DSGE Modelling and Financial Frictions

Bayesian Estimation of the NK and the three banking models

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Full Estimation of the NK Model

- We present the full Bayesian estimation of model D presented on day 1.
- NK model with positive inflation and growth in steady state with a stochastic trend shock.
- the measurement equations used for this model are the following:

$$\begin{aligned} Y_t^{obs} &= \Delta y_t^c + g^* + \epsilon_{A,t} \\ \Pi_t^{obs} &= \pi_t^c + \Pi^* \\ R_{n,t}^{obs} &= r_{n,t}^c + R_n^* \end{aligned}$$

- g^*, Π^* and R_n^* which are estimated using the average of the observables prior to the structural parameters estimation.

Full Estimation of the NK Model

- The mod files for this analysis are:
  - **NK_RES_Course_est_PI.mod** (provides results from posterior optimization)
  - **NK_RES_Course_est_PI_MCMC.mod** (provides posterior distribution from posterior simulation - MCMC).
- In Dynare, observed variables are declared after *varobs* (i.e. *varobs VARIABLE_NAME...;*) and must be available in the data file.
- Estimated parameters are declared in the *estim_params; ... end;* block. For each estimated parameter, declare the initial value and, optionally, a lower and upper bound for the ML estimation. If their prior distributions are further declared, Dynare chooses to perform the Bayesian estimation.
- Computing the estimation is triggered by the keyword *estimation* and the only required option in brackets after *estimation* is *datafile=FILENAME*.

Full Estimation of the NK Model

- *first_obs* = # sets the starting point of the data to be used in the estimation.
- *presample* = # sets the number of observation to be used to initialize the Kalman filter.
- *prefilter=0 or 1.* tells Dynare whether or not to demean the data (0 default; 1 demean).

Full Estimation of the NK Model

- A few structural parameters are kept fixed or calibrated based on some parameters being estimated in the estimation procedure

Calibrated parameter	Symbol	Value
Nominal Interest rate	$\bar{R}_n$	1.013142
Depreciation rate	δ	0.025
Growth rate	g	0.0046
Inflation rate	Π	1.0063
Hours	h	0.35
Substitution elasticity of goods	ζ	7
Fixed cost	c	$\frac{1}{\zeta} = 0.1429$
Implied steady state relationship		
Government expenditure-output ratio	g_y	0.2
Preference parameter	ϱ	$\frac{1-h}{1+h(c_y(1-\chi)/\alpha-1)}$
discount rate	β	$R^{ex} = \frac{\bar{R}_n}{\Pi}$

Table: Calibrated Parameters

Full Estimation of the NK Model

- The choice of priors for the estimated parameters is usually determined by the theoretical implications of the model and evidence from previous studies.
- In general, inverse gamma distributions are used as priors when non-negativity constraints are necessary, and beta distributions for fractions or probabilities. Normal distributions are used when more informative priors seem to be necessary.

Priors used in Estimation

Parameters		prior	mean	stdev
AR TFP	ρ^A	beta	0.5	0.2
AR gov spe	ρ^G	beta	0.5	0.2
AR mark-up	ρ^{MS}	beta	0.5	0.2
inv adj cost	ϕ^X	norm	2	1.5
preferences	σ_c	norm	1.5	0.375
habits	χ	beta	0.7	0.1
calvo	ξ	beta	0.5	0.1
indexation	γ_p	beta	0.5	0.15
infl weight in MPR	α_π	norm	2	0.25
interest rate smoothing	α_r	beta	0.75	0.1
output gap in MPR	α_y	norm	0.125	0.05
average inflation	conspie	gamm	0.63	0.1
average growth rate	trend	norm	0.46	0.1
average interest rate	consr	norm	1.314	0.1
Shocks				
TFP	ϵ_a	invg	0.1	2
Gov sp.	ϵ_G	invg	0.5	2
Mon. Pol	ϵ_{MPS}	invg	0.1	2
Mark-up	ϵ_{MS}	invg	0.1	2
Stoch. Trend	ϵ_A	invg	0.1	2
Y mes err	ϵ_Y	invg	0.1	2
Rn mes err	ϵ_R	invg	0.1	2
Π mes err	ϵ_Π	invg	0.1	2

