National University

of Computer & Emerging Sciences FAST Peshawar Campus

Name: _____ Roll No: _____

Program: BS (CS)
Semester: Fall – 2017
Time Allowed: 3: 00 hour
Course: Discrete Structures

Examination: Final
Weightage 50% Total Marks: 62
Date: Dec, 2017
Instructor: Dr. Nouman Azam

NOTE: Attempt all questions.

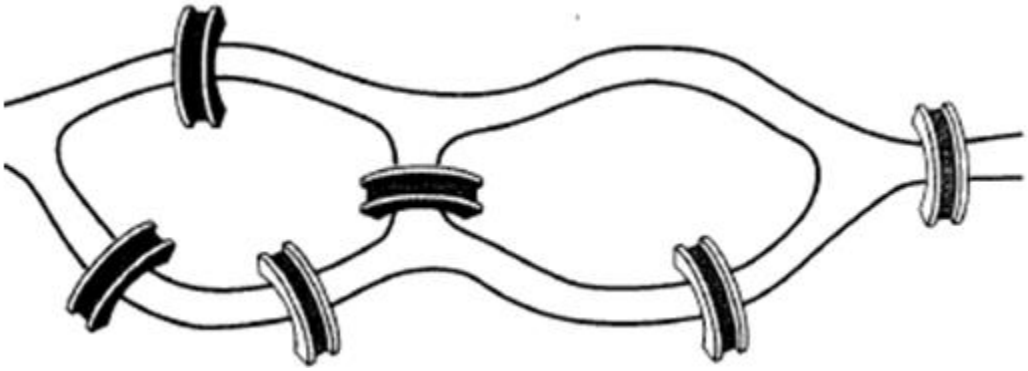
1. Draw Hasse Diagram for the poset ({1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}, |), i.e., the divides relation, (a,b) such that b divides a. Some sample tuples will be (1,2), (2,4) and so on.(**Points 2**)

2. Draw Hasse diagrams for all possible posets on a set containing three elements. Don't label the elements. (There are total of five possibilities) (**Points 5**)

3. Suppose the Hasse diagram of a poset is a disconnected Graph with two connected component. (**Points 1*4 = 4**)
   a. Can we ever have a minimal in this poset? provide arguments.
   b. Can we ever have a maximal in this poset? provide arguments.
   c. Can we ever have a greatest element in this poset? provide arguments.
   d. Can we ever have a least element in this poset? provide arguments.

4. Write the lexographic ordering of these n-tuples. (**Points 1*3 = 3**)
   a. (1,1,2), (1,2,1) _____
   b. (0,1,2,3), (0,1,3,2) _____
   c. (1,0,1,0,1), (0,1,1,1,0) _____

5. Consider the following variant of the famous Konigsberg problem shown below (**Points 3*2 = 6**)



   a. Represent this problem in using a graph form.

   b. What can we conclude from the degrees of the node.

   c. Construct an Euler circuit with starting node A. (write only the sequence of the nodes)

6. Hamilton and Euler paths. (**Points 3*2 = 6**)
    a. Draw a complete graph K_6 and a wheel W_6.

    b. Draw Hamilton path and circuit if any in K_6 and W_6 found in part a.

    c. Draw Euler path and circuit if any in K_6 and W_6 found in part a.

7. How many edges and vertices will be there in the following graphs (**Points 2*3 = 6**)
    a. C_n Edges _____ Vertices _____
    b. W_n Edges _____ Vertices _____
    c. $K_{m,n}$ Edges _____ Vertices _____

8. Starting from the least value of n, for which values of n are the graphs C_n and K_n bipartite. **Points 2**)

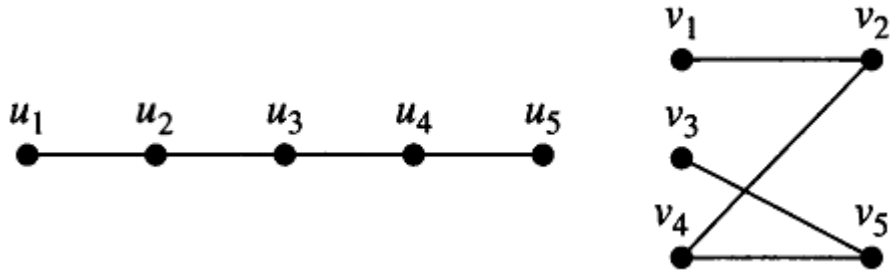
9. What is the chromatic number of (**Points 1*2 = 2**)
    a. K_n _____

