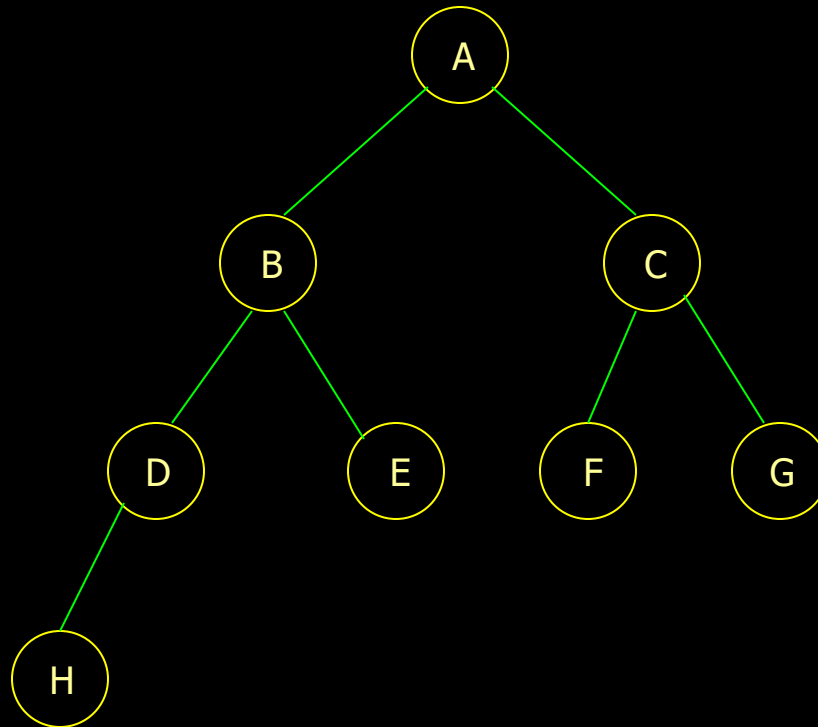


# Lecture # 17

## Heap

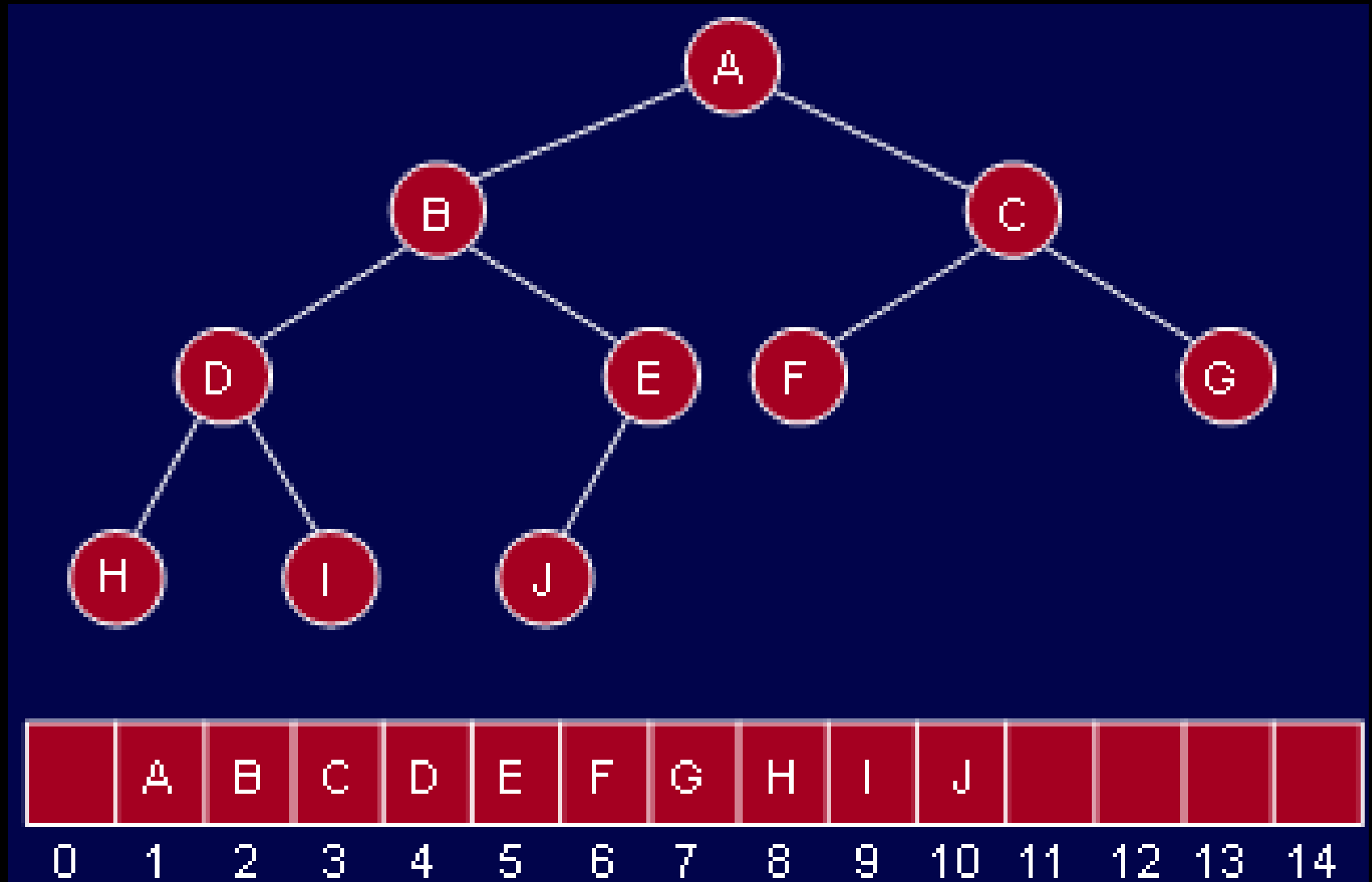
# Complete Binary Tree



# Complete Binary Tree

- Recall that such a tree of height  $h$  has between  $2^h$  to  $2^{h+1} - 1$  nodes.
- Because the tree is so regular, it can be stored in an *array*, no pointers are necessary.

# Complete Binary Tree

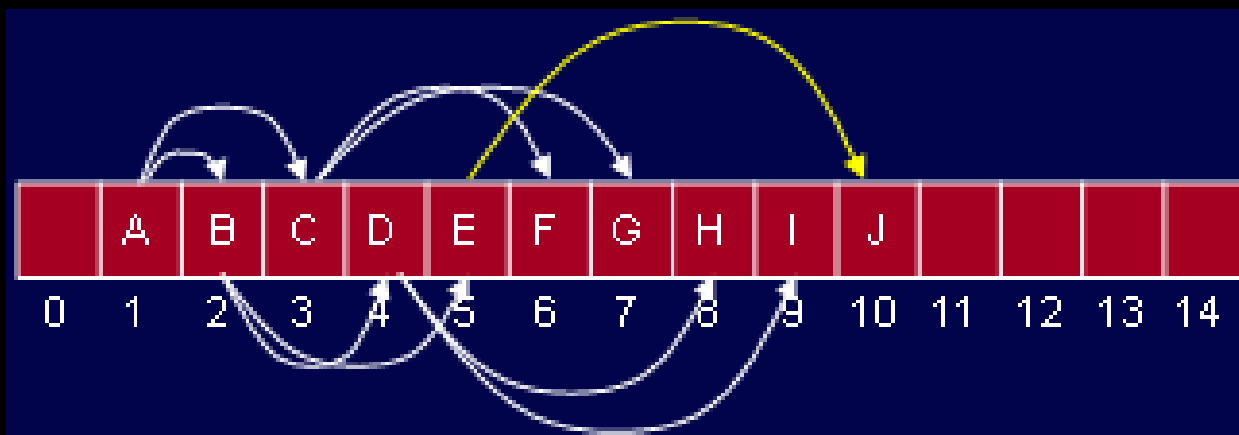
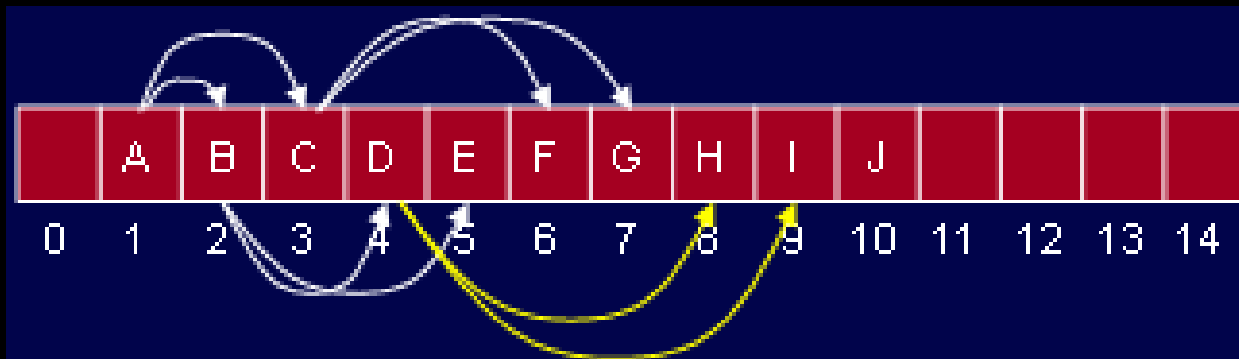
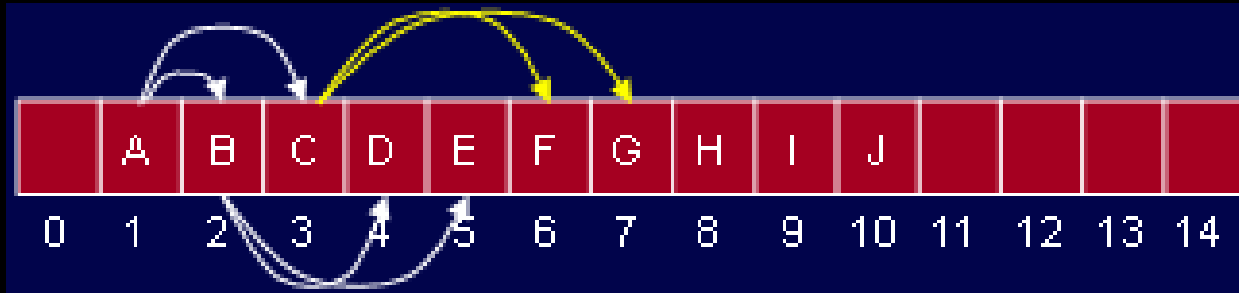


