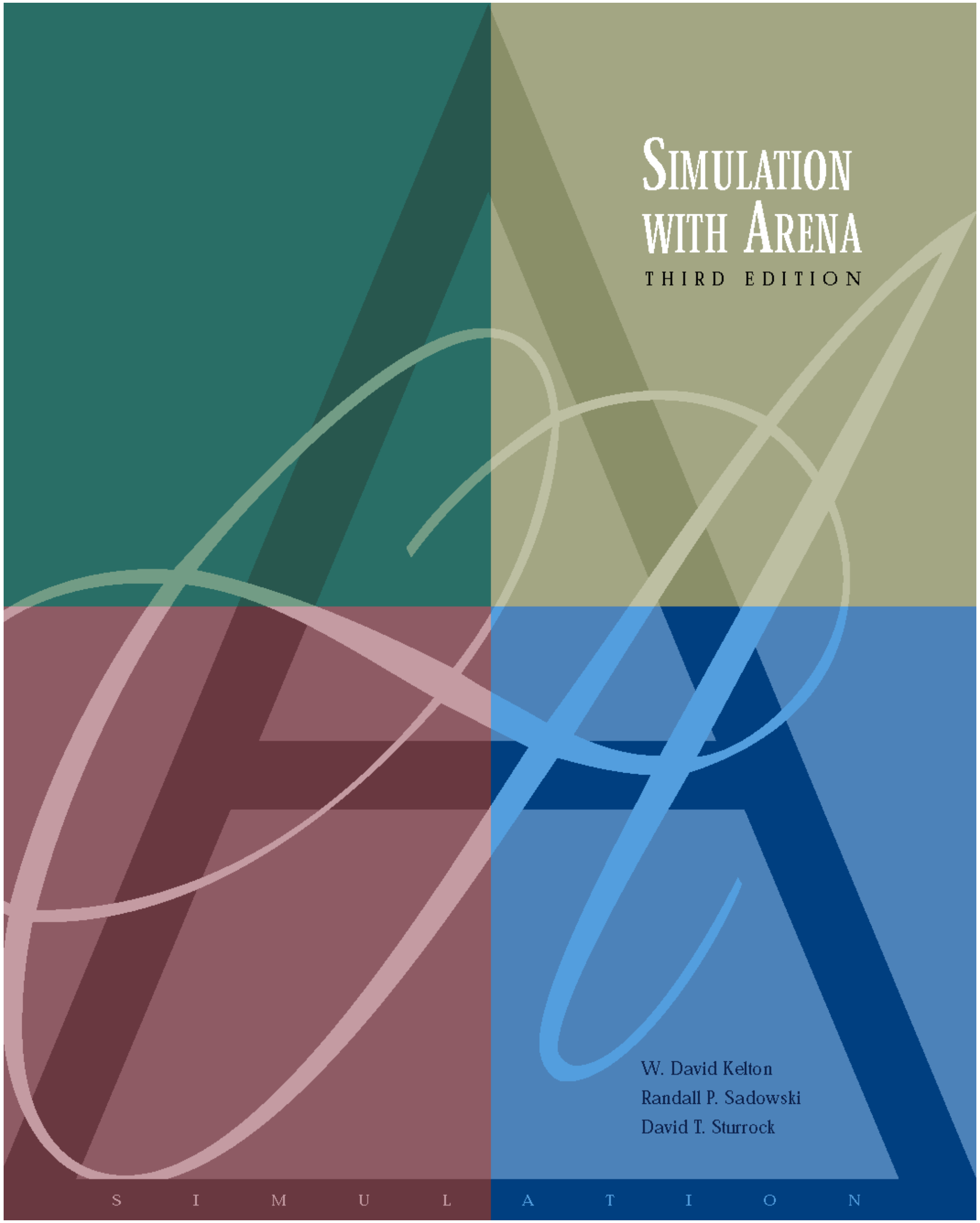


Fundamental Simulation Concepts

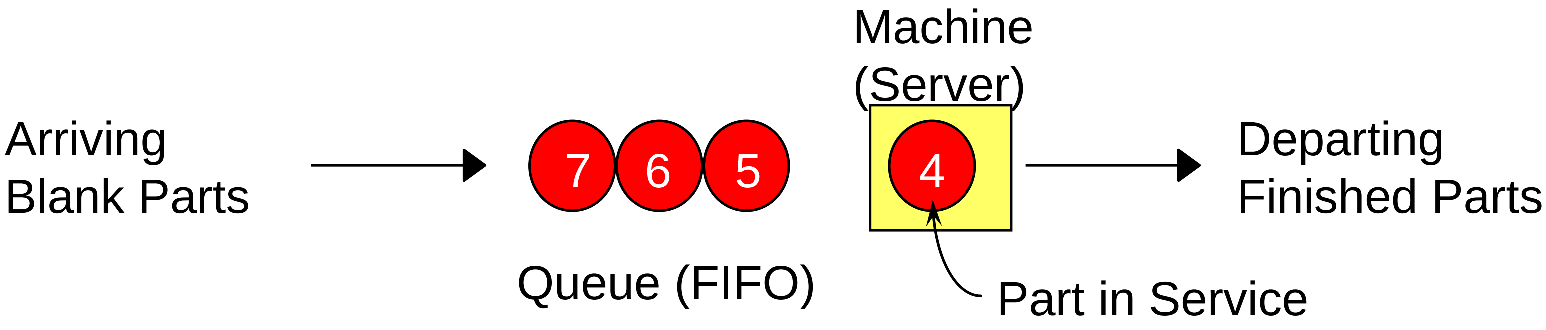
Chapter 2

Last revision June 7, 2003

What We'll Do ...

- **Underlying ideas, methods, and issues in simulation**
- **Software-independent (setting up for Arena)**
- **Centered around an example of a simple processing system**
    - Decompose the problem
    - Terminology
    - Simulation by hand
    - Some basic statistical issues
    - Overview of a simulation study

The System:
A Simple Processing System



- **General intent:**
  - Estimate expected production
  - Waiting time in queue, queue length, proportion of time machine is busy
- **Time units**
  - Can use different units in different places … must declare
  - Be careful to check the units when specifying inputs
  - Declare *base time units* for internal calculations, outputs
  - Be reasonable (interpretation, roundoff error)

Model Specifics

- **Initially (time 0) empty and idle**
- **Base time units: minutes**
- **Input data (assume given for now …), in minutes:**

Part Number	Arrival Time	Interarrival Time	Service Time
1	0.00	1.73	2.90
2	1.73	1.35	1.76
3	3.08	0.71	3.39
4	3.79	0.62	4.52
5	4.41	14.28	4.46
6	18.69	0.70	4.36
7	19.39	15.52	2.07
8	34.91	3.15	3.36
9	38.06	1.76	2.37
10	39.82	1.00	5.38
