DSGE Modelling and Financial Frictions

Cristiano Cantore, University of Surrey
Matteo Ghilardi, University of Surrey
Paul Levine, University of Surrey
Joseph Pearlman, City University

April 13, 2013

Four Policy Regimes

- Rational expectations introduces a *time-inconsistency problem*: with the mere passage of time policies that were initially optimal become sub-optimal
- Rules considered depend on whether the policymaker can *commit*, or she exercises *discretion* and engages in period-by-period optimization.
- With commitment the welfare-optimal policy is the solution to the *Ramsey problem*.
- This is not the same thing as the *social planner's problem* in any model with some market failure.
- In the absence of commitment the policymaker optimizes period-by-period - the *discretionary solution*. This can be very sub-optimal.
- Even with commitment the policymaker may be constrained to *simple rules* (e.g., Taylor-type rules)
- Rationale for simplicity: transparency, information available and ease of implementation

The Ramsey Problem: the Constraints

- Our models are all special cases of the following general setup recognized by Dynare in non-linear form

$$
\begin{aligned}
\mathrm{Z}_t &= J(\mathrm{Z}_{t-1}, \mathrm{X}_t, \mathrm{w}_t, \epsilon_t) && (1)\\
E_t \mathrm{X}_{t+1} &= K(\mathrm{Z}_t, \mathrm{X}_t, \mathrm{w}_t) && (2)
\end{aligned}
$$

  where $\mathrm{Z}_{t-1}, \mathrm{X}_t$ are $(n-m)\times 1$ and $m \times 1$ vectors of backward and forward-looking variables, respectively, ϵ_t is a $\ell \times 1$ i.i.d shock variable and w_t is an $r \times 1$ vector of instruments.
- Now define

$$
\mathrm{y}_t \equiv \begin{bmatrix} \mathrm{Z}_t \\ \mathrm{X}_t \end{bmatrix}
$$

- Then, as in Dynare User Guide, chapter 7, (1) and (2) can be written

$$
\begin{aligned}
E_t[f(\mathrm{y}_t, \mathrm{y}_{t+1}, \mathrm{y}_{t-1}, \mathrm{w}_t, \epsilon_t)] &= 0 && (3)\\
E_{t-1}[\epsilon_t] &= 0\\
E_{t-1}[\epsilon_t \epsilon_t'] &= \Sigma_\epsilon
\end{aligned}
$$

The Ramsey Problem: FOC

- The general problem is to maximize at time 0, $\Omega_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t u(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t)\right]$ subject to (3) given initial values Z_0. To carry out this problem write the Lagrangian

$$L = E_0 \left[\sum \beta^t [u(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t) + \lambda_t^T f(\mathrm{y}_t, \mathrm{y}_{t+1}, \mathrm{y}_{t-1}, \mathrm{w}_t, \epsilon_t] \right]$$

  where λ_t is a column vector of multipliers associated with the n constraints defining the model.

- First-order conditions are given by

$$E_0 \left[\frac{\partial L}{\partial \mathrm{w}_t} \right] = E_0 \left[u_3(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t) + \lambda_t^T f_4(\mathrm{y}_t, \mathrm{y}_{t+1}, \mathrm{y}_{t-1}, \mathrm{w}_t, \epsilon_t) \right] = 0$$

$$E_0 \left[\frac{\partial L}{\partial \mathrm{y}_t} \right] = E_0[u_1(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t) + \beta u_2(\mathrm{y}_{t+1}, \mathrm{y}_t, \mathrm{w}_{t+1}) + \lambda_t^T f_1(\mathrm{y}_t, \mathrm{y}_{t+1}, \mathrm{y}_{t-1}, \mathrm{w}_t, \epsilon_t) + \frac{1}{\beta} \lambda_{t-1}^T f_2(\mathrm{y}_{t-1}, \mathrm{y}_t, \mathrm{y}_{t-2}, \mathrm{w}_{t-1}, \epsilon_{t-1}) + \beta \lambda_{t+1}^T f_3(\mathrm{y}_{t+1}, \mathrm{y}_{t+2}, \mathrm{y}_t, \mathrm{w}_{t+1}, \epsilon_{t+1})] = 0$$

  where the subscripts in $\{u_i, f_i\}$ refer to the partial derivatives of the ith variable in u, f.

