ELEC 4700

Assignment #2 Report

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Due date : March 1st, 2022

Carleton University

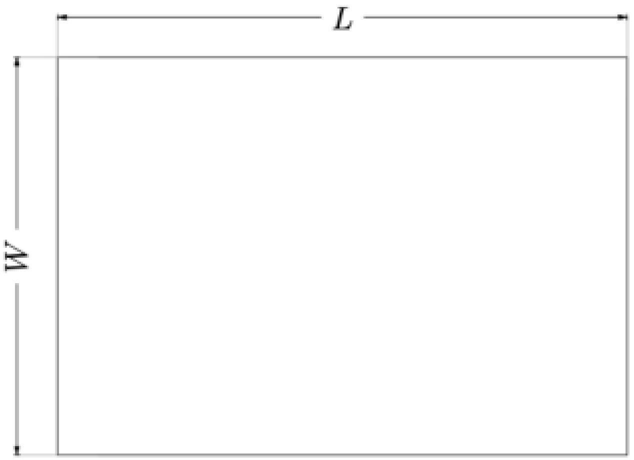
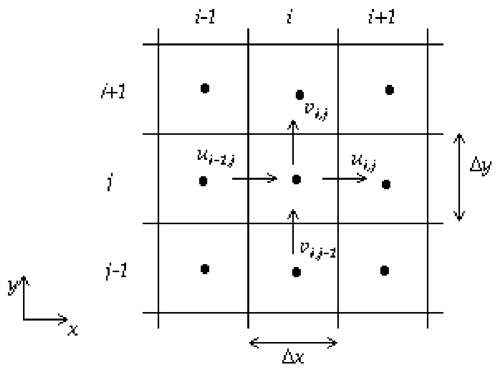


Figure 1: Rectangular region with isolated conducting sides

1- Using Finite Difference Method

Discretizarion :



$\nabla^2 V = 0$

$$\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta^2} + \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{\Delta^2} = 0$$

a) 1D case

$$\frac{\partial^2 V}{\partial x^2} = 0$$

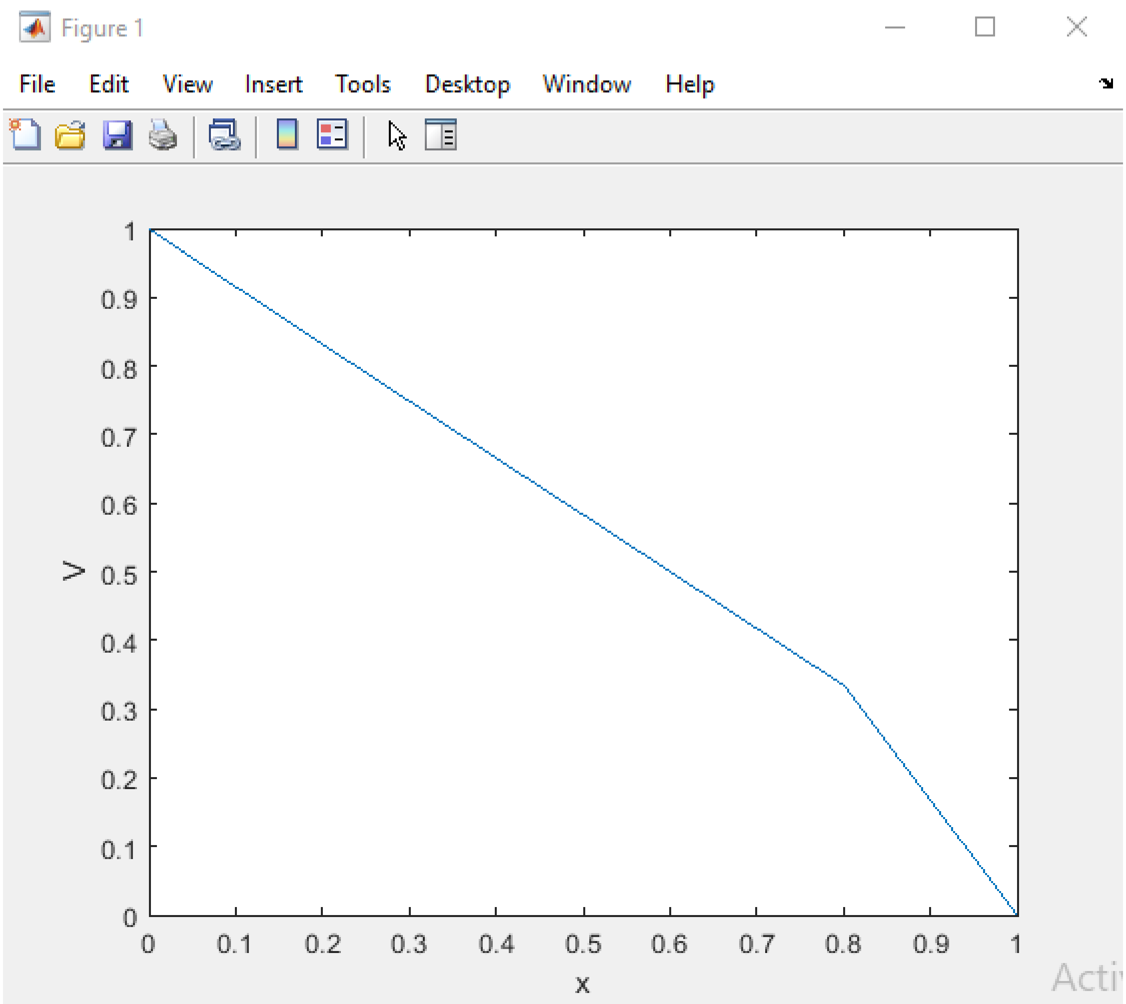
$$\frac{V_{i+1} - 2V_i + V_{i-1}}{\Delta^2} = 0$$

