still retain zero inflation in their steady state.
- We know from the work of [Ascari and Ropele(2007a)] and [Ascari and Ropele(2007b)] that this omission may be important.

The impact of positive inflation in Steady State

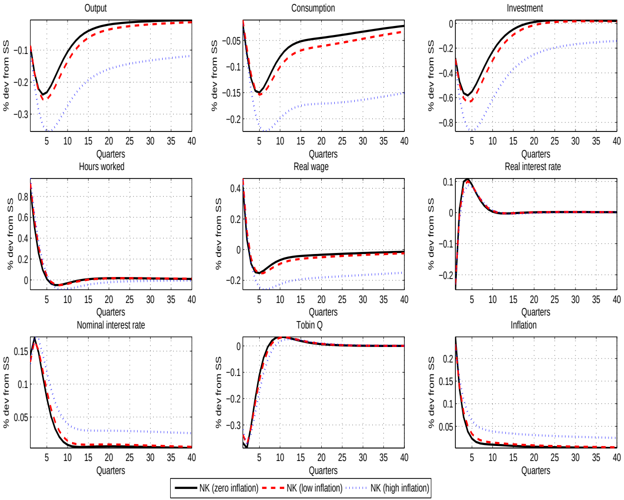

Figure: IRFs for Different Steady State Inflation Rates (low inflation rate= 2.5% per annum and a high rate= 5% per annum.) Technology Shock

The impact of positive inflation in Steady State

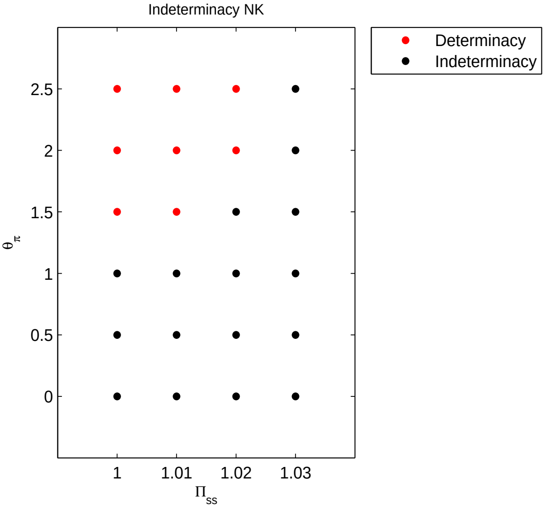

Figure: Indeterminacy Region with Positive Inflation, with ($\theta_y = 0$)and ($\alpha_r = 0$).

Preparing the Data

- We use a subsample of the same data set as in [Smets and Wouters(2007)] in first difference at quarterly frequency.
- The observable variables are the log difference of real GDP, the log difference of the GDP deflator and the federal funds rate. All series are seasonally adjusted.
- The time sample is 1984Q2-2008Q2.
- In Appendix D of the notes we provide all the details about the data sources and transformations.
- Since the variables in the model state space are measured as deviations from a constant steady state, we take the first difference of the real GDP. The data are stored in **us_data.mat**. Inflation and nominal interest rates are used as they are in percentage terms.

Measurement Equations

- Given that the data are in percentage we need to transform the measurement equations presented above to account for this.
- The measurement equations used below will then be:

$$
\begin{aligned}
Y_t^{obs} &= \Delta y_t^c + g^* + \epsilon_{A,t} + \Delta\epsilon_{Y,t} \\
\Pi_t^{obs} &= \pi_t^c + \Pi^* + \epsilon_{\Pi,t} \\
R_{n,t}^{obs} &= r_{n,t}^c + R_n^* + \epsilon_{R_n,t}
\end{aligned}
$$

- where $g^* = 100 * g$, $\Pi^* = (\Pi - 1) * 100$ and $R_n^* = (R_n - 1) * 100$.
- Those transformations of the constants in the measurement equations take into account the fact that the data are in percentage (and net for inflation and interest rate) while we have proportions (and gross rates) in the model.

First Stage Estimation Results

- Now we want to test the validity and desirability of our proposed estimation technique using positive growth and inflation in steady state and the introduction of a stochastic trend shock and measurement errors.
- We also want to compare with the *data inconsistent* estimation procedure used by [Smets and Wouters(2007)].
- For now we confine ourselves to the first stage Bayesian maximum-likelihood estimation of modes of parameters reporting on the approximate computation of the marginal likelihoods.

First Stage Estimation Results

The various exercises performed below are:

1. **Model A** (NK as in SW03). We drop trend shock and measurement errors ($\epsilon_{A,t} = \Delta\epsilon_{Y,t} = \cdot\cdot = 0$). g^*, Π^* and R_n^* jointly estimated with the rest of the parameters but use a steady state with *data-inconsistent* zero inflation and growth.
2. **Model B**. As Model A but now the steady state of inflation and growth rate are non zero and equal to data averages. g^*, Π^* and R_n^* are estimated as in Model A together with the structural parameters of the model.
3. **Model C**. As Model B with positive inflation and growth in steady state but g^*, Π^* and R_n^* which are estimated using the average of the observables prior to the structural parameters estimation.
4. **Model D**. Add trend shock
5. **Model E**. Add measurement error.