# Complete Binary Tree

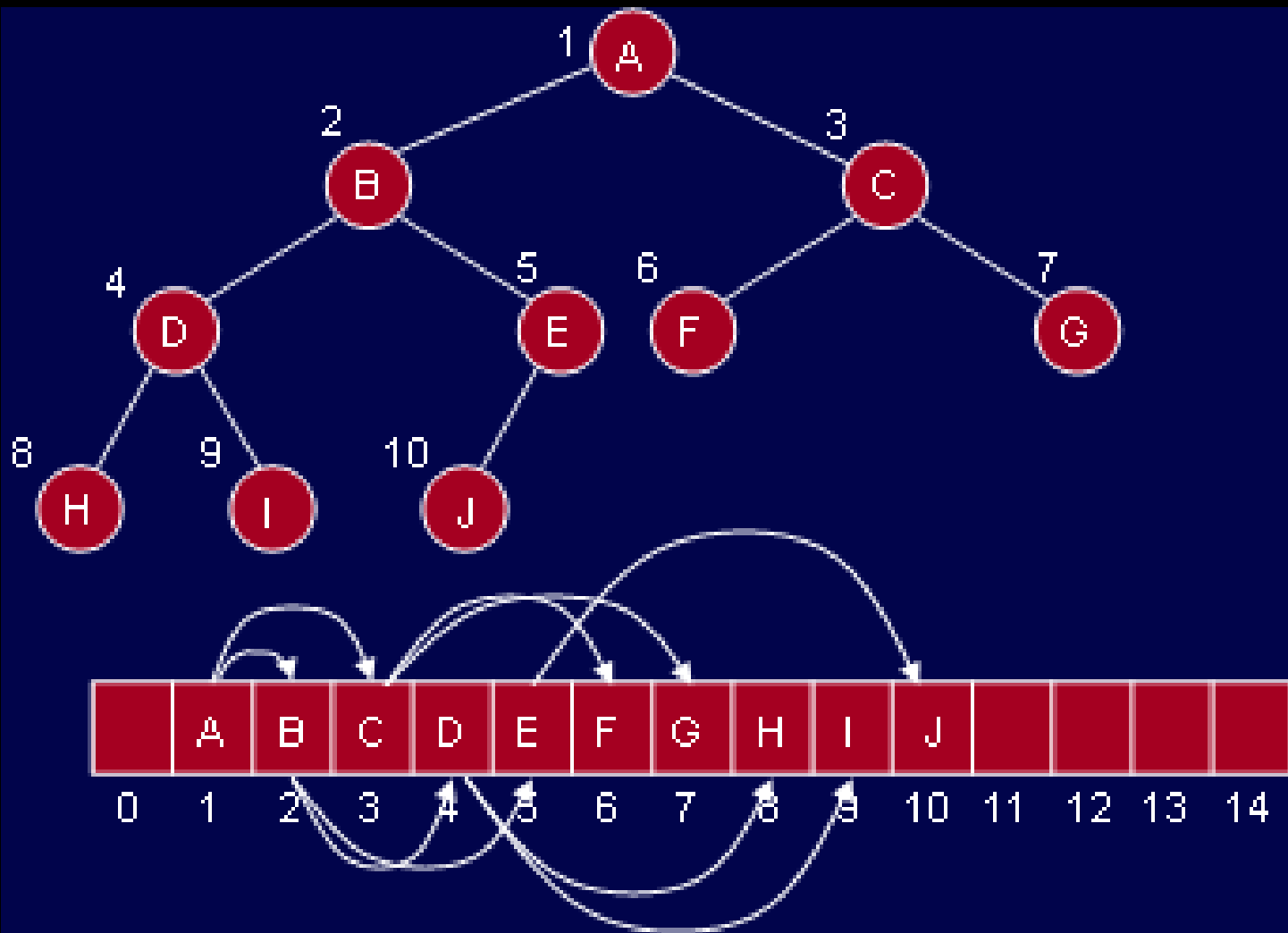
- For any array element at position  $i$ , the left child is at  $2i$ , the right child is at  $(2i+1)$  and the parent is at  $\lfloor i/2 \rfloor$ .



# Complete Binary Tree



# Complete Binary Tree



# Complete Binary Tree

- Question:

why don't we store all binary trees in arrays? Why use pointers?



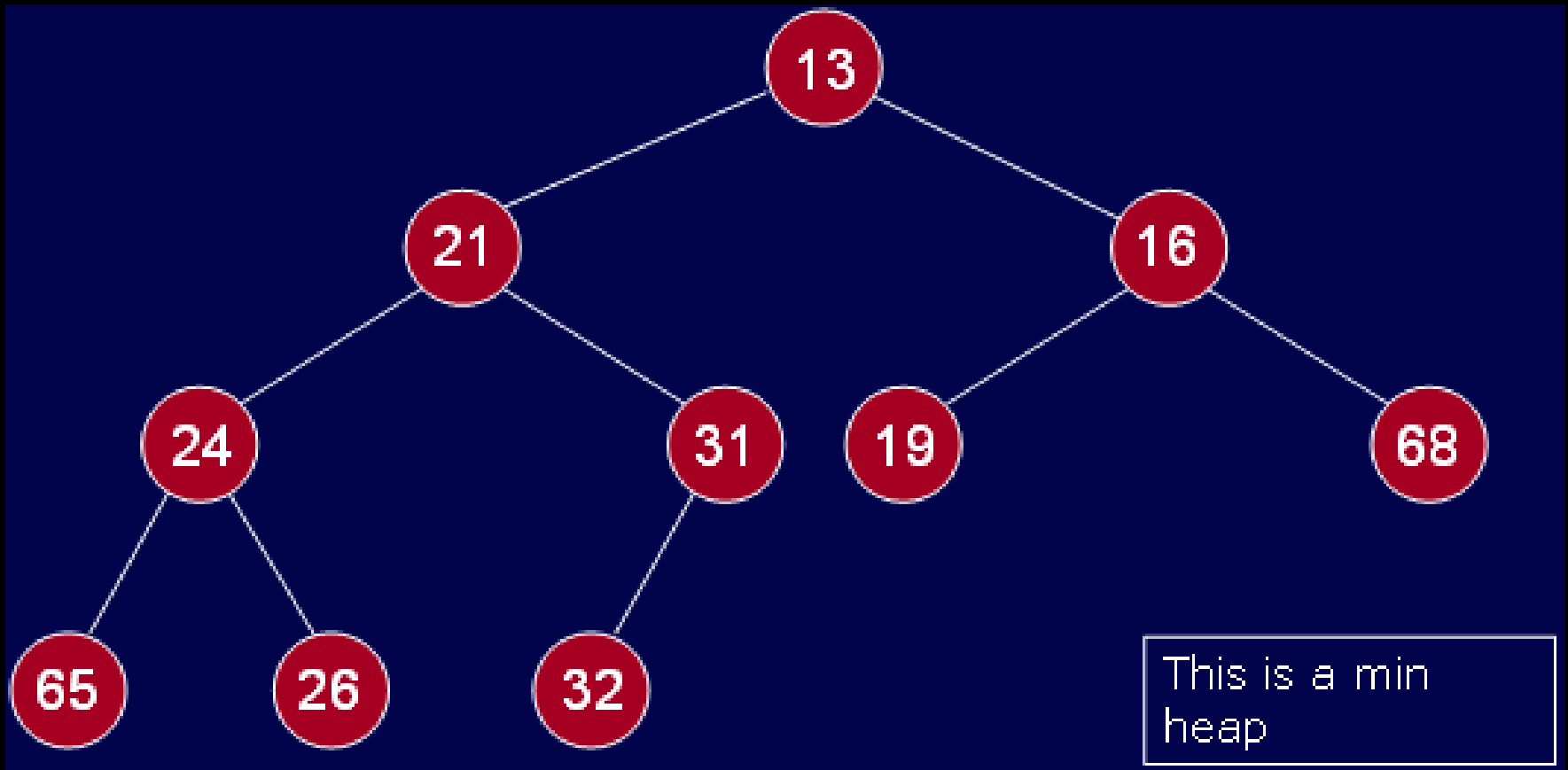
# The **Heap** ADT

- The major usage of heap is in **Priority Queues**.