Further Optimality Conditions

- Now partition $\lambda_t = [\lambda_{1,t}\, \lambda_{2,t}]$ so that $\lambda_{1,t}$, the co-state vector associated with the predetermined variables, is of dimension $(n-m)\times 1$ and $\lambda_{2,t}$, the co-state vector associated with the non-predetermined variables, is of dimension $m \times 1$.
- The terminal (tranversality) and initial optimal conditions are

$$
\begin{aligned}
\lim_{t\to\infty} \lambda_{1,t} &= 0 \\
\lambda_{2,0} &= 0; \text{ (ex ante optimal)} \\
\lambda_{2,0} &= \lambda_2; \text{ (`timeless' solution)}
\end{aligned}
$$

  where λ_2 is the deterministic steady state of $\lambda_{2,t}$.
- At any time $t > 0$ there then exists a gain from reneging by resetting $\lambda_{2,t} = 0$. Thus there is *an incentive to renege - the time-inconsistency problem*.
- The *timeless solution* imposes a *time-invariance* (not time-consistency!) on the solution.

Discretionary Policy

- To evaluate the time-consistent (discretionary) policy we write the expected loss Ω_t at time t in Bellman form as[1]

$$\Omega_t = E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t) \right] = u(\mathrm{y}_t, \mathrm{y}_{t-1}, \mathrm{w}_t) + \beta E_t \left[\Omega_{t+1} \right] \tag{4}$$

- The dynamic programming solution then seeks a stationary *Markov Perfect* solution of the form $\mathrm{w}_t = F(\mathrm{Z}_t)$, and $\mathrm{Z}_t = G(\mathrm{Z}_t)$. Ω_t is maximized at time t, subject to the model constraints, in the knowledge that a similar procedure will be used to minimize Ω_{t+1} at time $t+1$.[2]

[1] This applies only to the zero-growth steady state. To stationarize the problem for a trended balanced-growth steady state see Appendix F.

[2] See [Currie and Levine(1993)] and [Söderlind(1999)] for a LQ treatment of this problem.

Welfare-Optimal Simple Rules

- Optimal policy in the form of the Ramsey solution can be expressed as $\mathrm{w}_t = f(\mathrm{Z}_t, \lambda_{2,\mathrm{t}})$. This poses problems for the implementability of policy in terms of complexity and the observability of elements of Z_t (such as the technology process A_t, but more importantly $\lambda_{2,\mathrm{t}}$).
- The macroeconomic policy literature therefore focuses on simple rules, using the Ramsey solution as a benchmark. All our rules take the log-linear form[3]

$$\log \mathrm{w}_t = D \log \mathrm{y}_t$$

  where we define $\log \mathrm{w}_t \equiv [\log w_{1,t} \log w_{2,t}, \cdots, \log w_{r,t}]^T$ over r instruments, and similarly for $\log \mathrm{y}_t$, and the matrix D selects a subset of y_t from which to feedback.
- This is quite general in that y_t can be enlarged to include lagged and forward-looking variables.

[3]But in our implementation below rules can be non-linear.

Computing Welfare-Optimal Simple Rules in Dynare

- The optimized simple rules then defines the inter-temporal welfare loss at time t in Bellman form (4) (again ignoring long-term trend growth for now), sets steady-state values for instruments w_t, denoted by w, computes a second-order solution for a particular setting of w and solves the maximization problem at $t = 0$,

$$\max_{\mathrm{w},D} \Omega_0(\mathrm{Z}_0, \mathrm{w}, D)$$

  given initial values Z_0.
- In a purely stochastic problem we put $\mathrm{Z}_0 = \mathrm{Z}$, the steady state of Z_t, maximizing the conditional welfare at the steady state.
- In a purely deterministic problem there is no exogenous uncertainty and the optimization problem is driven by the need to return from Z_0 to its steady state, Z.