Table: Priors used in estimation

Estimation options

- In order to avoid stochastic singularity when evaluating the likelihood function, Dynare requires at least as many shocks or measurement errors in the models as observable variables (i.e. requires that the covariance matrix of endogenous variables is nonsingular).
- In the process of parameter estimation, the mode of the posterior is first estimated using Chris Sim's csminwel after the models' log-prior densities and log-likelihood functions have been obtained by running the Kalman recursion and maximized (this is triggered by the option *mode_compute=4*).
- Then a sample from the posterior distribution is obtained with the Metropolis-Hastings (MH) algorithm using the inverse Hessian at the estimated posterior mode as the covariance matrix of the jumping distribution.
- The covariance matrix needs to be adjusted in order to obtain reasonable acceptance rates. Thus, for example, the scale used for the jumping distribution in the MH is set to 0.40 (option *mh_jscale=0.40*), allowing good acceptance rates (around 20%-30%).

Estimation options

- Two parallel Markov chains of 150000 runs each are run from the posterior kernel for the MH (*mh_replic=INTEGER*: sets the number of draws.
- The first 20% of iterations (initial burn-in period) are discarded in order to remove any dependence of the chain from its starting values (use *mh_drop*: sets the percentage of discarded draws).
- The estimation outputs report the Bayesian inference which summarizes the prior distribution - posterior mean and 95% confident interval. The marginal data density of the model is computed using [Geweke(1999)] modified harmonic-mean estimator.

Estimation Results

ESTIMATION RESULTS

Log data density is -44.294618.

parameters

	prior mean	post. mean	conf. interval		prior	pstdev
rhoA	0.500	0.8315	0.7624	0.9031	beta	0.2000
rhoG	0.500	0.5243	0.1892	0.8503	beta	0.2000
rhoMS	0.500	0.5051	0.1690	0.8323	beta	0.2000
phiX	2.000	5.0800	3.3983	6.7202	norm	1.5000
sigma_c	1.500	1.6177	1.0014	2.2327	norm	0.3750
hab	0.700	0.8424	0.7351	0.9502	beta	0.1000
xi	0.500	0.7384	0.6640	0.8173	beta	0.1000
gamp	0.500	0.2277	0.0726	0.3813	beta	0.1500
alpha_pie	2.000	2.1697	1.8039	2.5367	norm	0.2500
alpha_r	0.750	0.8241	0.7873	0.8624	beta	0.1000
alpha_y	0.125	0.1291	0.0506	0.2095	norm	0.0500

standard deviation of shocks

	prior mean	post. mean	conf. interval		prior	pstdev
epsA	0.100	1.3174	0.7731	1.8505	invg	2.0000
epsG	0.500	0.3453	0.1236	0.5849	invg	2.0000
epsMPS	0.100	0.1474	0.1254	0.1689	invg	2.0000
epsMS	0.100	0.1895	0.0198	0.5997	invg	2.0000
epsAtrend	0.100	0.9807	0.8537	1.1081	invg	2.0000