In matrix form for n=5

$$\begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} -V_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using matlab :

```
%%question 1 a)
n=5;
L=1;
delta=(L/n);
D0=-2*diag(ones(n,1));
D1=diag(ones(n-1,1),1);
D2=diag(ones(n-1,1),-1);
D=D0+D1+D2;
V=zeros(1,n+1);
V0=1;
F=zeros(1,n);
V(1)=V0;
V(end)=0;
F(1)=-V0;
F(end)=0;
Q=inv(D)*F';
for i=1:n-1
    V(i+1)=Q(i);
end
N=0:n;
x=delta*N;
plot(x,V)
xlabel('x')
ylabel('V')
```

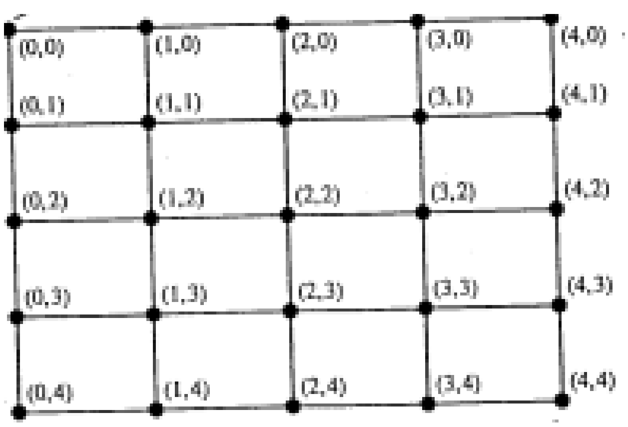


b) 2D case

$$\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{\Delta x^2} + \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{\Delta y^2} = 0$$