# Heap

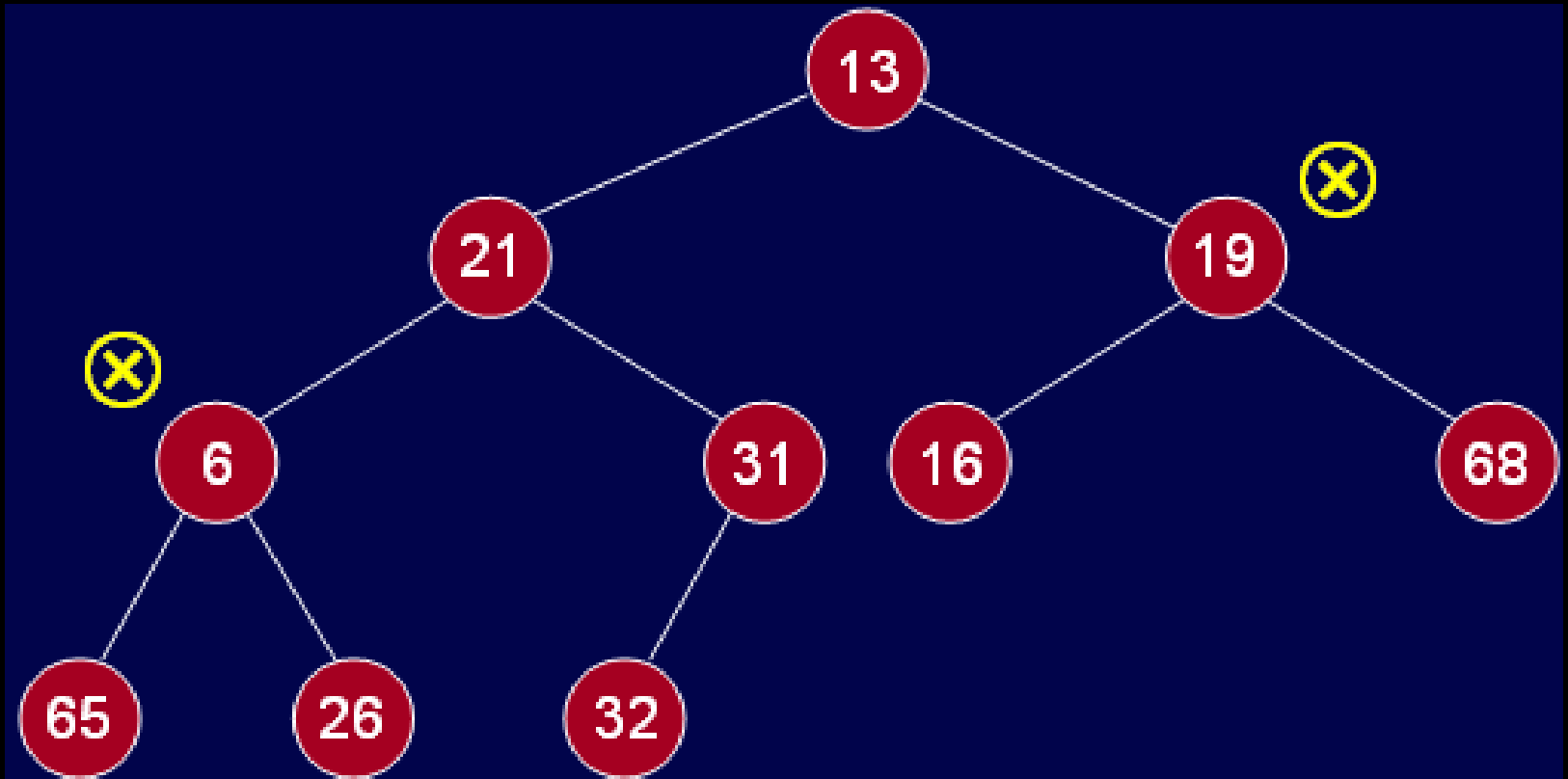
- A heap is a complete binary tree that conforms to the heap order.
- The heap order property: in a (min) heap, for every node  $X$ , the key in the parent is smaller than (or equal to) the key in  $X$ .
- Or, the parent node has key smaller than or equal to both of its children nodes.