Priors and Posteriors

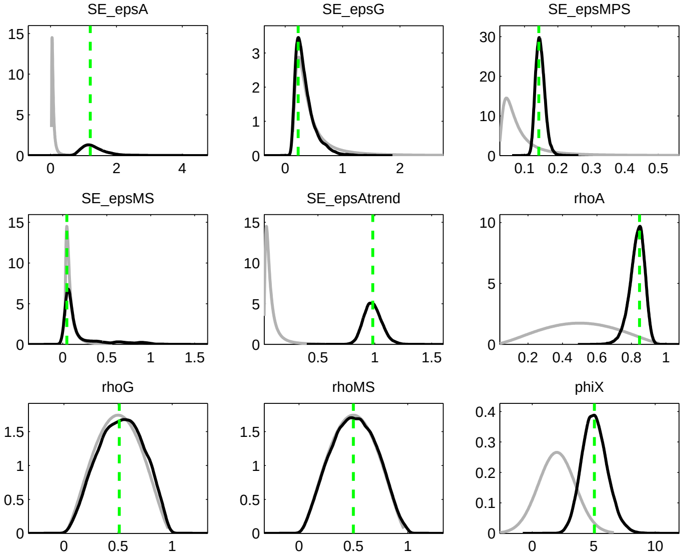

Figure: NK estimation - priors and posteriors

Priors and Posteriors

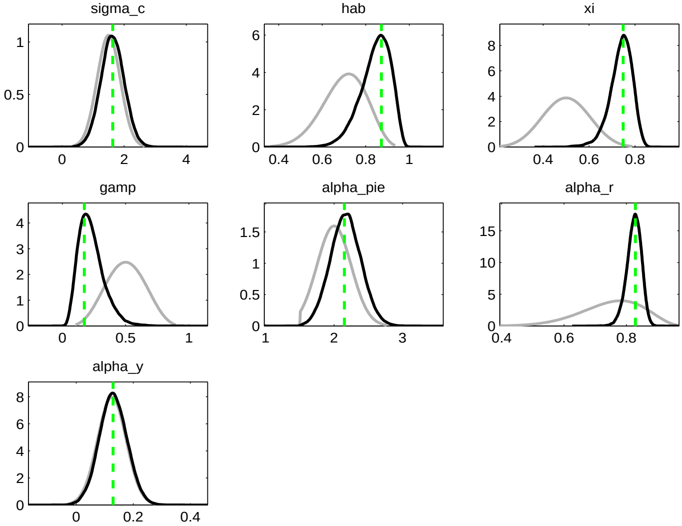

Figure: NK estimation - priors and posteriors

Second Moments Comparison

- To further evaluate the absolute performance of one particular model against data, one can compare the model's implied characteristics with those of the actual data.
- Hence in this section we carry out a traditional RBC second moments comparison.
- For the simulation and computation of moments, Dynare assumes that the shocks follow a normal distribution.
- we compute these model-implied statistics by solving the model at the posterior means obtained from estimation.
- The results of the model's second moments are compared with the second moments in the actual data to evaluate the model empirical performance.
- the command used straight after the estimation is *stoch_simul(irf=20,ar=10,conditional_variance_decomposition=[1 4 10 100]) dy pinfobs robs;*

Second Moments Comparison

Standard Deviation			
Model	Output	Inflation	Interest rate
Data	0.5432	0.2392	0.5952
NK Model	0.5935	0.2886	0.3050
Cross-correlation with Output			
Data	1.0000	-0.2013	0.1140
NK Model	1.0000	-0.2430	-0.2396
Autocorrelations (Order=1)			
Data	0.1526	0.5364	0.9462
NK Model	0.2179	0.6446	0.8670

Table: Selected Second Moments of the Model Variants

Autocorrelations

- We have so far considered autocorrelation only up to order 1. To further illustrate how the estimated models capture the data statistics and persistence in particular, we now plot the autocorrelations up to order 10 of the actual data and those of the endogenous variables generated by the model.

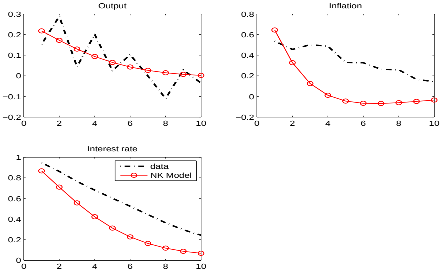

Figure: Autocorrelations of Observables in the Actual Data and in the Estimated NK Model

Variance Decomposition

VARIANCE DECOMPOSITION (in percent)

	epsA	epsG	epsMS	epsMPS	epsAtrend
dy	10.45	0.79	0.00	0.68	88.08
pinfobs	75.27	0.05	0.02	21.51	3.16
robs	74.25	0.02	0.01	17.86	7.86

- This section examines the contribution of each shock to the overall variance of each observable.
- Dynare provide the variance decomposition and the conditional variance decomposition for # periods ahead specified by the user. Here for brevity we only show the first.