Assuming Δx = Δy

$$V_{i,j+1} + V_{i,j-1} + V_{i+1,j} + V_{i-1,j} - 4V_{i,j} = 0$$



$$V_{1,2} + 0 + V_{2,1} + V0 - 4V_{1,1} = 0$$

$$\mathrm{V}_{2,2} + 0 + \mathrm{V}_{3,1} + \mathrm{V}_{1,1} - 4\mathrm{V}_{2,1} = 0$$
$$\mathrm{V}_{3,2} + 0 + \mathrm{V0} + \mathrm{V}_{2,1} - 4\mathrm{V}_{3,1} = 0$$
$$\mathrm{V}_{1,3} + \mathrm{V}_{1,1} + \mathrm{V}_{2,2} + \mathrm{V0} - 4\mathrm{V}_{1,2} = 0$$
$$\mathrm{V}_{2,3} + \mathrm{V}_{2,1} + \mathrm{V}_{3,2} + \mathrm{V}_{1,2} - 4\mathrm{V}_{2,2} = 0'$$
$$\mathrm{V}_{3,3} + \mathrm{V}_{3,1} + \mathrm{V0} + \mathrm{V}_{2,2} - 4\mathrm{V}_{3,2} = 0$$
$$0 + \mathrm{V}_{1,2} + \mathrm{V0} + \mathrm{V}_{2,3} - 4\mathrm{V}_{1,3} = 0$$
$$0 + \mathrm{V}_{2,2} + \mathrm{V}_{3,3} + \mathrm{V}_{1,3} - 4\mathrm{V}_{2,3} = 0$$
$$0 + \mathrm{V}_{3,2} + 0 + \mathrm{V}_{2,3} - 4\mathrm{V}_{3,3} = 0$$

Symmetry (we got only 4 equations)

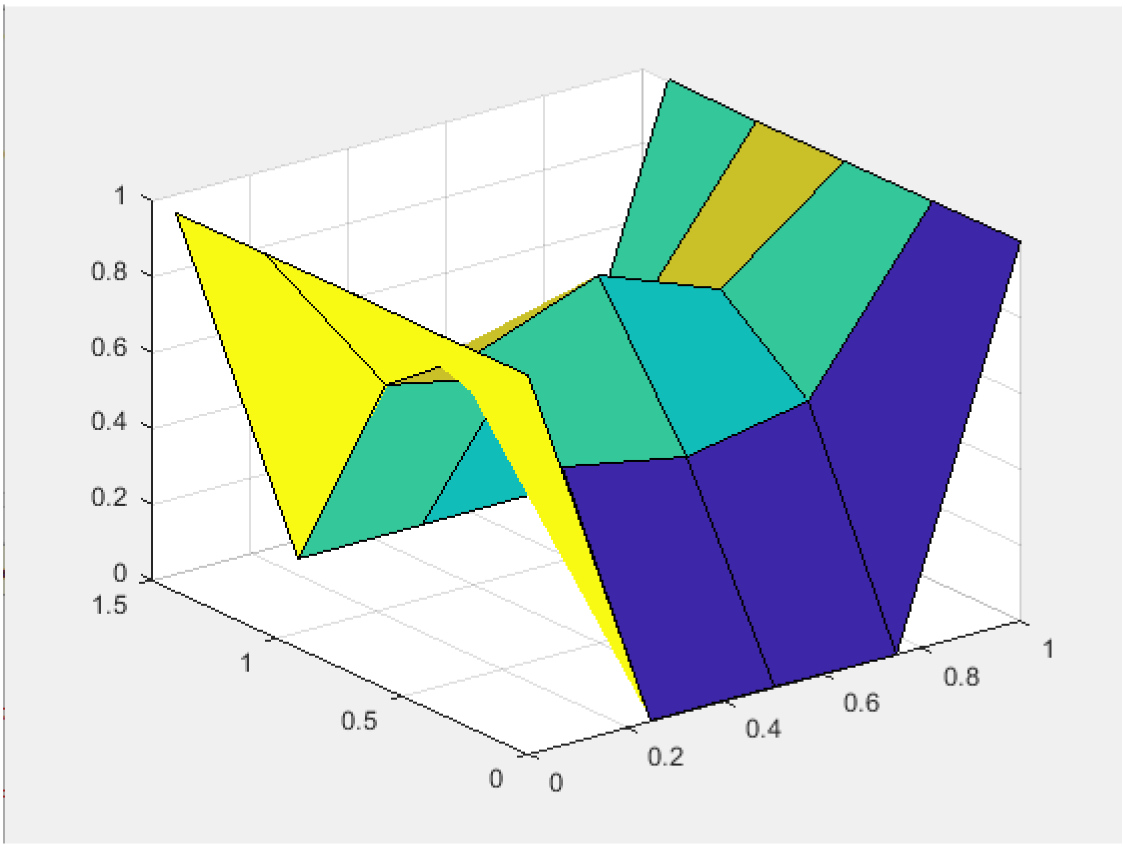
$$\mathrm{V}_{1,3} = \mathrm{V}_{1,1} = \mathrm{V}_{3,1} = \mathrm{V}_{3,3}$$