# Heap



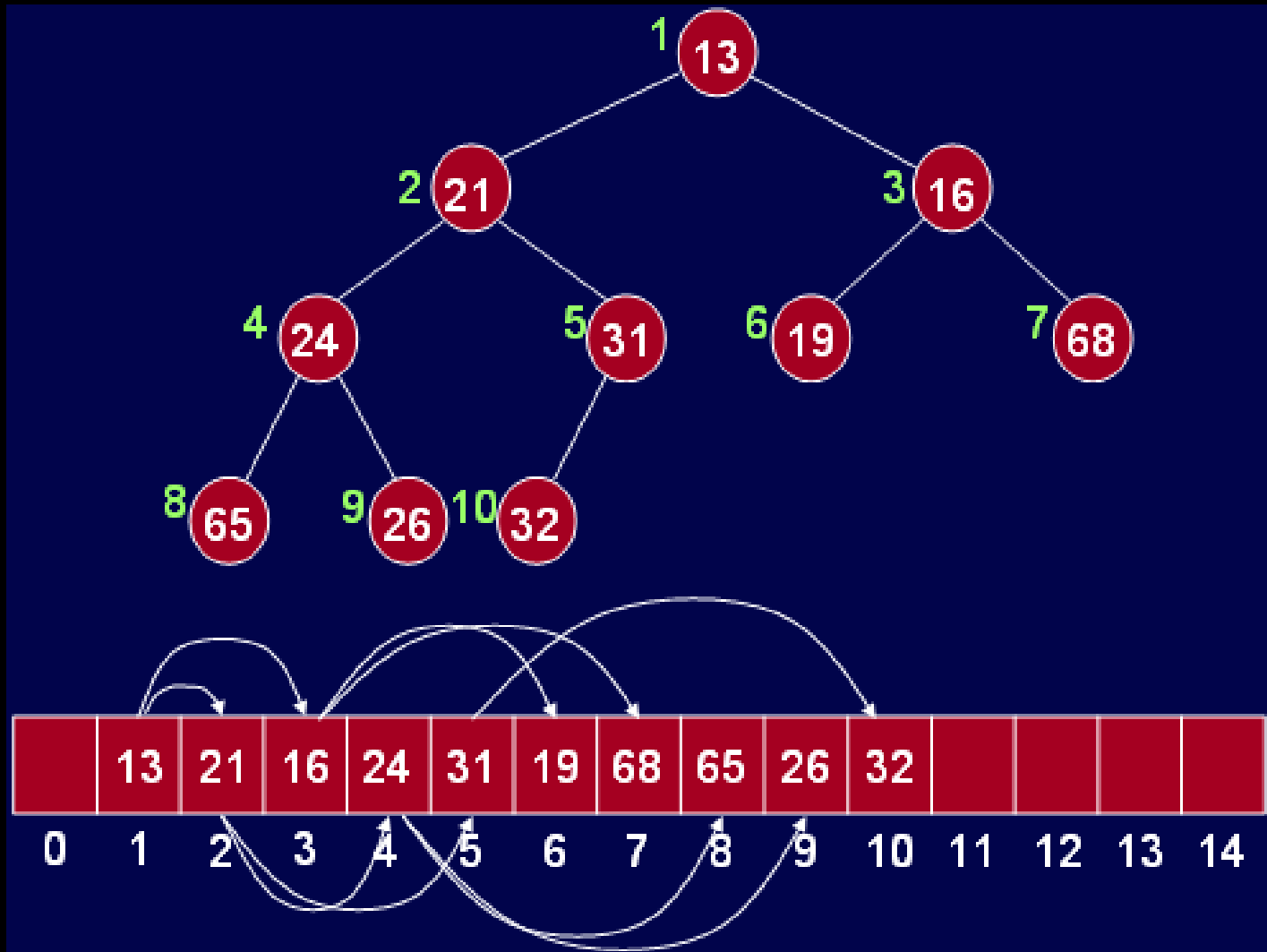
# Heap

- Not a heap: heap property violated



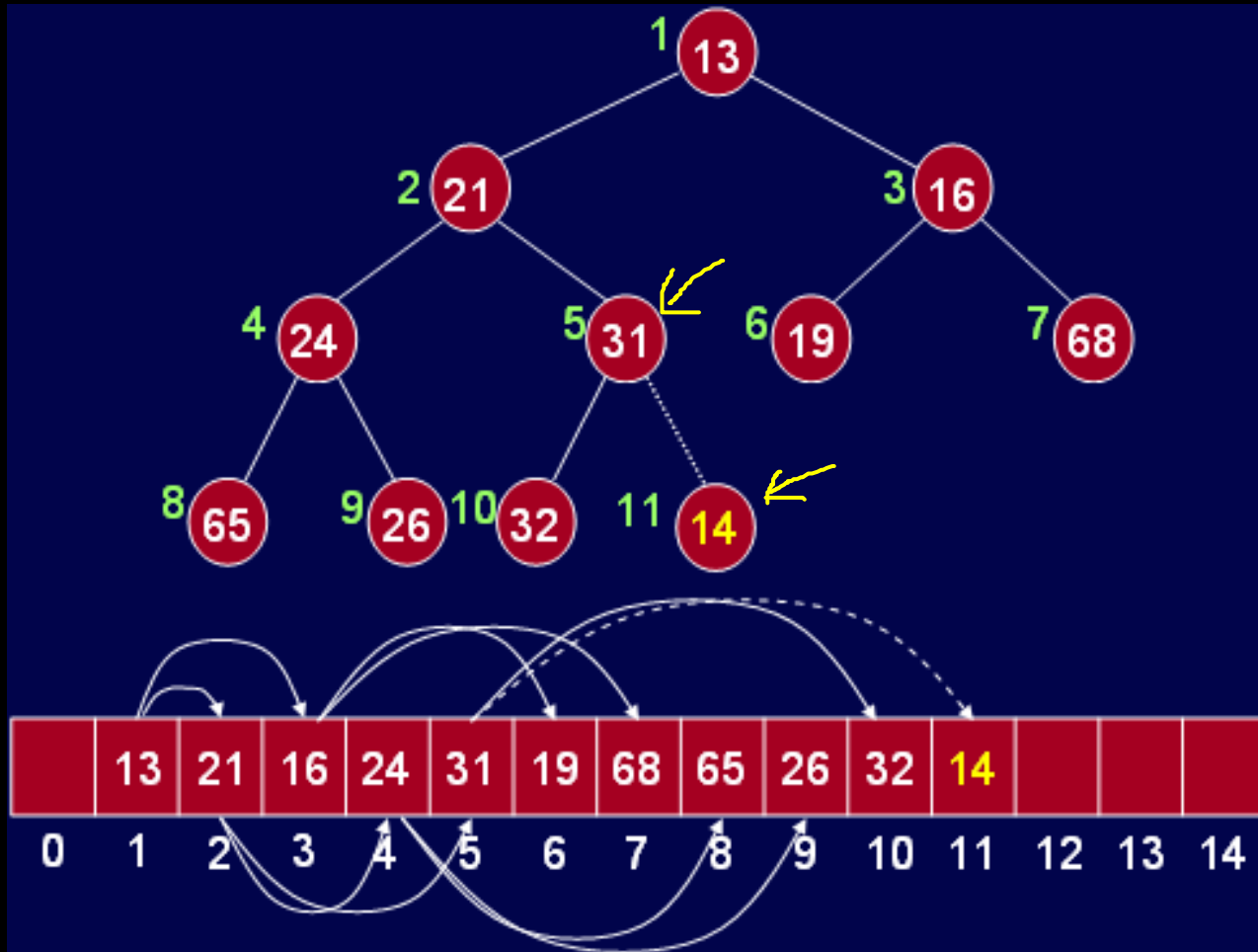
# Inserting into a **Heap**

Assume this  
existing  
heap



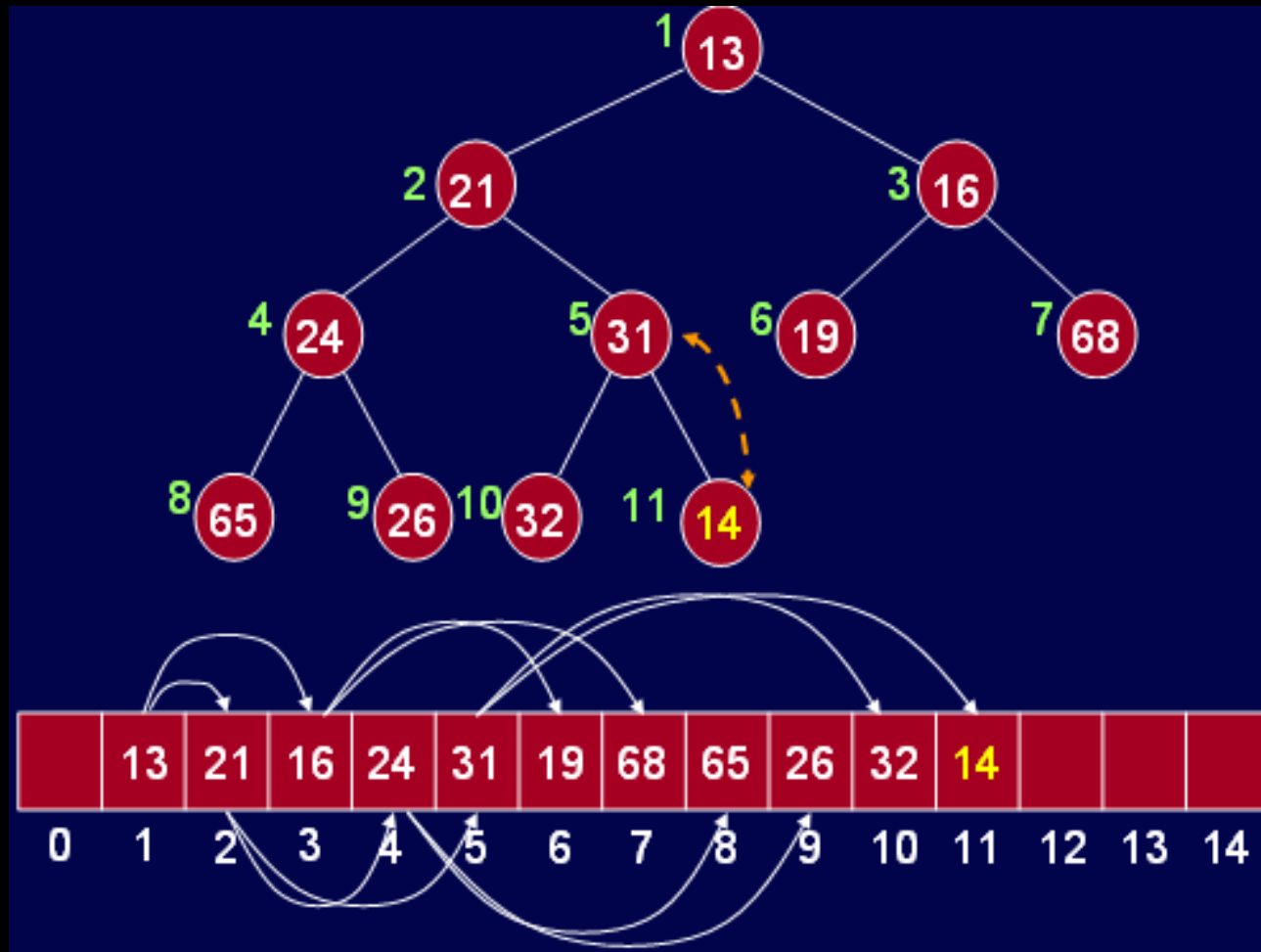
# Inserting into a **Heap**

- `insert(14)`

