Discrete Probability Distributions

Name	Parameters	Probability Mass Function	cdf	E(X)	V(X)	Equation Nos	Note
Bernoulli	p $(q = 1 - p)$	$p(x) = \begin{cases} p, & x = 1 \\ q, & x = 0 \\ 0, & \text{otherwise} \end{cases}$		p	$1 - p$	11	
Binomial	n, p $(q = 1 - p)$	$p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0, 1, 2, \dots, n \\ 0, & \text{otherwise} \end{cases}$		np	npq	12, 13, 14	1, 2
Geometric	p $(q = 1 - p)$	$p(x) = \begin{cases} q^{x-1}p, & x = 1,2, \dots \\ 0, & \text{otherwise} \end{cases}$		$\frac{1}{p}$	$\frac{q}{p^2}$	15, 16, 17	
Negative Binomial	k, p $(q = 1 - p)$	$p(y) = \begin{cases} \binom{y-1}{k-1} q^{y-k}p^k, & y = k, k+1, k+2, \dots \\ 0, & \text{otherwise} \end{cases}$		$\frac{k}{p}$	$\frac{kq}{p^2}$	18	
Poisson	α	$p(x) = \begin{cases} \frac{e^{-\alpha}\alpha^x}{x!}, & x = 0, 1, \dots \\ 0, & \text{otherwise} \end{cases}$	Table A.4	α	α	19, 20	3

Notes

1. Poisson distribution can be used to approximate binomial distribution. https://online.stat.psu.edu/stat414/lesson/12/12.4
2. Normal distribution can be used to approximate binomial distribution. https://online.stat.psu.edu/stat414/lesson/28/28.1
3. Normal distribution can be used to approximate Poisson distribution. https://online.stat.psu.edu/stat414/lesson/28/28.2

Continuous Probability Distributions

Name	Parameters	Probability Density Function / Cumulative Distribution Function	E(X)	V(X)	Equation Nos	Note
Uniform	a, b	$f(x) = \begin{cases} \frac{1}{b-a}, a \leq x \leq b \\ 0, \text{otherwise} \end{cases}$ $F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x \geq b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	22 23 24 25	
Exponential	λ	$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$ $F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x \geq 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	26 27 28	1
Gamma	β, θ	$f(x) = \begin{cases} \frac{\beta\theta}{\Gamma(\beta)}(\beta\theta x)^{\beta-1}e^{-\beta\theta x}, x > 0 \\ 0, & \text{otherwise} \end{cases}$ $F(x) = \begin{cases} 1 - \int_x^\infty \frac{\beta\theta}{\Gamma(\beta)}(\beta\theta t)^{\beta-1}e^{-\beta\theta t}dt, & x > 0 \\ 0, & x \leq 0 \end{cases}$	$\frac{1}{\theta}$	$\frac{1}{\beta\theta^2}$	34 35 36 37	2

Name	Parameters	Probability Density Function / Cumulative Distribution Function	E(X)	V(X)	Equation Nos	Note
Erlang	k, θ $k\ integer$ k: Shape θ: Scale	$f(x) = \begin{cases} \frac{k\theta}{\Gamma(k)}(k\theta x)^{k-1}e^{-k\theta x}, x > 0 \\ 0, \quad \text{otherwise} \end{cases}$ $F(x) = \begin{cases} 1 - \sum_{i=0}^{k-1} \frac{e^{-k\theta x}(k\theta x)^i}{i!}, & x > 0 \\ 0, & x \leq 0 \end{cases}$	$\frac{1}{\theta}$	$\frac{1}{k\theta^2}$	39	3 4
Normal	μ, σ	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), -\infty < x < \infty$ $F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right) dt$ Use Table A.3	μ	σ^2	41 42	5 6
Weibull	ν, α, β ν: Location α: Scale β: Shape	$f(x) = \begin{cases} \frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1} exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], & x \geq \nu \\ 0, & \text{otherwise} \end{cases}$ $F(x) = \begin{cases} 0, & x < \nu \\ 1 - exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], & x \geq \nu \end{cases}$	See Text	See Text	46 50	

Name	Parameters	Probability Density Function / Cumulative Distribution Function	E(X)	V(X)	Equation Nos	Note
Triangular	a, b, c	$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)}, & a \le x \le b \\ \frac{2(c-x)}{(c-b)(c-a)}, & b \le x \le c \\ 0, & \text{elsewhere} \end{cases}$ $F(x) = \begin{cases} 0, & x \le a \\ \frac{(x-a)^2}{(b-a)(c-a)}, & a < x \le b \\ 1 - \frac{(c-x)^2}{(c-b)(c-a)}, & b < x \le c \\ 1, & x < c \end{cases}$	$\frac{a+b+c}{3}$	See note 2	51 52 54	7 8

Notes

1. Exponential distribution exhibits memoryless property: $P(X > s + t|X > s) = P(X > t)$
2. If in a gamma distribution β is an <u>integer</u>, the gamma random variable is sum of β exponential random variables each with parameter $\beta\theta$. When β = 1, the gamma distribution is an exponential distribution with parameter θ.
3. Erlang distribution is a gamma distribution where $\beta = k$ where k is an integer.
4. The $\sum_{i=0}^{k-1} \frac{e^{-k\theta x}(k\theta x)^i}{i!}$ term in Erlang cdf is sum of Poisson terms with mean $\alpha = k\theta x$. Tables of cumulative Poisson distribution may be used to calculate cdf of Erlang distribution.
5. If $X \sim N(\mu, \sigma)$, a transformation $Z = \frac{x-\mu}{\sigma}$ is required before Table A.3 can be used.
6. Table A.3 does not list the values of $z < 0$. Use symmetry of normal distribution to obtain the value of cdf for $z < 0$.
7. For triangular distribution Mode = b = 3E(X) – (a + c). Also $\frac{2a+c}{3} \le E(X) \le \frac{a+2c}{3}$
8. Variance of triangular distribution $V(X) = \left[\frac{(a+b+c)^2}{18}\right] - \left[\frac{(ab+ac+bc)}{6}\right]$