Conventional Monetary Policy

- Policy is conducted in terms of one of two simple Taylor rules:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right)$$

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y_t^F}\right) +$$

- Write the inter-temporal welfare loss at time t as

$$\Omega_t = E_t\left[(1-\beta)\sum_{\tau=0}^{\infty}\beta^\tau \Lambda_{t+\tau}\right]$$

where the $\Lambda_t = U_t(C_t, h_t)$ is the household's single-period utility function. Program in Bellman form as

$$\Omega_t = (1-\beta)\Lambda_t + \beta E_t\left[\Omega_{t+1}\right]$$

- Optimized rules set $\Pi = 1$ and optimize *a second-order approximation of the mean of* Ω_t over ρ_r, θ_π and $\theta_{r,y}$
- In a bgp Ω_t must be expressed in terms of stationarized consumption, C_t^c (See Appendix F of notes).

The 'Approximate Ramsey' Rule

- The simple rules framework can be used to assess the cost of simplicity by comparing our optimized rules with one that responds (hypothetically) to all current realizations of the five exogenous shock processes; namely (for the implementable rule) with an optimized rule of the form:

$$\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + \alpha_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \alpha_y \log\left(\frac{Y_t}{Y}\right) \\
&+ \alpha_A \log\left(\frac{A_t}{A}\right) + \alpha_G \log\left(\frac{G_t}{G}\right) + \alpha_{MS} \log\left(\frac{MS_t}{MS}\right) \\
&+ \alpha_{trend} \log\left(\frac{trend_t}{trend}\right) + \alpha_{capqual} \log\left(\frac{capqual_t}{capqual}\right)
\end{aligned}$$

- This is not the Ramsey rule, but it will mimic the latter closely. We therefore refer to it as the '*approximate Ramsey rule*'.

Consumption Equivalent Calculation (CE)

- Given a particular equilibrium for C_t and h_t, compute ce, the increase in the single-period utility, $\Lambda_t = \Lambda_t(C_t, C_{t-1}, h_t)$ given by a 1% increase in consumption, by defining the variable:

$$CE_t \equiv \Lambda_t(1.01\, C_t, 1.01\, C_{t-1}, h_t) - \Lambda_t(C_t, C_{t-1}, h_t)$$

- Then we use the deterministic steady state of CE_t, CE, to compare welfare outcomes.
- For our calibrated NK model, $CE = 0.00218$.

Optimized Current Inflation and Output Targeting Rules in the NK Model

Rule	σ_r	ρ_r	α_π	α_y	Welfare	ce(%)
Implem	0.91	0.857	5.000	0.134	-185.4140	0.018
Implem (Ramsey)	0.44	1.000	0.965	0.000	-185.4077	0
Taylor	1.05	0.369	2.752	1.674	-185.4143	0.026
Taylor (Ramsey)	0.44	1.000	0.960	0.000	-185.4077	0.004
Price Level	0.85	1.00	5.00	0	-185.4143	0.023
Minimal Feedback	6.69	0	1.001	0	-187.1576	8.79

Table: **Optimized Current Inflation and Output Rules.** $\alpha_\pi = (1-\rho_r)\theta_\pi$, $\alpha_y = (1-\rho_r)\theta_y$

Optimized Forward-looking Targeting Rules in the NK Model

- We can generalize to *forward-looking* rules as follows:

$$
\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\theta_\pi E_t\left[\log\left(\frac{\Pi_{t+j}}{\Pi}\right)\right] \\
&+ (1-\rho_r)\theta_y E_t\left[\log\left(\frac{Y_{t+j}}{Y}\right)\right] + \epsilon_{MPS,t} \\
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\theta_\pi E_t\left[\log\left(\frac{\Pi_{t+j}}{\Pi}\right))\right] \\
&+ (1-\rho_r)\theta_y E_t\left[\log\left(\frac{Y_{t+j}}{Y_t^F}\right))\right] + \epsilon_{MPS,t}
\end{aligned}
$$

for $j = 0, 1, 2, 3, \cdots$.

- The case $j = 0$, a current inflation rule gets us back to previous rules. Putting $j = -1, -2, -3, \cdots$ models the case of *backward-looking* rules.
- To implement in Dynare define new variables $\Pi L1_t \equiv \Pi_{t+1}$, $\Pi L2_t \equiv \Pi L1_{t+1}$ etc.