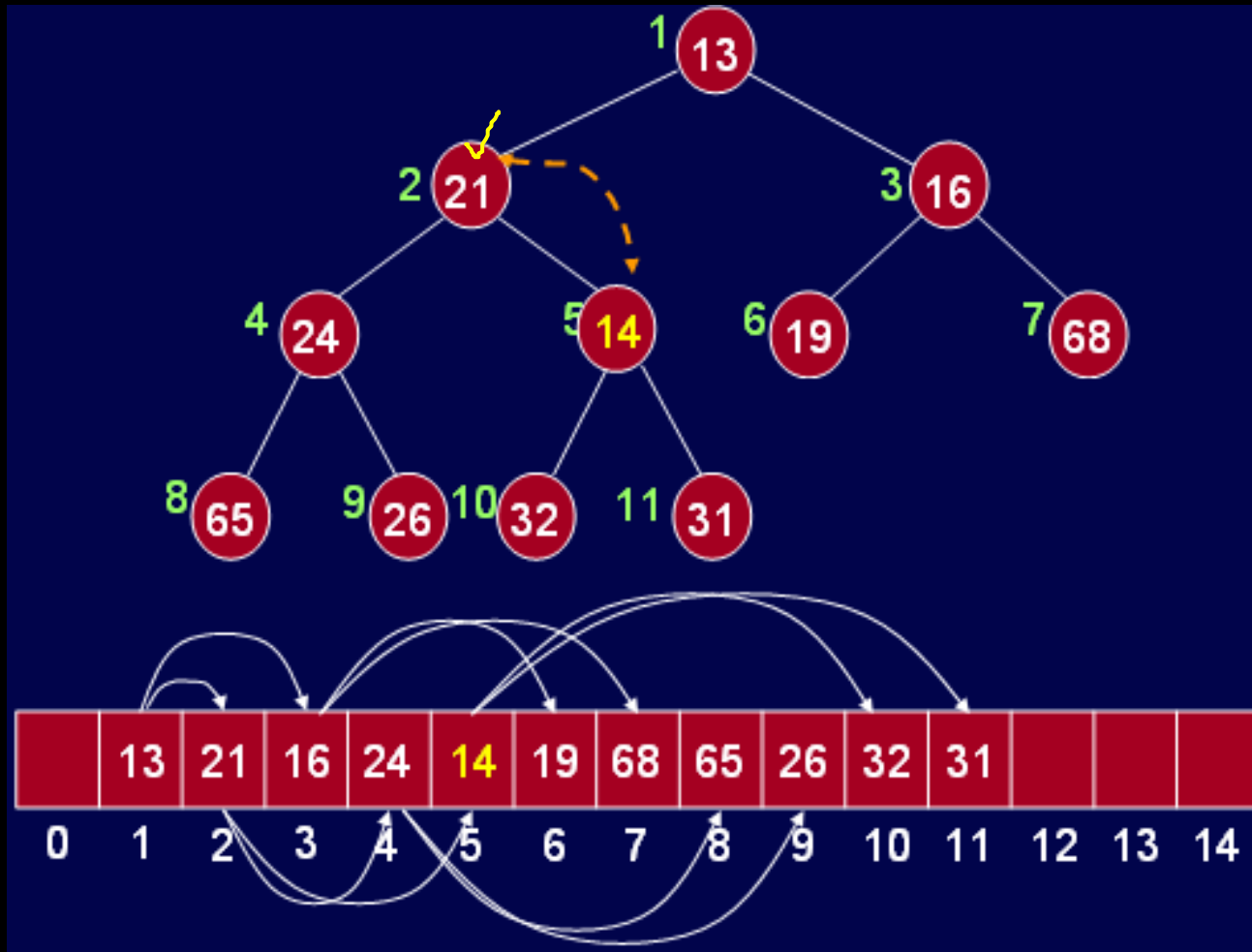


# Inserting into a Heap

- insert(14) with exchange

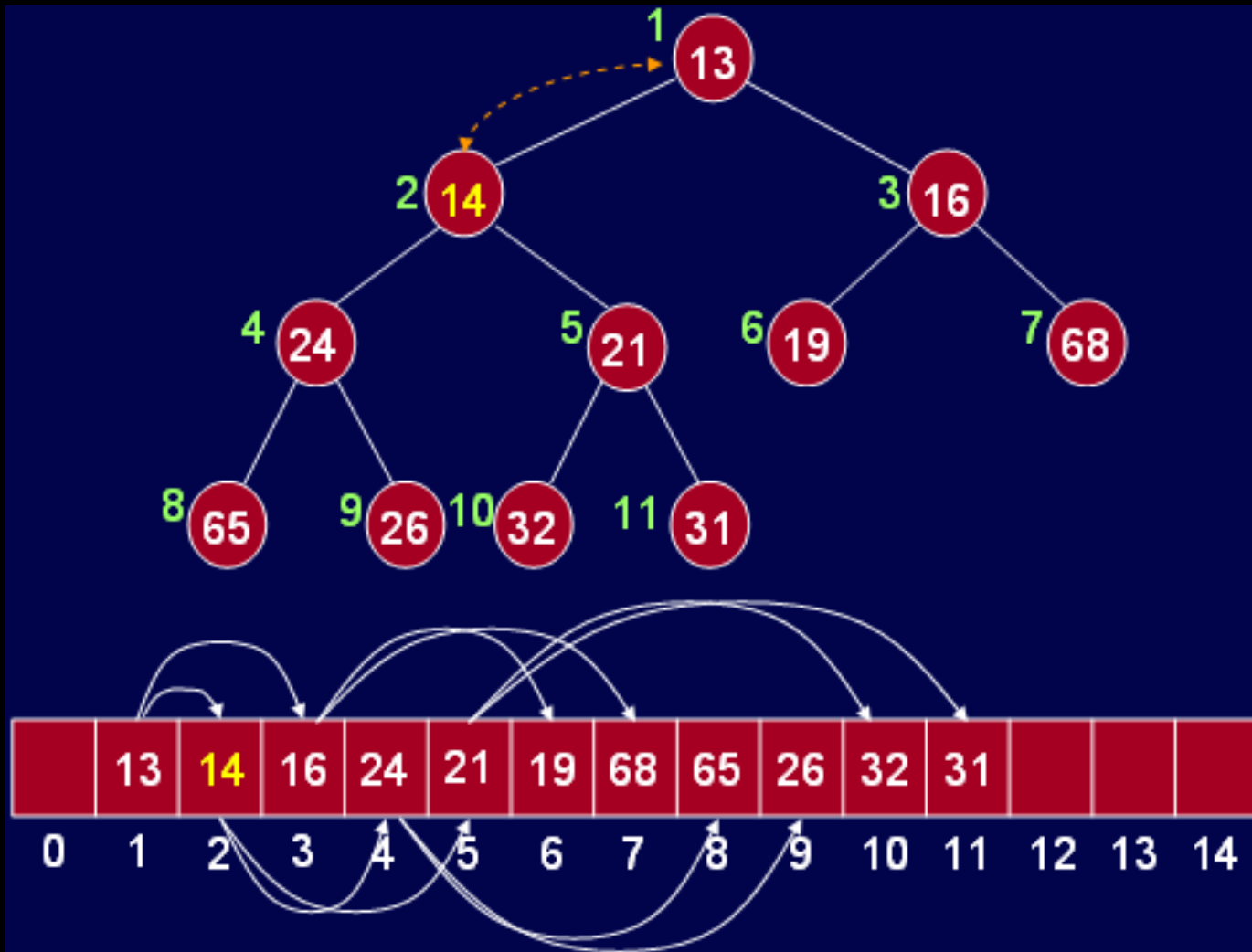


# Inserting into a **Heap**



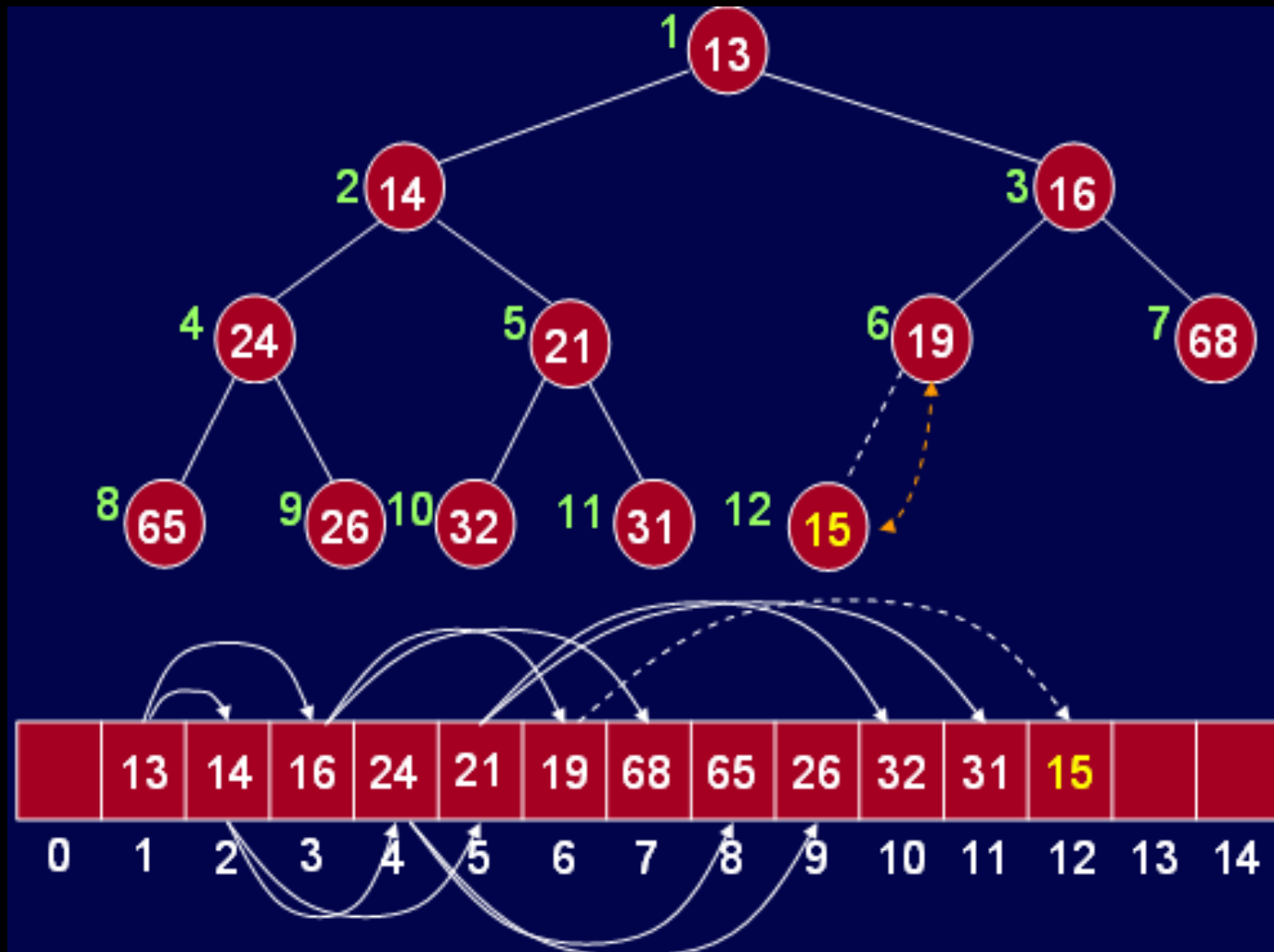


# Inserting into a **Heap**

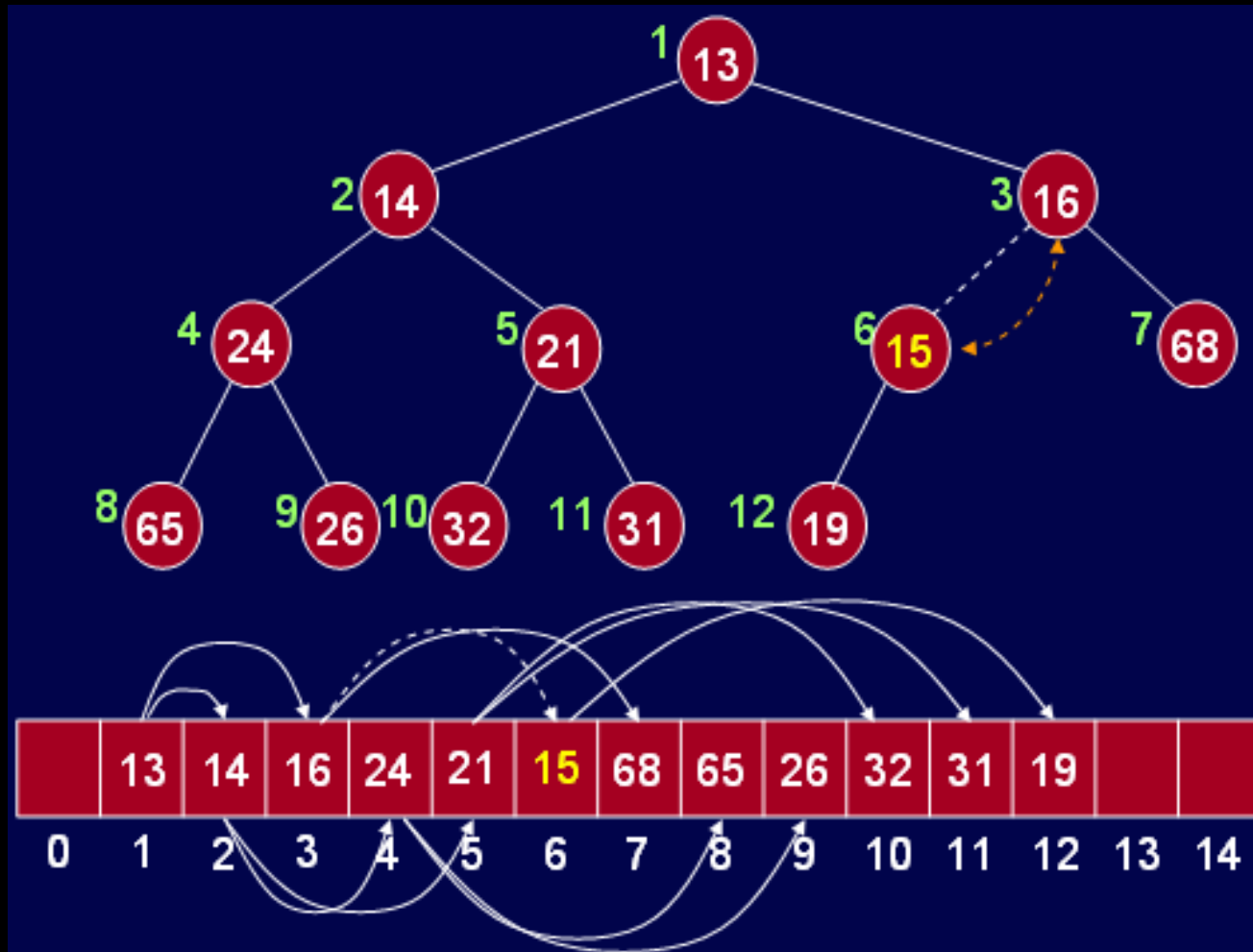