Empirical Comparison of GK, BGG and KM

- In this section we compare the three models with financial frictions presented above form an empirical point of view.
- With respect to the estimation of the NK model just presented here we introduce data for the nominal rental rate of capital as in [Christiano *et al.*(2010)] and compare the banking models by adding this extra observable in the information set for estimation.
- The priors used in the estimations are exactly the same as the one used for the NK.
- As for the NK estimation, we present results using 2 MCMC chains of 150000 draws each.

Empirical Comparison of GK, BGG and KM

Parameters	Models		
	GK	BGG	KM
ρ_A	0.6839	0.7593	0.8325
ρ_G	0.9299	0.9478	0.6747
ρ_{MS}	0.5634	0.5228	0.5476
ρ_ψ	0.9540	0.9667	0.9183
ϕ_X	2.2024	**3.6473**	**0.3290**
σ_c	2.3966	**1.9606**	**0.7400**
χ	0.7738	**0.8806**	0.7073
ξ	0.3350	0.3655	**0.1984**
γ_p	0.3699	0.3428	0.3986
α_π	2.7220	2.4866	2.8692
α_r	0.6487	0.6556	0.4492
α_y	0.1520	0.1840	**0.0603**
Shocks			
ϵ_a	0.0979	0.1777	**0.4952**
ϵ_G	1.6710	2.1118	1.5062
ϵ_{MPS}	0.2525	0.2266	**0.4052**
ϵ_{MS}	0.0833	0.0746	0.0647
ϵ_A	0.6807	0.6503	0.8814
ϵ_ψ	0.0665	0.0650	**0.1357**
ML	-127.338556	-93.612833	-251.341045

Table: Estimation Results using financial data

Model Comparison

- Bayesian inference also allows a framework for comparing alternative and potentially misspecified models based on their marginal likelihood.
- For a given model $m_i \in M$ and common dataset, the latter is obtained by integrating out vector θ,

$$L(y|m_i) = \int_{\Theta} L(y|\theta, m_i)\, p(\theta|m_i)\, d\theta$$

where $p_i(\theta|m_i)$ is the prior density for model m_i, and $L(y|m_i)$ is the data density for model m_i given parameter vector θ.

Model Comparison

- To compare models (say, m_i and m_j) we calculate the posterior odds ratio which is the ratio of their posterior model probabilities (or Bayes Factor when the prior odds ratio, $\frac{p(m_i)}{p(m_j)}$, is set to unity):

$$PO_{i,j} = \frac{p(m_i|y)}{p(m_j|y)} = \frac{L(y|m_i)p(m_i)}{L(y|m_j)p(m_j)} \quad (1)$$

$$BF_{i,j} = \frac{L(y|m_i)}{L(y|m_j)} = \frac{\exp(LL(y|m_i))}{\exp(LL(y|m_j))} \quad (2)$$

  in terms of the log-likelihoods.

- Components (1) and (2) provide a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. Such comparisons are important in the assessment of rival models.

Model Comparison

- Given Bayes factors we can easily compute the model probabilities $p_1, p_2, \cdots p_n$ for n models. Since $\sum_{i=1}^{n} p_i = 1$ we have that

$$\frac{1}{p_1} = \sum_{i=2}^{n} BF_{i,1}$$

  from which p_1 is obtained.

- Then $p_i = p_1 BF(i, 1)$ gives the remaining model probabilities.
- The MATLAB programme, **model_odds.m**, computes these probabilities give the the log-likelihood values from the competing models.

Model Comparison

- To interpret the marginal log-likelihood (LL) differences we appeal to [Jeffries(1996)] who judges that a BF of 3-10 is "slight evidence" in favour of model *i* over *j*. This corresponds to a LL difference in the range [ln 3, ln 10]= [1.10,2.30]. A BF of 10-100 or a LL range of [2.30, 4.61] is "strong to very strong evidence"; a BF over 100 (LL over 4.61) is "decisive evidence".