11	40.82	.	.
.	.	.	.
.	.	.	.

- **Stop when 20 minutes of (simulated) time have passed**

Goals of the Study: Output Performance Measures

- ***Total production*** **of parts over the run (*P*)**
- ***Average waiting time*** **of parts in queue:**

$$\frac{\sum_{i=1}^{N} WQ_i}{N}$$

N **= no. of parts completing queue wait**
WQ_i **= waiting time in queue of *i*th part**
Know: $WQ_1 = 0$ **(why?)**
$N \geq 1$ **(why?)**

- ***Maximum waiting time*** **of parts in queue:**

$$\max_{i=1,\ldots,N} WQ_i$$

Goals of the Study: Output Performance Measures (cont'd.)

- ***Time-average number of parts in queue***:

$$\frac{\int_0^{20} Q(t)\,dt}{20}$$

$Q(t)$ = number of parts in queue at time t

- ***Maximum number of parts in queue***: $\max_{0 \le t \le 20} Q(t)$
- ***Average*** **and** ***maximum total time in system*** **of parts (a.k.a.** ***cycle time*****):**

$$\frac{\sum_{i=1}^{P} TS_i}{P}, \qquad \max_{i=1,\ldots,P} TS_i$$

TS_i = time in system of part i

Goals of the Study: Output Performance Measures (cont'd.)

- *Utilization* of the machine (proportion of time busy)

$$\frac{\int_0^{20} B(t)\,dt}{20}, \quad B(t) = \begin{cases} 1 & \text{if the machine is busy at time } t \\ 0 & \text{if the machine is idle at time } t \end{cases}$$

- Many others possible (information overload?)