First Stage Estimation Results

Parameters		**Models**				
		A	B	C	D	E
AR TFP	ρ^A	0.9091	0.9012	0.9028	0.8473	0.8474
AR gov spe	ρ^G	0.9869	0.9854	0.9841	**0.5112**	**0.5111**
AR mark-up	ρ^{MS}	0.4997	0.4998	0.4997	0.4998	0.4999
inv adj cost	ϕ^X	4.4396	5.0158	5.0505	5.0539	5.0538
preferences	σ_c	1.3887	1.3817	1.3878	1.632	1.632
habits	χ	0.9568	0.9519	0.9521	**0.8718**	**0.8718**
calvo	ξ	0.6532	0.6465	0.6568	**0.7484**	**0.7484**
indexation	γ_p	0.1911	0.1987	0.1942	0.1737	0.1737
infl weight in MPR	α_π	2.3256	2.3148	2.3107	**2.1509**	**2.1509**
interest rate smoothing	α_r	0.8277	0.8233	0.8237	0.8288	0.8288
output gap in MPR	α_y	0.1285	0.1273	0.1296	0.1294	0.1294
average inflation	conspie	0.6272	0.6227	-	-	-
average growth rate	trend	0.4583	0.4646	-	-	-
average interest rate	consr	1.2088	1.2031	-	-	-
Shocks						
TFP	ϵ_a	0.8626	0.8608	0.899	**1.2053**	**1.2052**
Gov sp.	ϵ_G	2.6991	2.7701	2.7797	**0.2277**	**0.2277**
Mon. Pol	ϵ_{MPS}	0.1429	0.1444	0.1456	0.1429	0.1429
Mark-up	ϵ_{MS}	0.046	0.046	0.0461	0.0461	0.0461
Stoch. Trend	ϵ_A	-	-	-	**0.9815**	**0.9814**
Y mes err	ϵ_Y	-	-	-	-	0.0461
Rn mes err	ϵ_R	-	-	-	-	0.0461
Π mes err	ϵ_Π	-	-	-	-	0.0461
	ML	-52.069756	-55.268248	-54.691324	**-44.07426**	-45.208858

Table: Estimation results

First Stage Estimation Results

- Model D performs better in terms of Marginal Likelihood. This means that the introduction of the stochastic trend shock ϵ_A helps in fitting the data much better while the introduction of measurement errors does not make much difference. Actually we can see how the estimated parameters and structural shock are almost identical under models specifications D and E.
- Regarding the issue of estimating the constant in the measurement equations either together with the structural parameters or separately by calibrating them to the average of the observables it seems that either way does not affect much the fit of the data (comparison between ML of models A, B and C) although model A is 'slightly' preferred. The three models produce similar results in terms of ML and parameters estimates.
- Following this exercise in the rest of the estimation exercises we drop measurement errors from the measurement equations but we keep positive inflation and growth in steady state and the stochastic trend shock.

First Stage Estimation Results

Parameters		prior	mean	stdev
AR TFP	ρ^A	beta	0.5	0.2
AR gov spe	ρ^G	beta	0.5	0.2
AR mark-up	ρ^{MS}	beta	0.5	0.2
inv adj cost	ϕ^X	norm	2	1.5
preferences	σ_c	norm	1.5	0.375
habits	χ	beta	0.7	0.1
calvo	ξ	beta	0.5	0.1
indexation	γ_p	beta	0.5	0.15
infl weight in MPR	α_π	norm	2	0.25
interest rate smoothing	α_r	beta	0.75	0.1
output gap in MPR	α_y	norm	0.125	0.05
average inflation	conspie	gamm	0.63	0.1
average growth rate	trend	norm	0.46	0.1
average interest rate	consr	norm	1.314	0.1
Shocks				
TFP	ϵ_a	invg	0.1	2
Gov sp.	ϵ_G	invg	0.5	2
Mon. Pol	ϵ_{MPS}	invg	0.1	2
Mark-up	ϵ_{MS}	invg	0.1	2
Stoch. Trend	ϵ_A	invg	0.1	2
Y mes err	ϵ_Y	invg	0.1	2
Rn mes err	ϵ_R	invg	0.1	2
Π mes err	ϵ_Π	invg	0.1	2

Table: Priors used in estimation

Ascari, G. and Ropele, T. (2007a).
Optimal Monetary Policy under Low Trend Inflation.
Journal of Monetary Economics, **54**(8), 2568–2583.

Ascari, G. and Ropele, T. (2007b).
Trend Inflation, Taylor Principle and Indeterminacy.
Kiel Working Paper 1332.

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An estimated Stochastic Dynamic General Equilibrium Model of the Euro Area.
Journal of the European Economic Association, **1(5)**, 1123–1175.

Smets, F. and Wouters, R. (2007).
Shocks and Frictions in US business cycles: A Bayesian DSGE approach.
American Economic Review, **97**(3), 586–606.