# Inserting into a Heap

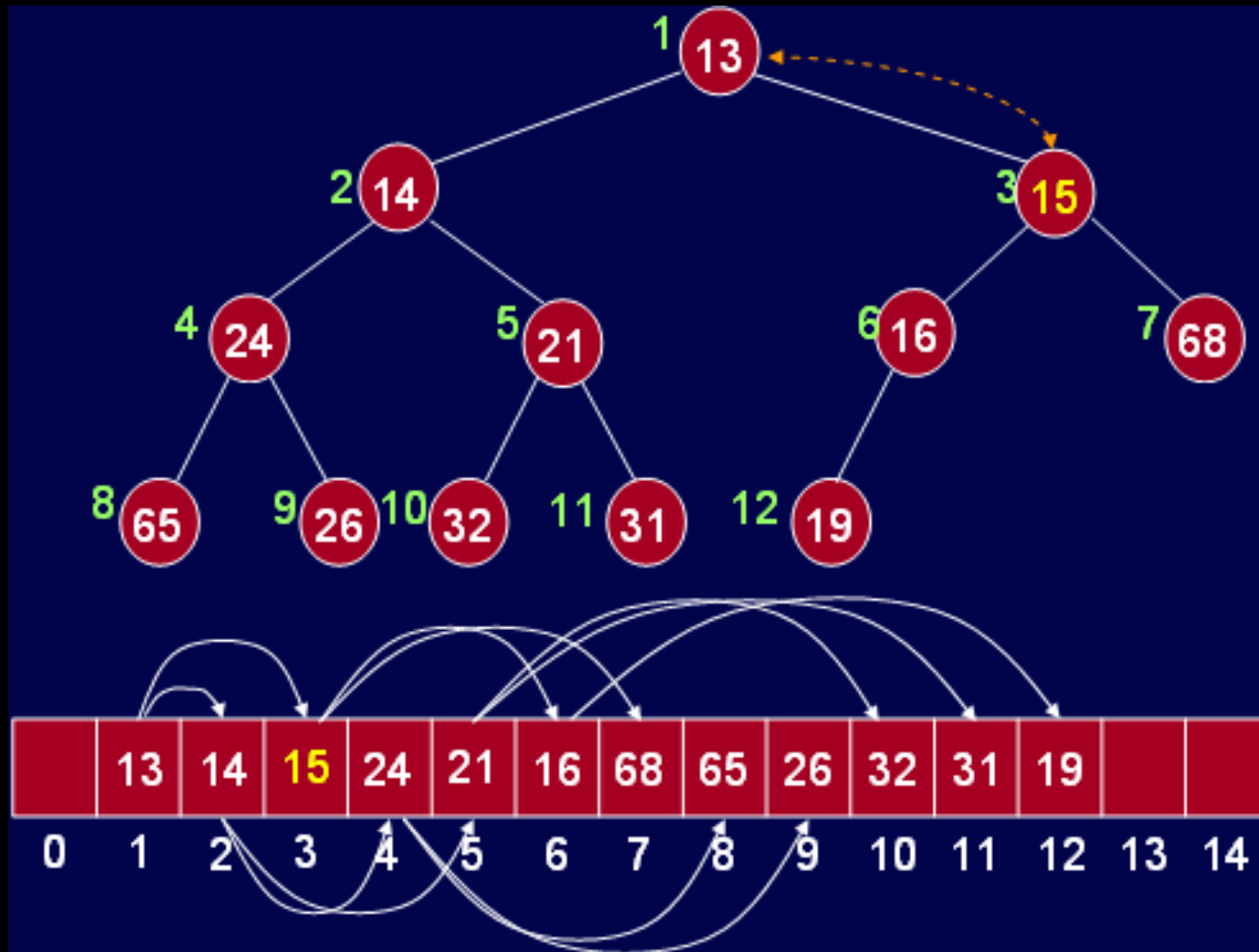
- insert(15) with exchange



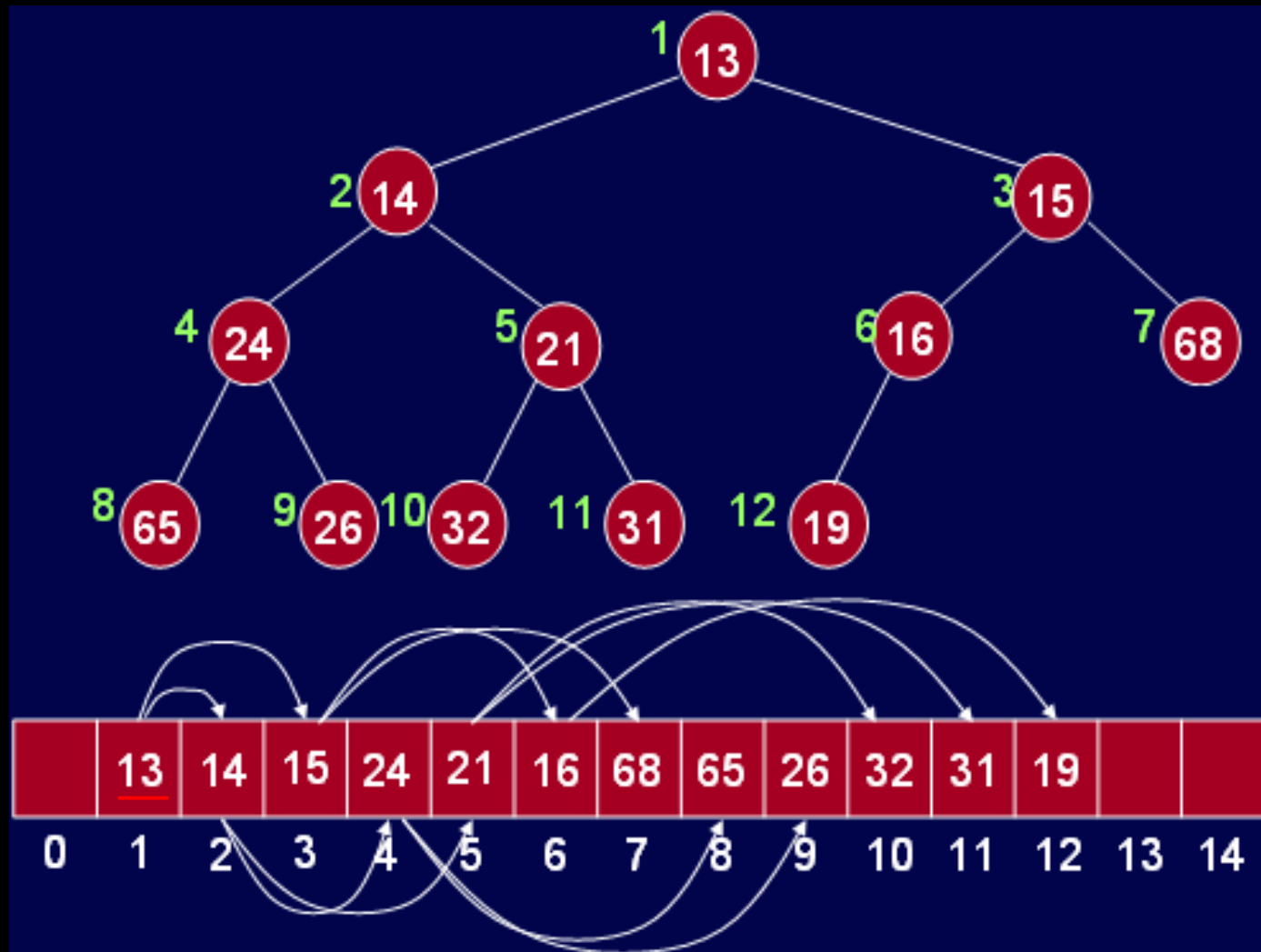
# Inserting into a **Heap**



# Inserting into a **Heap**



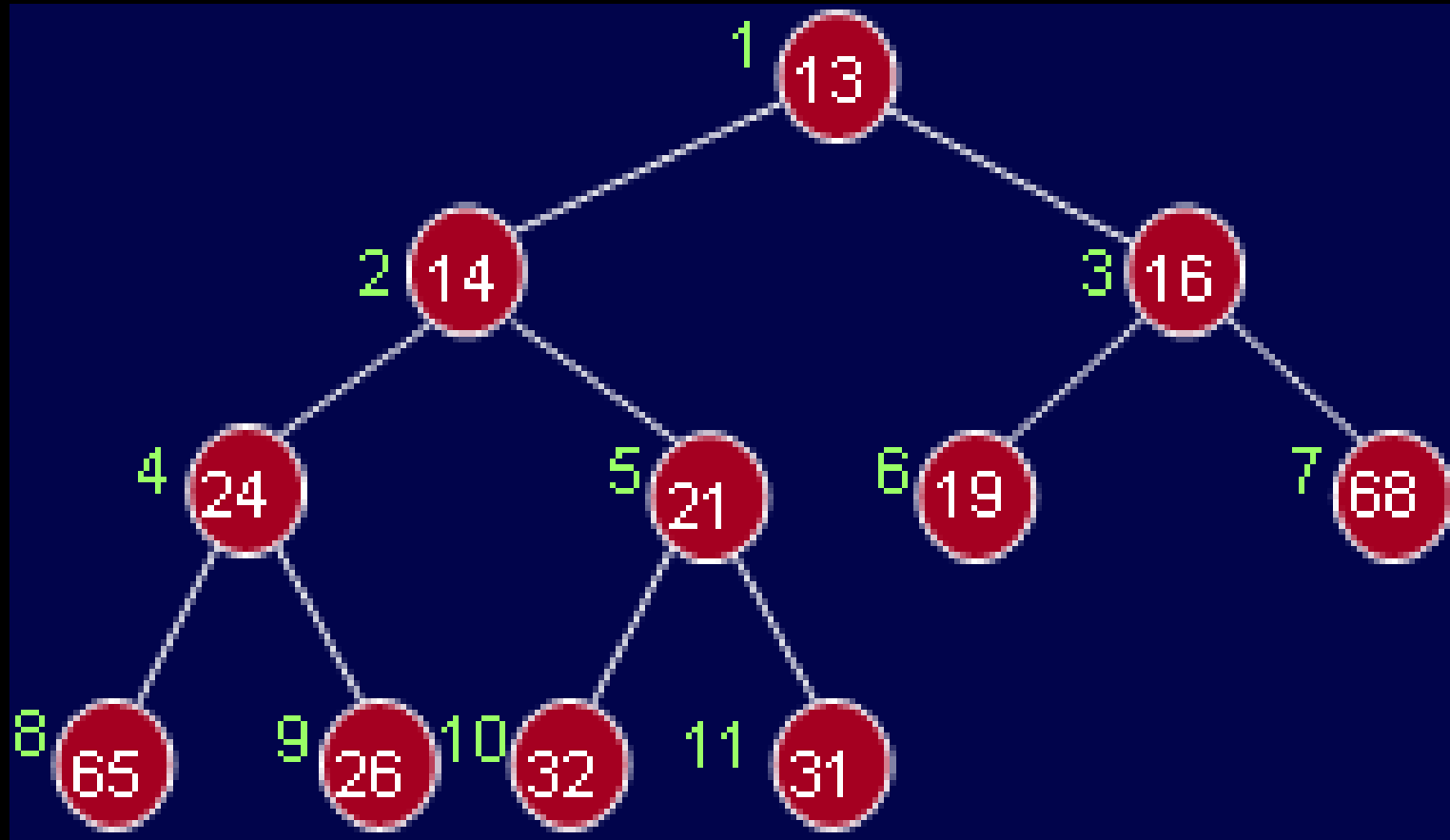
# Inserting into a **Heap**



# DeleteMin from Heap

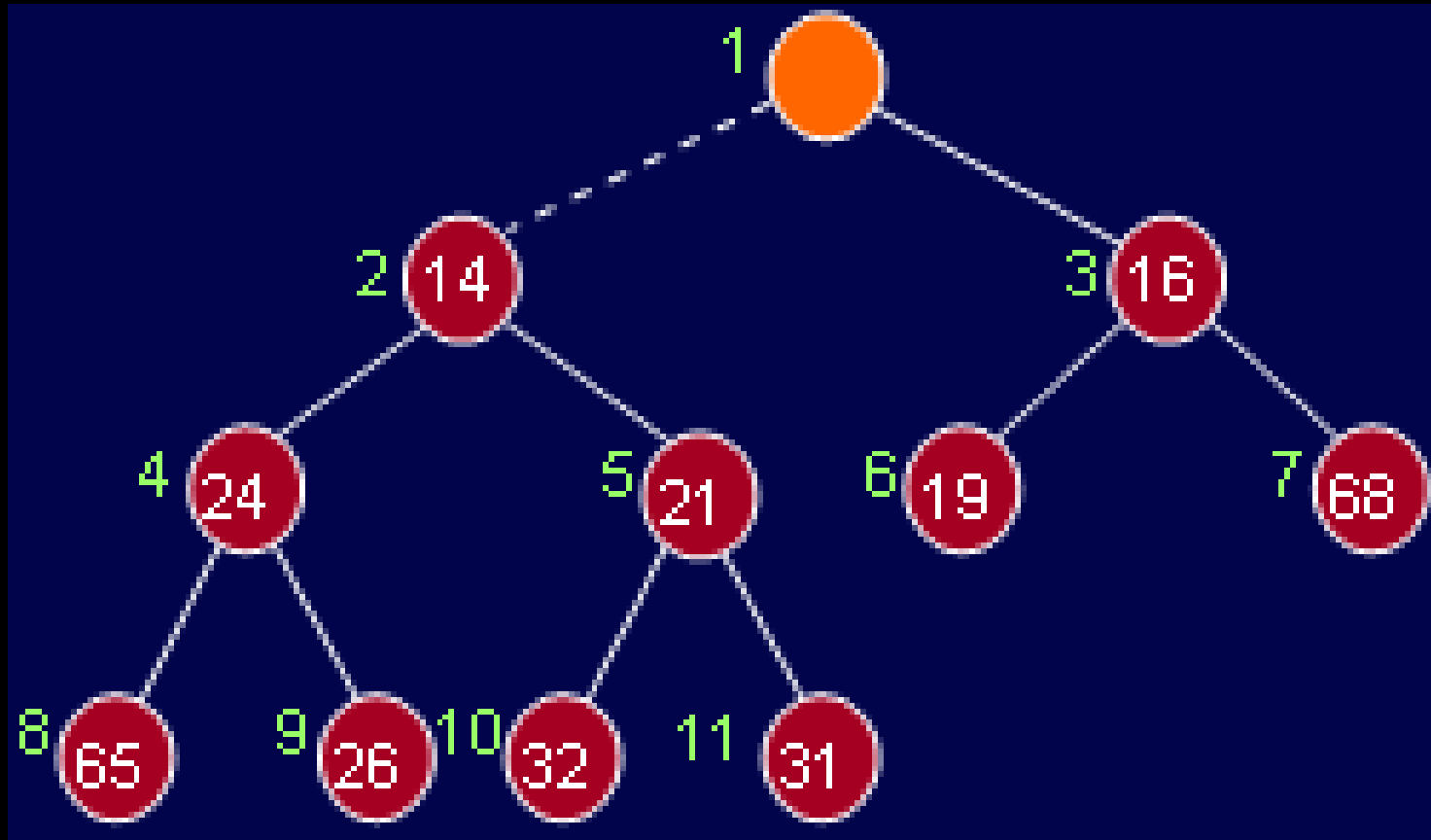
