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Roll No:- P20-0560

Class:- 3A

Assignment #2
Due: 15th of October, 2021

NOTE: Read this please

1 - Take a print of this assignment (If there is no nearby printer then write it again in the same format along with the spaces provided for answer).
2 - Some questions may need you to do some rough work. Do it on a blank sheet and write on the top of the sheet 'Rough Work'
3 - Solve it using pen
4 – In case you are not on campus and not come then take snaps and make a pdf of the solved assignment and submit via slate.

Q1: Use the principle of resolution to show that the hypothesis "Chohan works hard", "If Chohan works hard then he is a dull boy", "if Chohan is a dull boy, he will not get a job" imply the conclusion "Chohan will not get the job". **(Marks 3)**

Solution:

P1: P
P2: $P \rightarrow q$
P3: $q \rightarrow \neg r$
C: $\therefore \neg r$

C1: P
C2: $\neg P \rightarrow q$
C3: $\neg q \rightarrow \neg r$
C4: r
C5: $\therefore q$
C6: $\neg q$
C7:

From C1 and C2
From C3 and C4
From C6 and C7

Q2: Write the negation of the following statements in English using the logical equivalence of $\sim\forall x\ P(x) = \exists x \sim P(x)$ and $\sim\exists x\ P(x) = \forall x \sim P(x)$. No credit will be given if you didn't use these logical equivalences. **(6 marks)**

a. $\forall x \forall y\ (P(x,y) \rightarrow \sim Q(x,y))$

Solution:
$\exists x \exists y\ (P(x,y) \wedge Q(x,y))$

There exist x and y such that P(x,y) and Q(x,y).

b. $\exists x \forall y\ (P(x,y) \vee \sim Q(x,y))$

Solution:
$\forall x \exists y\ (\neg P(x,y) \wedge Q(x,y))$

For all x there exist y such that not P(x,y) and Q(x,y)

Note: When you are done with simplification of the quantifiers then also use the equivalences of $P \rightarrow Q = \sim P \vee Q$ and Demorgan law to simplify further your answer. I will deduct marks if you ignore this.

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Q3: Write a logical expression corresponding to the following using only the predicates, logical conjunctions, disjunction and nothing else. Assume the domain of x,y = {1,2,3} **(Marks 9)**

$\neg\forall x\,\forall y\,P(x,y) = \exists x\,\neg\forall y\,P(x,y) \Rightarrow \exists x\,\exists y\,P(x,y) \Rightarrow \exists x\,[\neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3)]$

$= [\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \vee [\neg P(2,1) \vee \neg P(2,2) \neg P(2,3)] \vee [\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$

$\exists x\,\neg\forall y\,P(x,y) = \exists x\,\exists y\,\neg P(x,y) \Rightarrow \exists x\,[\neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3)]$

$= [\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \vee [\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \vee [\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$

$\forall x\,\exists y\,\neg P(x,y) = \forall x\,[\neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3)]$

$= [\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \wedge [\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \wedge [\neg P(3,1) \vee \neg P(3,2) \neg P(3,3)]$

Q4: The following question relates to the inhabitants of the island of knights and knaves created by Smullyan. **The knights only speak the truth when they are happy. The knaves always speak lie regardless they are happy or sad.** You encounter two people A and B. Determine if possible what A and B are if they address you in the way. If you cannot determine what these two people are, can you draw any conclusions? **(Marks 20)**

A says "The two of us are both knights" and
B says "A is a knave"

Solution:

P = A is a knight ~P = A is a knave
Q = B is a knight ~Q = B is a knave

Scenario 1: Assume all knights to be happy

CASE 1: Knight , Knight
1) $P \wedge q = T \Rightarrow T \wedge T = T$.
2) $\neg P = T \Rightarrow F \neq T$

q	P	
T	T	T
¬q	¬P	
F	F	F

CASE 2: Knight , Knave
1) $P \wedge q = T \Rightarrow F \neq T$
2) $\neg P = F \Rightarrow T \neq F$

q	P	
F	T	F
¬q	¬P	
T	F	F

CASE 3: Knave , Knave
1) $P \wedge q = F \Rightarrow F = F$
2) $\neg P = F \Rightarrow T \neq F$

q	P	
F	F	F
¬q	¬P	
T	T	T

CASE 4: Knave , Knight
1) $P \wedge q = F \Rightarrow F = F$
2) $\neg P = T \Rightarrow T = T$

q	P	
T	F	F
¬q	¬P	
F	T	F

Conclusion: A is Knave , B is Knight

Scenario 2: Assume all knights to be sad (means the knights will speak lies now)

CASE 1: knight , knight
1) $P \wedge q = F$ ⇒ $F = F$
2) $\neg p = F$ ⇒ $T \neq F$

q	P	
F	F	F
¬q	¬P	
T	T	T

CASE 2: knight , knave
1) $P \wedge q = F$ $F = F$
2) $\neg p = F$ $T \neq F$

q	P	
F	F	F
¬q	¬P	
T	T	T

CASE 3: knave , knave
1) $P \wedge q = F$ $F = F$
2) $\neg p = F$ ⇒ $T \neq F$

q	P	
F	F	F
¬q	¬P	T
T	T	

CASE 4: knave , knight
1) $P \wedge q = F$ ⇒ $F = F$
2) $\neg p = F$ ⇒ $T \neq F$

q	P	
F	F	F
¬q	¬P	
T	T	T

Conclusion: A is NO Conclusion, B is = NO Conclusion .

Q5: Assume that the statement "if it is sunny day then I will not go to beach" is in contrapositive form. Make the following forms of this statement using English sentences **(3 marks)**

Converse: $q \rightarrow p$

If I will not go to beach then it is sunny day.

Contrapositive: $\neg q \rightarrow \neg p$

If I will go to beach then it is not sunny day.

Inverse:

$\neg p \rightarrow \neg q$

If it is not sunny day, then I will go to beach.

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