$$-4\mathrm{V}_{1,1} + \mathrm{V}_{1,2} + \mathrm{V}_{2,1} = -\mathrm{V0}$$
$$2\mathrm{V}_{1,1} + \mathrm{V}_{2,2} + -4\mathrm{V}_{2,1} = 0$$
$$-4\mathrm{V}_{1,1} + \mathrm{V}_{1,2} + \mathrm{V}_{2,1} = -\mathrm{V0}$$
$$2\mathrm{V}_{1,2} + 2\mathrm{V}_{2,1} - 4\mathrm{V}_{2,2} = 0$$

$$\begin{pmatrix} -4 & 1 & 1 & 0 \\ 2 & -4 & 0 & 1 \\ 2 & 0 & -4 & 1 \\ 0 & 2 & 2 & -4 \end{pmatrix} \begin{pmatrix} V_{1,1} \\ V_{2,1} \\ V_{1,2} \\ V_{2,2} \end{pmatrix} = \begin{pmatrix} -V_0 \\ 0 \\ -V_0 \\ -V_0 \end{pmatrix}$$

```
A=[-4 1 1 0;2 -4 0 1;2 0 -4 1;0 2 2 -4];
B=[-V0 0 -V0 -V0]';
vv=inv(A)*B;
VV=zeros(5,5);
VV(1,:)=0;
VV(end,:)=0;
VV(:,1)=V0;
VV(:,end)=V0;
VV(2,2)=vv(1);
VV(2,4)=vv(1);
VV(4,2)=vv(1);
VV(4,4)=vv(1);
VV(2,3)=vv(2);
VV(4,3)=vv(2);
VV(3,2)=vv(3);
VV(3,4)=vv(3);
VV(3,3)=vv(4);
```

```
VV =

    1.0000         0         0         0    1.0000
    1.0000    0.5625    0.5000    0.5625    1.0000
    1.0000    0.7500    0.8750    0.7500    1.0000
    1.0000    0.5625    0.5000    0.5625    1.0000
    1.0000         0         0         0    1.0000
```



Exact solution

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n} \frac{\cosh\left(\frac{n\pi x}{a}\right)}{\cosh\left(\frac{n\pi b}{a}\right)} \sin\left(\frac{n\pi y}{a}\right)$$

```
syms f(x,y)
n=1:2:100;
f(x,y) =
(4*V0/pi)*(sum((1./n).*((cosh(n*pi*x/1.5))./(cosh(n*pi/1.5))).*sin(n*pi*y/1
.5)));
i=1;
V1=zeros(5,5);
```

```
for xeval=0:0.25:1
yeval=0:0.35:1.5;
V1(:,i)=vpa(f(xeval,yeval));
i=i+1;
end
V1=vpa(V1,2)
```

```
V1 =

[     0,     0,    0,    0,    0]
[  0.21,  0.24, 0.34, 0.58,  1.0]
[  0.31,  0.35, 0.48,  0.7,  1.0]
[  0.25,  0.29,  0.4, 0.64, 0.99]
[ 0.065, 0.076, 0.11, 0.24,  1.0]
```