Forward-Looking Optimized Rules in the NK Model

j	σ_r	ρ_r	α_π	α_y	Welfare	CE (%)
0	1.05	0.350	2.901	1.7738	−185.4143	0.010
1	0.76	0.902	5.00	2.993	−185.41226	0
2	1.093	0.000	5.000	2.1451	−185.4279	0.003
3	1.10	0.166	1.650	1.100	−185.4237	0.057
4	0.75	1.000	2.000	2.000	−185.4526	0.202
5	0.79	1.000	0.500	0.500	−185.7755	1.816

Table: **Forward-Looking Optimized Rules (Conventional Taylor)**

NK and GK Optimized Simple Rules Compared

Model	Rule	σ_r	$\rho_r,$	α_π	α_y	Welfare	ce
NK	Implem	0.91	0.857	5.000	0.134	-185.4140	0
NK	Taylor	1.05	0.369	2.752	1.674	-185.4143	0
NK	Price Level	0.85	1.00	5.00	0	-185.4143	0
NK	Minimal	6.69	0	1.001	0	-187.1576	6.69
GK	Implem	1.08	0.471	3.193	0.042	-183.5956	0
GK	Taylor	1.10	0.411	2.976	0.083	-183.5956	0
GK	Price Level	0.90	1.00	4.729	0	-183.5959	0
GK	Minimal	7.19	0	1.001	0	-185.9825	11.93

Table: **Optimized Current Inflation and Output Rules: NK and GK Compared. ce=consumption equivalent % difference.**

Steady states of NK and GK Compared

- We can compare steady states of NK and GK

Variable	GK	NK
C	0.3758	0.3831
Y	0.6944	0.7356
h	0.3417	0.3504
Λ	-183.646	-185.446

Table: **Steady States of NK and GK Compared**

- Thus financial frictions are welfare-enhancing in the steady state! Why? External Habit.
- Similarly inflation is welfare-enhancing in the steady state.

ZLB Considerations

- Rather than requiring that $R_{n,t} \geq 0$ for *any* realization of shocks, we impose the constraint that the mean rate should be at least k standard deviations above the ZLB.
- Define $\bar{R}_n \equiv E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_{n,t}\right]$ to be the discounted future average of the nominal interest rate path $\{R_{n,t}\}$.
- Our 'approximate form' of the ZLB constraint is a requirement that $\bar{R}_n$ is at least k_r standard deviations above the zero lower bound; i.e., using discounted averages that

$$\bar{R}_n \geq k_r \sqrt{\overline{(R_{n,t} - \bar{R}_n)^2}} = k_r \sqrt{\overline{R_{n,t}^2} - (\bar{R}_n)^2} \tag{5}$$

- Squaring both sides of (5) and letting $K_r = 1 + k_r^{-2} > 1$ we arrive at

$$E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_{n,t}^2\right] \quad \leq \quad K_r \left[E_0 \left[(1-\beta)\sum_{t=0}^{\infty} \beta^t R_{n,t}\right]\right]^2 \tag{6}$$

ZLB Considerations

- We now maximize $\sum_{t=0}^{\infty} \beta^t \Lambda_t$ subject to the additional constraint (6) alongside the other dynamic constraints in the Ramsey problem.
- Using the Kuhn-Tucker theorem, this results in an additional term $(1-\beta)\sum_{t=0}^{\infty} \beta^t w_r \left(K_r R_{n,t} - R_{n,t}^2)\right) = -(1-\beta)\sum_{t=0}^{\infty} \beta^t w_r \left(R_{n,t} - \frac{1}{2}K_r)\right)^2 +$ a constant in the Lagrangian to incorporate this extra constraint, where $w_r > 0$ is a Lagrangian multiplier.
- Thus imposing the constraint that the ZLB is hit with only a given low probability per period p is equivalent to *shifting the steady state interest rate to a new higher target* $R_{n,t}^* \equiv \frac{1}{2}K_r$ and *lowering the variance* about this new steady state by increasing w_r. Appendix F describes an algorithm to implement this procedure for the NK model.
- The next Table sets out results for the NK model keeping steady-state net inflation at zero.

