S

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Complex Variable & Transform

3rd Semester (BE Mechanical)

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Q1) Find the values of z for which sin z = zi

Solution:

$\text{Sin } z = \dfrac{e^{iz} - e^{-iz}}{zi}$

Given, sin z = zi

$= \quad \dfrac{e^{iz} - e^{-iz}}{zi} = zi$

$= \quad e^{iz} - e^{-iz} = zi \times zi$

$= \quad e^{iz} \underline{-\ 1} \atop e^{iz} \quad = \ -4$

$= \quad (e^{iz}) - 1 = -4\, e^{iz}$

$= \quad (e^{iz})^2 - 1 = -4\, e^{iz}$

$= \quad (e^{iz})^2 + 4\, e^{iz} - 1 = 0$

$= \quad (e^{iz})^2 + 4\, e^{iz} + 4 = 5$

$= \quad (e^{iz} + 2)^2 = 5$

$= \quad e^{iz} = 2 + \sqrt{5}$

$= \quad iz = 1n \ (2+ \sqrt{5})$

$= \quad z = -I\ 1n \ (2+ \sqrt{5})$

Q2) Find all Roots of Equation Sin z = cos hy

Solution:

Sin z = cos hy

Sin (x + y) – cos hy

Sin x coshy – cos x sin hy – cos hy

This will hold if:

Sin x cos hy – cos hy

Cos x sin hy = 0

Now if sin hy = 0, then cos hy = 1

Hence,

Sin x = cos hy = 1

So, no solution in this case

Now if cos x = 0

$$X = \frac{(2x + 1)}{2}\pi$$

So, for the values of x, sin x = ± 1

Now cos hy > 0 and cos hy > 0 then we must have sin x = ± 1 and cos hy = cos hy

So, we get

$$y = \pm 4$$

$$X = \frac{(4n + 1)}{2}\pi$$

Hence, solution is

$$X = \frac{(4n + 1)}{2}\pi \pm 4i$$