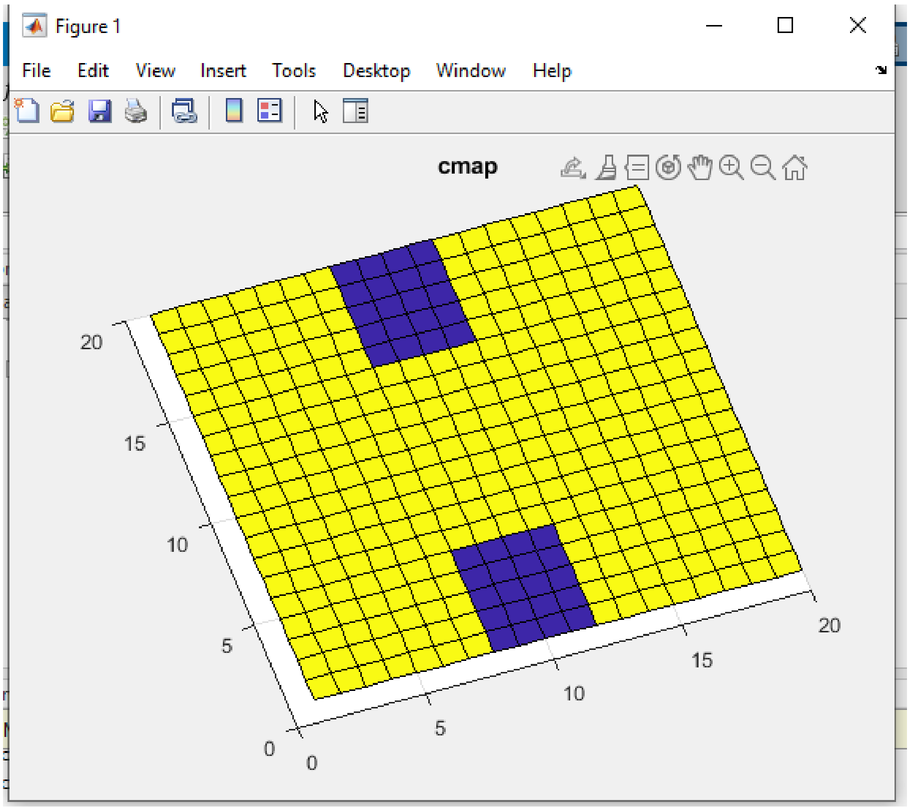
Conclusion : Exact solution and numerical solution don't match 100%.some error on the values. This is due by meshing system. We took a big space intervalle so bad quality on meshing.

Q2

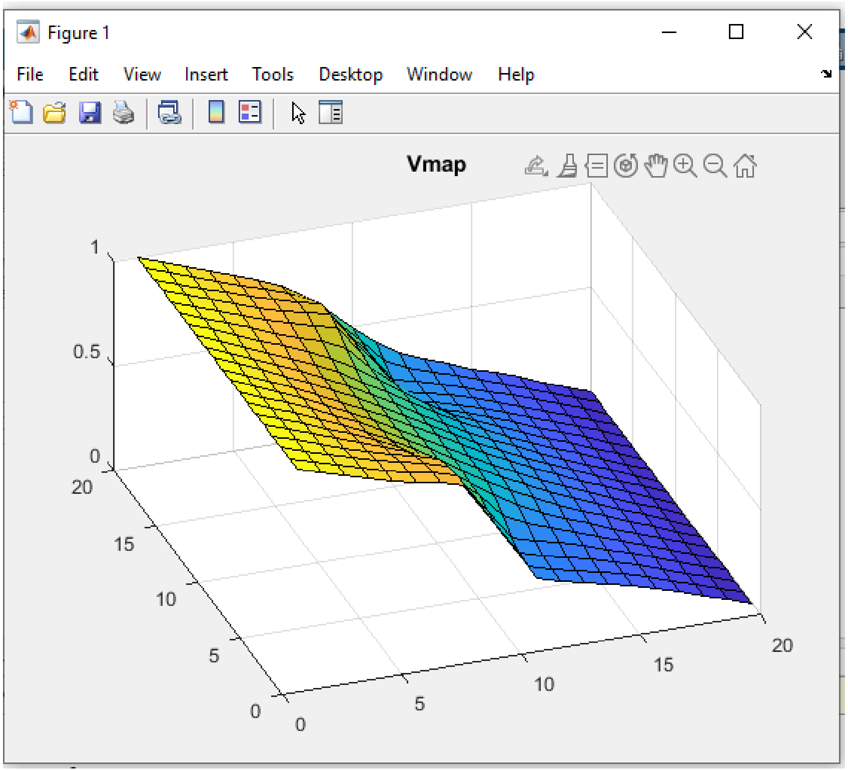
a)

$$\frac{\partial}{\partial x}\left(\sigma_x(x,y)\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left(\sigma_y(x,y)\frac{\partial V}{\partial y}\right)=0$$