    b. $K_{m,n}$ _____

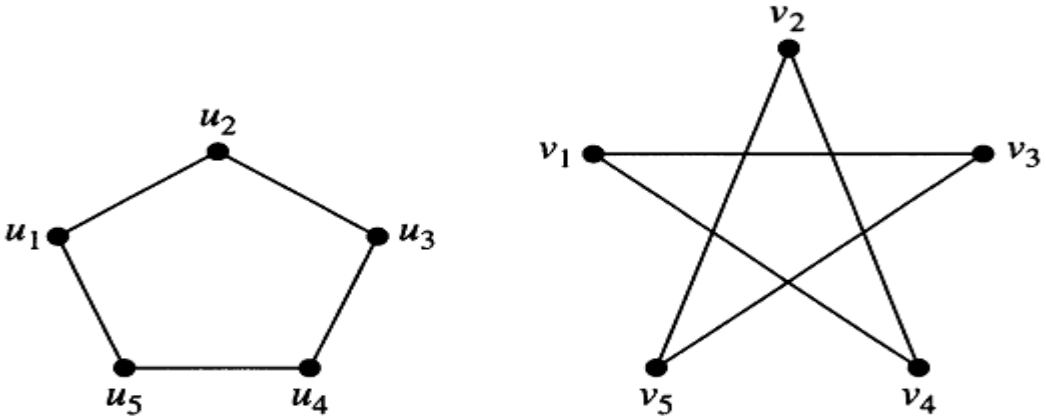
10. We need to schedule the final exam using the least number of time slots for the courses named as A, B, C, D, E, F, G, H. Moreover, we are given that there are **no** common students in the courses A &H, B & H, D & E, D & F, A & B, A & C, C & D. There are however common students in the remaining combination of the courses. Using Graph coloring, show the minimum number of time slots that are needed to conduct the exam. (**Points 3**)

11. Determine whether the given pair of graphs exhibit an isomorphism or provide a rigorous argument that none exists (follow all the steps). If they exhibit isomorphism than provide the assignment details of the nodes. Finally after making the assignment, double check that the assignment is correct by constructing adjacency matrices (**Points 2*2 = 4**)

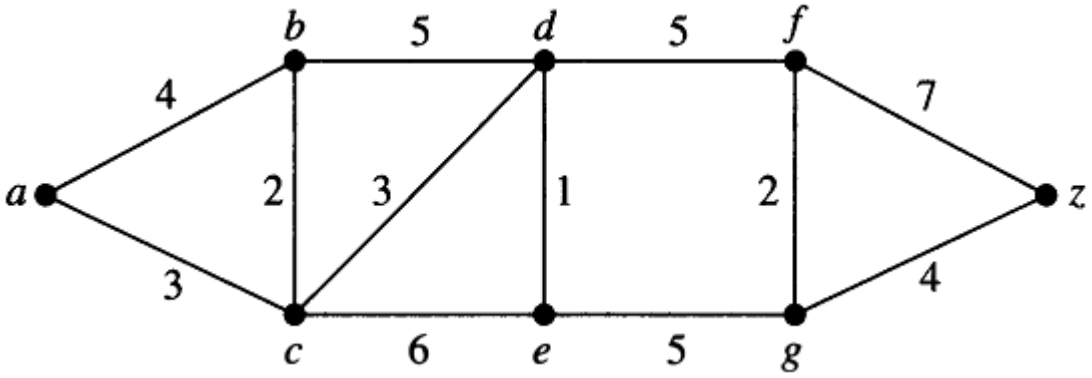
a.



b.



12. Using Dijistra algorithm, find the minimum path from node **d** to all other nodes. Show all the steps and the tables. (**Points 3**)



13. Notations. (**Points 1*4 = 4**)
    a. Represent (A ∩ B) - (A U (B - A)) using an ordered rooted tree.

    b. Write this expression in prefix notation.

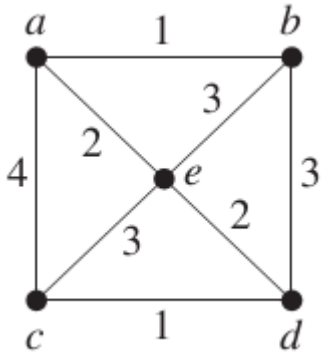
    c. Write this expression postfix notation.

    d. Write this expression infix notation.

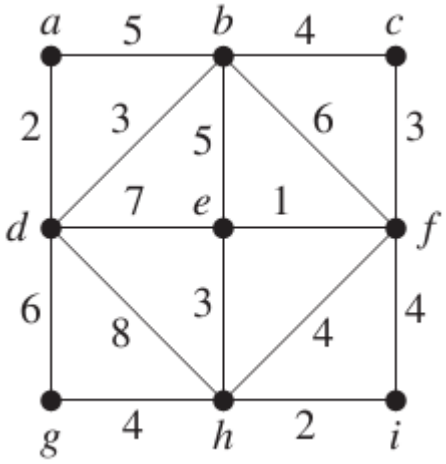
14. First draw W_5 and $K_{2,3}$ and then construct their respective spanning trees using depth first search. (**Points 2*2 = 4**)

15. Use Prim's algorithm to find a minimum spanning tree for the given weighted graphs. (Note: show detailed work by listing the edges you considered and the corresponding weights. Also show the total weights for the spanning tree) (**points 2*2 = 4**)

a.



b.



16. Use Krushal's algorithm to find a minimum spanning tree for the weighted graphs in Question 9 (a) and (b). (Note: show detailed work by listing the edges you considered and the corresponding weights) (**points 2*2 = 4**)

    a.

    b.

Good Luck