- Finding the minimum is **easy**; it is at the top of the heap.
- Deleting it (or removing it) causes a **hole** which needs to be filled.

# DeleteMin from Heap



# DeleteMin from Heap

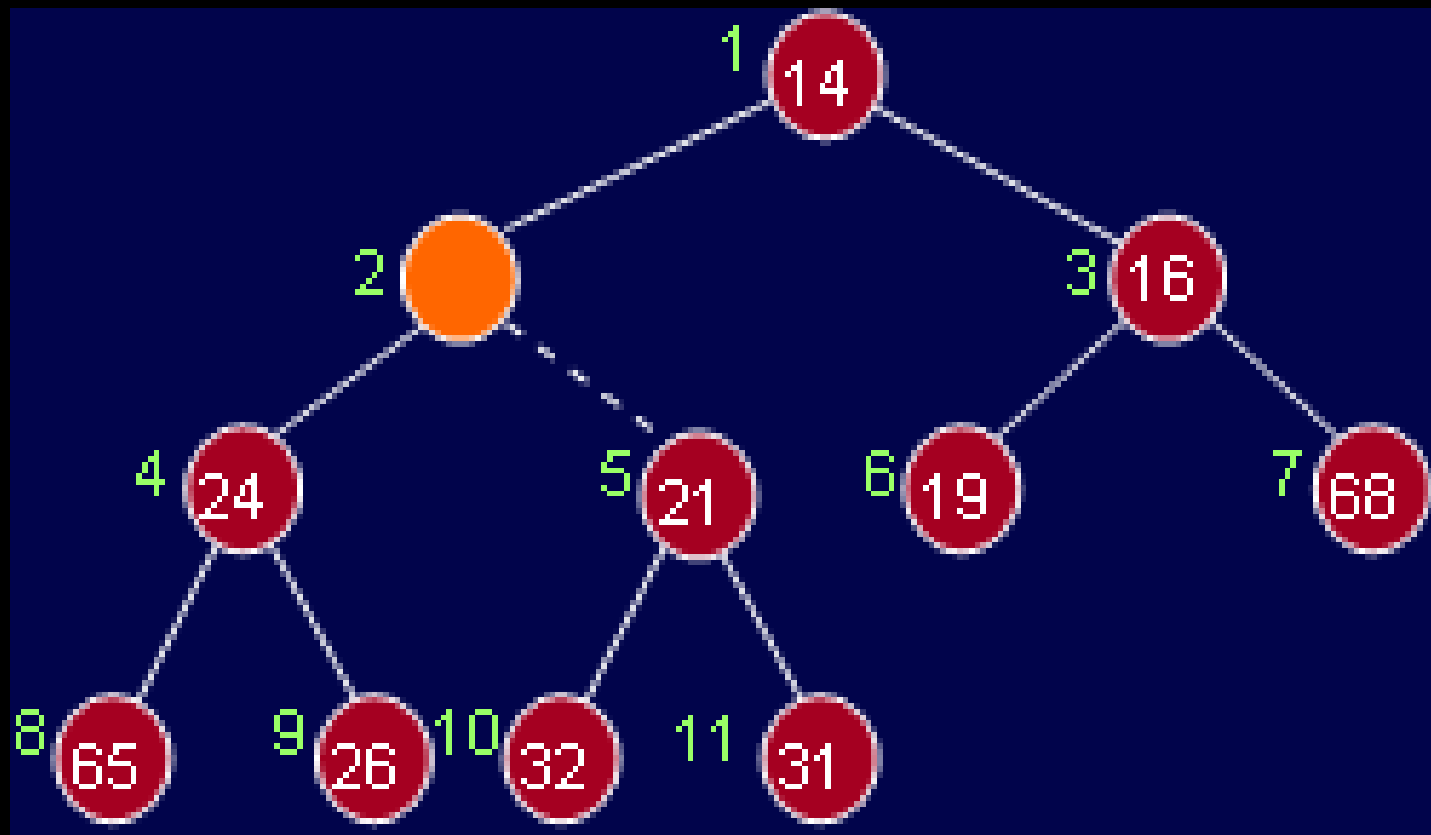
- deleteMin()