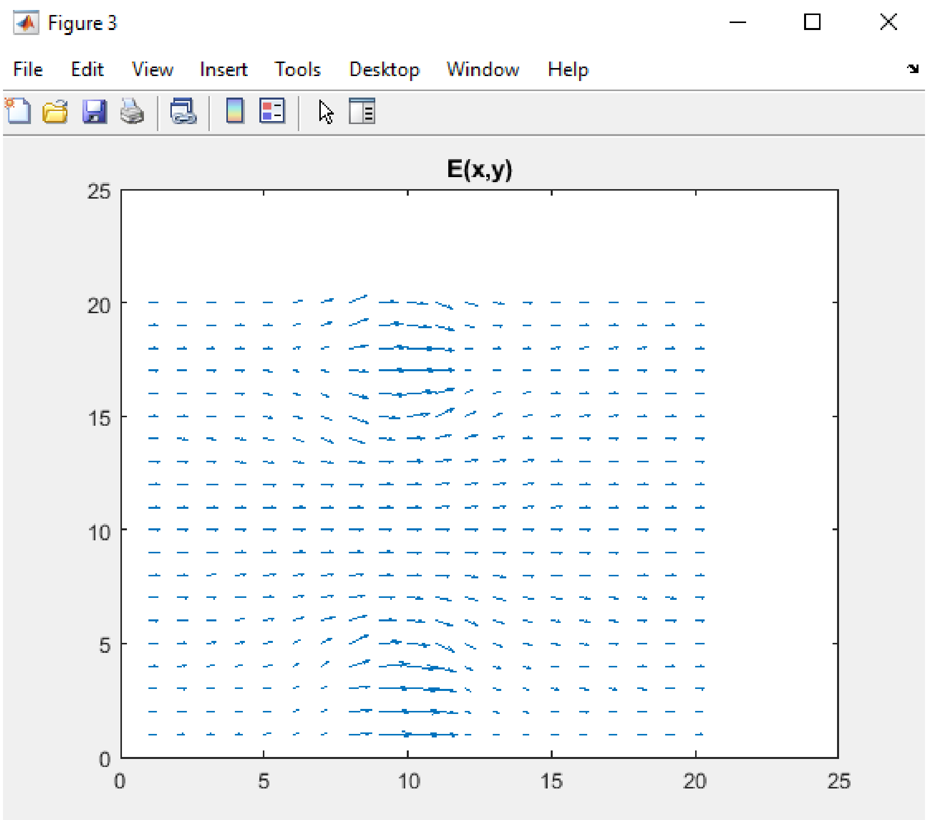
Plottins Cmap



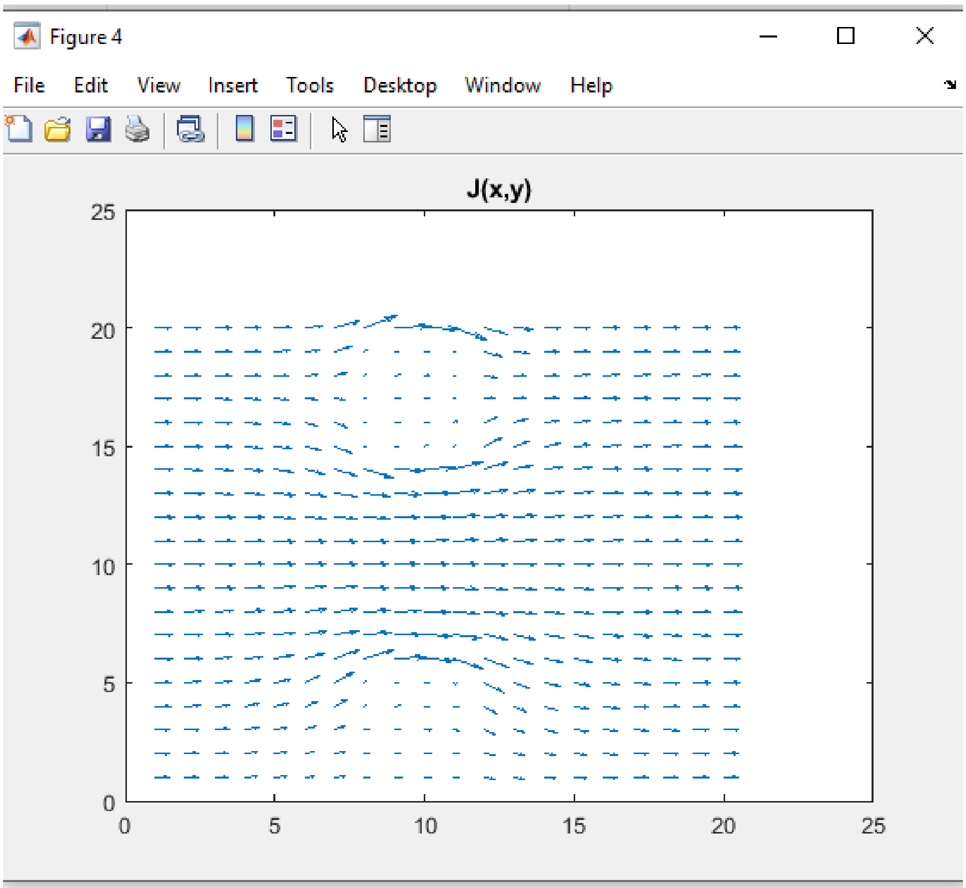
plotting V(x,y)



plotting E(x,y)



Plotting J(x,y)



