Source: Ross-Simulation

3.2 Using random numbers to evaluate integrals

One of the earliest applications of random numbers was in the computation of integrals. Let $g(x)$ be a function and suppose we wanted to compute θ where

$$\theta = \int_0^1 g(x)\,dx$$

To compute the value of θ, note that if U is uniformly distributed over (0, 1), then we can express θ as

$$\theta = E\,[g(U)]$$

If $U_1, \ldots, U_k$ are independent uniform (0, 1) random variables, it thus follows that the random variables $g(U_1), \ldots, g(U_k)$ are independent and identically distributed random variables having mean θ. Therefore, by the strong law of large numbers, it follows that, with probability 1,

$$\sum_{i=1}^{k} \frac{g(U_i)}{k} \to E\,[g(U)] = \theta \quad \text{as } k \to \infty$$

Hence we can approximate θ by generating a large number of random numbers u_i and taking as our approximation the average value of $g(u_i)$. This approach to approximating integrals is called the *Monte Carlo* approach.