My Application ID Number: 19615.

Quiz Answers:

1. The apparent paradox in the puzzle arises from a visual illusion created by the way the pieces are rearranged. Although it may seem that the four pieces fit together perfectly to form a larger rectangle, upon closer inspection, there is a hidden gap or overlap between the pieces. This gap or overlap results in an illusion of a larger total area without actually adding any extra material. This phenomenon is similar to other paradoxes such as the 'Chessboard paradox' or 'Missing square puzzle,' which exploit our perception of shapes and sizes to create apparent contradictions.

2. In a problem with 2n boxes of n different colors, labeled with n different uppercase and lowercase letters, Kayla can determine the secret numbers if and only if each color has one uppercase letter and one lowercase letter assigned to it. This ensures that the differences and sums reported by Maya and Nathan provide enough information to solve for the secret numbers in each color group.

3. a. To cause the universe to explode in both cases, you can ask the following question:

"Would you say that the sky is blue?"

This question puts both the number theorist and the analyst in a difficult situation because neither can truthfully answer 'yes' or 'no' to it. If the number theorist tries to answer truthfully, they would have to affirm something they don't know is true, leading to a paradox. Similarly, if the analyst tries to lie, they would also affirm something they don't know is true, also leading to a paradox.

4. For the scenario where the value of k is unknown, we can follow a different strategy:

1. Put the Mathcampers in a random order.
2. Serve slices of cake in that order until the cake runs out.

By ensuring that the Mathcampers are randomly ordered, regardless of the value of k, each Mathcamper will still have a probability of exactly kx_i of getting a slice of cake. This is because the probability of each Mathcamper receiving a slice of cake is independent of the order in which they are served, as long as each slice is served randomly.

5. To find the minimum length of thread needed to stitch every square in an $m \times n$ rectangular grid, we can observe the following:

Each square requires two stitches, one from the top-right corner to the bottom-left corner and one from the top-left corner to the bottom-right corner. These stitches form the diagonal of the square. The length of each diagonal stitch is 2$2$ units. There are $(m-1)$ vertical stitches and $(n-1)$ horizontal stitches in the grid. Therefore, the total length of thread needed is:

$$\text{Length}=2\times2\times((m-1)+(n-1))$$

$$=22\times(m+n-2)$$

$$=22(m+n)-42$$

$$=mn(2+1)-42$$

This provides a general formula for the minimum length of thread needed for an $m\times n$ rectangular grid.

6. To prove that every system of unchanging rings has an unchanging constant n, such that every circle contains exactly n marked points, and every marked point is contained in exactly n circles, we can use the following reasoning:

1. Let N be the total number of marked points in the system.
2. Consider any circle in the system. By the first rule, if two circles meet at a marked point, they share exactly two marked points. Therefore, each circle contributes 2$2$ marked points for every marked point it intersects.
3. Similarly, by the second rule, if two marked points lie on a common circle, they must share exactly two common circles. Therefore, each marked point is contained in 2$2$ circles for every circle it is part of.
4. Using these observations, we can form the equation $N=2n\cdot C=2n\cdot M$, where C is the total number of circles and M is the total number of marked points.
5. Solving for n, we get $n=2CN=2MN$. This value of n is constant for the entire system.

Therefore, every system of unchanging rings has an unchanging constant n, such that every circle contains exactly n marked points, and every marked point is contained in exactly n circles.