b)

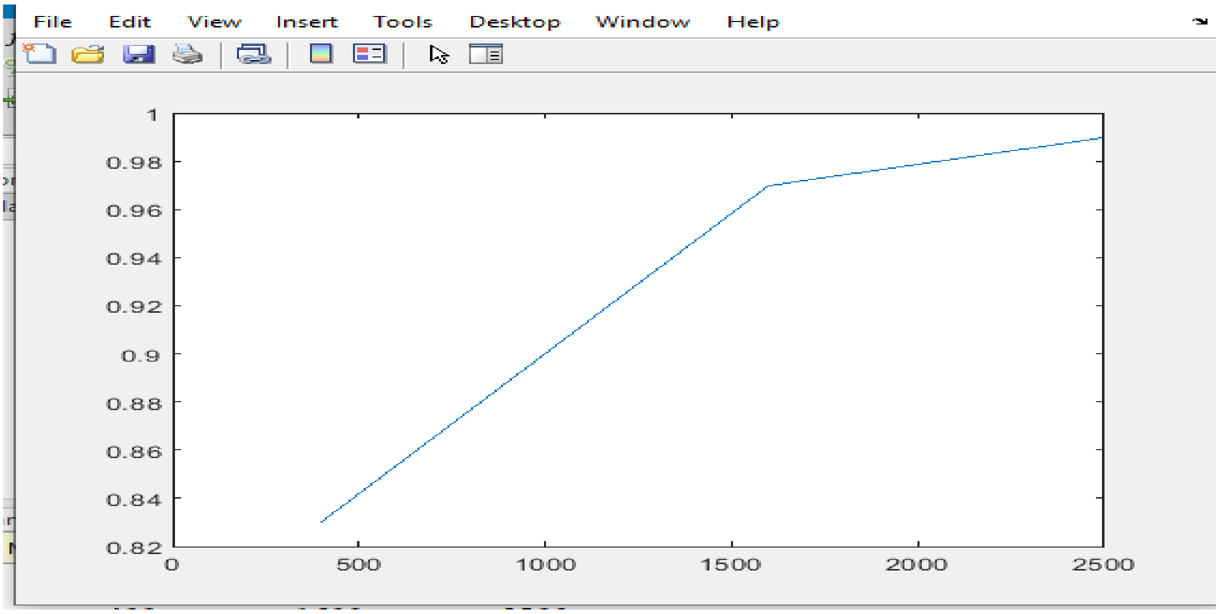
Current :

```
C0 = sum(eFlowx(1, :));
Cnx = sum(eFlowx(nx, :));
Curr = (C0 + Cnx) * 0.5;
```