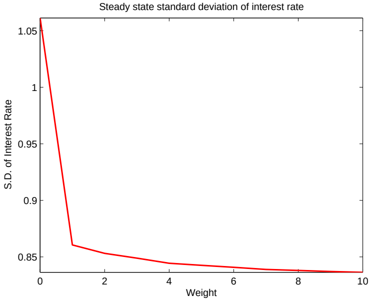
ZLB Considerations

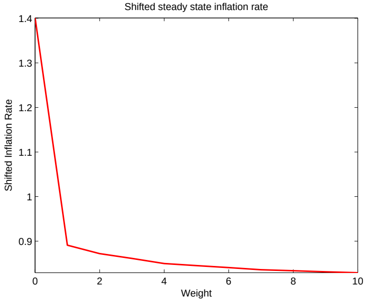
w_r	σ_r	z_0	Prob	ρ_r	θ_π	θ_y	Wel (actual)	ce(%)
0	1.05	1.251	0.125	0.369	2.752	1.693	-185.4143	0
1.0	0.86	1.528	0.076	1.000	5.000	0.000	-185.4143	0.000
2.0	0.85	1.546	0.061	1.000	4.681	0.000	-185.4290	0.000
5.0	0.84	1.564	0.060	1.000	3.582	0.000	-185.4505	0.005
10.0	0.83	1.583	0.057	1.000	2.884	0.000	-185.4163	0.100
50.0	0.82	1.602	0.055	1.000	1.817	0.000	-185.4210	0.034
100	0.82	1.602	0.055	1.000	1.545	0.000	-185.4238	0.048

Table: **Imposing the ZLB without raising Π (Conventional Taylor)**

- The probability of $R_{n,t} < 1$ or $Z_t \equiv \frac{R_{n,t}-R_n}{\sigma_r/100} < -\frac{100(R_n-1)}{\sigma_r} \equiv -z_0$ for a standard normal distribution is shown.
- At this point σ_r is almost stationary and the ZLB is reached with a frequency of about 1/18 quarters or about 1/4.5 years with a modest consumption equivalent cost.
- But to get below this frequency we need to increase $\Pi > 1$.

ZLB Considerations: Raising Steady State Inflation





ZLB Considerations: Raising Steady State Inflation

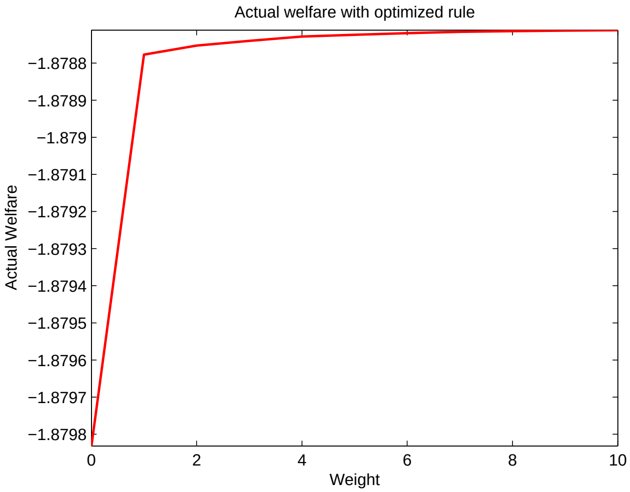


Figure: Imposition of ZLB for Implementable Rule

Unconventional Monetary Policy: Direct Lending

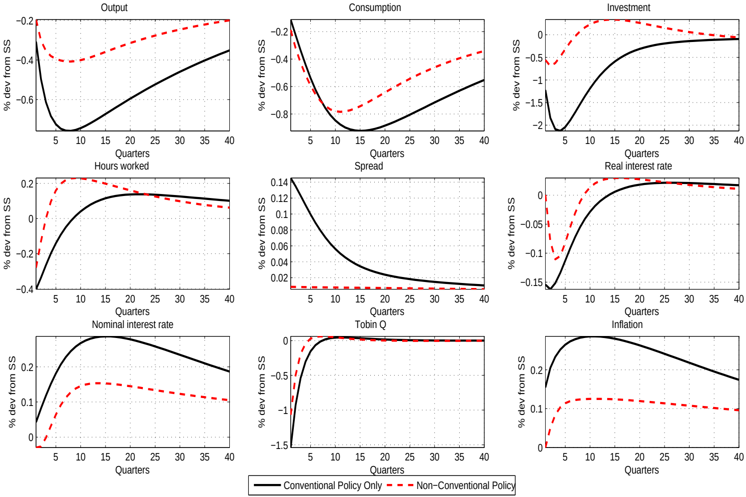
- Now suppose the CB facilitates direct lending to firms. The total value of intermediated assets is now

$$Q_t S_t = \underbrace{Q_t S_{p,t}}_{\text{private}} + \underbrace{Q_t S_{g,t}}_{\text{CB}}$$