	GK	BGG	KM
LLs	-127.338556	-93.612833	-251.341045
prob.	0.0000	1.0000	0.0000

Table: Marginal Log-likelihood Values and Posterior Model Odds Across Model Variants

Second Moments

Model	Output	Inflation	Interest rate	Baa
Standard Deviation				
Data	0.5432	0.2392	0.5952	0.5199
GK Model	0.5921	0.2416	0.3059	0.9007
BGG Model	0.6378	0.2704	0.3126	0.6862
KM Model	0.6591	0.2643	0.3024	3.4386
Cross-correlation with Output				
Data	1.0000	-0.2013	0.1140	0.1352
GK Model	1.0000	-0.1448	-0.1842	0.4371
BGG Model	1.000	-0.1353	-0.1410	0.5166
KM Model	1.0000	-0.0472	-0.2113	0.4124
Autocorrelations (Order=1)				
Data	0.1526	0.5364	0.9462	0.9517
GK Model	0.1690	0.4122	0.8122	-0.0908
BGG Model	0.1743	0.5481	0.8509	0.0705
KM Model	0.1191	0.4112	0.7720	-0.4970

Table: Selected Second Moments of the Model Variants

Autocorrelations

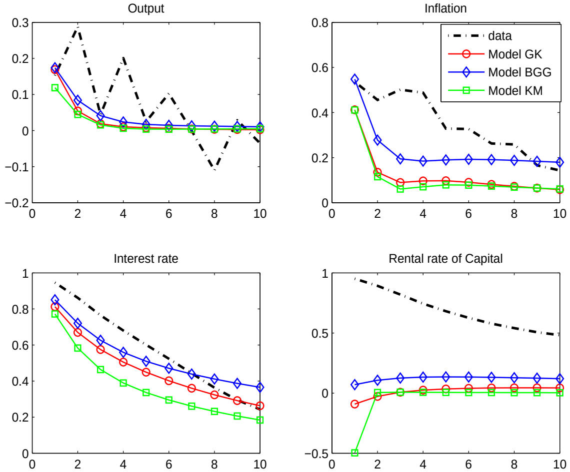

Figure: Autocorrelations of Observables in the Actual Data and in the three Estimated models

Variance Decomposition

VARIANCE DECOMPOSITION (in percent) GK Model

	epsA	epsG	epsMS	epsMPS	epsAtrend	epscapqual
dy	6.35	0.60	0.03	1.67	90.26	1.09
pinfobs	30.54	0.91	0.17	50.53	6.34	11.51
robs	28.29	0.74	0.11	11.01	15.03	44.82
rkn_obs	23.56	0.33	0.06	37.59	9.21	29.26

VARIANCE DECOMPOSITION (in percent) BGG Model

	epsA	epsG	epsMS	epsMPS	epsAtrend	epscapqual
dy	7.70	0.57	0.01	2.58	86.65	2.49
pinfobs	39.84	0.33	0.08	37.03	6.02	16.70
robs	40.53	0.24	0.05	10.27	13.21	35.70
rkn_obs	20.00	0.07	0.03	31.77	11.09	37.04

VARIANCE DECOMPOSITION (in percent) KM Model

	epsA	epsG	epsMS	epsMPS	epsAtrend	epscapqual
dy	9.78	0.50	0.04	6.00	80.34	3.34
pinfobs	25.93	0.32	0.09	58.04	5.37	10.25
robs	48.10	0.46	0.09	8.15	11.49	31.71
rkn_obs	11.97	0.00	0.12	51.41	3.67	32.82

Bayesian IRFs

- In this section we present Bayesian IRFs for the three models computed at the estimated posterior means of the structural parameters and shocks.

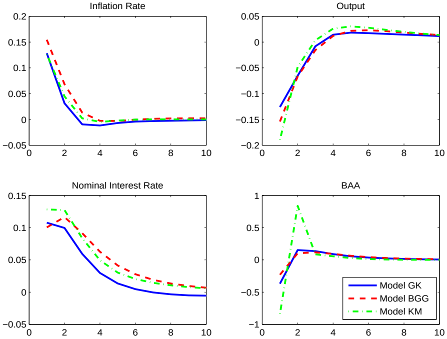

Figure: Bayesian IRF - technology shock

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Working Paper Series 1192, European Central Bank.

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