Analysis Options

- **Educated guessing**
  - Average interarrival time = 4.08 minutes
  - Average service time = 3.46 minutes
  - So (on average) parts are being processed faster than they arrive
    - System has a chance of operating in a stable way in the long run, i.e., might not “explode”
    - If all interarrivals and service times were exactly at their mean, there would never be a queue
    - But the data clearly exhibit variability, so a queue could form
  - If we’d had average interarrival < average service time, and this persisted, then queue would explode
  - Truth — between these extremes
  - Guessing has its limits …

Analysis Options (cont'd.)

- **Queueing theory**
  - Requires additional assumptions about the model
  - Popular, simple model: *M/M/1 queue*
    - Interarrival times ~ exponential
    - Service times ~ exponential, indep. of interarrivals
    - Must have E(service) < E(interarrival)
    - Steady-state (long-run, forever)
    - Exact analytic results; e.g., average waiting time in queue is

$$\frac{\mu_S^2}{\mu_A - \mu_S}, \qquad \begin{aligned} \mu_A &= \text{E(interarrival time)} \\ \mu_S &= \text{E(service time)} \end{aligned}$$

  - Problems: validity, estimating means, time frame
  - Often useful as first-cut approximation

Mechanistic Simulation

- **Individual operations (arrivals, service times) will occur exactly as in reality**
- **Movements, changes occur at the right “time,” in the right order**
- **Different pieces interact**
- **Install “observers” to get output performance measures**
- **Concrete, “brute-force” analysis approach**
- **Nothing mysterious or subtle**
  - But a lot of details, bookkeeping
  - Simulation software keeps track of things for you

Pieces of a Simulation Model

- ***Entities***
  - "Players" that move around, change status, affect and are affected by other entities
  - *Dynamic objects* — get created, move around, leave (maybe)
  - Usually represent "real" things
    - Our model: entities are the parts
  - Can have "fake" entities for modeling "tricks"
    - Breakdown demon, break angel

      Though Arena has built-in ways to model these examples directly
  - Usually have multiple *realizations* floating around
  - Can have different types of entities concurrently
  - Usually, identifying the types of entities is the first thing to do in building a model

Pieces of a Simulation Model (cont'd.)

- ***Attributes***
  - Characteristic of all entities: describe, differentiate
  - All entities have same attribute "slots" but different values for different entities, for example:
    - Time of arrival
    - Due date
    - Priority
    - Color
  - Attribute value tied to a specific entity
  - Like "local" (to entities) variables
  - Some automatic in Arena, some you define

Pieces of a Simulation Model (cont'd.)

- **(Global) *Variables***
  - Reflects a characteristic of the whole model, not of specific entities
  - Used for many different kinds of things
    - Travel time between all station pairs
    - Number of parts in system
    - Simulation clock (built-in Arena variable)
  - Name, value of which there's only one copy for the whole model
  - Not tied to entities
  - Entities can access, change variables
  - Writing on the wall (rewriteable)
  - Some built-in by Arena, you can define others

Pieces of a Simulation Model (cont'd.)

- ***Resources***
  - What entities compete for
    - People
    - Equipment
    - Space
  - Entity *seizes* a resource, uses it, *releases* it
  - Think of a *resource being assigned to an entity*, rather than an entity "belonging to" a resource
  - "A" resource can have several *units* of capacity
    - Seats at a table in a restaurant
    - Identical ticketing agents at an airline counter
  - Number of units of resource can be changed during the simulation

Pieces of a Simulation Model (cont'd.)

- ***Queues***
  - Place for entities to wait when they can't move on (maybe since the resource they want to seize is not available)
  - Have names, often tied to a corresponding resource
  - Can have a finite capacity to model limited space — have to model what to do if an entity shows up to a queue that's already full
  - Usually watch the length of a queue, waiting time in it

Pieces of a Simulation Model (cont'd.)

- ***Statistical accumulators***
  - Variables that "watch" what's happening
  - Depend on output performance measures desired
  - "Passive" in model — don't participate, just watch
  - Many are automatic in Arena, but some you may have to set up and maintain during the simulation
  - At end of simulation, used to compute final output performance measures

Pieces of a Simulation Model (cont'd.)

- **Statistical accumulators for the simple processing system**
  - Number of parts produced so far
  - Total of the waiting times spent in queue so far
  - No. of parts that have gone through the queue
  - Max time in queue we've seen so far
  - Total of times spent in system
  - Max time in system we've seen so far
  - Area so far under queue-length curve $Q(t)$
  - Max of $Q(t)$ so far
  - Area so far under server-busy curve $B(t)$