- The CB issues government debt to household who pay the riskless real interest rate R^{ex}_{t+1} and lends to firms at the market rate of return $R_{k,t+1}$.
- This involves a efficiency cost τ per unit of credit to add to resource constraint.
- Suppose $Q_t S_{g,t} = \gamma_t Q_t S_t$
- It issues government bonds $B_{g,t} = \gamma_t Q_t S_t$ and earns $(R_{k,t} - R^{ex}_t) B_{g,t-1}$ from period $t-1$ which enters the GBC. But with lump-sum taxes this plays no role.
- Unconventional policy conducted in terms of a Taylor-type rule with $\gamma_t = \gamma = 0$ in the steady state:

$$\gamma_t = \rho_\gamma \gamma_{t-1} + (1-\rho_\gamma)\theta_{\text{sp}}(E_t[R_{k,t+1}] - R_k - (E_t[R^{ex}_{t+1}] - R^{ex}))$$

Conventional and Unconventional Policy with Capital Quality Shock



Unconventional Monetary Policy: Liquidity Facilities

- Now suppose the CB lends to banks who in turn lend to firms. Must go back to the banking model.
- The balance sheet of the banks is now

$$Q_t s_t = n_t + d_t = n_t + d_{p,t} + d_{g,t}$$

- The IC constraint becomes

$$V_t \geq \Theta_t (Q_t s_t - \omega_g d_{g,t})$$

  where $\omega_g \in (0, 1]$ indicates the CB's advantage in retrieving assets.
- Must rework problem (in progress)
- Also can have equity injections by government - see [Gertler and Kiyotaki(2009)].

Dynare Codes

- All the codes for the material of this section are in the folder **Optimal Policy**.
- For the NK model, Dynare codes for optimized current inflation rules (*NK_optpol_RES_Course*), the approximate Ramsey rule (*NK_optpol_Ramsey_RES_Course*), forward-looking rule (*NK_optpol_FL_RES_Course*) are to be found in sub-folder *NK_optpol_Ramsey_RES_Course*.
- Subfolders *GK_optpol_RES_Course* and *GK_optpol_unconven_RES_Course* contain codes the the GK model for conventional and unconventional policy respectively.

Exercises

1. Examine optimal backward-looking rules ($j = -1, j = -2, etc$) corresponding to Table 2. What do you notice?
2. Rework the forward-looking Table 2 for the GK model. What do you notice?
3. Compute the optimized unconventional policy rule (22) for different settings of the efficiency cost parameter τ.

CIMS Summer Course (9 - 13 Sept, 2013)

1. Two parts: part I, basics (days 1 – 2); part II (days 3 – 5), advanced.
2. The construction of a DSGE model describing the first-order conditions for economic agents in the form of a set of non-linear difference equations
3. The solution of the steady state to be used for both solution and calibration
4. Bayesian estimation methods
5. Model comparisons between different models or variants of same model
6. Further model validation by comparison with second moments
7. **DSGE-VAR modelling**
8. **Solution Techniques - Perturbation and projection computational methods**
9. Optimal policy analysis. **Zero Lower Bound. Robust Rules.**
10. The rational expectations solution, estimation under imperfect information

Currie, D. and Levine, P. (1993).
Rules, Reputation and Macroeconomic Policy Coordination.
Cambridge University Press.

Gertler, M. and Kiyotaki, N. (2009).
Financial Intermediation and Credit Policy in Business Cycle Analysis.
Mimeo and forthcoming chapter in the Handbook of Macroeconomics, Elsevier, 2010.

Söderlind, P. (1999).
Solution and Estimation of RE Macromodels with Optimal Policy.
European Economic Review, **43**, 813–823.