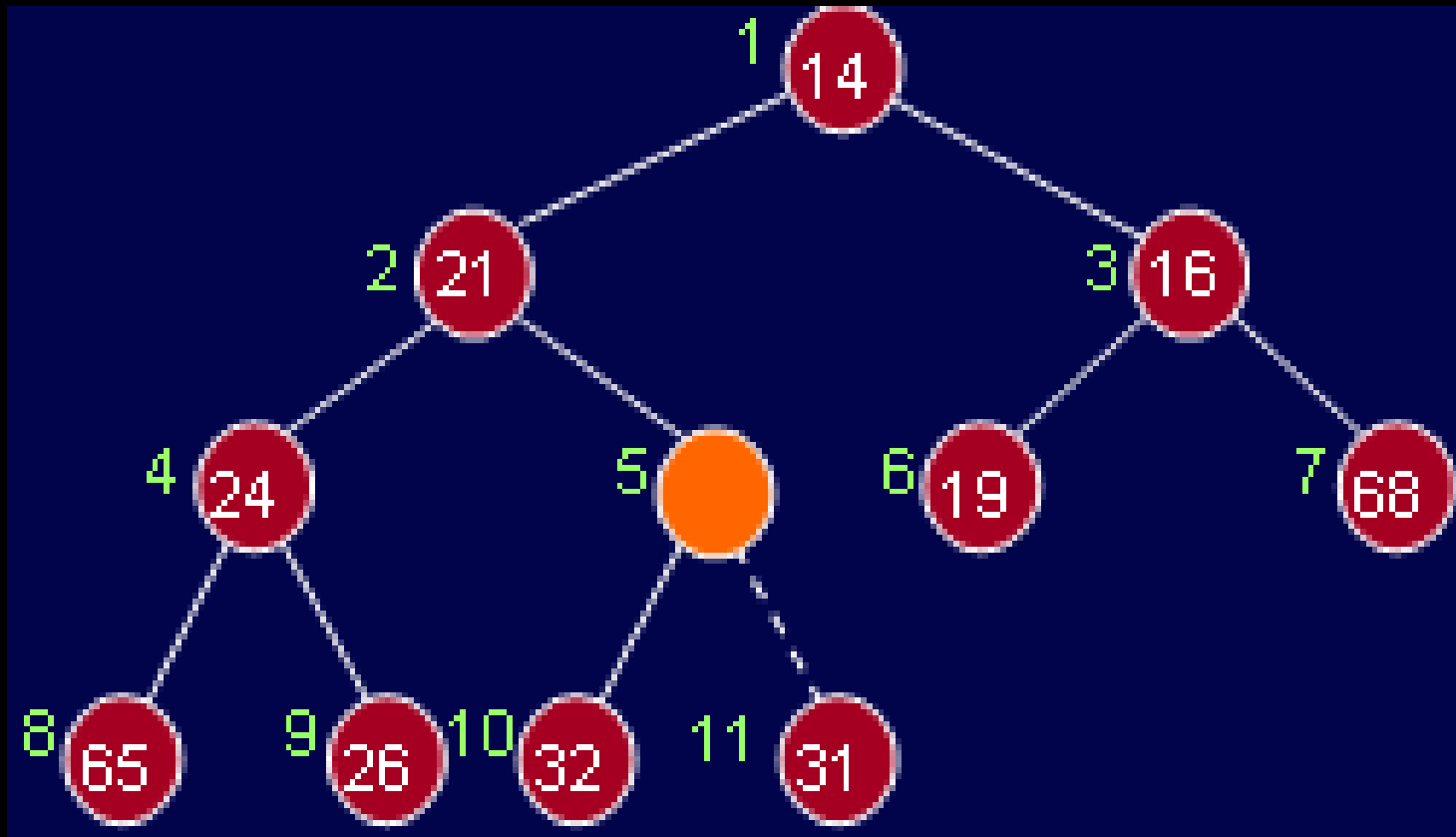
# DeleteMin from Heap

- deleteMin()



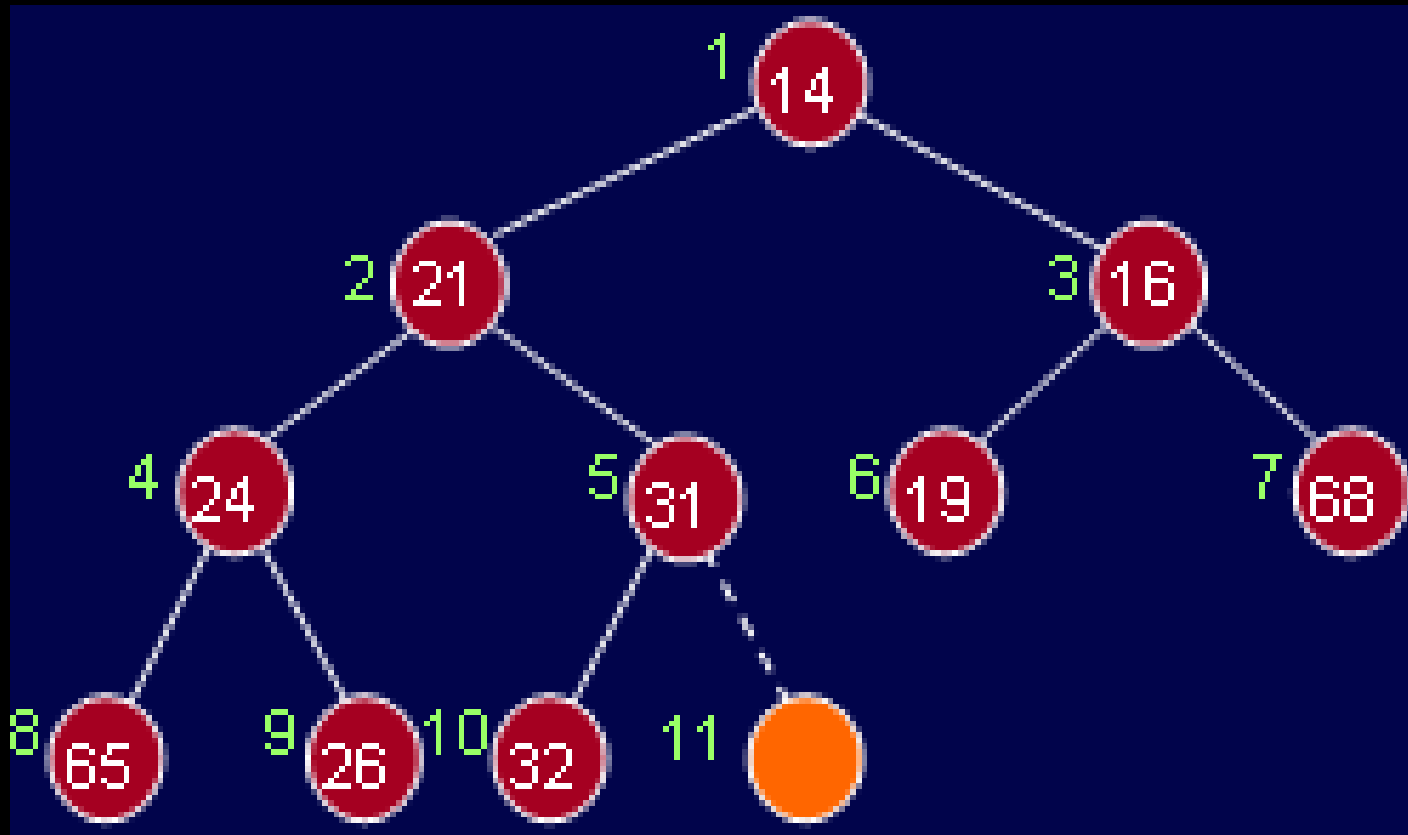
# DeleteMin from Heap

- deleteMin()



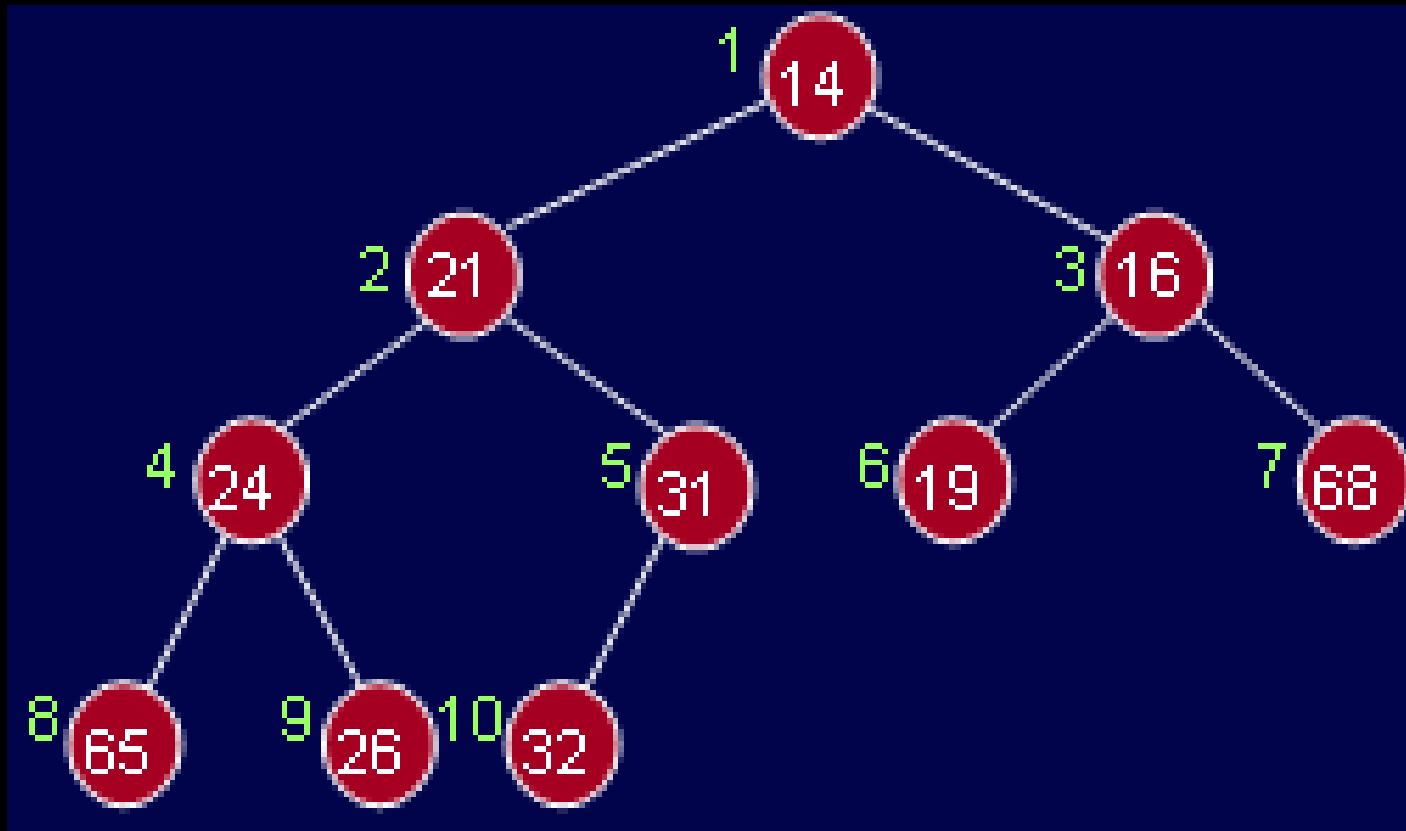
# DeleteMin from Heap

- deleteMin()



# DeleteMin from Heap

- deleteMin(): heap size is reduced by 1.



Thank You ...