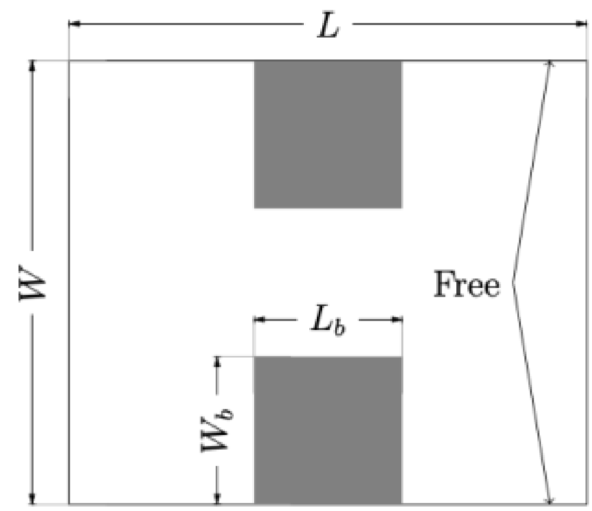
```
Curr =

     0.8386 A
```

<u>Mesh size Vs current</u>

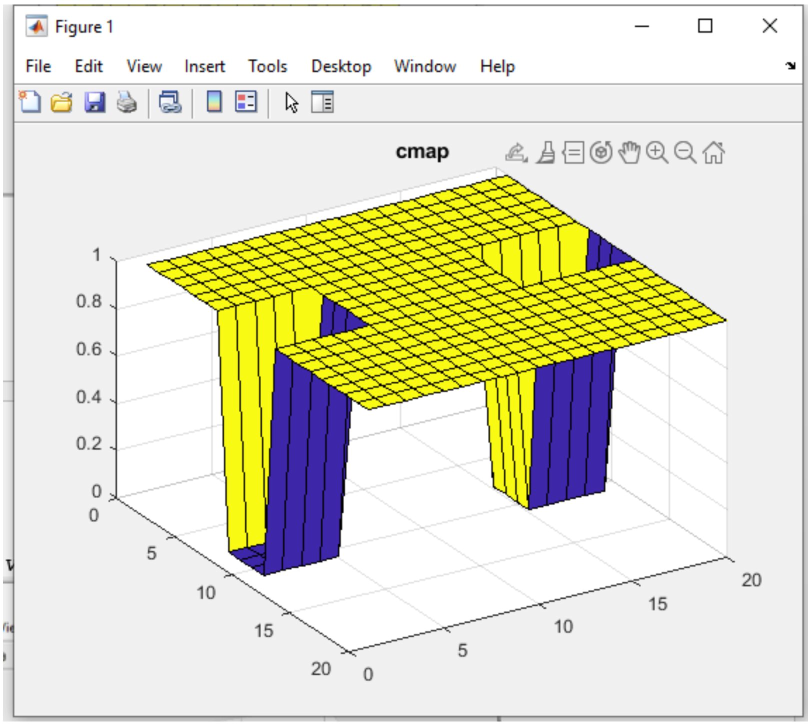
mesh	400	1600	2500
current	0.83	0.97	0.99



c)



: Rectangular region with isolated conducting sides and "bottle-neck".



<u>**d)**</u>

<u>**sigma Vs current**</u>

sigma	**0.01**	**0.05**	**0.1**	**0.15**	**0.2**
current	**0.77**	**0.8**	**0.83**	**0.86**	**0.88**